axiom™

The 30 Year Horizon

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Volume 10: Axiom Algebra: Categories

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Contents

1 Categories **1**

2 Category Layer 1 **3**
2.1 Category (CATEGORY) 3
2.2 ArcHyperbolicFunctionCategory (AHYP) 5
2.3 ArcTrigonometricFunctionCategory (ATRIG) 7
2.4 AttributeRegistry (ATTREG) 9
2.5 BasicType (BASTYPE) 13
2.6 CoercibleTo (KOERCE) 15
2.7 CombinatorialFunctionCategory (CFCAT) 17
2.8 ConvertibleTo (KONVERT) 19
2.9 ElementaryFunctionCategory (ELEMFUN) 23
2.10 Eltable (ELTAB) 25
2.11 HyperbolicFunctionCategory (HYPCAT) 27
2.12 InnerEvalable (IEVALAB) 29
2.13 OpenMath (OM) 32
2.14 PartialTranscendentalFunctions (PTRANFN) 34
2.15 Patternable (PATAB) 38
2.16 PrimitiveFunctionCategory (PRIMCAT) 40
2.17 RadicalCategory (RADCAT) 42
2.18 RetractableTo (RETRACT) 44
2.19 SpecialFunctionCategory (SPFCAT) 48
2.20 TrigonometricFunctionCategory (TRIGCAT) 51
2.21 Type (TYPE) 53

3 Category Layer 2 **55**
3.1 Aggregate (AGG) 55
3.2 CombinatorialOpsCategory (COMBOPC) 59
3.3 EltableAggregate (ELTAGG) 62
3.4 Evalable (EVALAB) 65
3.5 FortranProgramCategory (FORTCAT) 68
3.6 FullyRetractableTo (FRETRCT) 71
3.7 FullyPatternMatchable (FPATMAB) 74
3.8 Logic (LOGIC) 77

3.9 PlottablePlaneCurveCategory (PPCURVE) . . . 79
3.10 PlottableSpaceCurveCategory (PSCURVE) . . . 82
3.11 RealConstant (REAL) . . . 85
3.12 SegmentCategory (SEGCAT) . . . 88
3.13 SetCategory (SETCAT) . . . 91
3.14 TranscendentalFunctionCategory (TRANFUN) . . . 94

4 Category Layer 3 99
4.1 AbelianSemiGroup (ABELSG) . . . 99
4.2 FortranFunctionCategory (FORTFN) . . . 103
4.3 FortranMatrixCategory (FMC) . . . 107
4.4 FortranMatrixFunctionCategory (FMFUN) . . . 110
4.5 FortranVectorCategory (FVC) . . . 114
4.6 FortranVectorFunctionCategory (FVFUN) . . . 117
4.7 FullyEvalableOver (FEVALAB) . . . 121
4.8 FileCategory (FILECAT) . . . 125
4.9 Finite (FINITE) . . . 129
4.10 FileNameCategory (FNCAT) . . . 132
4.11 GradedModule (GRMOD) . . . 135
4.12 HomogeneousAggregate (HOAGG) . . . 139
4.13 IndexedDirectProductCategory (IDPC) . . . 145
4.14 LiouvillianFunctionCategory (LFCAT) . . . 148
4.15 Monad (MONAD) . . . 152
4.16 NumericalIntegrationCategory (NUMINT) . . . 156
4.17 NumericalOptimizationCategory (OPTCAT) . . . 160
4.18 OrdinaryDifferentialEquationsSolverCategory (ODECAT) . . . 164
4.19 OrderedSet (ORDSET) . . . 168
4.20 PartialDifferentialEquationsSolverCategory (PDECAT) . . . 172
4.21 PatternMatchable (PATMAB) . . . 176
4.22 RealRootCharacterizationCategory (RRCC) . . . 179
4.23 SegmentExpansionCategory (SEGXCAT) . . . 183
4.24 SemiGroup (SGROUP) . . . 186
4.25 SExpressionCategory (SEXCAT) . . . 189
4.26 StepThrough (STEP) . . . 193
4.27 ThreeSpaceCategory (SPACEC) . . . 196

5 Category Layer 4 207
5.1 AbelianMonoid (ABELMON) . . . 207
5.2 BagAggregate (BGAGG) . . . 211
5.3 CachableSet (CACHSET) . . . 215
5.4 Collection (CLAGG) . . . 218
5.5 DifferentialVariableCategory (DVARCAT) . . . 224
5.6 ExpressionSpace (ES) . . . 230
5.7 GradedAlgebra (GRALG) . . . 242
5.8 IndexedAggregate (IXAGG) . . . 246
5.9 MonadWithUnit (MONADWU) . . . 252

5.10 Monoid (MONOID) . . . 256
5.11 OrderedFinite (ORDFIN) . . . 260
5.12 RecursiveAggregate (RCAGG) . . . 263
5.13 TwoDimensionalArrayCategory (ARR2CAT) . . . 268

6 Category Layer 5 281
6.1 BinaryRecursiveAggregate (BRAGG) . . . 282
6.2 CancellationAbelianMonoid (CABMON) . . . 289
6.3 DictionaryOperations (DIOPS) . . . 293
6.4 DoublyLinkedAggregate (DLAGG) . . . 299
6.5 Group (GROUP) . . . 304
6.6 LinearAggregate (LNAGG) . . . 308
6.7 MatrixCategory (MATCAT) . . . 315
6.8 OrderedAbelianSemiGroup (OASGP) . . . 364
6.9 OrderedMonoid (ORDMON) . . . 368
6.10 PolynomialSetCategory (PSETCAT) . . . 373
6.11 PriorityQueueAggregate (PRQAGG) . . . 386
6.12 QueueAggregate (QUAGG) . . . 390
6.13 SetAggregate (SETAGG) . . . 395
6.14 StackAggregate (SKAGG) . . . 402
6.15 UnaryRecursiveAggregate (URAGG) . . . 407

7 Category Layer 6 417
7.1 AbelianGroup (ABELGRP) . . . 418
7.2 BinaryTreeCategory (BTCAT) . . . 423
7.3 Dictionary (DIAGG) . . . 428
7.4 DequeueAggregate (DQAGG) . . . 433
7.5 ExtensibleLinearAggregate (ELAGG) . . . 438
7.6 FiniteLinearAggregate (FLAGG) . . . 444
7.7 FreeAbelianMonoidCategory (FAMONC) . . . 451
7.8 MultiDictionary (MDAGG) . . . 456
7.9 OrderedAbelianMonoid (OAMON) . . . 461
7.10 PermutationCategory (PERMCAT) . . . 464
7.11 StreamAggregate (STAGG) . . . 470
7.12 TriangularSetCategory (TSETCAT) . . . 478

8 Category Layer 7 497
8.1 FiniteDivisorCategory (FDIVCAT) . . . 498
8.2 FiniteSetAggregate (FSAGG) . . . 503
8.3 KeyedDictionary (KDAGG) . . . 510
8.4 LazyStreamAggregate (LZSTAGG) . . . 516
8.5 LeftModule (LMODULE) . . . 533
8.6 ListAggregate (LSAGG) . . . 536
8.7 MultisetAggregate (MSETAGG) . . . 548
8.8 NonAssociativeRng (NARNG) . . . 552
8.9 OneDimensionalArrayAggregate (A1AGG) . . . 556

8.10 OrderedCancellationAbelianMonoid (OCAMON) . . . 567
8.11 RegularTriangularSetCategory (RSETCAT) . . . 570
8.12 RightModule (RMODULE) . . . 584
8.13 Rng (RNG) . . . 587

9 Category Layer 8 **591**
9.1 BiModule (BMODULE) . . . 592
9.2 BitAggregate (BTAGG) . . . 596
9.3 NonAssociativeRing (NASRING) . . . 604
9.4 NormalizedTriangularSetCategory (NTSCAT) . . . 608
9.5 OrderedAbelianGroup (OAGROUP) . . . 617
9.6 OrderedAbelianMonoidSup (OAMONS) . . . 620
9.7 OrderedMultisetAggregate (OMSAGG) . . . 623
9.8 Ring (RING) . . . 628
9.9 SquareFreeRegularTriangularSetCategory (SFRTCAT) . . . 632
9.10 StringAggregate (SRAGG) . . . 640
9.11 TableAggregate (TBAGG) . . . 649
9.12 VectorCategory (VECTCAT) . . . 659

10 Category Layer 9 **667**
10.1 AssociationListAggregate (ALAGG) . . . 667
10.2 CharacteristicNonZero (CHARNZ) . . . 677
10.3 CharacteristicZero (CHARZ) . . . 681
10.4 CommutativeRing (COMRING) . . . 685
10.5 DifferentialRing (DIFRING) . . . 690
10.6 EntireRing (ENTIRER) . . . 694
10.7 FreeModuleCat (FMCAT) . . . 699
10.8 LeftAlgebra (LALG) . . . 704
10.9 LinearlyExplicitRingOver (LINEXP) . . . 707
10.10Module (MODULE) . . . 711
10.11OrderedRing (ORDRING) . . . 715
10.12PartialDifferentialRing (PDRING) . . . 720
10.13PointCategory (PTCAT) . . . 726
10.14RectangularMatrixCategory (RMATCAT) . . . 732
10.15SquareFreeNormalizedTriangularSetCategory (SNTSCAT) . . . 740
10.16StringCategory (STRICAT) . . . 747
10.17UnivariateSkewPolynomialCategory (OREPCAT) . . . 754
10.18XAlgebra (XALG) . . . 765

11 Category Layer 10 **771**
11.1 Algebra (ALGEBRA) . . . 771
11.2 DifferentialExtension (DIFEXT) . . . 777
11.3 FullyLinearlyExplicitRingOver (FLINEXP) . . . 782
11.4 LieAlgebra (LIECAT) . . . 787
11.5 LinearOrdinaryDifferentialOperatorCategory (LODOCAT) . . . 791
11.6 NonAssociativeAlgebra (NAALG) . . . 798

11.7 VectorSpace (VSPACE) . . . 803
11.8 XFreeAlgebra (XFALG) . . . 807

12 Category Layer 11 **815**
12.1 DirectProductCategory (DIRPCAT) . . . 815
12.2 DivisionRing (DIVRING) . . . 825
12.3 FiniteRankNonAssociativeAlgebra (FINAALG) . . . 830
12.4 FreeLieAlgebra (FLALG) . . . 852
12.5 IntegralDomain (INTDOM) . . . 857
12.6 MonogenicLinearOperator (MLO) . . . 862
12.7 OctonionCategory (OC) . . . 868
12.8 QuaternionCategory (QUATCAT) . . . 878
12.9 SquareMatrixCategory (SMATCAT) . . . 887
12.10XPolynomialsCat (XPOLYC) . . . 898

13 Category Layer 12 **905**
13.1 AbelianMonoidRing (AMR) . . . 905
13.2 FortranMachineTypeCategory (FMTC) . . . 913
13.3 FramedNonAssociativeAlgebra (FRNAALG) . . . 918
13.4 GcdDomain (GCDDOM) . . . 932
13.5 OrderedIntegralDomain (OINTDOM) . . . 937

14 Category Layer 13 **941**
14.1 FiniteAbelianMonoidRing (FAMR) . . . 941
14.2 IntervalCategory (INTCAT) . . . 950
14.3 PowerSeriesCategory (PSCAT) . . . 958
14.4 PrincipalIdealDomain (PID) . . . 965
14.5 UniqueFactorizationDomain (UFD) . . . 969

15 Category Layer 14 **975**
15.1 EuclideanDomain (EUCDOM) . . . 975
15.2 MultivariateTaylorSeriesCategory (MTSCAT) . . . 983
15.3 PolynomialFactorizationExplicit (PFECAT) . . . 990
15.4 UnivariatePowerSeriesCategory (UPSCAT) . . . 996

16 Category Layer 15 **1005**
16.1 Field (FIELD) . . . 1005
16.2 IntegerNumberSystem (INS) . . . 1011
16.3 PAdicIntegerCategory (PADICCT) . . . 1021
16.4 PolynomialCategory (POLYCAT) . . . 1027
16.5 UnivariateTaylorSeriesCategory (UTSCAT) . . . 1046

17 Category Layer 16 **1061**
17.1 AlgebraicallyClosedField (ACF) . . . 1061
17.2 DifferentialPolynomialCategory (DPOLCAT) . . . 1069
17.3 FieldOfPrimeCharacteristic (FPC) . . . 1084

17.4 FiniteRankAlgebra (FINRALG) . . . 1089
17.5 FunctionSpace (FS) . . . 1095
17.6 QuotientFieldCategory (QFCAT) . . . 1121
17.7 RealClosedField (RCFIELD) . . . 1132
17.8 RealNumberSystem (RNS) . . . 1141
17.9 RecursivePolynomialCategory (RPOLCAT) . . . 1148
17.10 UnivariateLaurentSeriesCategory (ULSCAT) . . . 1186
17.11 UnivariatePuiseuxSeriesCategory (UPXSCAT) . . . 1195
17.12 UnivariatePolynomialCategory (UPOLYC) . . . 1204

18 Category Layer 17 1225
18.1 AlgebraicallyClosedFunctionSpace (ACFS) . . . 1225
18.2 ExtensionField (XF) . . . 1237
18.3 FiniteFieldCategory (FFIELDC) . . . 1244
18.4 FloatingPointSystem (FPS) . . . 1254
18.5 FramedAlgebra (FRAMALG) . . . 1261
18.6 UnivariateLaurentSeriesConstructorCategory (ULSCCAT) . . . 1267
18.7 UnivariatePuiseuxSeriesConstructorCategory (UPXSCCA) . . . 1280

19 Category Layer 18 1291
19.1 FiniteAlgebraicExtensionField (FAXF) . . . 1291
19.2 MonogenicAlgebra (MONOGEN) . . . 1305

20 Category Layer 19 1315
20.1 ComplexCategory (COMPCAT) . . . 1315
20.2 FunctionFieldCategory (FFCAT) . . . 1334

21 The bootstrap code 1355
21.1 ABELGRP.lsp BOOTSTRAP . . . 1355
21.2 ABELGRP-.lsp BOOTSTRAP . . . 1357
21.3 ABELMON.lsp BOOTSTRAP . . . 1359
21.4 ABELMON-.lsp BOOTSTRAP . . . 1361
21.5 ABELSG.lsp BOOTSTRAP . . . 1363
21.6 ABELSG-.lsp BOOTSTRAP . . . 1364
21.7 ALAGG.lsp BOOTSTRAP . . . 1366
21.8 CABMON.lsp BOOTSTRAP . . . 1368
21.9 CLAGG.lsp BOOTSTRAP . . . 1370
21.10 CLAGG-.lsp BOOTSTRAP . . . 1372
21.11 COMRING.lsp BOOTSTRAP . . . 1377
21.12 DIFRING.lsp BOOTSTRAP . . . 1378
21.13 DIFRING-.lsp BOOTSTRAP . . . 1379
21.14 DIVRING.lsp BOOTSTRAP . . . 1381
21.15 DIVRING-.lsp BOOTSTRAP . . . 1383
21.16 ES.lsp BOOTSTRAP . . . 1386
21.17 ES-.lsp BOOTSTRAP . . . 1389
21.18 EUCDOM.lsp BOOTSTRAP . . . 1406

21.18.1 The Lisp Implementation 1406
21.19EUCDOM-.lsp BOOTSTRAP 1409
21.19.1 The Lisp Implementation 1409
21.20ENTIRER.lsp BOOTSTRAP 1426
21.21FFIELDC.lsp BOOTSTRAP 1427
21.22FFIELDC-.lsp BOOTSTRAP 1429
21.23FPS.lsp BOOTSTRAP 1442
21.24FPS-.lsp BOOTSTRAP 1444
21.25GCDDOM.lsp BOOTSTRAP 1446
21.26GCDDOM-.lsp BOOTSTRAP 1448
21.27HOAGG.lsp BOOTSTRAP 1454
21.28HOAGG-.lsp BOOTSTRAP 1456
21.29INS.lsp BOOTSTRAP 1463
21.30INS-.lsp BOOTSTRAP 1465
21.31INTDOM.lsp BOOTSTRAP 1474
21.32INTDOM-.lsp BOOTSTRAP 1476
21.33LNAGG.lsp BOOTSTRAP 1479
21.34LNAGG-.lsp BOOTSTRAP 1481
21.35LSAGG.lsp BOOTSTRAP 1484
21.36LSAGG-.lsp BOOTSTRAP 1486
21.37MONOID.lsp BOOTSTRAP 1505
21.38MONOID-.lsp BOOTSTRAP 1506
21.39MTSCAT.lsp BOOTSTRAP 1508
21.40OINTDOM.lsp BOOTSTRAP 1510
21.41ORDRING.lsp BOOTSTRAP 1511
21.42ORDRING-.lsp BOOTSTRAP 1513
21.43POLYCAT.lsp BOOTSTRAP 1515
21.44POLYCAT-.lsp BOOTSTRAP 1518
21.45PSETCAT.lsp BOOTSTRAP 1552
21.46PSETCAT-.lsp BOOTSTRAP 1555
21.47QFCAT.lsp BOOTSTRAP 1574
21.48QFCAT-.lsp BOOTSTRAP 1576
21.49RCAGG.lsp BOOTSTRAP 1585
21.50RCAGG-.lsp BOOTSTRAP 1587
21.51RING.lsp BOOTSTRAP 1589
21.52RING-.lsp BOOTSTRAP 1590
21.53RNG.lsp BOOTSTRAP 1591
21.54RNS.lsp BOOTSTRAP 1592
21.55RNS-.lsp BOOTSTRAP 1594
21.56SETAGG.lsp BOOTSTRAP 1599
21.57SETAGG-.lsp BOOTSTRAP 1601
21.58SETCAT.lsp BOOTSTRAP 1603
21.59SETCAT-.lsp BOOTSTRAP 1605
21.60STAGG.lsp BOOTSTRAP 1607
21.61STAGG-.lsp BOOTSTRAP 1609
21.62TSETCAT.lsp BOOTSTRAP 1616

21.63TSETCAT-.lsp BOOTSTRAP 1620
21.64UFD.lsp BOOTSTRAP 1642
21.65UFD-.lsp BOOTSTRAP 1644
21.66ULSCAT.lsp BOOTSTRAP 1647
21.67UPOLYC.lsp BOOTSTRAP 1649
21.68UPOLYC-.lsp BOOTSTRAP 1653
21.69URAGG.lsp BOOTSTRAP 1684
21.70URAGG-.lsp BOOTSTRAP 1686

22 Chunk collections **1701**

New Foreword

On October 1, 2001 Axiom was withdrawn from the market and ended life as a commercial product. On September 3, 2002 Axiom was released under the Modified BSD license, including this document. On August 27, 2003 Axiom was released as free and open source software available for download from the Free Software Foundation's website, Savannah.

Work on Axiom has had the generous support of the Center for Algorithms and Interactive Scientific Computation (CAISS) at City College of New York. Special thanks go to Dr. Gilbert Baumslag for his support of the long term goal.

The online version of this documentation is roughly 1000 pages. In order to make printed versions we've broken it up into three volumes. The first volume is tutorial in nature. The second volume is for programmers. The third volume is reference material. We've also added a fourth volume for developers. All of these changes represent an experiment in print-on-demand delivery of documentation. Time will tell whether the experiment succeeded.

Axiom has been in existence for over thirty years. It is estimated to contain about three hundred man-years of research and has, as of September 3, 2003, 143 people listed in the credits. All of these people have contributed directly or indirectly to making Axiom available. Axiom is being passed to the next generation. I'm looking forward to future milestones.

With that in mind I've introduced the theme of the "30 year horizon". We must invent the tools that support the Computational Mathematician working 30 years from now. How will research be done when every bit of mathematical knowledge is online and instantly available? What happens when we scale Axiom by a factor of 100, giving us 1.1 million domains? How can we integrate theory with code? How will we integrate theorems and proofs of the mathematics with space-time complexity proofs and running code? What visualization tools are needed? How do we support the conceptual structures and semantics of mathematics in effective ways? How do we support results from the sciences? How do we teach the next generation to be effective Computational Mathematicians?

The "30 year horizon" is much nearer than it appears.

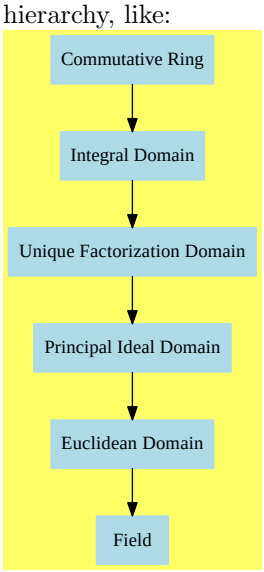
Tim Daly
CAISS, City College of New York
November 10, 2003 ((iHy))

Chapter 1

Categories

Axiom has 3 main algebra components, Categories, Domains, and Packages. If we make an analogy to dressmaking, you can consider the Categories to be hierarchies of properties of things, like patterns, colors, or fabrics. Domains are instances of things based on category choices, such as a dress with a particular style, fabric, color, etc. Packages are tools that work with dresses such as irons, sewing machines, etc.

Axiom is based on abstract algebra and uses it as a scaffolding for constructing well-formed algebra. For instance, in abstract algebra there is a strict subset hierarchy, like:



⟨*algebrahierarchy.dotpic*⟩≡

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```

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"Integral Domain"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=INTDOM"];
"Unique Factorization Domain"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=UFD"];
"Principal Ideal Domain"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=PID"];
"Euclidean Domain"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=EUCDOM"];
"Field"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=FIELD"];

"Commutative Ring" -> "Integral Domain"
"Integral Domain" -> "Unique Factorization Domain"
"Unique Factorization Domain" -> "Principal Ideal Domain"
"Principal Ideal Domain" -> "Euclidean Domain"
"Euclidean Domain" -> "Field"

}
```

Chapter 2

Category Layer 1

In general, we use several colors in the graph images. The "lightblue" color indicates a category that is in the direct inheritance path. The "green" (#00EE00) color indicates a category or domain used in the exports. The "seagreen" (a dark green, indicates a category or domain which is used but does not correspond to the signature of an existing category. The system can infer that this "subsumption node" matches the category. A "yellow" color indicates a domain.

2.1 Category (CATEGORY)

This is the root of the category hierarchy and is not represented by code.

See:

⇒ "ArcHyperbolicFunctionCategory" (AHYP) 2.2 on page 5
⇒ "ArcTrigonometricFunctionCategory" (ATRIG) 2.3 on page 7
⇒ "BasicType" (BASTYPE) 2.5 on page 13
⇒ "CoercibleTo" (KOERCE) 2.6 on page 15
⇒ "CombinatorialFunctionCategory" (CFCAT) 2.7 on page 17
⇒ "ConvertibleTo" (KONVERT) 2.8 on page 19
⇒ "ElementaryFunctionCategory" (ELEMFUN) 2.9 on page 23
⇒ "Eltable" (ELTAB) 2.10 on page 25
⇒ "FullyEvalableOver" (FEVALAB) 4.7 on page 121
⇒ "HyperbolicFunctionCategory" (HYPCAT) 2.11 on page 27
⇒ "InnerEvalable" (IEVALAB) 2.12 on page 29
⇒ "Logic" (LOGIC) 3.8 on page 77

⇒ “OpenMath” (OM) 2.13 on page 32
⇒ “PartialTranscendentalFunctions” (PTRANFN) 2.14 on page 34
⇒ “Patternable” (PATAB) 2.15 on page 38
⇒ “PrimitiveFunctionCategory” (PRIMCAT) 2.16 on page 40
⇒ “RadicalCategory” (RADCAT) 2.17 on page 42
⇒ “RetractableTo” (RETRACT) 2.18 on page 44
⇒ “SpecialFunctionCategory” (SPFCAT) 2.19 on page 48
⇒ “TrigonometricFunctionCategory” (TRIGCAT) 2.20 on page 51
⇒ “Type” (TYPE) 2.21 on page 53

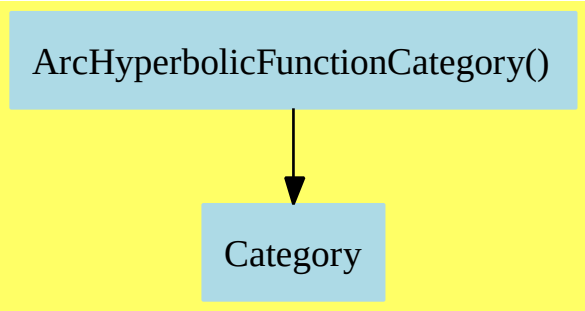
⟨*CATEGORY.dotabb*⟩≡
```
"CATEGORY"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CATEGORY"];
```

⟨*CATEGORY.dotfull*⟩≡
```
"Category"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CATEGORY"];
```

⟨*CATEGORY.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Category" [color=lightblue];
}
```

2.2 ArcHyperbolicFunctionCategory (AHYP)



See:

⇒ "TranscendentalFunctionCategory" (TRANFUN) 3.14 on page 94
⇐ "Category" (CATEGORY) 2.1 on page 3

Exports:
acosh acoth acsch asech asinh atanh

These are directly exported but not implemented:

```
acosh : % -> %
acoth : % -> %
acsch : % -> %
asech : % -> %
asinh : % -> %
atanh : % -> %
```

⟨*category AHYP ArcHyperbolicFunctionCategory*⟩≡

```
)abbrev category AHYP ArcHyperbolicFunctionCategory
++ Category for the inverse hyperbolic trigonometric functions
++ Author: ???
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description:
++ Category for the inverse hyperbolic trigonometric functions;
ArcHyperbolicFunctionCategory(): Category == with
    acosh: $ -> $ ++ acosh(x) returns the hyperbolic arc-cosine of x.
    acoth: $ -> $ ++ acoth(x) returns the hyperbolic arc-cotangent of x.
    acsch: $ -> $ ++ acsch(x) returns the hyperbolic arc-cosecant of x.
    asech: $ -> $ ++ asech(x) returns the hyperbolic arc-secant of x.
    asinh: $ -> $ ++ asinh(x) returns the hyperbolic arc-sine of x.
    atanh: $ -> $ ++ atanh(x) returns the hyperbolic arc-tangent of x.
```

⟨*AHYP.dotabb*⟩≡
```
"AHYP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=AHYP"];
"AHYP" -> "CATEGORY"
```

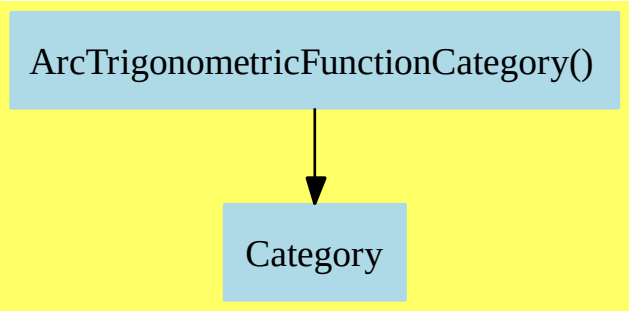
⟨*AHYP.dotfull*⟩≡
```
"ArcHyperbolicFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=AHYP"];
"ArcHyperbolicFunctionCategory()" -> "Category"
```

⟨*AHYP.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ArcHyperbolicFunctionCategory()" [color=lightblue];
"ArcHyperbolicFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

2.3 ArcTrigonometricFunctionCategory (ATRIG)



The `asec` and `acsc` functions were modified to include an intermediate test to check that the argument has a reciprocal values.

See:

⇒ "TranscendentalFunctionCategory" (TRANFUN) 3.14 on page 94
⇐ "Category" (CATEGORY) 2.1 on page 3

Exports:
acos acot acsc asec asin atan

These are directly exported but not implemented:

```
acos : % -> %
acot : % -> %
asin : % -> %
atan : % -> %
```

These are implemented by this category:

```
acsc : % -> %
asec : % -> %
```

⟨*category ATRIG ArcTrigonometricFunctionCategory*⟩≡

```
)abbrev category ATRIG ArcTrigonometricFunctionCategory
++ Category for the inverse trigonometric functions
++ Author: ???
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description: Category for the inverse trigonometric functions;
ArcTrigonometricFunctionCategory(): Category == with
    acos: $ -> $       ++ acos(x) returns the arc-cosine of x.
    acot: $ -> $       ++ acot(x) returns the arc-cotangent of x.
    acsc: $ -> $       ++ acsc(x) returns the arc-cosecant of x.
    asec: $ -> $       ++ asec(x) returns the arc-secant of x.
    asin: $ -> $       ++ asin(x) returns the arc-sine of x.
```

```
      atan: $ -> $       ++ atan(x) returns the arc-tangent of x.
   add
      if $ has Ring then
         asec(x) ==
           (a := recip x) case "failed" => error "asec: no reciprocal"
           acos(a::$)
         acsc(x) ==
           (a := recip x) case "failed" => error "acsc: no reciprocal"
           asin(a::$)
```

⟨*ATRIG.dotabb*⟩≡

```
  "ATRIG"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ATRIG"];
  "ATRIG" -> "CATEGORY"
```

⟨*ATRIG.dotfull*⟩≡

```
  "ArcTrigonometricFunctionCategory()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ATRIG"];
  "ArcTrigonometricFunctionCategory()" -> "Category"
```

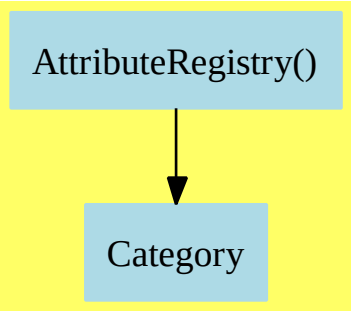
⟨*ATRIG.dotpic*⟩≡

```
  digraph pic {
   fontsize=10;
   bgcolor="#FFFF66";
   node [shape=box, color=white, style=filled];

  "ArcTrigonometricFunctionCategory()" [color=lightblue];
  "ArcTrigonometricFunctionCategory()" -> "Category"

  "Category" [color=lightblue];
  }
```

2.4 AttributeRegistry (ATTREG)



See:

⇐ "Category" (CATEGORY) 2.1 on page 3

Exports: Nothing

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **arbitraryPrecision** means the user can set the precision for subsequent calculations.

- **partiallyOrderedSet** is true if a set with $<$ which is transitive, but not($a < b$ or $a = b$) does not necessarily imply $b < a$.
- **central** is true if, given an algebra over a ring R, the image of R is the center of the algebra, i.e. the set of members of the algebra which commute with all others is precisely the image of R in the algebra.
- **noetherian** is true if all of its ideals are finitely generated.
- **additiveValuation** implies `euclideanSize(a*b)=euclideanSize(a)+euclideanSize(b)`.
- **multiplicativeValuation** implies `euclideanSize(a*b)=euclideanSize(a)*euclideanSize(b)`.
- **NullSquare** means that $[x, x] = 0$ holds. See `LieAlgebra`.
- **JacobiIdentity** means that $[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$ holds. See `LieAlgebra`.
- **canonical** is true if and only if distinct elements have distinct data structures. For example, a domain of mathematical objects which has the `canonical` attribute means that two objects are mathematically equal if and only if their data structures are equal.
- **approximate** means "is an approximation to the real numbers".
- **complex** means that this domain has $\sqrt{-1}$

⟨*category ATTREG AttributeRegistry*⟩≡

```
)abbrev category ATTREG AttributeRegistry

++ This category exports the attributes in the AXIOM Library
AttributeRegistry(): Category == with
  finiteAggregate
    ++ \spad{finiteAggregate} is true if it is an aggregate with a
    ++ finite number of elements.
  commutative("*")
    ++ \spad{commutative("*")} is true if it has an operation
    ++ \spad{"*": (D,D) -> D} which is commutative.
  shallowlyMutable
    ++ \spad{shallowlyMutable} is true if its values
    ++ have immediate components that are updateable (mutable).
    ++ Note: the properties of any component domain are irrevelant to the
    ++ \spad{shallowlyMutable} proper.
  unitsKnown
    ++ \spad{unitsKnown} is true if a monoid (a multiplicative semigroup
    ++ with a 1) has \spad{unitsKnown} means that
    ++ the operation \spadfun{recip} can only return "failed"
```

```
  ++ if its argument is not a unit.
leftUnitary
  ++ \spad{leftUnitary} is true if \spad{1 * x = x} for all x.
rightUnitary
  ++ \spad{rightUnitary} is true if \spad{x * 1 = x} for all x.
noZeroDivisors
  ++ \spad{noZeroDivisors} is true if \spad{x * y \~~= 0} implies
  ++ both x and y are non-zero.
canonicalUnitNormal
  ++ \spad{canonicalUnitNormal} is true if we can choose a canonical
  ++ representative for each class of associate elements, that is
  ++ \spad{associates?(a,b)} returns true if and only if
  ++ \spad{unitCanonical(a) = unitCanonical(b)}.
canonicalsClosed
  ++ \spad{canonicalsClosed} is true if
  ++ \spad{unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)}.
arbitraryPrecision
  ++ \spad{arbitraryPrecision} means the user can set the
  ++ precision for subsequent calculations.
partiallyOrderedSet
  ++ \spad{partiallyOrderedSet} is true if
  ++ a set with \spadop{<} which is transitive,
  ++ but \spad{not(a < b or a = b)}
  ++ does not necessarily imply \spad{b<a}.
central
  ++ \spad{central} is true if, given an algebra over a ring R,
  ++ the image of R is the center
  ++ of the algebra, i.e. the set of members of the algebra which commute
  ++ with all others is precisely the image of R in the algebra.
noetherian
  ++ \spad{noetherian} is true if all of its ideals are finitely generated.
additiveValuation
  ++ \spad{additiveValuation} implies
  ++ \spad{euclideanSize(a*b)=euclideanSize(a)+euclideanSize(b)}.
multiplicativeValuation
  ++ \spad{multiplicativeValuation} implies
  ++ \spad{euclideanSize(a*b)=euclideanSize(a)*euclideanSize(b)}.
NullSquare
  ++ \axiom{NullSquare} means that \axiom{[x,x] = 0} holds.
  ++ See \axiomType{LieAlgebra}.
JacobiIdentity
  ++ \axiom{JacobiIdentity} means that
  ++ \axiom{[x,[y,z]]+[y,[z,x]]+[z,[x,y]] = 0} holds.
  ++ See \axiomType{LieAlgebra}.
canonical
  ++ \spad{canonical} is true if and only if distinct elements have
```

```
      ++ distinct data structures. For example, a domain of mathematical
      ++ objects  which has the \spad{canonical} attribute means that two
      ++ objects are mathematically equal if and only if their data
      ++ structures are equal.
    approximate
      ++ \spad{approximate} means "is an approximation to the real numbers".
```

⟨*ATTREG.dotabb*⟩≡

```
  "ATTREG"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ATTREG"];
  "ATTREG" -> "CATEGORY"
```

⟨*ATTREG.dotfull*⟩≡

```
  "AttributeRegistry()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ATTREG"];
  "AttributeRegistry()" -> "Category"
```

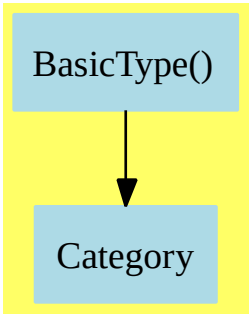
⟨*ATTREG.dotpic*⟩≡

```
  digraph pic {
   fontsize=10;
   bgcolor="#FFFF66";
   node [shape=box, color=white, style=filled];

  "AttributeRegistry()" [color=lightblue];
  "AttributeRegistry()" -> "Category"

  "Category" [color=lightblue];
  }
```

2.5 BasicType (BASTYPE)



See:

⇒ "SetCategory" (SETCAT) 3.13 on page 91
⇐ "Category" (CATEGORY) 2.1 on page 3

Exports:
?=? ?~=?

These are directly exported but not implemented:

```
?=? : (%,%) -> Boolean
```

These are implemented by this category:

```
?~=? : (%,%) -> Boolean
```

⟨*category BASTYPE BasicType*⟩≡

```
)abbrev category BASTYPE BasicType
--% BasicType
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \spadtype{BasicType} is the basic category for describing a collection
++ of elements with \spadop{=} (equality).
BasicType(): Category == with
      "=": (%,%) -> Boolean    ++ x=y tests if x and y are equal.
      "~=": (%,%) -> Boolean   ++ x~=y tests if x and y are not equal.
   add
```

```
        _~_=(x:%,y:%) : Boolean == not(x=y)
```

⟨*BASTYPE.dotabb*⟩≡

```
  "BASTYPE"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=BASTYPE"];
  "BASTYPE" -> "CATEGORY"
```

⟨*BASTYPE.dotfull*⟩≡

```
  "BasicType()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=BASTYPE"];
  "BasicType()" -> "Category"
```

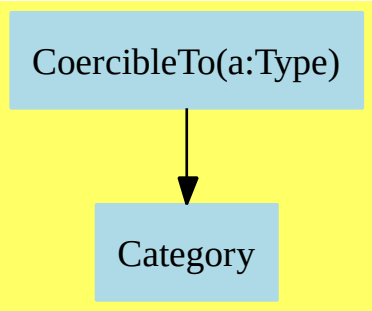
⟨*BASTYPE.dotpic*⟩≡

```
  digraph pic {
   fontsize=10;
   bgcolor="#FFFF66";
   node [shape=box, color=white, style=filled];

  "BasicType()" [color=lightblue];
  "BasicType()" -> "Category"

  "Category" [color=lightblue];
  }
```

2.6 CoercibleTo (KOERCE)



See:

⇒ "DirectProductCategory" (DIRPCAT) 12.1 on page 815
⇒ "FortranProgramCategory" (FORTCAT) 3.5 on page 68
⇒ "PlottablePlaneCurveCategory" (PPCURVE) 3.9 on page 79
⇒ "PlottableSpaceCurveCategory" (PSCURVE) 3.10 on page 82
⇒ "PolynomialSetCategory" (PSETCAT) 6.10 on page 373
⇒ "SetCategory" (SETCAT) 3.13 on page 91
⇐ "Category" (CATEGORY) 2.1 on page 3

Exports:
coerce

This is directly exported but not implemented:

```
coerce : % -> S
```

⟨*category KOERCE CoercibleTo*⟩≡

```
)abbrev category KOERCE CoercibleTo
++ Category for coerce
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description:
++ A is coercible to B means any element of A can automatically be
++ converted into an element of B by the interpreter.
CoercibleTo(S:Type): Category == with
    coerce: % -> S
      ++ coerce(a) transforms a into an element of S.
```

⟨*KOERCE.dotabb*⟩≡
```
"KOERCE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=KOERCE"];
"KOERCE" -> "CATEGORY"
```

⟨*KOERCE.dotfull*⟩≡
```
"CoercibleTo(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=KOERCE"];
"CoercibleTo(a:Type)" -> "Category"

"CoercibleTo(OutputForm)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KOERCE"];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

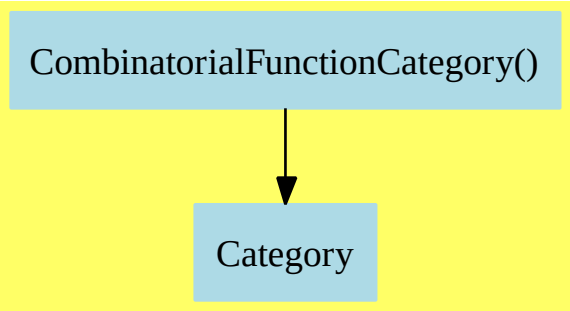
"CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),Orde
 [color=seagreen,href="bookvol10.2.pdf#nameddest=KOERCE"];
"CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),Orde
  -> "CoercibleTo(a:Type)"
```

⟨*KOERCE.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

2.7 CombinatorialFunctionCategory (CFCAT)



See:

⇒ "CombinatorialOpsCategory" (COMBOPC) 3.2 on page 59
⇒ "IntegerNumberSystem" (INS) 16.2 on page 1011
⇐ "Category" (CATEGORY) 2.1 on page 3

Exports:
binomial factorial permutation

These are directly exported but not implemented:

```
binomial : (%,%) -> %
factorial : % -> %
permutation : (%,%) -> %
```

⟨*category CFCAT CombinatorialFunctionCategory*⟩≡

```
)abbrev category CFCAT CombinatorialFunctionCategory
++ Category for the usual combinatorial functions
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description: Category for the usual combinatorial functions;
CombinatorialFunctionCategory(): Category == with
    binomial   : ($, $) -> $
      ++ binomial(n,r) returns the \spad{(n,r)} binomial coefficient
      ++ (often denoted in the literature by \spad{C(n,r)}).
      ++ Note: \spad{C(n,r) = n!/(r!(n-r)!)} where \spad{n >= r >= 0}.
      ++
      ++X [binomial(5,i) for i in 0..5]
    factorial  : $ -> $
      ++ factorial(n) computes the factorial of n
      ++ (denoted in the literature by \spad{n!})
      ++ Note: \spad{n! = n (n-1)! when n > 0}; also, \spad{0! = 1}.
    permutation: ($, $) -> $
      ++ permutation(n, m) returns the number of
```

```
        ++ permutations of n objects taken m at a time.
        ++ Note: \spad{permutation(n,m) = n!/(n-m)!}.
```

⟨*CFCAT.dotabb*⟩≡

```
  "CFCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=CFCAT"];
  "CFCAT" -> "CATEGORY"
```

⟨*CFCAT.dotfull*⟩≡

```
  "CombinatorialFunctionCategory()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=CFCAT"];
  "CombinatorialFunctionCategory()" -> "Category"
```

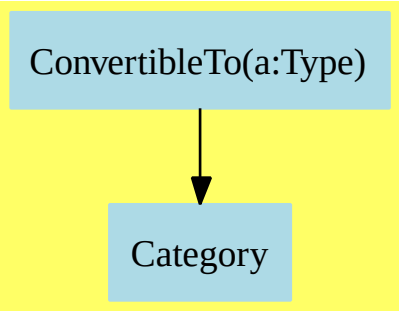
⟨*CFCAT.dotpic*⟩≡

```
  digraph pic {
   fontsize=10;
   bgcolor="#FFFF66";
   node [shape=box, color=white, style=filled];

  "CombinatorialFunctionCategory()" [color=lightblue];
  "CombinatorialFunctionCategory()" -> "Category"

  "Category" [color=lightblue];
  }
```

2.8 ConvertibleTo (KONVERT)



See:

⇒ "Collection" (CLAGG) 5.4 on page 218
⇒ "FunctionSpace" (FS) 17.5 on page 1095
⇒ "IntegerNumberSystem" (INS) 16.2 on page 1011
⇒ "MonogenicAlgebra" (MONOGEN) 19.2 on page 1305
⇒ "QuotientFieldCategory" (QFCAT) 17.6 on page 1121
⇒ "RealConstant" (REAL) 3.11 on page 85
⇒ "RealNumberSystem" (RNS) 17.8 on page 1141
⇐ "Category" (CATEGORY) 2.1 on page 3

Exports:
convert

This is directly exported but not implemented:

```
convert : % -> S
```

⟨*category KONVERT ConvertibleTo*⟩≡

```
)abbrev category KONVERT ConvertibleTo
++ Category for convert
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description:
++ A is convertible to B means any element of A
++ can be converted into an element of B,
++ but not automatically by the interpreter.
ConvertibleTo(S:Type): Category == with
    convert: % -> S
      ++ convert(a) transforms a into an element of S.
```

⟨*KONVERT.dotabb*⟩≡

```
"KONVERT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=KONVERT"];
"KONVERT" -> "CATEGORY"
```

⟨*KONVERT.dotfull*⟩≡

```
  "ConvertibleTo(a:Type)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=KONVERT"];
  "ConvertibleTo(a:Type)" -> "Category"

  "ConvertibleTo(DoubleFloat)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
  "ConvertibleTo(DoubleFloat)" -> "ConvertibleTo(a:Type)"

  "ConvertibleTo(Float)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
  "ConvertibleTo(Float)" -> "ConvertibleTo(a:Type)"

  "ConvertibleTo(InputForm)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
  "ConvertibleTo(InputForm)" -> "ConvertibleTo(a:Type)"

  "ConvertibleTo(Integer)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
  "ConvertibleTo(Integer)" -> "ConvertibleTo(a:Type)"

  "ConvertibleTo(Pattern(Integer))"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
  "ConvertibleTo(Pattern(Integer))" -> "ConvertibleTo(a:Type)"

  "ConvertibleTo(Pattern(Float))"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
  "ConvertibleTo(Pattern(Float))" -> "ConvertibleTo(a:Type)"

  "ConvertibleTo(Complex(Float))"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
  "ConvertibleTo(Complex(Float))" ->  "ConvertibleTo(a:Type)"

  "ConvertibleTo(Complex(DoubleFloat))"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
  "ConvertibleTo(Complex(DoubleFloat))" ->  "ConvertibleTo(a:Type)"

  "ConvertibleTo(String)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
  "ConvertibleTo(String)" ->  "ConvertibleTo(a:Type)"

  "ConvertibleTo(Symbol)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
  "ConvertibleTo(Symbol)" ->  "ConvertibleTo(a:Type)"

  "ConvertibleTo(SExpression)"
```

```
   [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
  "ConvertibleTo(SExpression)" ->  "ConvertibleTo(a:Type)"

  "ConvertibleTo(Pattern(Base))"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
  "ConvertibleTo(Pattern(Base))" ->  "ConvertibleTo(a:Type)"

  "ConvertibleTo(List(Integer))"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
  "ConvertibleTo(List(Integer))" ->  "ConvertibleTo(a:Type)"

  "ConvertibleTo(List(Character))"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
  "ConvertibleTo(List(Character))" ->  "ConvertibleTo(a:Type)"

  "ConvertibleTo(UnivariatePolynomialCategory(CommutativeRing))"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=KONVERT"];
  "ConvertibleTo(UnivariatePolynomialCategory(CommutativeRing))" ->
      "ConvertibleTo(a:Type)"
```

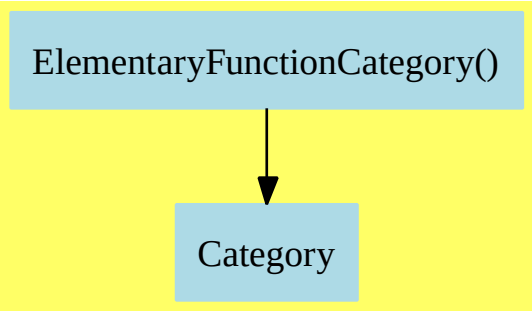
⟨*KONVERT.dotpic*⟩≡

```
  digraph pic {
   fontsize=10;
   bgcolor="#FFFF66";
   node [shape=box, color=white, style=filled];

  "ConvertibleTo(a:Type)" [color=lightblue];
  "ConvertibleTo(a:Type)" -> "Category"

  "Category" [color=lightblue];
  }
```

2.9 ElementaryFunctionCategory (ELEMFUN)



See:

⇒ "TranscendentalFunctionCategory" (TRANFUN) 3.14 on page 94
⇐ "Category" (CATEGORY) 2.1 on page 3

Exports:
?**? exp log

These are directly exported but not implemented:

```
exp : % -> %
log : % -> %
```

These are implemented by this category:

```
?**? : (%,%) -> %
```

⟨*category ELEMFUN ElementaryFunctionCategory*⟩≡

```
)abbrev category ELEMFUN ElementaryFunctionCategory
++ Category for the elementary functions
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description: Category for the elementary functions;
ElementaryFunctionCategory(): Category == with
    log : $ -> $       ++ log(x) returns the natural logarithm of x.
    exp : $ -> $       ++ exp(x) returns %e to the power x.
    "**": ($, $) -> $  ++ x**y returns x to the power y.
 add
   if $ has Monoid then
     x ** y == exp(y * log x)
```

⟨*ELEMFUN.dotabb*⟩≡
```
"ELEMFUN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ELEMFUN"];
"ELEMFUN" -> "CATEGORY"
```

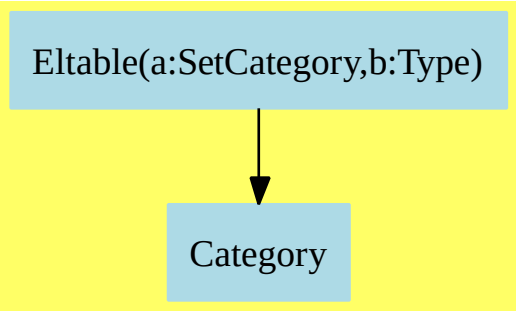
⟨*ELEMFUN.dotfull*⟩≡
```
"ElementaryFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ELEMFUN"];
"ElementaryFunctionCategory()" -> "Category"
```

⟨*ELEMFUN.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ElementaryFunctionCategory()" [color=lightblue];
"ElementaryFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

2.10 Eltable (ELTAB)



See:

⇒ "EltableAggregate" (ELTAGG) 3.3 on page 62
⇒ "LinearOrdinaryDifferentialOperatorCategory" (LODOCAT) 11.5 on page 791

⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.12 on page 1204
⇐ "Category" (CATEGORY) 2.1 on page 3

Exports:
?.?

This syntax for elt is supported by the interpreter and compiler.

This is directly exported but not implemented:

```
?.? : (%,S) -> Index
```

⟨*category ELTAB Eltable*⟩≡

```
)abbrev category ELTAB Eltable
++ Author: Michael Monagan; revised by Manuel Bronstein and Manuel Bronstein
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ An eltable over domains D and I is a structure which can be viewed
++ as a function from D to I.
++ Examples of eltable structures range from data structures, e.g. those
++ of type \spadtype{List}, to algebraic structures like
++ \spadtype{Polynomial}.
Eltable(S:SetCategory, Index:Type): Category == with
  elt : (%, S) -> Index
```

```
       ++ elt(u,i) (also written: u . i) returns the element of u indexed by i.
       ++ Error: if i is not an index of u.
```

⟨*ELTAB.dotabb*⟩≡

```
  "ELTAB" [color=lightblue,href="bookvol10.2.pdf#nameddest=ELTAB"];
  "ELTAB" -> "CATEGORY"
```

⟨*ELTAB.dotfull*⟩≡

```
  "Eltable(a:SetCategory,b:Type)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ELTAB"];
  "Eltable(a:SetCategory,b:Type)" -> "Category"

  "Eltable(a:UnivariatePolynomialCategory(a:Ring),b:UnivariatePolynomialCategory(a:
   [color=seagreen,href="bookvol10.2.pdf#nameddest=ELTAB"];
  "Eltable(a:UnivariatePolynomialCategory(a:Ring),b:UnivariatePolynomialCategory(a:
     "Eltable(a:SetCategory,b:Type)"

  "Eltable(a:Ring,b:Ring)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=ELTAB"];
  "Eltable(a:Ring,b:Ring)" ->
     "Eltable(a:SetCategory,b:Type)"

  "Eltable(a:SetCategory,b:SetCategory)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=ELTAB"];
  "Eltable(a:SetCategory,b:SetCategory)" ->
     "Eltable(a:SetCategory,b:Type)"
```

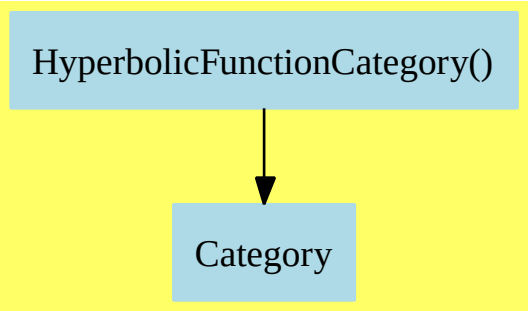
⟨*ELTAB.dotpic*⟩≡

```
  digraph pic {
   fontsize=10;
   bgcolor="#FFFF66";
   node [shape=box, color=white, style=filled];

  "Eltable(a:SetCategory,b:Type)" [color=lightblue];
  "Eltable(a:SetCategory,b:Type)" -> "Category"

  "Category" [color=lightblue];
  }
```

2.11 HyperbolicFunctionCategory (HYPCAT)



The csch and sech functions were modified to include an intermediate test to check that the argument has a reciprocal values.

See:

⇒ "TranscendentalFunctionCategory" (TRANFUN) 3.14 on page 94
⇐ "Category" (CATEGORY) 2.1 on page 3

Exports:
cosh coth csch sech sinh tanh

These are implemented by this category:

```
cosh : % -> %
coth : % -> %
csch : % -> %
sech : % -> %
sinh : % -> %
tanh : % -> %
```

⟨*category HYPCAT HyperbolicFunctionCategory*⟩≡

```
)abbrev category HYPCAT HyperbolicFunctionCategory
++ Category for the hyperbolic trigonometric functions
++ Author: ???
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description: Category for the hyperbolic trigonometric functions;
HyperbolicFunctionCategory(): Category == with
    cosh: $ -> $       ++ cosh(x) returns the hyperbolic cosine of x.
    coth: $ -> $       ++ coth(x) returns the hyperbolic cotangent of x.
    csch: $ -> $       ++ csch(x) returns the hyperbolic cosecant of x.
    sech: $ -> $       ++ sech(x) returns the hyperbolic secant of x.
    sinh: $ -> $       ++ sinh(x) returns the hyperbolic sine of x.
    tanh: $ -> $       ++ tanh(x) returns the hyperbolic tangent of x.
 add
    if $ has Ring then
```

```
        csch x ==
          (a := recip(sinh x)) case "failed" => error "csch: no reciprocal"
          a::$
        sech x ==
          (a := recip(cosh x)) case "failed" => error "sech: no reciprocal"
          a::$
        tanh x == sinh x * sech x
        coth x == cosh x * csch x
        if $ has ElementaryFunctionCategory then
          cosh x ==
            e := exp x
            (e + recip(e)::$) * recip(2::$)::$
          sinh(x):$ ==
            e := exp x
            (e - recip(e)::$) * recip(2::$)::$
```

⟨*HYPCAT.dotabb*⟩≡

```
"HYPCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=HYPCAT"];
"HYPCAT" -> "CATEGORY"
```

⟨*HYPCAT.dotfull*⟩≡

```
"HyperbolicFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=HYPCAT"];
"HyperbolicFunctionCategory()" -> "Category"
```

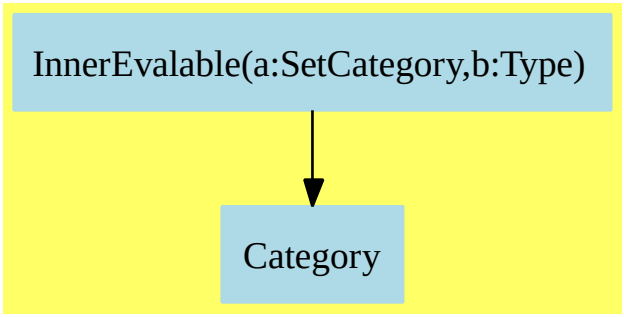
⟨*HYPCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"HyperbolicFunctionCategory()" [color=lightblue];
"HyperbolicFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

2.12 InnerEvalable (IEVALAB)



See:

⇒ "Evalable" (EVALAB) 3.4 on page 65
⇒ "ExpressionSpace" (ES) 5.6 on page 230
⇒ "MultivariateTaylorSeriesCategory" (MTSCAT) 15.2 on page 983
⇒ "PolynomialCategory" (POLYCAT) 16.4 on page 1027
⇐ "Category" (CATEGORY) 2.1 on page 3

Exports:
eval

These are directly exported but not implemented:

```
eval : (%,A,B) -> %
```

These are implemented by this category:

```
eval : (%,List A,List B) -> %
```

⟨*category IEVALAB InnerEvalable*⟩≡

```
)abbrev category IEVALAB InnerEvalable
-- FOR THE BENEFIT OF LIBAXO GENERATION
++ Author:
++ Date Created:
++ Date Last Updated: June 3, 1991
++ Basic Operations:
++ Related Domains:
++ Also See: Evalable
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
++   This category provides \spadfun{eval} operations.
++   A domain may belong to this category if it is possible to make
```

```
++   ``evaluation'' substitutions.  The difference between this
++   and \spadtype{Evalable} is that the operations in this category
++   specify the substitution as a pair of arguments rather than as
++   an equation.
InnerEvalable(A:SetCategory, B:Type): Category == with
    eval: ($, A, B) -> $
       ++ eval(f, x, v) replaces x by v in f.
    eval: ($, List A, List B) -> $
       ++ eval(f, [x1,...,xn], [v1,...,vn]) replaces xi by vi in f.
 add
    eval(f:$, x:A, v:B) == eval(f, [x], [v])
```

⟨*IEVALAB.dotabb*⟩≡

```
"IEVALAB"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=IEVALAB"];
"IEVALAB" -> "CATEGORY"
```

⟨*IEVALAB.dotfull*⟩≡
```
  "InnerEvalable(a:SetCategory,b:Type)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=IEVALAB"];
  "InnerEvalable(a:SetCategory,b:Type)" -> "Category"

  "InnerEvalable(a:SetCategory,b:SetCategory)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=IEVALAB"];
  "InnerEvalable(a:SetCategory,b:SetCategory)" ->
      "InnerEvalable(a:SetCategory,b:Type)"

  "InnerEvalable(a:OrderedSet,b:Ring)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=IEVALAB"];
  "InnerEvalable(a:OrderedSet,b:Ring)" ->
      "InnerEvalable(a:SetCategory,b:Type)"

  "InnerEvalable(a:OrderedSet,b:PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedS
   [color=seagreen,href="bookvol10.2.pdf#nameddest=IEVALAB"];
  "InnerEvalable(a:OrderedSet,b:PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedS
      "InnerEvalable(a:SetCategory,b:Type)"

  "InnerEvalable(a:Ring,MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet))"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=IEVALAB"];
  "InnerEvalable(a:Ring,MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet))"
       -> "InnerEvalable(a:SetCategory,b:Type)"

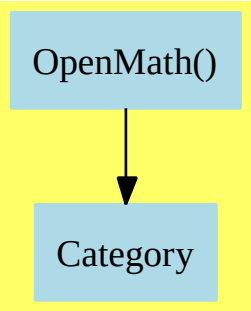
  "InnerEvalable(Kernal(ExpressionSpace),ExpressionSpace)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=IEVALAB"];
  "InnerEvalable(Kernal(ExpressionSpace),ExpressionSpace)" ->
      "InnerEvalable(a:SetCategory,b:Type)"
```

⟨*IEVALAB.dotpic*⟩≡
```
  digraph pic {
   fontsize=10;
   bgcolor="#FFFF66";
   node [shape=box, color=white, style=filled];

  "InnerEvalable(a:SetCategory,b:Type)" [color=lightblue];
  "InnerEvalable(a:SetCategory,b:Type)" -> "Category"

  "Category" [color=lightblue];
  }
```

2.13 OpenMath (OM)



See:

⇒ "StringCategory" (STRICAT) 10.16 on page 747
⇐ "Category" (CATEGORY) 2.1 on page 3

Exports:
OMwrite

These are directly exported but not implemented:

```
OMwrite : % -> String
OMwrite : (%,Boolean) -> String
OMwrite : (OpenMathDevice,%) -> Void
OMwrite : (OpenMathDevice,%,Boolean) -> Void
```

⟨*category OM OpenMath*⟩≡

```
)abbrev category OM OpenMath
++ Author: Mike Dewar & Vilya Harvey
++ Basic Functions: OMwrite
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \spadtype{OpenMath} provides operations for exporting an object
++ in OpenMath format.

OpenMath(): Category == with
  OMwrite  : % -> String
  ++ OMwrite(u) returns the OpenMath XML encoding of \axiom{u} as a
  ++ complete OpenMath object.
  OMwrite  : (%, Boolean) -> String
  ++ OMwrite(u, true) returns the OpenMath XML encoding of \axiom{u}
  ++ as a complete OpenMath object; OMwrite(u, false) returns the
```

```
    ++ OpenMath XML encoding of \axiom{u} as an OpenMath fragment.
    OMwrite  : (OpenMathDevice, %) -> Void
    ++ OMwrite(dev, u) writes the OpenMath form of \axiom{u} to the
    ++ OpenMath device \axiom{dev} as a complete OpenMath object.
    OMwrite  : (OpenMathDevice, %, Boolean) -> Void
    ++ OMwrite(dev, u, true) writes the OpenMath form of \axiom{u} to
    ++ the OpenMath device \axiom{dev} as a complete OpenMath object;
    ++ OMwrite(dev, u, false) writes the object as an OpenMath fragment.
```

⟨*OM.dotabb*⟩≡

```
  "OM"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=OM"];
  "OM" -> "CATEGORY"
```

⟨*OM.dotfull*⟩≡

```
  "OpenMath()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=OM"];
  "OpenMath()" -> "Category"
```

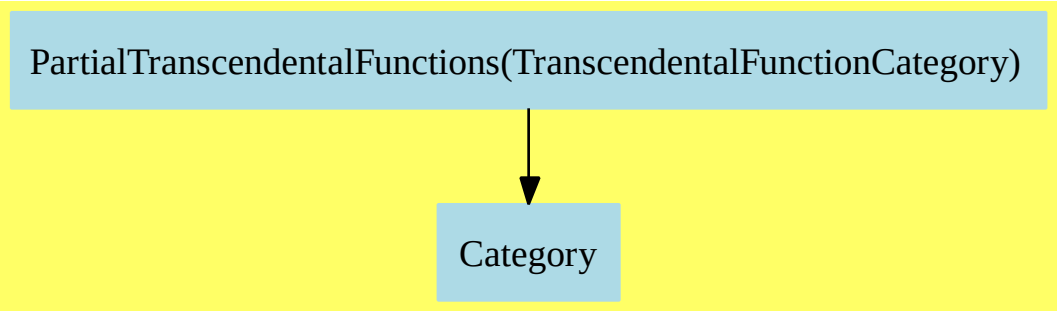
⟨*OM.dotpic*⟩≡

```
  digraph pic {
   fontsize=10;
   bgcolor="#FFFF66";
   node [shape=box, color=white, style=filled];

  "OpenMath()" [color=lightblue];
  "OpenMath()" -> "Category"

  "Category" [color=lightblue];
  }
```

2.14 PartialTranscendentalFunctions (PTRANFN)



See:

⇐ "Category" (CATEGORY) 2.1 on page 3

Exports:

acosIfCan	acoshIfCan	acotIfCan	acothIfCan	acscIfCan
acschIfCan	asecIfCan	asechIfCan	asinIfCan	asinhIfCan
atanIfCan	atanhIfCan	cosIfCan	coshIfCan	cotIfCan
cothIfCan	cscIfCan	cschIfCan	expIfCan	logIfCan
nthRootIfCan	secIfCan	sechIfCan	sinIfCan	sinhIfCan
tanIfCan	tanhIfCan			

These are directly exported but not implemented:

```
acosIfCan : K -> Union(K,"failed")
acoshIfCan : K -> Union(K,"failed")
acotIfCan : K -> Union(K,"failed")
acothIfCan : K -> Union(K,"failed")
acscIfCan : K -> Union(K,"failed")
acschIfCan : K -> Union(K,"failed")
asecIfCan : K -> Union(K,"failed")
asechIfCan : K -> Union(K,"failed")
asinIfCan : K -> Union(K,"failed")
asinhIfCan : K -> Union(K,"failed")
atanIfCan : K -> Union(K,"failed")
atanhIfCan : K -> Union(K,"failed")
cosIfCan : K -> Union(K,"failed")
coshIfCan : K -> Union(K,"failed")
cotIfCan : K -> Union(K,"failed")
cothIfCan : K -> Union(K,"failed")
cscIfCan : K -> Union(K,"failed")
cschIfCan : K -> Union(K,"failed")
expIfCan : K -> Union(K,"failed")
logIfCan : K -> Union(K,"failed")
nthRootIfCan : (K,NonNegativeInteger) -> Union(K,"failed")
secIfCan : K -> Union(K,"failed")
```

```
sechIfCan : K -> Union(K,"failed")
sinIfCan : K -> Union(K,"failed")
sinhIfCan : K -> Union(K,"failed")
tanIfCan : K -> Union(K,"failed")
tanhIfCan : K -> Union(K,"failed")
```

⟨*category PTRANFN PartialTranscendentalFunctions*⟩≡

```
)abbrev category PTRANFN PartialTranscendentalFunctions
++ Description of a package which provides partial transcendental
++ functions, i.e. functions which return an answer or "failed"
++ Author: Clifton J. Williamson
++ Date Created: 12 February 1990
++ Date Last Updated: 14 February 1990
++ Keywords:
++ Examples:
++ References:
++ Description:
++ This is the description of any package which provides partial
++ functions on a domain belonging to TranscendentalFunctionCategory.

PartialTranscendentalFunctions(K): Category == Definition where
  K :     TranscendentalFunctionCategory
  NNI ==> NonNegativeInteger

  Definition ==> with

--% Exponentials and Logarithms

    nthRootIfCan: (K,NNI) -> Union(K,"failed")
      ++ nthRootIfCan(z,n) returns the nth root of z if possible,
      ++ and "failed" otherwise.
    expIfCan: K -> Union(K,"failed")
      ++ expIfCan(z) returns exp(z) if possible, and "failed" otherwise.
    logIfCan: K -> Union(K,"failed")
      ++ logIfCan(z) returns log(z) if possible, and "failed" otherwise.

--% TrigonometricFunctionCategory

    sinIfCan : K -> Union(K,"failed")
      ++ sinIfCan(z) returns sin(z) if possible, and "failed" otherwise.
    cosIfCan: K -> Union(K,"failed")
      ++ cosIfCan(z) returns cos(z) if possible, and "failed" otherwise.
    tanIfCan: K -> Union(K,"failed")
      ++ tanIfCan(z) returns tan(z) if possible, and "failed" otherwise.
    cotIfCan: K -> Union(K,"failed")
      ++ cotIfCan(z) returns cot(z) if possible, and "failed" otherwise.
    secIfCan: K -> Union(K,"failed")
```

```
      ++ secIfCan(z) returns sec(z) if possible, and "failed" otherwise.
    cscIfCan: K -> Union(K,"failed")
      ++ cscIfCan(z) returns csc(z) if possible, and "failed" otherwise.

  --% ArcTrigonometricFunctionCategory

    asinIfCan: K -> Union(K,"failed")
      ++ asinIfCan(z) returns asin(z) if possible, and "failed" otherwise.
    acosIfCan: K -> Union(K,"failed")
      ++ acosIfCan(z) returns acos(z) if possible, and "failed" otherwise.
    atanIfCan: K -> Union(K,"failed")
      ++ atanIfCan(z) returns atan(z) if possible, and "failed" otherwise.
    acotIfCan: K -> Union(K,"failed")
      ++ acotIfCan(z) returns acot(z) if possible, and "failed" otherwise.
    asecIfCan: K -> Union(K,"failed")
      ++ asecIfCan(z) returns asec(z) if possible, and "failed" otherwise.
    acscIfCan: K -> Union(K,"failed")
      ++ acscIfCan(z) returns acsc(z) if possible, and "failed" otherwise.

  --% HyperbolicFunctionCategory

    sinhIfCan: K -> Union(K,"failed")
      ++ sinhIfCan(z) returns sinh(z) if possible, and "failed" otherwise.
    coshIfCan: K -> Union(K,"failed")
      ++ coshIfCan(z) returns cosh(z) if possible, and "failed" otherwise.
    tanhIfCan: K -> Union(K,"failed")
      ++ tanhIfCan(z) returns tanh(z) if possible, and "failed" otherwise.
    cothIfCan: K -> Union(K,"failed")
      ++ cothIfCan(z) returns coth(z) if possible, and "failed" otherwise.
    sechIfCan: K -> Union(K,"failed")
      ++ sechIfCan(z) returns sech(z) if possible, and "failed" otherwise.
    cschIfCan: K -> Union(K,"failed")
      ++ cschIfCan(z) returns csch(z) if possible, and "failed" otherwise.

  --% ArcHyperbolicFunctionCategory

    asinhIfCan: K -> Union(K,"failed")
      ++ asinhIfCan(z) returns asinh(z) if possible, and "failed" otherwise.
    acoshIfCan: K -> Union(K,"failed")
      ++ acoshIfCan(z) returns acosh(z) if possible, and "failed" otherwise.
    atanhIfCan: K -> Union(K,"failed")
      ++ atanhIfCan(z) returns atanh(z) if possible, and "failed" otherwise.
    acothIfCan: K -> Union(K,"failed")
      ++ acothIfCan(z) returns acoth(z) if possible, and "failed" otherwise.
    asechIfCan: K -> Union(K,"failed")
      ++ asechIfCan(z) returns asech(z) if possible, and "failed" otherwise.
```

```
      acschIfCan: K -> Union(K,"failed")
        ++ acschIfCan(z) returns acsch(z) if possible, and "failed" otherwise.
```

⟨*PTRANFN.dotabb*⟩≡

```
"PTRANFN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PTRANFN"];
"PTRANFN" -> "CATEGORY"
```

⟨*PTRANFN.dotfull*⟩≡

```
"PartialTranscendentalFunctions(TranscendentalFunctionCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PTRANFN"];
"PartialTranscendentalFunctions(TranscendentalFunctionCategory)" ->
   "Category()"
```

⟨*PTRANFN.dotpic*⟩≡

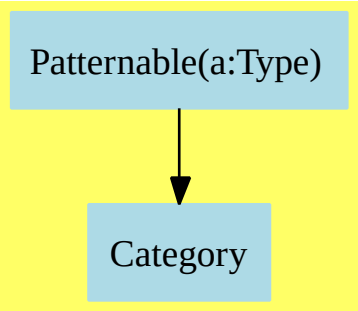
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PartialTranscendentalFunctions(TranscendentalFunctionCategory)"
 [color=lightblue];
"PartialTranscendentalFunctions(TranscendentalFunctionCategory)" ->
   "Category"

"Category" [color=lightblue];

}
```

2.15 Patternable (PATAB)



See:

⇒ "ComplexCategory" (COMPCAT) 20.1 on page 1315
⇒ "FunctionSpace" (FS) 17.5 on page 1095
⇒ "IntegerNumberSystem" (INS) 16.2 on page 1011
⇒ "QuotientFieldCategory" (QFCAT) 17.6 on page 1121
⇐ "Category" (CATEGORY) 2.1 on page 3

Exports:
convert

These exports come from (p19) ConvertibleTo(Pattern(Integer)):

```
convert : % -> Pattern Integer if R has KONVERT PATTERN INT
```

These exports come from (p19) ConvertibleTo(Pattern(Float)):

```
convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT
```

⟨*category PATAB Patternable*⟩≡

```
)abbrev category PATAB Patternable
++ Category of sets that can be converted to useful patterns
++ Author: Manuel Bronstein
++ Date Created: 29 Nov 1989
++ Date Last Updated: 29 Nov 1989
++ Description:
++   An object S is Patternable over an object R if S can
++   lift the conversions from R into \spadtype{Pattern(Integer)} and
++   \spadtype{Pattern(Float)} to itself;
++ Keywords: pattern, matching.
Patternable(R:Type): Category == with
  if R has ConvertibleTo Pattern Integer then
           ConvertibleTo Pattern Integer
  if R has ConvertibleTo Pattern Float then
           ConvertibleTo Pattern Float
```

⟨*PATAB.dotabb*⟩≡
```
"PATAB"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PATAB"];
"PATAB" -> "CATEGORY"
```

⟨*PATAB.dotfull*⟩≡
```
"Patternable(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PATAB"];
"Patternable(a:Type)" -> "Category"

"Patternable(IntegralDomain)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PATAB"];
"Patternable(IntegralDomain)" -> "Patternable(a:Type)"

"Patternable(OrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PATAB"];
"Patternable(OrderedSet)" -> "Patternable(a:Type)"

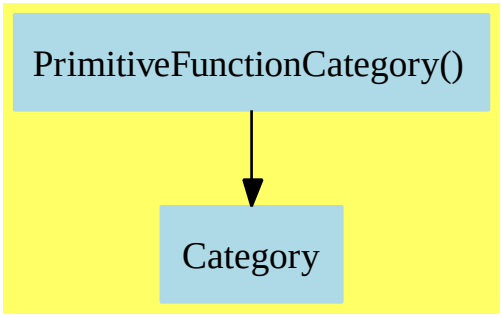
"Patternable(CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PATAB"];
"Patternable(CommutativeRing)" -> "Patternable(a:Type)"
```

⟨*PATAB.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Patternable(a:Type)" [color=lightblue];
"Patternable(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

2.16 PrimitiveFunctionCategory (PRIMCAT)



See:

⇒ "LiouvillianFunctionCategory" (LFCAT) 4.14 on page 148
⇐ "Category" (CATEGORY) 2.1 on page 3

Exports:
integral

These are directly exported but not implemented:

```
integral : (%,Symbol) -> %
integral : (%,SegmentBinding %) -> %
```

⟨*category PRIMCAT PrimitiveFunctionCategory*⟩≡

```
)abbrev category PRIMCAT PrimitiveFunctionCategory
++ Category for the integral functions
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description: Category for the functions defined by integrals;
PrimitiveFunctionCategory(): Category == with
    integral: ($, Symbol) -> $
      ++ integral(f, x) returns the formal integral of f dx.
    integral: ($, SegmentBinding $) -> $
      ++ integral(f, x = a..b) returns the formal definite integral
      ++ of f dx for x between \spad{a} and b.
```

⟨*PRIMCAT.dotabb*⟩≡

```
"PRIMCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PRIMCAT"];
"PRIMCAT" -> "CATEGORY"
```

⟨*PRIMCAT.dotfull*⟩≡

```
"PrimitiveFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PRIMCAT"];
"PrimitiveFunctionCategory()" -> "Category"
```

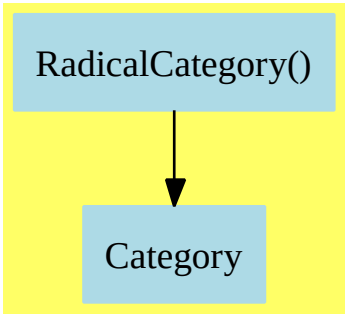
⟨*PRIMCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PrimitiveFunctionCategory()" [color=lightblue];
"PrimitiveFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

2.17 RadicalCategory (RADCAT)



See:

⇒ "AlgebraicallyClosedField" (ACF) 17.1 on page 1061
⇒ "IntervalCategory" (INTCAT) 14.2 on page 950
⇒ "RealClosedField" (RCFIELD) 17.7 on page 1132
⇒ "RealNumberSystem" (RNS) 17.8 on page 1141
⇒ "UnivariateLaurentSeriesCategory" (ULSCAT) 17.10 on page 1186
⇒ "UnivariatePuiseuxSeriesCategory" (UPXSCAT) 17.11 on page 1195
⇒ "UnivariateTaylorSeriesCategory" (UTSCAT) 16.5 on page 1046
⇐ "Category" (CATEGORY) 2.1 on page 3

Exports:
nthRoot sqrt ?**?

These are directly exported but not implemented:

```
?**? : (%,Fraction Integer) -> %
```

These are implemented by this category:

```
nthRoot : (%,Integer) -> %
sqrt : % -> %
```

⟨*category RADCAT RadicalCategory*⟩≡

```
)abbrev category RADCAT RadicalCategory
++ Author:
++ Date Created:
++ Change History:
++ Basic Operations: nthRoot, sqrt, **
++ Related Constructors:
++ Keywords: rational numbers
++ Description: The \spad{RadicalCategory} is a model for the
++              rational numbers.
RadicalCategory(): Category == with
  sqrt   : % -> %
```

```
        ++ sqrt(x) returns the square root of x.
    nthRoot: (%, Integer) -> %
        ++ nthRoot(x,n) returns the nth root of x.
    _*_*   : (%, Fraction Integer) -> %
        ++ x ** y is the rational exponentiation of x by the power y.
   add
    sqrt x        == x ** inv(2::Fraction(Integer))
    nthRoot(x, n) == x ** inv(n::Fraction(Integer))
```

⟨*RADCAT.dotabb*⟩≡

```
  "RADCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=RADCAT"];
  "RADCAT" -> "CATEGORY"
```

⟨*RADCAT.dotfull*⟩≡

```
  "RadicalCategory()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=RADCAT"];
  "RadicalCategory()" -> "Category"
```

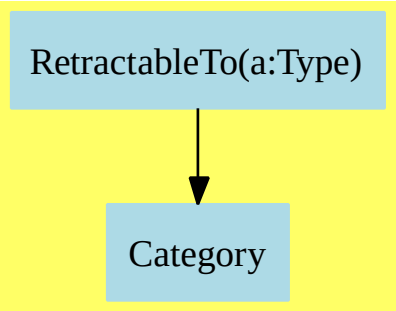
⟨*RADCAT.dotpic*⟩≡

```
  digraph pic {
   fontsize=10;
   bgcolor="#FFFF66";
   node [shape=box, color=white, style=filled];

  "RadicalCategory()" [color=lightblue];
  "RadicalCategory()" -> "Category"

  "Category" [color=lightblue];
  }
```

2.18 RetractableTo (RETRACT)



See:

⇒ “DifferentialPolynomialCategory” (DPOLCAT) 17.2 on page 1069
⇒ “DifferentialVariableCategory” (DVARCAT) 5.5 on page 224
⇒ “ExtensionField” (XF) 18.2 on page 1237
⇒ “ExpressionSpace” (ES) 5.6 on page 230
⇒ “FiniteAlgebraicExtensionField” (FAXF) 19.1 on page 1291
⇒ “FortranMachineTypeCategory” (FMTC) 13.2 on page 913
⇒ “FreeAbelianMonoidCategory” (FAMONC) 7.7 on page 451
⇒ “FreeModuleCat” (FMCAT) 10.7 on page 699
⇒ “FullyRetractableTo” (FRETRCT) 3.6 on page 71
⇒ “FunctionSpace” (FS) 17.5 on page 1095
⇒ “GradedAlgebra” (GRALG) 5.7 on page 242
⇒ “IntegerNumberSystem” (INS) 16.2 on page 1011
⇒ “IntervalCategory” (INTCAT) 14.2 on page 950
⇒ “PolynomialCategory” (POLYCAT) 16.4 on page 1027
⇒ “QuotientFieldCategory” (QFCAT) 17.6 on page 1121
⇒ “RealNumberSystem” (RNS) 17.8 on page 1141
⇒ “UnivariateLaurentSeriesConstructorCategory” (ULSCCAT) 18.6 on page 1267

⇒ “UnivariatePuiseuxSeriesConstructorCategory” (UPXSCCA) 18.7 on page 1280

⇒ “XFreeAlgebra” (XFALG) 11.8 on page 807
⇐ “Category” (CATEGORY) 2.1 on page 3

Exports:
coerce retract retractIfCan

These are directly exported but not implemented:

```
coerce : S -> %
retractIfCan : % -> Union(S,"failed")
```

These are implemented by this category:

```
retract : % -> S
```

⟨*category RETRACT RetractableTo*⟩≡

```
)abbrev category RETRACT RetractableTo
++ Category for retract
++ Author: ???
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description:
++ A is retractable to B means that some elementsif A can be converted
++ into elements of B and any element of B can be converted into an
++ element of A.
RetractableTo(S: Type): Category == with
    coerce:       S -> %
      ++ coerce(a) transforms a into an element of %.
    retractIfCan: % -> Union(S,"failed")
      ++ retractIfCan(a) transforms a into an element of S if possible.
      ++ Returns "failed" if a cannot be made into an element of S.
    retract:      % -> S
      ++ retract(a) transforms a into an element of S if possible.
      ++ Error: if a cannot be made into an element of S.
  add
    retract(s) ==
      (u:=retractIfCan s) case "failed" => error "not retractable"
      u
```

⟨*RETRACT.dotabb*⟩≡

```
"RETRACT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RETRACT" -> "CATEGORY"
```

⟨*RETRACT.dotfull*⟩≡

```
"RetractableTo(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(a:Type)" -> "Category"

"RetractableTo(SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(SetCategory)" -> "RetractableTo(a:Type)"

"RetractableTo(OrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(OrderedSet)" -> "RetractableTo(a:Type)"

"RetractableTo(Symbol)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Symbol)" -> "RetractableTo(a:Type)"

"RetractableTo(Integer)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Integer)" -> "RetractableTo(a:Type)"

"RetractableTo(NonNegativeInteger)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(NonNegativeInteger)" -> "RetractableTo(a:Type)"

"RetractableTo(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Fraction(Integer))" -> "RetractableTo(a:Type)"

"RetractableTo(Float)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Float)" -> "RetractableTo(a:Type)"

"RetractableTo(Kernel(ExpressionSpace))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Kernel(ExpressionSpace))" -> "RetractableTo(a:Type)"

"RetractableTo(CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(CommutativeRing)" -> "RetractableTo(a:Type)"

"RetractableTo(UnivariatePuiseuxSeriesCategory(Ring))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(UnivariatePuiseuxSeriesCategory(Ring))"
  -> "RetractableTo(a:Type)"
```

```
  "RetractableTo(Field)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
  "RetractableTo(Field)" -> "RetractableTo(a:Type)"

  "RetractableTo(IntegralDomain)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
  "RetractableTo(IntegralDomain)" -> "RetractableTo(a:Type)"

  "RetractableTo(OrderedFreeMonoid(OrderedSet))"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
  "RetractableTo(OrderedFreeMonoid(OrderedSet))" -> "RetractableTo(a:Type)"
```

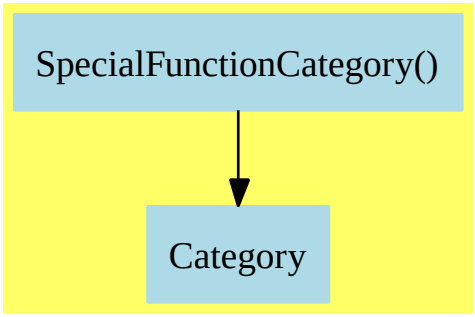
⟨*RETRACT.dotpic*⟩≡

```
  digraph pic {
   fontsize=10;
   bgcolor="#FFFF66";
   node [shape=box, color=white, style=filled];

  "RetractableTo(a:Type)" [color=lightblue];
  "RetractableTo(a:Type)" -> "Category"

  "Category" [color=lightblue];
  }
```

2.19 SpecialFunctionCategory (SPFCAT)



See:

⇐ "Category" (CATEGORY) 2.1 on page 3

Exports:

abs	airyAi	airyBi	besselI	besselJ
besselK	besselY	Beta	digamma	Gamma
polygamma				

These are directly exported but not implemented:

```
abs : % -> %
airyAi : % -> %
airyBi : % -> %
besselI : (%,%) -> %
besselJ : (%,%) -> %
besselK : (%,%) -> %
besselY : (%,%) -> %
Beta : (%,%) -> %
digamma : % -> %
Gamma : % -> %
Gamma : (%,%) -> %
polygamma : (%,%) -> %
```

⟨*category SPFCAT SpecialFunctionCategory*⟩≡

```
)abbrev category SPFCAT SpecialFunctionCategory
++ Category for the other special functions
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 11 May 1993
++ Description: Category for the other special functions;
SpecialFunctionCategory(): Category == with
    abs :      $ -> $
        ++ abs(x) returns the absolute value of x.
    Gamma:     $ -> $
        ++ Gamma(x) is the Euler Gamma function.
```

```
Beta:      ($,$)->$
    ++ Beta(x,y) is \spad{Gamma(x) * Gamma(y)/Gamma(x+y)}.
digamma:   $ -> $
    ++ digamma(x) is the logarithmic derivative of \spad{Gamma(x)}
    ++ (often written \spad{psi(x)} in the literature).
polygamma: ($, $) -> $
    ++ polygamma(k,x) is the \spad{k-th} derivative of \spad{digamma(x)},
    ++ (often written \spad{psi(k,x)} in the literature).
Gamma:     ($, $) -> $
    ++ Gamma(a,x) is the incomplete Gamma function.
besselJ:   ($,$) -> $
    ++ besselJ(v,z) is the Bessel function of the first kind.
besselY:   ($,$) -> $
    ++ besselY(v,z) is the Bessel function of the second kind.
besselI:   ($,$) -> $
    ++ besselI(v,z) is the modified Bessel function of the first kind.
besselK:   ($,$) -> $
    ++ besselK(v,z) is the modified Bessel function of the second kind.
airyAi:    $ -> $
    ++ airyAi(x) is the Airy function \spad{Ai(x)}.
airyBi:    $ -> $
    ++ airyBi(x) is the Airy function \spad{Bi(x)}.
```

⟨*SPFCAT.dotabb*⟩≡

```
"SPFCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SPFCAT"];
"SPFCAT" -> "CATEGORY"
```

⟨*SPFCAT.dotfull*⟩≡

```
"SpecialFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SPFCAT"];
"SpecialFunctionCategory()" -> "Category"
```

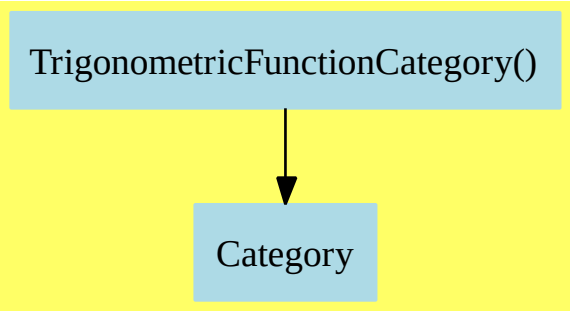
⟨*SPFCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"SpecialFunctionCategory()" [color=lightblue];
"SpecialFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

2.20 TrigonometricFunctionCategory (TRIGCAT)



The csc and sec functions were modified to include an intermediate test to check that the argument has a reciprocal values.

See:

⇒ "TranscendentalFunctionCategory" (TRANFUN) 3.14 on page 94
⇐ "Category" (CATEGORY) 2.1 on page 3

Exports:
cos cot csc sec sin tan

These are directly exported but not implemented:

```
cos : % -> %
sin : % -> %
```

These are implemented by this category:

```
cot : % -> %
csc : % -> %
sec : % -> %
tan : % -> %
```

⟨*category TRIGCAT TrigonometricFunctionCategory*⟩≡

```
)abbrev category TRIGCAT TrigonometricFunctionCategory
++ Category for the trigonometric functions
++ Author: ???
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description: Category for the trigonometric functions;
TrigonometricFunctionCategory(): Category == with
    cos: $ -> $        ++ cos(x) returns the cosine of x.
    cot: $ -> $        ++ cot(x) returns the cotangent of x.
    csc: $ -> $        ++ csc(x) returns the cosecant of x.
    sec: $ -> $        ++ sec(x) returns the secant of x.
    sin: $ -> $        ++ sin(x) returns the sine of x.
```

```
      tan: $ -> $        ++ tan(x) returns the tangent of x.
   add
      if $ has Ring then
         csc x ==
           (a := recip(sin x)) case "failed" => error "csc: no reciprocal"
           a::$
         sec x ==
           (a := recip(cos x)) case "failed" => error "sec: no reciprocal"
           a::$
         tan x == sin x * sec x
         cot x == cos x * csc x
```

⟨*TRIGCAT.dotabb*⟩≡

```
  "TRIGCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=TRIGCAT"];
  "TRIGCAT" -> "CATEGORY"
```

⟨*TRIGCAT.dotfull*⟩≡

```
  "TrigonometricFunctionCategory()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=TRIGCAT"];
  "TrigonometricFunctionCategory()" -> "Category"
```

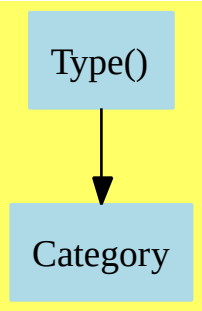
⟨*TRIGCAT.dotpic*⟩≡

```
  digraph pic {
   fontsize=10;
   bgcolor="#FFFF66";
   node [shape=box, color=white, style=filled];

  "TrigonometricFunctionCategory()" [color=lightblue];
  "TrigonometricFunctionCategory()" -> "Category"

  "Category" [color=lightblue];
  }
```

2.21 Type (TYPE)



See:

⇒ “Aggregate” (AGG) 3.1 on page 55
⇒ “FortranProgramCategory” (FORTCAT) 3.5 on page 68
⇒ “FullyPatternMatchable” (FPATMAB) 3.7 on page 74
⇒ “SegmentCategory” (SEGCAT) 3.12 on page 88
⇐ “Category” (CATEGORY) 2.1 on page 3

Attributes exported:

- **nil**

⟨*category TYPE Type*⟩≡
```
)abbrev category TYPE Type
++ The new fundamental Type (keeping Object for 1.5 as well)
++ Author: Richard Jenks
++ Date Created: 14 May 1992
++ Date Last Updated: 14 May 1992
++ Description: The fundamental Type;
Type(): Category == with nil
```

⟨*TYPE.dotabb*⟩≡
```
"TYPE" [color=lightblue,href="bookvol10.2.pdf#nameddest=TYPE"];
"TYPE" -> "CATEGORY"
```

⟨*TYPE.dotfull*⟩≡
```
"Type()" [color=lightblue,href="bookvol10.2.pdf#nameddest=TYPE"];
"Type()" -> "Category"
```

⟨*TYPE.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

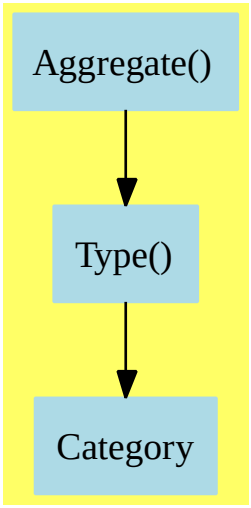
"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

Chapter 3

Category Layer 2

3.1 Aggregate (AGG)



See:

⇒ "HomogeneousAggregate" (HOAGG) 4.12 on page 139
⇐ "Type" (TYPE) 2.21 on page 53

Attributes exported:

- **nil**

Exports:
empty? eq? less? more? sample size?

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are implemented by this category:

```
empty? : % -> Boolean
eq? : (%,%) -> Boolean
less? : (%,NonNegativeInteger) -> Boolean
more? : (%,NonNegativeInteger) -> Boolean
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
```

⟨*category AGG Aggregate*⟩≡

```
)abbrev category AGG Aggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The notion of aggregate serves to model any data structure aggregate,
++ designating any collection of objects,
++ with heterogenous or homogeneous members,
++ with a finite or infinite number
++ of members, explicitly or implicitly represented.
++ An aggregate can in principle
++ represent everything from a string of characters to abstract sets such
++ as "the set of x satisfying relation {\em r(x)}"
++ An attribute \spadatt{finiteAggregate} is used to assert that a domain
++ has a finite number of elements.
Aggregate: Category == Type with
   eq?: (%,%) -> Boolean
     ++ eq?(u,v) tests if u and v are same objects.
   copy: % -> %
     ++ copy(u) returns a top-level (non-recursive) copy of u.
     ++ Note: for collections, \axiom{copy(u) == [x for x in u]}.
   empty: () -> %
     ++ empty()$D creates an aggregate of type D with 0 elements.
     ++ Note: The {\em $D} can be dropped if understood by context,
     ++ e.g. \axiom{u: D := empty()}.
   empty?: % -> Boolean
```

```
      ++ empty?(u) tests if u has 0 elements.
    less?: (%,NonNegativeInteger) -> Boolean
      ++ less?(u,n) tests if u has less than n elements.
    more?: (%,NonNegativeInteger) -> Boolean
      ++ more?(u,n) tests if u has greater than n elements.
    size?: (%,NonNegativeInteger) -> Boolean
      ++ size?(u,n) tests if u has exactly n elements.
    sample: constant -> %
      ++ sample yields a value of type %
    if % has finiteAggregate then
      "#": % -> NonNegativeInteger
        ++ # u returns the number of items in u.
  add
   eq?(a,b) == EQ(a,b)$Lisp
   sample() == empty()
   if % has finiteAggregate then
     empty? a   == #a = 0
     less?(a,n) == #a < n
     more?(a,n) == #a > n
     size?(a,n) == #a = n
```

⟨*AGG.dotabb*⟩≡

```
"AGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=AGG"];
"AGG" -> "TYPE"
```

⟨*AGG.dotfull*⟩≡

```
"Aggregate()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=AGG"];
"Aggregate()" -> "Type()"
```

⟨*AGG.dotpic*⟩≡

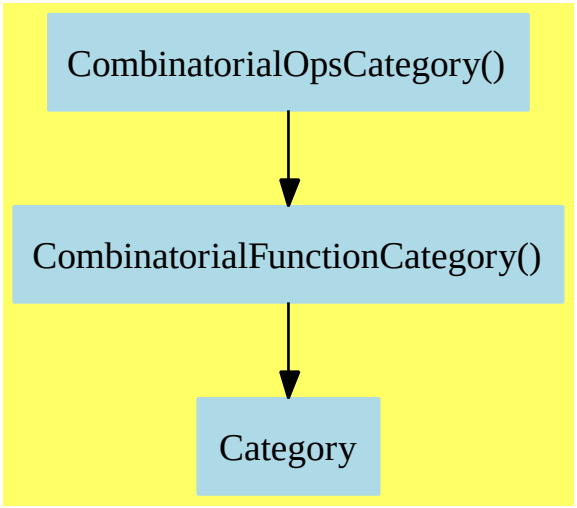
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

3.2 CombinatorialOpsCategory (COMBOPC)



See:

⇐ "CombinatorialFunctionCategory" (CFCAT) 2.7 on page 17

Exports:
binomial factorial factorials permutation product summation

These are directly exported but not implemented:

```
factorials : % -> %
factorials : (%,Symbol) -> %
product : (%,Symbol) -> %
product : (%,SegmentBinding %) -> %
summation : (%,Symbol) -> %
summation : (%,SegmentBinding %) -> %
```

These exports come from (p17) CombinatorialFunctionCategory():

```
binomial : (%,%) -> %
factorial : % -> %
permutation : (%,%) -> %
```

⟨*category COMBOPC CombinatorialOpsCategory*⟩≡

```
)abbrev category COMBOPC CombinatorialOpsCategory
++ Category for summations and products
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 22 February 1993 (JHD/BMT)
++ Description:
```

```
++   CombinatorialOpsCategory is the category obtaining by adjoining
++   summations and products to the usual combinatorial operations;
CombinatorialOpsCategory(): Category ==
  CombinatorialFunctionCategory with
    factorials : $ -> $
      ++ factorials(f) rewrites the permutations and binomials in f
      ++ in terms of factorials;
    factorials : ($, Symbol) -> $
      ++ factorials(f, x) rewrites the permutations and binomials in f
      ++ involving x in terms of factorials;
    summation  : ($, Symbol)            -> $
      ++ summation(f(n), n) returns the formal sum S(n) which verifies
      ++ S(n+1) - S(n) = f(n);
    summation  : ($, SegmentBinding $)  -> $
      ++ summation(f(n), n = a..b) returns f(a) + ... + f(b) as a
      ++ formal sum;
    product    : ($, Symbol)            -> $
      ++ product(f(n), n) returns the formal product P(n) which verifies
      ++ P(n+1)/P(n) = f(n);
    product    : ($, SegmentBinding  $) -> $
      ++ product(f(n), n = a..b) returns f(a) * ... * f(b) as a
      ++ formal product;
```

⟨*COMBOPC.dotabb*⟩≡

```
"COMBOPC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=COMBOPC"];
"COMBOPC" -> "CFCAT"
```

⟨*COMBOPC.dotfull*⟩≡

```
"CombinatorialOpsCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=COMBOPC"];
"CombinatorialOpsCategory()" -> "CombinatorialFunctionCategory()"
```

⟨*COMBOPC.dotpic*⟩≡

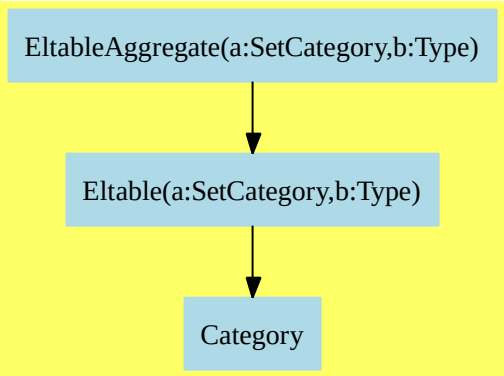
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"CombinatorialOpsCategory()" [color=lightblue];
"CombinatorialOpsCategory()" -> "CombinatorialFunctionCategory()"

"CombinatorialFunctionCategory()" [color=lightblue];
"CombinatorialFunctionCategory()" -> "Category"

"Category" [color=lightblue];
}
```

3.3 EltableAggregate (ELTAGG)



See:

⇒ "IndexedAggregate" (IXAGG) 5.8 on page 246
⇐ "Eltable" (ELTAB) 2.10 on page 25

Exports:
elt qelt qsetelt! setelt ?.?

Attributes Used:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are part of this category's direct exports:

```
elt : (%,Dom,Im) -> Im
setelt : (%,Dom,Im) -> Im if $ has shallowlyMutable
```

These are implemented by this category:

```
qelt : (%,Dom) -> Im
qsetelt! : (%,Dom,Im) -> Im if $ has shallowlyMutable
```

These exports come from (p25) Eltable():

```
?.? : (%,Dom) -> Im
```

⟨*category ELTAGG EltableAggregate*⟩≡

```
)abbrev category ELTAGG EltableAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
```

```
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ An eltable aggregate is one which can be viewed as a function.
++ For example, the list \axiom{[1,7,4]} can applied to 0,1, and 2
++ respectively will return the integers 1,7, and 4; thus this list may
++ be viewed as mapping 0 to 1, 1 to 7 and 2 to 4. In general, an aggregate
++ can map members of a domain {\em Dom} to an image domain {\em Im}.
EltableAggregate(Dom:SetCategory, Im:Type): Category ==
-- This is separated from Eltable
-- and series won't have to support qelt's and setelt's.
  Eltable(Dom, Im) with
    elt : (%, Dom, Im) -> Im
       ++ elt(u, x, y) applies u to x if x is in the domain of u,
       ++ and returns y otherwise.
       ++ For example, if u is a polynomial in \axiom{x} over the rationals,
       ++ \axiom{elt(u,n,0)} may define the coefficient of \axiom{x}
       ++ to the power n, returning 0 when n is out of range.
    qelt: (%, Dom) -> Im
       ++ qelt(u, x) applies \axiom{u} to \axiom{x} without checking whether
       ++ \axiom{x} is in the domain of \axiom{u}.  If \axiom{x} is not
       ++ in the domain of \axiom{u} a memory-access violation may occur.
       ++ If a check on whether \axiom{x} is in the domain of \axiom{u}
       ++ is required, use the function \axiom{elt}.
    if % has shallowlyMutable then
       setelt : (%, Dom, Im) -> Im
           ++ setelt(u,x,y) sets the image of x to be y under u,
           ++ assuming x is in the domain of u.
           ++ Error: if x is not in the domain of u.
           -- this function will soon be renamed as setelt!.
       qsetelt_!: (%, Dom, Im) -> Im
           ++ qsetelt!(u,x,y) sets the image of \axiom{x} to be \axiom{y}
           ++ under \axiom{u}, without checking that \axiom{x} is in
           ++ the domain of \axiom{u}.
           ++ If such a check is required use the function \axiom{setelt}.
 add
  qelt(a, x) == elt(a, x)
  if % has shallowlyMutable then
    qsetelt_!(a, x, y) == (a.x := y)
```

⟨*ELTAGG.dotabb*⟩≡
```
"ELTAGG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ELTAGG"];
"ELTAGG" -> "ELTAB"
```

⟨*ELTAGG.dotfull*⟩≡
```
"EltableAggregate(a:SetCategory,b:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ELTAGG"];
"EltableAggregate(a:SetCategory,b:Type)" -> "Eltable(a:SetCategory,b:Type)"
```

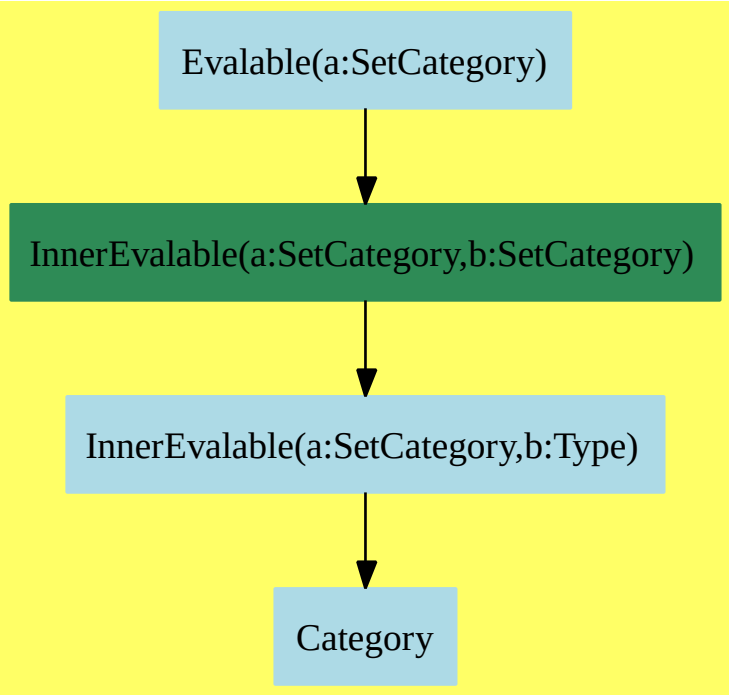
⟨*ELTAGG.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"EltableAggregate(a:SetCategory,b:Type)" [color=lightblue];
"EltableAggregate(a:SetCategory,b:Type)" -> "Eltable(a:SetCategory,b:Type)"

"Eltable(a:SetCategory,b:Type)" [color=lightblue];
"Eltable(a:SetCategory,b:Type)" -> "Category"

"Category" [color=lightblue];
}
```

3.4 Evalable (EVALAB)



See:

⇒ "ExpressionSpace" (ES) 5.6 on page 230
⇒ "MultivariateTaylorSeriesCategory" (MTSCAT) 15.2 on page 983
⇒ "PolynomialCategory" (POLYCAT) 16.4 on page 1027
⇐ "InnerEvalable" (IEVALAB) 2.12 on page 29

Exports:
eval

These are directly exported but not implemented:

```
eval : (%,List Equation R) -> %
```

These are implemented by this category:

```
eval : (%,Equation R) -> %
eval : (%,List R,List R) -> %
```

These exports come from (p29) InnerEvalable(R:SetCategory,R:SetCategory):

```
eval : (%,R,R) -> %
```

⟨*category EVALAB Evalable*⟩≡
```
)abbrev category EVALAB Evalable
++ Author:
++ Date Created:
++ Date Last Updated: June 3, 1991
++ Basic Operations:
++ Related Domains:
++ Also See: FullyEvalable
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
++   This category provides \spadfun{eval} operations.
++   A domain may belong to this category if it is possible to make
++   ‘‘evaluation’’ substitutions.
Evalable(R:SetCategory): Category == InnerEvalable(R,R) with
    eval: ($, Equation R) -> $
       ++ eval(f,x = v) replaces x by v in f.
    eval: ($, List Equation R) -> $
       ++ eval(f, [x1 = v1,...,xn = vn]) replaces xi by vi in f.
 add
    eval(f:$, eq:Equation R) == eval(f, [eq])
    eval(f:$, xs:List R,vs:List R) == eval(f,[x=v for x in xs for v in vs])
```

⟨*EVALAB.dotabb*⟩≡
```
"EVALAB"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=EVALAB"];
"EVALAB" -> "IEVALAB"
```

⟨*EVALAB.dotfull*⟩≡
```
  "Evalable(a:SetCategory)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=EVALAB"];
  "Evalable(a:SetCategory)" -> "InnerEvalable(a:SetCategory,b:SetCategory)"

  "Evalable(MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet))"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=EVALAB"];
  "Evalable(MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet))" ->
      "Evalable(a:SetCategory)"

  "Evalable(ExpressionSpace)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=EVALAB"];
  "Evalable(ExpressionSpace)" -> "Evalable(a:SetCategory)"

  "Evalable(PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet))"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=EVALAB"];
  "Evalable(PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet))"
    -> "Evalable(a:SetCategory)"
```

⟨*EVALAB.dotpic*⟩≡
```
  digraph pic {
   fontsize=10;
   bgcolor="#FFFF66";
   node [shape=box, color=white, style=filled];

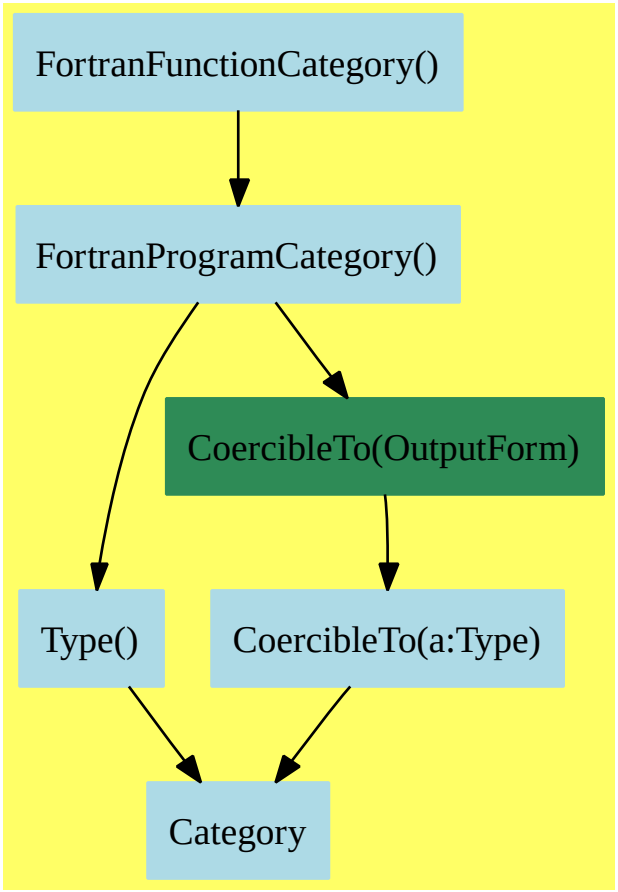
  "Evalable(a:SetCategory)" [color=lightblue];
  "Evalable(a:SetCategory)" -> "InnerEvalable(a:SetCategory,b:SetCategory)"

  "InnerEvalable(a:SetCategory,b:SetCategory)" [color=seagreen];
  "InnerEvalable(a:SetCategory,b:SetCategory)" ->
      "InnerEvalable(a:SetCategory,b:Type)"

  "InnerEvalable(a:SetCategory,b:Type)" [color=lightblue];
  "InnerEvalable(a:SetCategory,b:Type)" -> "Category"

  "Category" [color=lightblue];
  }
```

3.5 FortranProgramCategory (FORTCAT)



See:

⇒ "FortranFunctionCategory" (FORTFN) 4.2 on page 103
⇒ "FortranMatrixCategory" (FMC) 4.3 on page 107
⇒ "FortranMatrixFunctionCategory" (FMFUN) 4.4 on page 110
⇒ "FortranVectorCategory" (FVC) 4.5 on page 114
⇒ "FortranVectorFunctionCategory" (FVFUN) 4.6 on page 117
⇐ "CoercibleTo" (KOERCE) 2.6 on page 15
⇐ "Type" (TYPE) 2.21 on page 53

Exports:
coerce outputAsFortran

Attributes:

- **nil**

These are directly exported but not implemented:

```
outputAsFortran : % -> Void
```

These exports come from (p15) CoercibleTo(OutputForm):

```
coerce : % -> OutputForm
```

⟨*category FORTCAT FortranProgramCategory*⟩≡

```
)abbrev category FORTCAT FortranProgramCategory
++ Author: Mike Dewar
++ Date Created: November 1992
++ Date Last Updated:
++ Basic Operations:
++ Related Constructors: FortranType, FortranCode, Switch
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \axiomType{FortranProgramCategory} provides various models of
++ FORTRAN subprograms.  These can be transformed into actual FORTRAN
++ code.
FortranProgramCategory():Category == Join(Type,CoercibleTo OutputForm) with
    outputAsFortran : $ -> Void
    ++ \axiom{outputAsFortran(u)} translates \axiom{u} into a legal FORTRAN
    ++ subprogram.
```

⟨*FORTCAT.dotabb*⟩≡

```
"FORTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FORTCAT"];
"FORTCAT" -> "KOERCE"
"FORTCAT" -> "TYPE"
```

⟨*FORTCAT.dotfull*⟩≡

```
"FortranProgramCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FORTCAT"];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"
```

⟨*FORTCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FortranProgramCategory()" [color=lightblue];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"

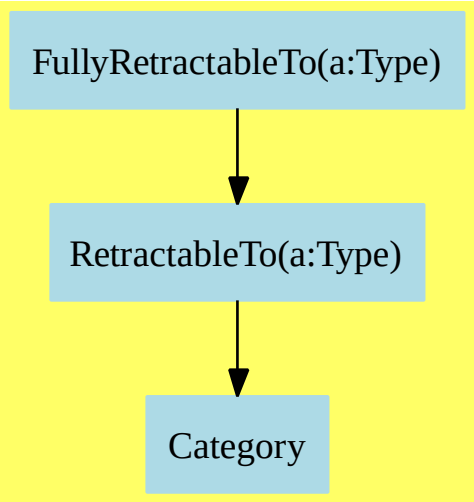
"Type()" [color=lightblue];
"Type()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

3.6 FullyRetractableTo (FRETRCT)



See:

⇒ "ComplexCategory" (COMPCAT) 20.1 on page 1315
⇒ "DirectProductCategory" (DIRPCAT) 12.1 on page 815
⇒ "FiniteAbelianMonoidRing" (FAMR) 14.1 on page 941
⇒ "FunctionSpace" (FS) 17.5 on page 1095
⇒ "MonogenicAlgebra" (MONOGEN) 19.2 on page 1305
⇒ "OctonionCategory" (OC) 12.7 on page 868
⇒ "QuaternionCategory" (QUATCAT) 12.8 on page 878
⇒ "RealClosedField" (RCFIELD) 17.7 on page 1132
⇒ "SquareMatrixCategory" (SMATCAT) 12.9 on page 887
⇒ "UnivariateSkewPolynomialCategory" (OREPCAT) 10.17 on page 754
⇐ "RetractableTo" (RETRACT) 2.18 on page 44

Exports:
coerce retract retractIfCan

These are implemented by this category:

```
coerce : Integer -> %
    if S has RETRACT INT
coerce : Fraction Integer -> %
    if S has RETRACT FRAC INT
retract : % -> Integer
    if S has RETRACT INT
retract : % -> Fraction Integer
    if S has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
    if S has RETRACT INT
```

```
retractIfCan : % -> Union(Fraction Integer,"failed")
    if S has RETRACT FRAC INT
```

These exports come from (p44) RetractableTo(S:Type):

```
coerce : S -> %
retract : % -> S
retractIfCan : % -> Union(S,"failed")
```

⟨*category FRETRCT FullyRetractableTo*⟩≡

```
)abbrev category FRETRCT FullyRetractableTo
++ Author: Manuel Bronstein
++ Description:
++   A is fully retractable to B means that A is retractable to B, and,
++   in addition, if B is retractable to the integers or rational
++   numbers then so is A.
++   In particular, what we are asserting is that there are no integers
++   (rationals) in A which don't retract into B.
++ Date Created: March 1990
++ Date Last Updated: 9 April 1991
FullyRetractableTo(S: Type): Category == RetractableTo(S) with
    if (S has RetractableTo Integer) then RetractableTo Integer
    if (S has RetractableTo Fraction Integer) then
              RetractableTo Fraction Integer
  add
    if not(S is Integer) then
      if (S has RetractableTo Integer) then    -- induction
        coerce(n:Integer):%  == n::S::%
        retract(r:%):Integer == retract(retract(r)@S)

        retractIfCan(r:%):Union(Integer, "failed") ==
          (u:= retractIfCan(r)@Union(S,"failed")) case "failed"=> "failed"
          retractIfCan(u::S)

    if not(S is Fraction Integer) then
      if (S has RetractableTo Fraction Integer) then   -- induction
        coerce(n:Fraction Integer):%  == n::S::%
        retract(r:%):Fraction(Integer) == retract(retract(r)@S)

        retractIfCan(r:%):Union(Fraction Integer, "failed") ==
          (u:=retractIfCan(r)@Union(S,"failed")) case "failed"=>"failed"
          retractIfCan(u::S)
```

⟨*FRETRCT.dotabb*⟩≡
```
"FRETRCT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FRETRCT" -> "RETRACT"
```

⟨*FRETRCT.dotfull*⟩≡
```
"FullyRetractableTo(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(a:Type)" -> "RetractableTo(a:Type)"

"FullyRetractableTo(a:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(a:Ring)" -> "FullyRetractableTo(a:Type)"

"FullyRetractableTo(a:CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(a:CommutativeRing)" -> "FullyRetractableTo(a:Type)"

"FullyRetractableTo(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(a:SetCategory)" -> "FullyRetractableTo(a:Type)"

"FullyRetractableTo(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(Fraction(Integer))" -> "FullyRetractableTo(a:Type)"
```

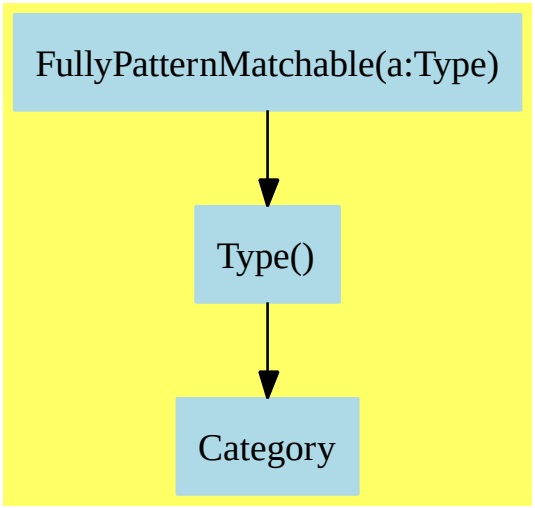
⟨*FRETRCT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FullyRetractableTo(a:Type)" [color=lightblue];
"FullyRetractableTo(a:Type)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

3.7 FullyPatternMatchable (FPATMAB)



See:

⇒ "ComplexCategory" (COMPCAT) 20.1 on page 1315
⇒ "FunctionSpace" (FS) 17.5 on page 1095
⇒ "QuotientFieldCategory" (QFCAT) 17.6 on page 1121
⇐ "Type" (TYPE) 2.21 on page 53

Attributes exported:

- **nil**

Exports:
coerce hash latex patternMatch ?=?
?~=?

These exports come from (p176) PatternMatchable(Integer):

```
coerce : % -> OutputForm
    if R has PATMAB INT
    or R has PATMAB FLOAT
hash : % -> SingleInteger
    if R has PATMAB INT
    or R has PATMAB FLOAT
latex : % -> String
    if R has PATMAB INT
    or R has PATMAB FLOAT
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%))
     -> PatternMatchResult(Integer,%) if R has PATMAB INT
?=? : (%,%) -> Boolean
```

```
    if R has PATMAB INT
    or R has PATMAB FLOAT
?~=? : (%,%) -> Boolean
    if R has PATMAB INT
    or R has PATMAB FLOAT
```

These exports come from (p176) PatternMatchable(Float):

```
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%))
     -> PatternMatchResult(Float,%) if R has PATMAB FLOAT
```

These exports come from (p53) Type():

⟨*category FPATMAB FullyPatternMatchable*⟩≡

```
)abbrev category FPATMAB FullyPatternMatchable
++ Category of sets that can be pattern-matched on
++ Author: Manuel Bronstein
++ Date Created: 28 Nov 1989
++ Date Last Updated: 29 Nov 1989
++ Description:
++   A set S is PatternMatchable over R if S can lift the
++   pattern-matching functions of S over the integers and float
++   to itself (necessary for matching in towers).
++ Keywords: pattern, matching.
FullyPatternMatchable(R:Type): Category == Type with
  if R has PatternMatchable Integer then PatternMatchable Integer
  if R has PatternMatchable Float   then PatternMatchable Float
```

⟨*FPATMAB.dotabb*⟩≡

```
"FPATMAB"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FPATMAB"];
"FPATMAB" -> "TYPE"
```

⟨*FPATMAB.dotfull*⟩≡

```
  "FullyPatternMatchable(a:Type)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FPATMAB"];
  "FullyPatternMatchable(a:Type)" -> "Type()"

  "FullyPatternMatchable(IntegralDomain)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=FPATMAB"];
  "FullyPatternMatchable(IntegralDomain)" ->
    "FullyPatternMatchable(a:Type)"

  "FullyPatternMatchable(OrderedSet)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=FPATMAB"];
  "FullyPatternMatchable(OrderedSet)" ->
    "FullyPatternMatchable(a:Type)"

  "FullyPatternMatchable(CommutativeRing)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=FPATMAB"];
  "FullyPatternMatchable(CommutativeRing)" ->
    "FullyPatternMatchable(a:Type)"
```

⟨*FPATMAB.dotpic*⟩≡

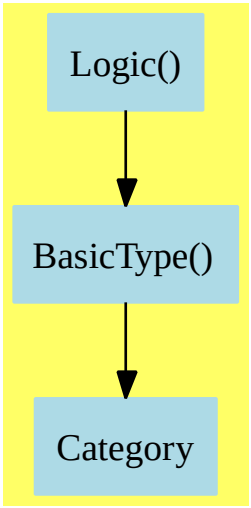
```
  digraph pic {
   fontsize=10;
   bgcolor="#FFFF66";
   node [shape=box, color=white, style=filled];

  "FullyPatternMatchable(a:Type)" [color=lightblue];
  "FullyPatternMatchable(a:Type)" -> "Type()"

  "Type()" [color=lightblue];
  "Type()" -> "Category"

  "Category" [color=lightblue];
  }
```

3.8 Logic (LOGIC)



See:

⇒ "BitAggregate" (BTAGG) 9.2 on page 596
⇐ "BasicType" (BASTYPE) 2.5 on page 13

Exports:
?/\? ?=? ?\/? ~? ?~=?

These are directly exported but not implemented:

```
~? : % -> %
?/\? : (%,%) -> %
```

These are implemented by this category:

```
?\/? : (%,%) -> %
```

These exports come from (p13) BasicType():

```
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category LOGIC Logic*⟩≡

```
)abbrev category LOGIC Logic
++ Author:
++ Date Created:
++ Change History:
++ Basic Operations: ~, /\, \/
++ Related Constructors:
++ Keywords: boolean
```

```
  ++ Description:
  ++ 'Logic' provides the basic operations for lattices,
  ++ e.g., boolean algebra.
  Logic: Category == BasicType with
         _~:        % -> %
          ++ ~(x) returns the logical complement of x.
         _/_\:       (%, %) -> %
          ++ \spadignore { /\ }returns the logical 'meet', e.g. 'and'.
         _\_/:       (%, %) -> %
          ++ \spadignore{ \/ } returns the logical 'join', e.g. 'or'.
    add
      _\_/(x: %,y: %) == _~( _/_\(_~(x), _~(y)))
```

⟨*LOGIC.dotabb*⟩≡

```
  "LOGIC"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=LOGIC"];
  "LOGIC" -> "BASTYPE"
```

⟨*LOGIC.dotfull*⟩≡

```
  "Logic()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=LOGIC"];
  "Logic()" -> "BasicType()"
```

⟨*LOGIC.dotpic*⟩≡

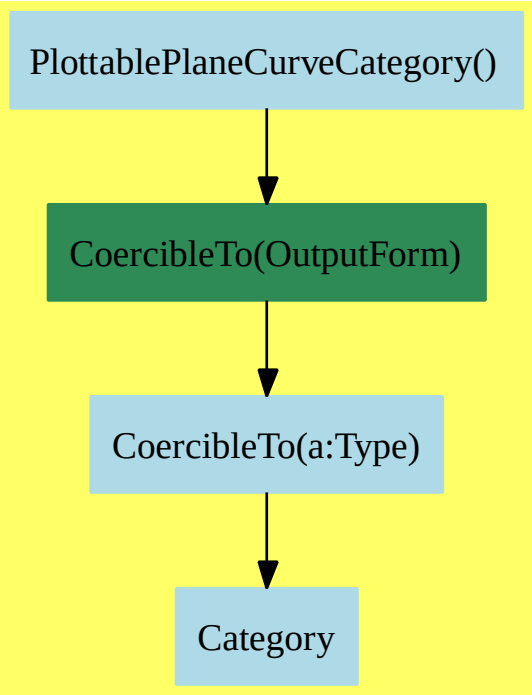
```
  digraph pic {
   fontsize=10;
   bgcolor="#FFFF66";
   node [shape=box, color=white, style=filled];

  "Logic()" [color=lightblue];
  "Logic()" -> "BasicType()"

  "BasicType()" [color=lightblue];
  "BasicType()" -> "Category"

  "Category" [color=lightblue];
  }
```

3.9 PlottablePlaneCurveCategory (PPCURVE)



See:

⇐ "CoercibleTo" (KOERCE) 2.6 on page 15

Exports:
coerce listBranches xRange yRange

These are directly exported but not implemented:

```
listBranches : % -> List List Point DoubleFloat
xRange : % -> Segment DoubleFloat
yRange : % -> Segment DoubleFloat
```

These exports come from (p15) CoercibleTo(OutputForm):

```
coerce : % -> OutputForm
```

⟨*category PPCURVE PlottablePlaneCurveCategory*⟩≡

```
)abbrev category PPCURVE PlottablePlaneCurveCategory
++ Author: Clifton J. Williamson
++ Date Created: 11 January 1990
++ Date Last Updated: 15 June 1990
++ Basic Operations: listBranches, xRange, yRange
```

```
  ++ Related Constructors:
  ++ Also See:
  ++ AMS Classifications:
  ++ Keywords: plot, graphics
  ++ References:
  ++ Description: PlottablePlaneCurveCategory is the category of curves in the
  ++ plane which may be plotted via the graphics facilities.  Functions are
  ++ provided for obtaining lists of lists of points, representing the
  ++ branches of the curve, and for determining the ranges of the
  ++ x-coordinates and y-coordinates of the points on the curve.

  PlottablePlaneCurveCategory(): Category == Definition where
    L     ==> List
    SEG   ==> Segment
    SF    ==> DoubleFloat
    POINT ==> Point DoubleFloat

    Definition ==> CoercibleTo OutputForm with

      listBranches: % -> L L POINT
        ++ listBranches(c) returns a list of lists of points, representing the
        ++ branches of the curve c.
      xRange: % -> SEG SF
        ++ xRange(c) returns the range of the x-coordinates of the points
        ++ on the curve c.
      yRange: % -> SEG SF
        ++ yRange(c) returns the range of the y-coordinates of the points
        ++ on the curve c.
```

⟨*PPCURVE.dotabb*⟩≡

```
  "PPCURVE"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=PPCURVE"];
  "PPCURVE" -> "KOERCE"
```

⟨*PPCURVE.dotfull*⟩≡

```
  "PlottablePlaneCurveCategory()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=PPCURVE"];
  "PlottablePlaneCurveCategory()" -> "CoercibleTo(OutputForm)"
```

⟨*PPCURVE.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

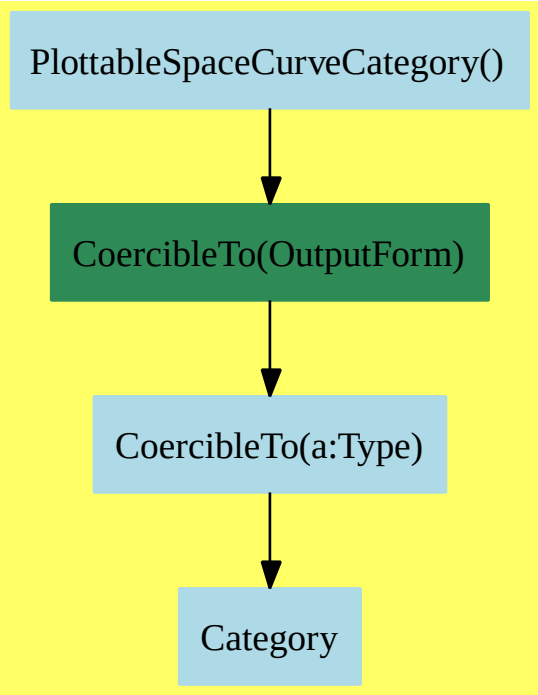
"PlottablePlaneCurveCategory()" [color=lightblue];
"PlottablePlaneCurveCategory()" -> "CoercibleTo(OutputForm)"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

3.10 PlottableSpaceCurveCategory (PSCURVE)



See:

⇐ "CoercibleTo" (KOERCE) 2.6 on page 15

Exports:
coerce listBranches xRange yRange zRange

These are directly exported but not implemented:

```
listBranches : % -> List List Point DoubleFloat
xRange : % -> Segment DoubleFloat
yRange : % -> Segment DoubleFloat
zRange : % -> Segment DoubleFloat
```

These exports come from (p15) CoercibleTo(OutputForm):

```
coerce : % -> OutputForm
```

⟨*category PSCURVE PlottableSpaceCurveCategory*⟩≡

```
)abbrev category PSCURVE PlottableSpaceCurveCategory
++ Author: Clifton J. Williamson
++ Date Created: 11 January 1990
++ Date Last Updated: 15 June 1990
++ Basic Operations: listBranches, xRange, yRange, zRange
```

```
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: plot, graphics
++ References:
++ Description: PlottableSpaceCurveCategory is the category of curves in
++ 3-space which may be plotted via the graphics facilities.  Functions are
++ provided for obtaining lists of lists of points, representing the
++ branches of the curve, and for determining the ranges of the
++ x-, y-, and z-coordinates of the points on the curve.

PlottableSpaceCurveCategory(): Category == Definition where
  L     ==> List
  SEG   ==> Segment
  SF    ==> DoubleFloat
  POINT ==> Point DoubleFloat

  Definition ==> CoercibleTo OutputForm with

    listBranches: % -> L L POINT
      ++ listBranches(c) returns a list of lists of points, representing the
      ++ branches of the curve c.
    xRange: % -> SEG SF
      ++ xRange(c) returns the range of the x-coordinates of the points
      ++ on the curve c.
    yRange: % -> SEG SF
      ++ yRange(c) returns the range of the y-coordinates of the points
      ++ on the curve c.
    zRange: % -> SEG SF
      ++ zRange(c) returns the range of the z-coordinates of the points
      ++ on the curve c.
```

⟨*PSCURVE.dotabb*⟩≡

```
"PSCURVE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PSCURVE"];
"PSCURVE" -> "KOERCE"
```

⟨*PSCURVE.dotfull*⟩≡

```
"PlottableSpaceCurveCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PSCURVE"];
"PlottableSpaceCurveCategory()" -> "CoercibleTo(OutputForm)"
```

⟨*PSCURVE.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

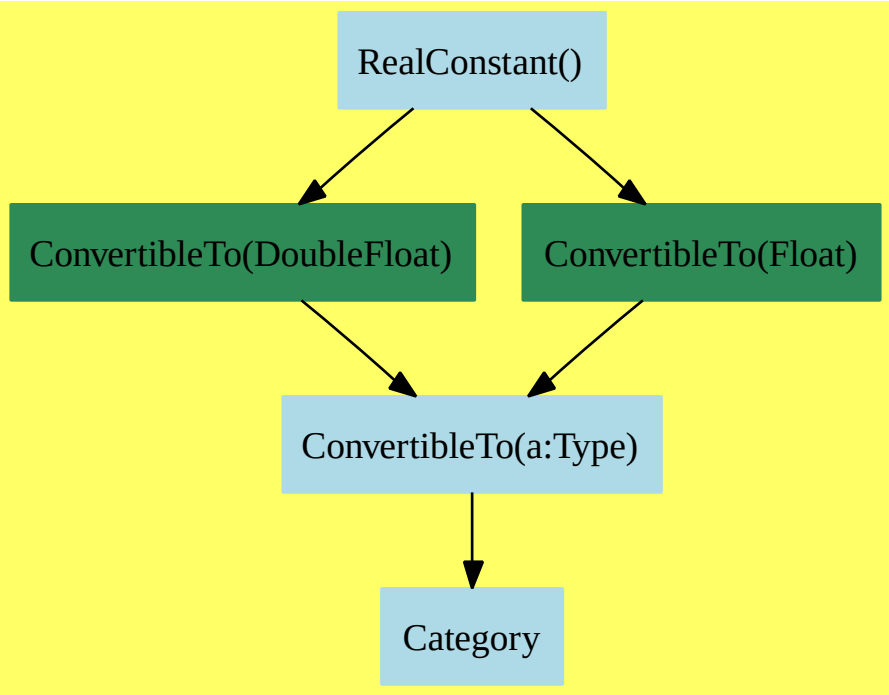
"PlottableSpaceCurveCategory()" [color=lightblue];
"PlottableSpaceCurveCategory()" -> "CoercibleTo(OutputForm)"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

3.11 RealConstant (REAL)



See:

⇒ "IntegerNumberSystem" (INS) 16.2 on page 1011
⇒ "QuotientFieldCategory" (QFCAT) 17.6 on page 1121
⇒ "RealNumberSystem" (RNS) 17.8 on page 1141
⇐ "ConvertibleTo" (KONVERT) 2.8 on page 19

Exports:
convert

These exports come from (p19) ConvertibleTo(DoubleFloat):

```
convert : % -> DoubleFloat
```

These exports come from (p19) ConvertibleTo(Float):

```
convert : % -> Float
```

⟨*category REAL RealConstant*⟩≡

```
)abbrev category REAL RealConstant
++ Author:
++ Date Created:
++ Date Last Updated:
```

```
  ++ Basic Functions:
  ++ Related Constructors:
  ++ Also See:
  ++ AMS Classifications:
  ++ Keywords:
  ++ References:
  ++ Description:
  ++ The category of real numeric domains, i.e. convertible to floats.
  RealConstant(): Category ==
    Join(ConvertibleTo DoubleFloat, ConvertibleTo Float)
```

⟨*REAL.dotabb*⟩≡

```
  "REAL"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=REAL"];
  "REAL" -> "KONVERT"
```

⟨*REAL.dotfull*⟩≡

```
  "RealConstant()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=REAL"];
  "RealConstant()" -> "ConvertibleTo(DoubleFloat)"
  "RealConstant()" -> "ConvertibleTo(Float)"
```

⟨*REAL.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"RealConstant()" [color=lightblue];
"RealConstant()" -> "ConvertibleTo(DoubleFloat)"
"RealConstant()" -> "ConvertibleTo(Float)"

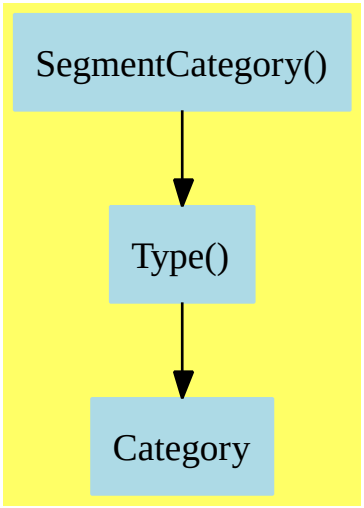
"ConvertibleTo(DoubleFloat)" [color=seagreen];
"ConvertibleTo(DoubleFloat)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(Float)" [color=seagreen];
"ConvertibleTo(Float)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(a:Type)" [color=lightblue];
"ConvertibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

3.12 SegmentCategory (SEGCAT)



See:

⇒ "SegmentExpansionCategory" (SEGXCAT) 4.23 on page 183
⇐ "Type" (TYPE) 2.21 on page 53

Exports:

BY	convert	hi	high	incr
lo	low	segment	?..?	

Attributes Exported:

- **nil**

These are directly exported but not implemented:

```
BY : (%,Integer) -> %
convert : S -> %
hi : % -> S
high : % -> S
incr : % -> Integer
lo : % -> S
low : % -> S
segment : (S,S) -> %
?..? : (S,S) -> %
```

⟨*category SEGCAT SegmentCategory*⟩≡

```
)abbrev category SEGCAT SegmentCategory
++ Author:  Stephen M. Watt
++ Date Created:  December 1986
++ Date Last Updated: June 3, 1991
```

```
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: range, segment
++ Examples:
++ References:
++ Description:
++   This category provides operations on ranges, or {\em segments}
++   as they are called.

SegmentCategory(S:Type): Category == Type with
    SEGMENT: (S, S) -> %
        ++ \spad{l..h} creates a segment with l and h as the endpoints.
    BY: (%, Integer) -> %
        ++ \spad{s by n} creates a new segment in which only every
        ++ \spad{n}-th element is used.
    lo: % -> S
        ++ lo(s) returns the first endpoint of s.
        ++ Note: \spad{lo(l..h) = l}.
    hi: % -> S
        ++ hi(s) returns the second endpoint of s.
        ++ Note: \spad{hi(l..h) = h}.
    low: % -> S
        ++ low(s) returns the first endpoint of s.
        ++ Note: \spad{low(l..h) = l}.
    high: % -> S
        ++ high(s) returns the second endpoint of s.
        ++ Note: \spad{high(l..h) = h}.
    incr: % -> Integer
        ++ incr(s) returns \spad{n}, where s is a segment in which every
        ++ \spad{n}-th element is used.
        ++ Note: \spad{incr(l..h by n) = n}.
    segment: (S, S) -> %
        ++ segment(i,j) is an alternate way to create the segment
        ++ \spad{i..j}.
    convert: S -> %
        ++ convert(i) creates the segment \spad{i..i}.
```

⟨*SEGCAT.dotabb*⟩≡

```
"SEGCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEGCAT"];
"SEGCAT" -> "TYPE"
```

⟨*SEGCAT.dotfull*⟩≡

```
"SegmentCategory(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEGCAT"];
"SegmentCategory(a:Type)" -> "Type()"

"SegmentCategory(OrderedRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=SEGCAT"];
"SegmentCategory(OrderedRing)" -> "SegmentCategory(a:Type)"
```

⟨*SEGCAT.dotpic*⟩≡

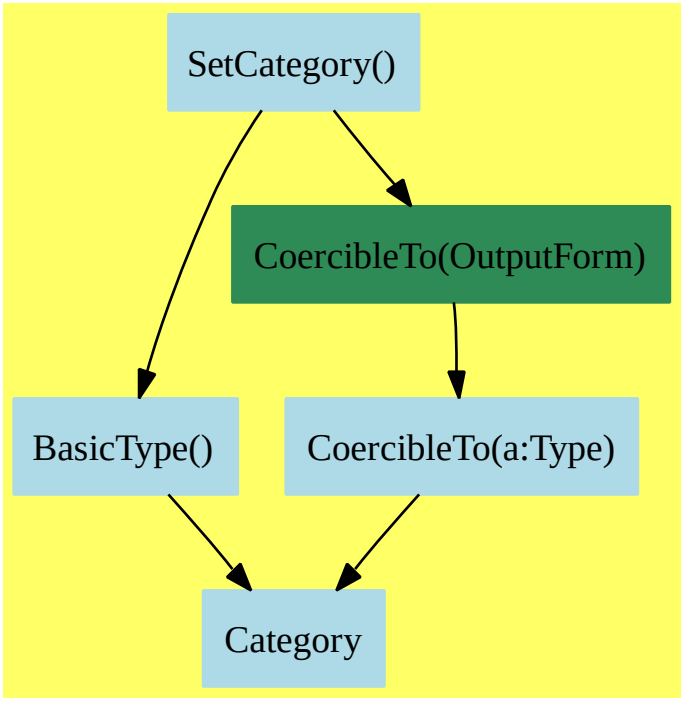
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"SegmentCategory(a:Type)" [color=lightblue];
"SegmentCategory(a:Type)" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

3.13 SetCategory (SETCAT)



See:

⇒ “AbelianSemiGroup” (ABELSG) 4.1 on page 99
⇒ “FileCategory” (FILECAT) 4.8 on page 125
⇒ “FileNameCategory” (FNCAT) 4.10 on page 132
⇒ “Finite” (FINITE) 4.9 on page 129
⇒ “GradedModule” (GRMOD) 4.11 on page 135
⇒ “HomogeneousAggregate” (HOAGG) 4.12 on page 139
⇒ “IndexedDirectProductCategory” (IDPC) 4.13 on page 145
⇒ “NumericalIntegrationCategory” (NUMINT) 4.16 on page 156
⇒ “NumericalOptimizationCategory” (OPTCAT) 4.17 on page 160
⇒ “OrderedSet” (ORDSET) 4.19 on page 168
⇒ “OrdinaryDifferentialEquationsSolverCategory” (ODECAT) 4.18 on page 164

⇒ “PartialDifferentialEquationsSolverCategory” (PDECAT) 4.20 on page 172
⇒ “PatternMatchable” (PATMAB) 4.21 on page 176
⇒ “PolynomialSetCategory” (PSETCAT) 6.10 on page 373
⇒ “RealRootCharacterizationCategory” (RRCC) 4.22 on page 179
⇒ “SemiGroup” (SGROUP) 4.24 on page 186
⇒ “SetAggregate” (SETAGG) 6.13 on page 395
⇒ “SExpressionCategory” (SEXCAT) 4.25 on page 189

⇒ "StepThrough" (STEP) 4.26 on page 193
⇒ "StringCategory" (STRICAT) 10.16 on page 747
⇒ "ThreeSpaceCategory" (SPACEC) 4.27 on page 196
⇐ "BasicType" (BASTYPE) 2.5 on page 13
⇐ "CoercibleTo" (KOERCE) 2.6 on page 15

Exports:
coerce hash latex ?=? ?~=?

These are implemented by this category:

```
hash : % -> SingleInteger
latex : % -> String
```

These exports come from (p13) BasicType():

```
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

These exports come from (p15) CoercibleTo(OutputForm):

```
coerce : % -> OutputForm
```

⟨*category SETCAT SetCategory*⟩≡

```
)abbrev category SETCAT SetCategory
++ Author:
++ Date Created:
++ Date Last Updated:
++   09/09/92   RSS   added latex and hash
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \spadtype{SetCategory} is the basic category for describing a collection
++ of elements with \spadop{=} (equality) and \spadfun{coerce} to
++ output form.
++
++ Conditional Attributes:
++    canonical\tab{15}data structure equality is the same as \spadop{=}
SetCategory(): Category == Join(BasicType,CoercibleTo OutputForm) with
      hash: % -> SingleInteger  ++ hash(s) calculates a hash code for s.
      latex: % -> String       ++ latex(s) returns a LaTeX-printable output
                               ++ representation of s.
  add
      hash(s : %):  SingleInteger == 0$SingleInteger
```

```
        latex(s : %): String       == "\mbox{\bf Unimplemented}"
```

⟨*SETCAT.dotabb*⟩≡
```
  "SETCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=SETCAT"];
  "SETCAT" -> "BASTYPE"
  "SETCAT" -> "KOERCE"
```

⟨*SETCAT.dotfull*⟩≡
```
  "SetCategory()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=SETCAT"];
  "SetCategory()" -> "BasicType()"
  "SetCategory()" -> "CoercibleTo(OutputForm)"
```

⟨*SETCAT.dotpic*⟩≡
```
  digraph pic {
   fontsize=10;
   bgcolor="#FFFF66";
   node [shape=box, color=white, style=filled];

  "SetCategory()" [color=lightblue];
  "SetCategory()" -> "BasicType()"
  "SetCategory()" -> "CoercibleTo(OutputForm)"

  "BasicType()" [color=lightblue];
  "BasicType()" -> "Category"

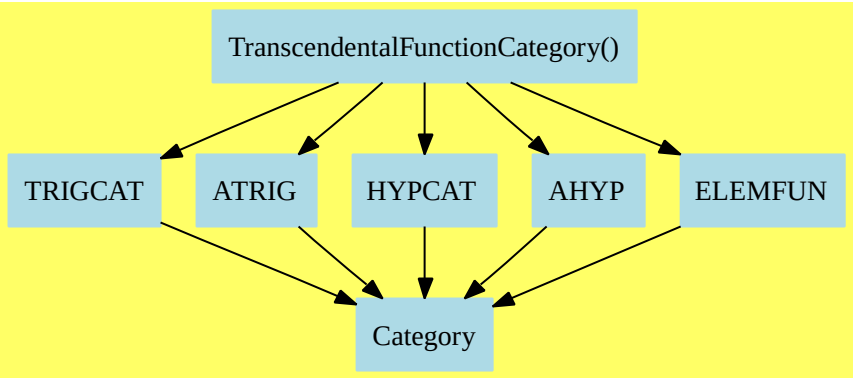
  "CoercibleTo(OutputForm)" [color=seagreen];
  "CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

  "CoercibleTo(a:Type)" [color=lightblue];
  "CoercibleTo(a:Type)" -> "Category"

  "Category" [color=lightblue];

  }
```

3.14 TranscendentalFunctionCategory (TRANFUN)



The `acsch`, `asech`, and `acoth` functions were modified to include an intermediate test to check that the argument has a reciprocal values.

See:

⇒ "IntervalCategory" (INTCAT) 14.2 on page 950
⇒ "LiouvillianFunctionCategory" (LFCAT) 4.14 on page 148
⇒ "UnivariateLaurentSeriesCategory" (ULSCAT) 17.10 on page 1186
⇒ "UnivariatePuiseuxSeriesCategory" (UPXSCAT) 17.11 on page 1195
⇒ "UnivariateTaylorSeriesCategory" (UTSCAT) 16.5 on page 1046
⇐ "ArcHyperbolicFunctionCategory" (AHYP) 2.2 on page 5
⇐ "ArcTrigonometricFunctionCategory" (ATRIG) 2.3 on page 7
⇐ "ElementaryFunctionCategory" (ELEMFUN) 2.9 on page 23
⇐ "HyperbolicFunctionCategory" (HYPCAT) 2.11 on page 27
⇐ "TrigonometricFunctionCategory" (TRIGCAT) 2.20 on page 51

Exports:

?**?	acos	acosh	acot	acoth
acsc	acsch	asec	asech	asin
asinh	atan	atanh	cos	cosh
cot	coth	csc	csch	exp
log	pi	sec	sech	sin
sinh	tan	tanh		

These are implemented by this category:

```
pi : () -> %
```

These exports come from (p51) TrigonometricFunctionCategory():

```
cos : % -> %
cot : % -> %
```

```
csc : % -> %
sec : % -> %
sin : % -> %
tan : % -> %
```

These exports come from (p7) ArcTrigonometricFunctionCategory():

```
acos : % -> %
acot : % -> %
acsc : % -> %
asec : % -> %
asin : % -> %
atan : % -> %
```

These exports come from (p27) HyperbolicFunctionCategory():

```
cosh : % -> %
coth : % -> %
csch : % -> %
sech : % -> %
sinh : % -> %
tanh : % -> %
```

These exports come from (p5) ArcHyperbolicFunctionCategory():

```
acosh : % -> %
acoth : % -> %
acsch : % -> %
asech : % -> %
asinh : % -> %
atanh : % -> %
```

These exports come from (p23) ElementaryFunctionCategory():

```
?**? : (%,%) -> %
exp : % -> %
log : % -> %
```

⟨*category TRANFUN TranscendentalFunctionCategory*⟩≡

```
)abbrev category TRANFUN TranscendentalFunctionCategory
++ Category for the transcendental elementary functions
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description: Category for the transcendental elementary functions;
TranscendentalFunctionCategory(): Category ==
    Join(TrigonometricFunctionCategory,ArcTrigonometricFunctionCategory,
         HyperbolicFunctionCategory,ArcHyperbolicFunctionCategory,
         ElementaryFunctionCategory) with
```

```
        pi : () -> $        ++ pi() returns the constant pi.
    add
      if $ has Ring then
        pi()   == 2*asin(1)
        acsch x ==
          (a := recip x) case "failed" => error "acsch: no reciprocal"
          asinh(a::$)
        asech x ==
          (a := recip x) case "failed" => error "asech: no reciprocal"
          acosh(a::$)
        acoth x ==
          (a := recip x) case "failed" => error "acoth: no reciprocal"
          atanh(a::$)
      if $ has Field and $ has sqrt: $ -> $ then
        asin x == atan(x/sqrt(1-x**2))
        acos x == pi()/2::$ - asin x
        acot x == pi()/2::$ - atan x
        asinh x == log(x + sqrt(x**2 + 1))
        acosh x == 2*log(sqrt((x+1)/2::$) + sqrt((x-1)/2::$))
        atanh x == (log(1+x)-log(1-x))/2::$
```

⟨*TRANFUN.dotabb*⟩≡

```
"TRANFUN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=TRANFUN"];
"TRANFUN" -> "TRIGCAT"
"TRANFUN" -> "ATRIG"
"TRANFUN" -> "HYPCAT"
"TRANFUN" -> "AHYP"
"TRANFUN" -> "ELEMFUN"
```

⟨*TRANFUN.dotfull*⟩≡

```
"TranscendentalFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=TRANFUN"];
"TranscendentalFunctionCategory()" ->
    "TrigonometricFunctionCategory()"
"TranscendentalFunctionCategory()" ->
    "ArcTrigonometricFunctionCategory()"
"TranscendentalFunctionCategory()" ->
    "HyperbolicFunctionCategory()"
"TranscendentalFunctionCategory()" ->
    "ArcHyperbolicFunctionCategory()"
"TranscendentalFunctionCategory()" ->
    "ElementaryFunctionCategory()"
```

⟨*TRANFUN.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"TranscendentalFunctionCategory()" [color=lightblue];
"TranscendentalFunctionCategory()" ->
    "TRIGCAT"
"TranscendentalFunctionCategory()" ->
    "ATRIG"
"TranscendentalFunctionCategory()" ->
    "HYPCAT"
"TranscendentalFunctionCategory()" ->
    "AHYP"
"TranscendentalFunctionCategory()" ->
    "ELEMFUN"

"TRIGCAT" [color=lightblue];
"TRIGCAT" -> "Category"

"ATRIG" [color=lightblue];
"ATRIG" -> "Category"

"HYPCAT" [color=lightblue];
"HYPCAT" -> "Category"

"AHYP" [color=lightblue];
"AHYP" -> "Category"

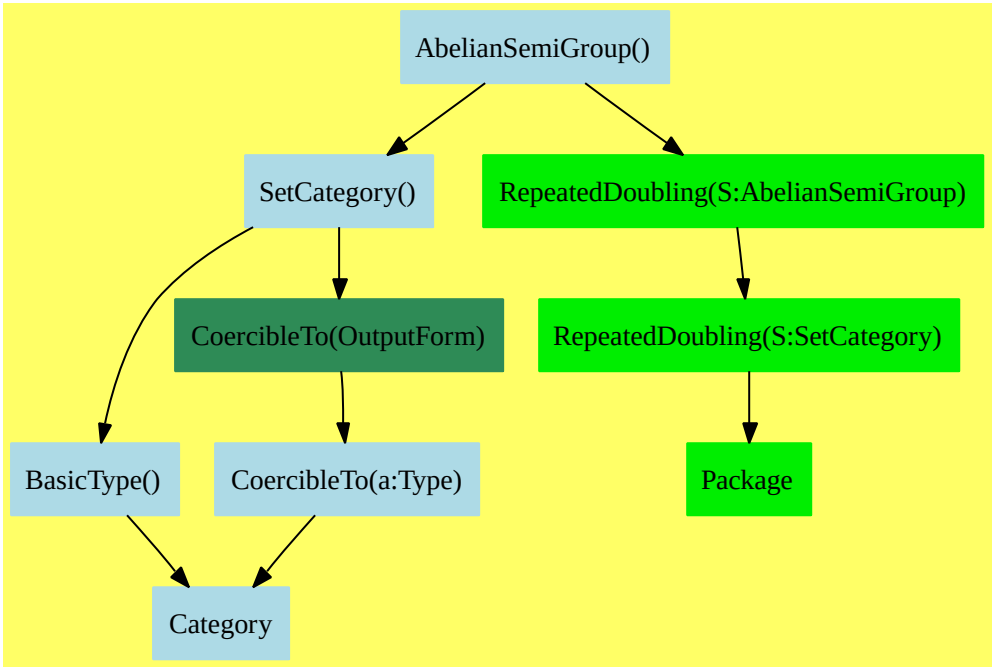
"ELEMFUN" [color=lightblue];
"ELEMFUN" -> "Category"

"Category" [color=lightblue];
}
```

Chapter 4

Category Layer 3

4.1 AbelianSemiGroup (ABELSG)



See:

⇒ "AbelianMonoid" (ABELMON) 5.1 on page 207
⇒ "FunctionSpace" (FS) 17.5 on page 1095
⇐ "SetCategory" (SETCAT) 3.13 on page 91

Exports:
coerce hash latex ?*? ?+?
?=? ?~=?

These are directly exported but not implemented:

```
?+? : (%,%) -> %
```

These are implemented by this category:

```
?*? : (PositiveInteger,%) -> %
```

These exports come from (p91) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category ABELSG AbelianSemiGroup*⟩≡

```
)abbrev category ABELSG AbelianSemiGroup
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ the class of all additive (commutative) semigroups, i.e.
++ a set with a commutative and associative operation \spadop{+}.
++
++ Axioms:
++   \spad{associative("+":(%,%)->%)}\tab{30}\spad{ (x+y)+z = x+(y+z) }
++   \spad{commutative("+":(%,%)->%)}\tab{30}\spad{ x+y = y+x }
AbelianSemiGroup(): Category == SetCategory with
      "+": (%,%) -> %                  ++ x+y computes the sum of x and y.
      "*": (PositiveInteger,%) -> %
        ++ n*x computes the left-multiplication of x by the positive
        ++ integer n. This is equivalent to adding x to itself n times.
    add
      import RepeatedDoubling(%)
      if not (% has Ring) then
        n:PositiveInteger * x:% == double(n,x)
```

⟨*ABELSG.dotabb*⟩≡
```
"ABELSG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ABELSG"];
"ABELSG" -> "SETCAT"
"ABELSG" -> "REPDB"
```

⟨*ABELSG.dotfull*⟩≡
```
"AbelianSemiGroup()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ABELSG"];
"AbelianSemiGroup()" -> "SetCategory()"
"AbelianSemiGroup()" -> "RepeatedDoubling(a:SetCategory)"
```

⟨*ABELSG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SetCategory()"
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

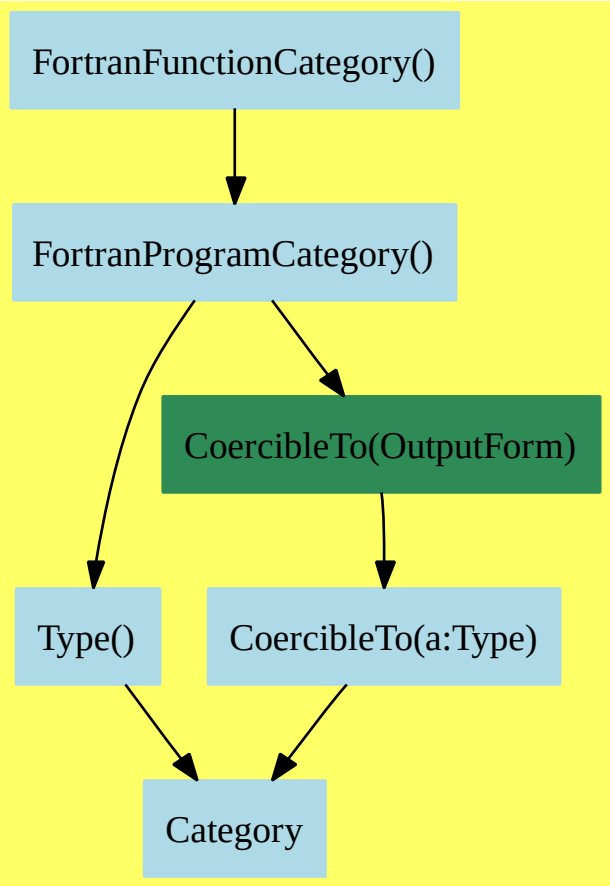
"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];
}
```

4.2 FortranFunctionCategory (FORTFN)



See:

⇐ "FortranProgramCategory" (FORTCAT) 3.5 on page 68

Exports:
coerce outputAsFortran retract retractIfCan

Attributes:

- **nil**

These are directly exported but not implemented:

```
coerce : FortranCode -> %
coerce : List FortranCode -> %
coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
retract : Expression Float -> %
```

```
retract : Expression Integer -> %
retract : Polynomial Float -> %
retract : Polynomial Integer -> %
retract : Fraction Polynomial Integer -> %
retract : Fraction Polynomial Float -> %
retractIfCan : Fraction Polynomial Integer -> Union(%,"failed")
retractIfCan : Fraction Polynomial Float -> Union(%,"failed")
retractIfCan : Polynomial Integer -> Union(%,"failed")
retractIfCan : Polynomial Float -> Union(%,"failed")
retractIfCan : Expression Integer -> Union(%,"failed")
retractIfCan : Expression Float -> Union(%,"failed")
```

These exports come from (p68) FortranProgramCategory():

```
coerce : % -> OutputForm
outputAsFortran : % -> Void
```

⟨*category FORTFN FortranFunctionCategory*⟩≡

```
)abbrev category FORTFN FortranFunctionCategory
++ Author: Mike Dewar
++ Date Created: 13 January 1994
++ Date Last Updated: 18 March 1994
++ Related Constructors: FortranProgramCategory.
++ Description:
++ \axiomType{FortranFunctionCategory} is the category of arguments to
++ NAG Library routines which return (sets of) function values.
FortranFunctionCategory():Category == FortranProgramCategory with
  coerce : List FortranCode -> $
    ++ coerce(e) takes an object from \spadtype{List FortranCode} and
    ++  uses it as the body of an ASP.
  coerce : FortranCode -> $
    ++ coerce(e) takes an object from \spadtype{FortranCode} and
    ++  uses it as the body of an ASP.
  coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $
    ++ coerce(e) takes the component of \spad{e} from
    ++ \spadtype{List FortranCode} and uses it as the body of the ASP,
    ++ making the declarations in the \spadtype{SymbolTable} component.
  retract : Expression Float -> $
    ++ retract(e) tries to convert \spad{e} into an ASP, checking that
    ++  legal Fortran-77 is produced.
  retractIfCan : Expression Float -> Union($,"failed")
    ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
    ++  legal Fortran-77 is produced.
  retract : Expression Integer -> $
    ++ retract(e) tries to convert \spad{e} into an ASP, checking that
    ++  legal Fortran-77 is produced.
  retractIfCan : Expression Integer -> Union($,"failed")
    ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
```

```
      ++  legal Fortran-77 is produced.
    retract : Polynomial Float -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retractIfCan : Polynomial Float -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retract : Polynomial Integer -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retractIfCan : Polynomial Integer -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retract : Fraction Polynomial Float -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retractIfCan : Fraction Polynomial Float -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retract : Fraction Polynomial Integer -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retractIfCan : Fraction Polynomial Integer -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    -- NB: These ASPs also have a coerce from an appropriate instantiation
    --     of FortranExpression.
```

⟨*FORTFN.dotabb*⟩≡

```
  "FORTFN"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FORTFN"];
  "FORTFN" -> "FORTCAT"
```

⟨*FORTFN.dotfull*⟩≡

```
  "FortranFunctionCategory()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FORTFN"];
  "FortranFunctionCategory()" -> "FortranProgramCategory()"
```

⟨*FORTFN.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FortranFunctionCategory()" [color=lightblue];
"FortranFunctionCategory()" -> "FortranProgramCategory()"

"FortranProgramCategory()" [color=lightblue];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"

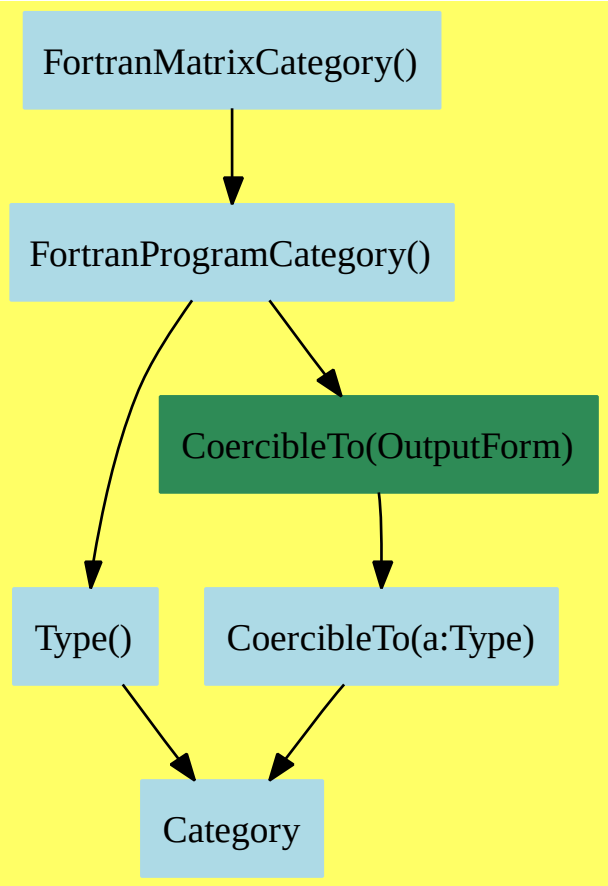
"Type()" [color=lightblue];
"Type()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

4.3 FortranMatrixCategory (FMC)



See:

⇐ "FortranProgramCategory" (FORTCAT) 3.5 on page 68

Exports:
coerce outputAsFortran

Attributes:

- **nil**

These are directly exported but not implemented:

```
coerce : Matrix MachineFloat -> %
coerce : List FortranCode -> %
coerce : FortranCode -> %
coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
```

These exports come from (p68) FortranProgramCategory():

```
coerce : % -> OutputForm
outputAsFortran : % -> Void
```

⟨*category FMC FortranMatrixCategory*⟩≡

```
)abbrev category FMC FortranMatrixCategory
++ Author: Mike Dewar
++ Date Created: 21 March 1994
++ Date Last Updated:
++ Related Constructors: FortranProgramCategory.
++ Description:
++ \axiomType{FortranMatrixCategory} provides support for
++ producing Functions and Subroutines when the input to these
++ is an AXIOM object of type \axiomType{Matrix} or in domains
++ involving \axiomType{FortranCode}.
FortranMatrixCategory():Category == FortranProgramCategory with
    coerce : Matrix MachineFloat -> $
      ++ coerce(v) produces an ASP which returns the value of \spad{v}.
    coerce : List FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{List FortranCode} and
      ++  uses it as the body of an ASP.
    coerce : FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{FortranCode} and
      ++  uses it as the body of an ASP.
    coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $
      ++ coerce(e) takes the component of \spad{e} from
      ++ \spadtype{List FortranCode} and uses it as the body of the ASP,
      ++ making the declarations in the \spadtype{SymbolTable} component.
```

⟨*FMC.dotabb*⟩≡

```
"FMC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMC"];
"FMC" -> "FORTCAT"
```

⟨*FMC.dotfull*⟩≡

```
"FortranMatrixCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMC"];
"FortranMatrixCategory()" -> "FortranProgramCategory()"
```

⟨*FMC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FortranMatrixCategory()" [color=lightblue];
"FortranMatrixCategory()" -> "FortranProgramCategory()"

"FortranProgramCategory()" [color=lightblue];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"

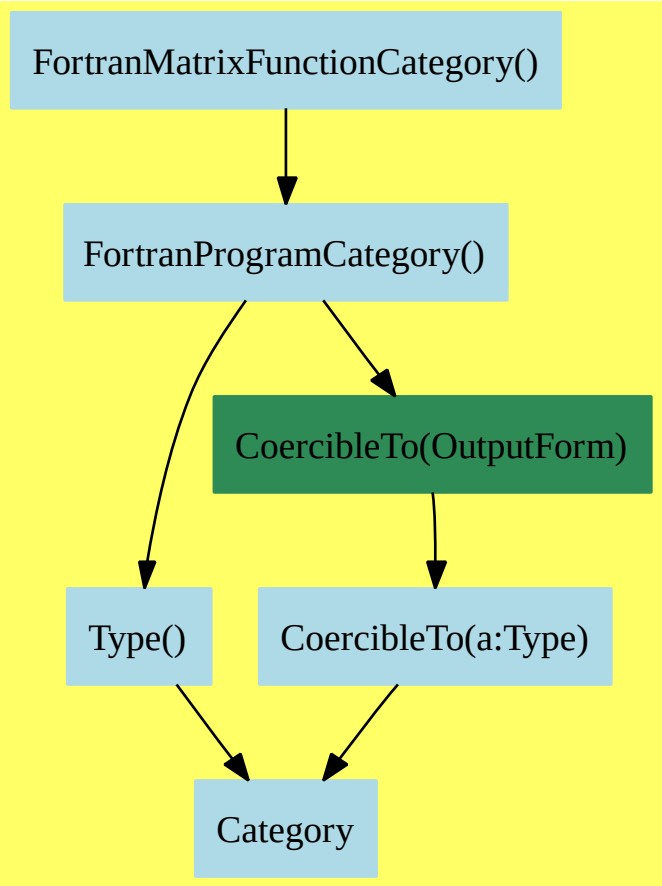
"Type()" [color=lightblue];
"Type()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

4.4 FortranMatrixFunctionCategory (FMFUN)



See:

⇐ "FortranProgramCategory" (FORTCAT) 3.5 on page 68

Exports:
coerce outputAsFortran retract retractIfCan

Attributes:

- **nil**

These are directly exported but not implemented:

```
coerce : List FortranCode -> %
coerce : FortranCode -> %
coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
retract : Matrix Expression Float -> %
```

```
retract : Matrix Expression Integer -> %
retract : Matrix Polynomial Float -> %
retract : Matrix Polynomial Integer -> %
retract : Matrix Fraction Polynomial Float -> %
retract : Matrix Fraction Polynomial Integer -> %
retractIfCan : Matrix Fraction Polynomial Integer -> Union(%,"failed")
retractIfCan : Matrix Fraction Polynomial Float -> Union(%,"failed")
retractIfCan : Matrix Polynomial Integer -> Union(%,"failed")
retractIfCan : Matrix Polynomial Float -> Union(%,"failed")
retractIfCan : Matrix Expression Integer -> Union(%,"failed")
retractIfCan : Matrix Expression Float -> Union(%,"failed")
```

These exports come from (p68) FortranProgramCategory():

```
coerce : % -> OutputForm
outputAsFortran : % -> Void
```

⟨*category FMFUN FortranMatrixFunctionCategory*⟩≡

```
)abbrev category FMFUN FortranMatrixFunctionCategory
++ Author: Mike Dewar
++ Date Created: March 18 1994
++ Date Last Updated:
++ Related Constructors: FortranProgramCategory.
++ Description:
++ \axiomType{FortranMatrixFunctionCategory} provides support for
++ producing Functions and Subroutines representing matrices of
++ expressions.

FortranMatrixFunctionCategory():Category == FortranProgramCategory with
    coerce : List FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{List FortranCode} and
      ++  uses it as the body of an ASP.
    coerce : FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{FortranCode} and
      ++  uses it as the body of an ASP.
    coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $
      ++ coerce(e) takes the component of \spad{e} from
      ++ \spadtype{List FortranCode} and uses it as the body of the ASP,
      ++ making the declarations in the \spadtype{SymbolTable} component.
    retract : Matrix Expression Float -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retractIfCan : Matrix Expression Float -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retract : Matrix Expression Integer -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
```

```
      retractIfCan : Matrix Expression Integer -> Union($,"failed")
        ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
        ++  legal Fortran-77 is produced.
      retract : Matrix Polynomial Float -> $
        ++ retract(e) tries to convert \spad{e} into an ASP, checking that
        ++  legal Fortran-77 is produced.
      retractIfCan : Matrix Polynomial Float -> Union($,"failed")
        ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
        ++  legal Fortran-77 is produced.
      retract : Matrix Polynomial Integer -> $
        ++ retract(e) tries to convert \spad{e} into an ASP, checking that
        ++  legal Fortran-77 is produced.
      retractIfCan : Matrix Polynomial Integer -> Union($,"failed")
        ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
        ++  legal Fortran-77 is produced.
      retract : Matrix Fraction Polynomial Float -> $
        ++ retract(e) tries to convert \spad{e} into an ASP, checking that
        ++  legal Fortran-77 is produced.
      retractIfCan : Matrix Fraction Polynomial Float -> Union($,"failed")
        ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
        ++  legal Fortran-77 is produced.
      retract : Matrix Fraction Polynomial Integer -> $
        ++ retract(e) tries to convert \spad{e} into an ASP, checking that
        ++  legal Fortran-77 is produced.
      retractIfCan : Matrix Fraction Polynomial Integer -> Union($,"failed")
        ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
        ++  legal Fortran-77 is produced.

      -- NB: These ASPs also have a coerce from an appropriate instantiation
      --     of Matrix FortranExpression.
```

⟨*FMFUN.dotabb*⟩≡

```
  "FMFUN"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FMFUN"];
  "FMFUN" -> "FORTCAT"
```

⟨*FMFUN.dotfull*⟩≡

```
  "FortranMatrixFunctionCategory()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FMFUN"];
  "FortranMatrixFunctionCategory()" -> "FortranProgramCategory()"
```

⟨*FMFUN.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FortranMatrixFunctionCategory()" [color=lightblue];
"FortranMatrixFunctionCategory()" -> "FortranProgramCategory()"

"FortranProgramCategory()" [color=lightblue];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"

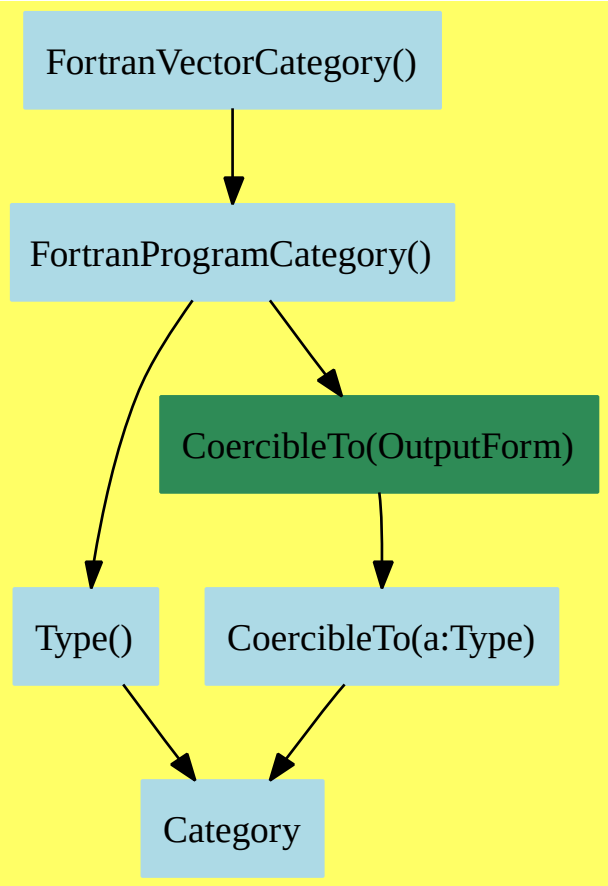
"Type()" [color=lightblue];
"Type()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

4.5 FortranVectorCategory (FVC)



See:

⇐ "FortranProgramCategory" (FORTCAT) 3.5 on page 68

Exports:

Attributes:

- **nil**

These are directly exported but not implemented:

```
coerce : FortranCode -> %
coerce : List FortranCode -> %
coerce : Vector MachineFloat -> %
coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
```

These exports come from (p68) FortranProgramCategory():

```
coerce : % -> OutputForm
outputAsFortran : % -> Void
```

⟨*category FVC FortranVectorCategory*⟩≡

```
)abbrev category FVC FortranVectorCategory
++ Author: Mike Dewar
++ Date Created: October 1993
++ Date Last Updated: 18 March 1994
++ Related Constructors: FortranProgramCategory.
++ Description:
++ \axiomType{FortranVectorCategory} provides support for
++ producing Functions and Subroutines when the input to these
++ is an AXIOM object of type \axiomType{Vector} or in domains
++ involving \axiomType{FortranCode}.
FortranVectorCategory():Category == FortranProgramCategory with
    coerce : Vector MachineFloat -> $
      ++ coerce(v) produces an ASP which returns the value of \spad{v}.
    coerce : List FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{List FortranCode} and
      ++  uses it as the body of an ASP.
    coerce : FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{FortranCode} and
      ++  uses it as the body of an ASP.
    coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $
      ++ coerce(e) takes the component of \spad{e} from
      ++ \spadtype{List FortranCode} and uses it as the body of the ASP,
      ++ making the declarations in the \spadtype{SymbolTable} component.
```

⟨*FVC.dotabb*⟩≡

```
"FVC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FVC"];
"FVC" -> "FORTCAT"
```

⟨*FVC.dotfull*⟩≡

```
"FortranVectorCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FVC"];
"FortranVectorCategory()" -> "FortranProgramCategory()"
```

⟨*FVC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FortranVectorCategory()" [color=lightblue];
"FortranVectorCategory()" -> "FortranProgramCategory()"

"FortranProgramCategory()" [color=lightblue];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"

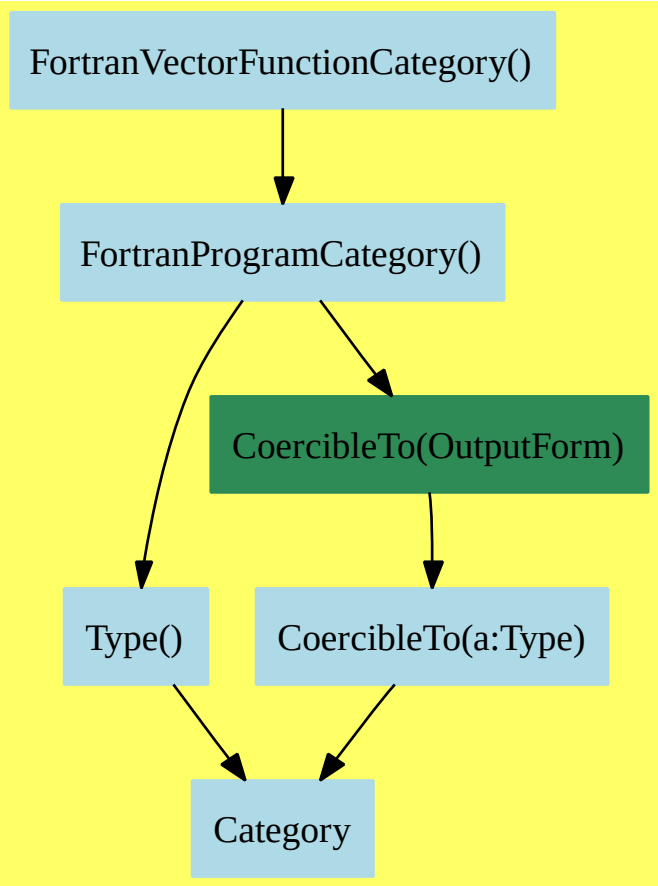
"Type()" [color=lightblue];
"Type()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

4.6 FortranVectorFunctionCategory (FVFUN)



See:

⇐ "FortranProgramCategory" (FORTCAT) 3.5 on page 68

Exports:
coerce outputAsFortran retract retractIfCan

Attributes:

- **nil**

These are directly exported but not implemented:

```
coerce : FortranCode -> %
coerce : List FortranCode -> %
coerce : Record(localSymbols: SymbolTable,code: List FortranCode) -> %
retract : Vector Fraction Polynomial Integer -> %
```

```
retract : Vector Fraction Polynomial Float -> %
retract : Vector Polynomial Integer -> %
retract : Vector Polynomial Float -> %
retract : Vector Expression Integer -> %
retract : Vector Expression Float -> %
retractIfCan : Vector Fraction Polynomial Integer -> Union(%,"failed")
retractIfCan : Vector Fraction Polynomial Float -> Union(%,"failed")
retractIfCan : Vector Polynomial Integer -> Union(%,"failed")
retractIfCan : Vector Polynomial Float -> Union(%,"failed")
retractIfCan : Vector Expression Integer -> Union(%,"failed")
retractIfCan : Vector Expression Float -> Union(%,"failed")
```

These exports come from (p68) FortranProgramCategory():

```
coerce : % -> OutputForm
outputAsFortran : % -> Void
```

⟨*category FVFUN FortranVectorFunctionCategory*⟩≡

```
)abbrev category FVFUN FortranVectorFunctionCategory
++ Author: Mike Dewar
++ Date Created: 11 March 1994
++ Date Last Updated: 18 March 1994
++ Related Constructors: FortranProgramCategory.
++ Description:
++ \axiomType{FortranVectorFunctionCategory} is the catagory of arguments
++ to NAG Library routines which return the values of vectors of functions.
FortranVectorFunctionCategory():Category == FortranProgramCategory with
    coerce : List FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{List FortranCode} and
      ++  uses it as the body of an ASP.
    coerce : FortranCode -> $
      ++ coerce(e) takes an object from \spadtype{FortranCode} and
      ++  uses it as the body of an ASP.
    coerce : Record(localSymbols:SymbolTable,code:List(FortranCode)) -> $
      ++ coerce(e) takes the component of \spad{e} from
      ++ \spadtype{List FortranCode} and uses it as the body of the ASP,
      ++ making the declarations in the \spadtype{SymbolTable} component.
    retract : Vector Expression Float -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retractIfCan : Vector Expression Float -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retract : Vector Expression Integer -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retractIfCan : Vector Expression Integer -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
```

```
      ++  legal Fortran-77 is produced.
    retract : Vector Polynomial Float -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retractIfCan : Vector Polynomial Float -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retract : Vector Polynomial Integer -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retractIfCan : Vector Polynomial Integer -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retract : Vector Fraction Polynomial Float -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retractIfCan : Vector Fraction Polynomial Float -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retract : Vector Fraction Polynomial Integer -> $
      ++ retract(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.
    retractIfCan : Vector Fraction Polynomial Integer -> Union($,"failed")
      ++ retractIfCan(e) tries to convert \spad{e} into an ASP, checking that
      ++  legal Fortran-77 is produced.

    -- NB: These ASPs also have a coerce from an appropriate instantiation
    --     of Vector FortranExpression.
```

⟨*FVFUN.dotabb*⟩≡

```
"FVFUN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FVFUN"];
"FVFUN" -> "FORTCAT"
```

⟨*FVFUN.dotfull*⟩≡

```
"FortranVectorFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FVFUN"];
"FortranVectorFunctionCategory()" -> "FortranProgramCategory()"
```

⟨*FVFUN.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FortranVectorFunctionCategory()" [color=lightblue];
"FortranVectorFunctionCategory()" -> "FortranProgramCategory()"

"FortranProgramCategory()" [color=lightblue];
"FortranProgramCategory()" -> "Type()"
"FortranProgramCategory()" -> "CoercibleTo(OutputForm)"

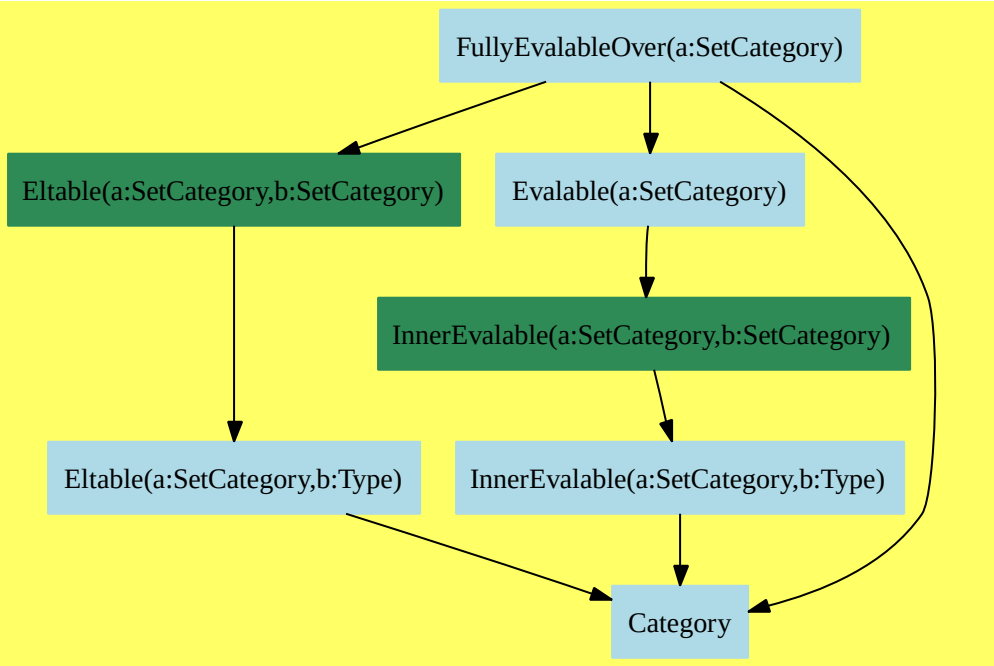
"Type()" [color=lightblue];
"Type()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

4.7 FullyEvalableOver (FEVALAB)



See:

⇒ "ComplexCategory" (COMPCAT) 20.1 on page 1315
⇒ "OctonionCategory" (OC) 12.7 on page 868
⇒ "QuaternionCategory" (QUATCAT) 12.8 on page 878
⇒ "QuotientFieldCategory" (QFCAT) 17.6 on page 1121
⇐ "Category" (CATEGORY) 2.1 on page 3

Exports:
eval map ?.?

These are directly exported but not implemented:

```
map : ((R -> R),%) -> %
```

These are implemented by this category:

```
?.? : (%,R) -> % if R has ELTAB(R,R)
eval : (%,Equation R) -> % if R has EVALAB R
eval : (%,List Symbol,List R) -> % if R has IEVALAB(SYMBOL,R)
```

These exports come from (p65) Evalable(a:Type):

```
eval : (%,List Equation R) -> % if R has EVALAB R
```

```
eval : (%,R,R) -> % if R has EVALAB R
eval : (%,List R,List R) -> % if R has EVALAB R
```

These exports come from (p29) InnerEvalable(a:Symbol,b:SetCategory):

```
eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
```

⟨*category FEVALAB FullyEvalableOver*⟩≡

```
)abbrev category FEVALAB FullyEvalableOver
++ Author:
++ Date Created:
++ Date Last Updated: June 3, 1991
++ Basic Operations:
++ Related Domains: Equation
++ Also See:
++ AMS Classifications:
++ Keywords: equation
++ Examples:
++ References:
++ Description:
++    This category provides a selection of evaluation operations
++    depending on what the argument type R provides.
FullyEvalableOver(R:SetCategory): Category == with
    map: (R -> R, $) -> $
        ++ map(f, ex) evaluates ex, applying f to values of type R in ex.
    if R has Eltable(R, R) then Eltable(R, $)
    if R has Evalable(R) then Evalable(R)
    if R has InnerEvalable(Symbol, R) then InnerEvalable(Symbol, R)
 add
    if R has Eltable(R, R) then
      elt(x:$, r:R) == map(y +-> y(r), x)

    if R has Evalable(R) then
      eval(x:$, l:List Equation R) == map(y +-> eval(y, l), x)

    if R has InnerEvalable(Symbol, R) then
      eval(x:$, ls:List Symbol, lv:List R) == map(y +-> eval(y, ls, lv), x)
```

⟨*FEVALAB.dotabb*⟩≡

```
"FEVALAB"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FEVALAB"];
"FEVALAB" -> "ELTAB"
"FEVALAB" -> "EVALAB"
"FEVALAB" -> "IEVALAB"
"FEVALAB" -> "CATEGORY"
```

⟨*FEVALAB.dotfull*⟩≡

```
  "FullyEvalableOver(a:SetCategory)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FEVALAB"];
  "FullyEvalableOver(a:SetCategory)" -> "Eltable(a:SetCategory,b:Type)"
  "FullyEvalableOver(a:SetCategory)" -> "Evalable(a:SetCategory)"
  "FullyEvalableOver(a:SetCategory)" -> "Category"

  "FullyEvalableOver(IntegralDomain)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=FEVALAB"];
  "FullyEvalableOver(IntegralDomain)" ->
    "FullyEvalableOver(a:SetCategory)"

  "FullyEvalableOver(CommutativeRing)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=FEVALAB"];
  "FullyEvalableOver(CommutativeRing)" ->
    "FullyEvalableOver(a:SetCategory)"
```

⟨*FEVALAB.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FullyEvalableOver(a:SetCategory)" [color=lightblue];
"FullyEvalableOver(a:SetCategory)" -> "Eltable(a:SetCategory,b:SetCategory)"
"FullyEvalableOver(a:SetCategory)" -> "Evalable(a:SetCategory)"
"FullyEvalableOver(a:SetCategory)" -> "Category"

"Eltable(a:SetCategory,b:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ELTAB"];
"Eltable(a:SetCategory,b:SetCategory)" ->
   "Eltable(a:SetCategory,b:Type)"

"Eltable(a:SetCategory,b:Type)" [color=lightblue];
"Eltable(a:SetCategory,b:Type)" -> "Category"

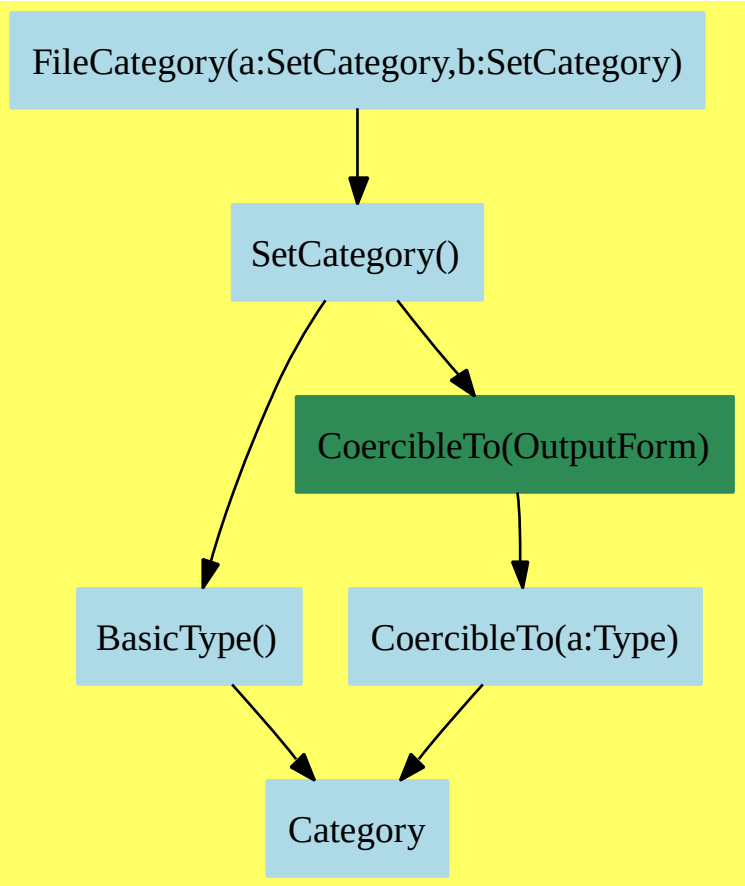
"Evalable(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=EVALAB"];
"Evalable(a:SetCategory)" -> "InnerEvalable(a:SetCategory,b:SetCategory)"

"InnerEvalable(a:SetCategory,b:SetCategory)" [color=seagreen];
"InnerEvalable(a:SetCategory,b:SetCategory)" ->
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:SetCategory,b:Type)" [color=lightblue];
"InnerEvalable(a:SetCategory,b:Type)" -> "Category"

"Category" [color=lightblue];
}
```

4.8 FileCategory (FILECAT)



See:

⇐ "SetCategory" (SETCAT) 3.13 on page 91

Exports:

close!	coerce	hash	iomode	latex
name	open	read!	reopen!	write!
?=?	?~=?			

These are directly exported but not implemented:

```
close! : % -> %
iomode : % -> String
name : % -> Name
open : Name -> %
open : (Name,String) -> %
read! : % -> S
```

```
reopen! : (%,String) -> %
write! : (%,S) -> S
```

These exports come from SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category FILECAT FileCategory*⟩≡

```
)abbrev category FILECAT FileCategory
++ Author: Stephen M. Watt, Victor Miller
++ Date Created:
++ Date Last Updated: June 4, 1991
++ Basic Operations:
++ Related Domains: File
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++   This category provides an interface to operate on files in the
++   computer's file system.  The precise method of naming files
++   is determined by the Name parameter.  The type of the contents
++   of the file is determined by S.

FileCategory(Name, S): Category == FCdefinition where
    Name:      SetCategory
    S:         SetCategory
    IOMode ==> String  -- Union("input", "output", "closed")

    FCdefinition == SetCategory with
        open: Name -> %
          ++ open(s) returns the file s open for input.

        open: (Name, IOMode) -> %
          ++ open(s,mode) returns a file s open for operation in the
          ++ indicated mode: "input" or "output".

        reopen_!: (%, IOMode) -> %
          ++ reopen!(f,mode) returns a file f reopened for operation in the
          ++ indicated mode: "input" or "output".
          ++ \spad{reopen!(f,"input")} will reopen the file f for input.
```

```
close_!: % -> %
  ++ close!(f) returns the file f closed to input and output.

name: % -> Name
  ++ name(f) returns the external name of the file f.

iomode: % -> IOMode
  ++ iomode(f) returns the status of the file f. The input/output
  ++ status of f may be "input", "output" or "closed" mode.

read_!: % -> S
  ++ read!(f) extracts a value from file f.  The state of f is
  ++ modified so a subsequent call to \spadfun{read!} will return
  ++ the next element.

write_!: (%,S) -> S
  ++ write!(f,s) puts the value s into the file f.
  ++ The state of f is modified so subsequents call to \spad{write!}
  ++ will append one after another.
```

⟨*FILECAT.dotabb*⟩≡

```
"FILECAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FILECAT"];
"FILECAT" -> "SETCAT"
```

⟨*FILECAT.dotfull*⟩≡

```
"FileCategory(a:SetCategory,b:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FILECAT"];
"FileCategory(a:SetCategory,b:SetCategory)" -> "SetCategory()"
```

⟨*FILECAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FileCategory(a:SetCategory,b:SetCategory)" [color=lightblue];
"FileCategory(a:SetCategory,b:SetCategory)" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

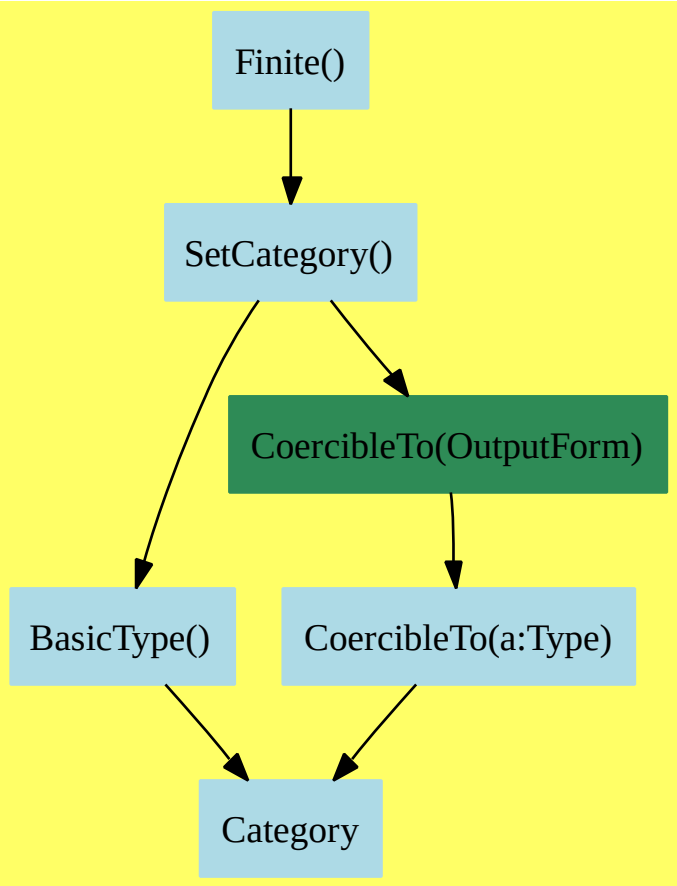
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.9 Finite (FINITE)



See:

⇒ "DirectProductCategory" (DIRPCAT) 12.1 on page 815
⇒ "FiniteFieldCategory" (FFIELDC) 18.3 on page 1244
⇒ "OrderedFinite" (ORDFIN) 5.11 on page 260
⇐ "SetCategory" (SETCAT) 3.13 on page 91

Exports:

coerce	hash	index	latex	lookup	random
size		?=?	?~=?		

These are directly exported but not implemented:

```
index : PositiveInteger -> %
lookup : % -> PositiveInteger
random : () -> %
size : () -> NonNegativeInteger
```

These exports come from (p91) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category FINITE Finite*⟩≡

```
)abbrev category FINITE Finite
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of domains composed of a finite set of elements.
++ We include the functions \spadfun{lookup} and \spadfun{index}
++ to give a bijection between the finite set and an initial
++ segment of positive integers.
++
++ Axioms:
++   \spad{lookup(index(n)) = n}
++   \spad{index(lookup(s)) = s}

Finite(): Category == SetCategory with
      size: () ->  NonNegativeInteger
        ++ size() returns the number of elements in the set.
      index: PositiveInteger -> %
        ++ index(i) takes a positive integer i less than or equal
        ++ to \spad{size()} and
        ++ returns the \spad{i}-th element of the set.
        ++ This operation establishs a bijection
        ++ between the elements of the finite set and \spad{1..size()}.
      lookup: % -> PositiveInteger
        ++ lookup(x) returns a positive integer such that
        ++ \spad{x = index lookup x}.
      random: () -> %
        ++ random() returns a random element from the set.
```

⟨*FINITE.dotabb*⟩≡
```
"FINITE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FINITE"];
"FINITE" -> "SETCAT"
```

⟨*FINITE.dotfull*⟩≡
```
"Finite()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FINITE"];
"Finite()" -> "SetCategory()"
```

⟨*FINITE.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Finite()" [color=lightblue];
"Finite()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

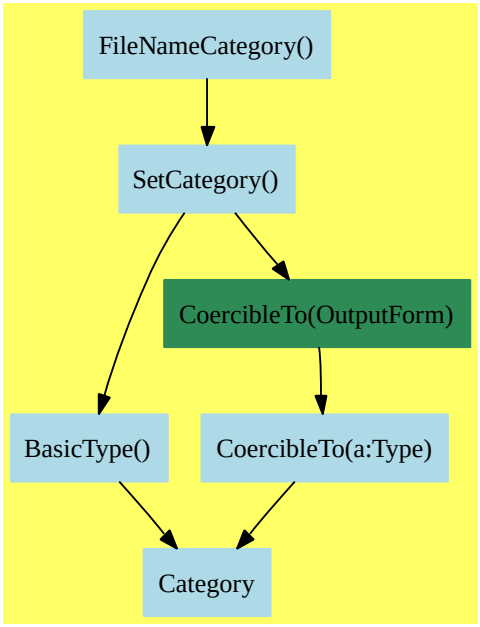
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.10 FileNameCategory (FNCAT)



See:

⇐ "SetCategory" (SETCAT) 3.13 on page 91

Exports:

coerce	directory	exists?	extension	filename
hash	latex	name	new	readable?
writable?	?=?	?~=?		

These are directly exported but not implemented:

```
coerce : String -> %
coerce : % -> String
directory : % -> String
exists? : % -> Boolean
extension : % -> String
filename : (String,String,String) -> %
name : % -> String
new : (String,String,String) -> %
readable? : % -> Boolean
writable? : % -> Boolean
```

These exports come from (p91) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
```

```
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category FNCAT FileNameCategory*⟩≡

```
)abbrev category FNCAT FileNameCategory
++ Author: Stephen M. Watt
++ Date Created: 1985
++ Date Last Updated: June 20, 1991
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++   This category provides an interface to names in the file system.
FileNameCategory(): Category == SetCategory with
   coerce: String -> %
     ++ coerce(s) converts a string to a file name
     ++ according to operating system-dependent conventions.
   coerce: % -> String
     ++ coerce(fn) produces a string for a file name
     ++ according to operating system-dependent conventions.
   filename: (String, String, String) -> %
     ++ filename(d,n,e) creates a file name with
     ++ d as its directory, n as its name and e as its extension.
     ++ This is a portable way to create file names.
     ++ When d or t is the empty string, a default is used.
   directory: % -> String
     ++ directory(f) returns the directory part of the file name.
   name: % -> String
     ++ name(f) returns the name part of the file name.
   extension: % -> String
     ++ extension(f) returns the type part of the file name.
   exists?: % -> Boolean
     ++ exists?(f) tests if the file exists in the file system.
   readable?: % -> Boolean
     ++ readable?(f) tests if the named file exist and can it be opened
     ++ for reading.
   writable?: % -> Boolean
     ++ writable?(f) tests if the named file be opened for writing.
     ++ The named file need not already exist.
   new: (String, String, String) -> %
     ++ new(d,pref,e) constructs the name of a new writable file with
```

```
      ++ d as its directory, pref as a prefix of its name and
      ++ e as its extension.
      ++ When d or t is the empty string, a default is used.
      ++ An error occurs if a new file cannot be written in the given
      ++ directory.
```

⟨*FNCAT.dotabb*⟩≡

```
"FNCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FNCAT"];
"FNCAT" -> "SETCAT"
```

⟨*FNCAT.dotfull*⟩≡

```
"FileNameCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FNCAT"];
"FileNameCategory()" -> "SetCategory()"
```

⟨*FNCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FileNameCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FNCAT"];
"FileNameCategory()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

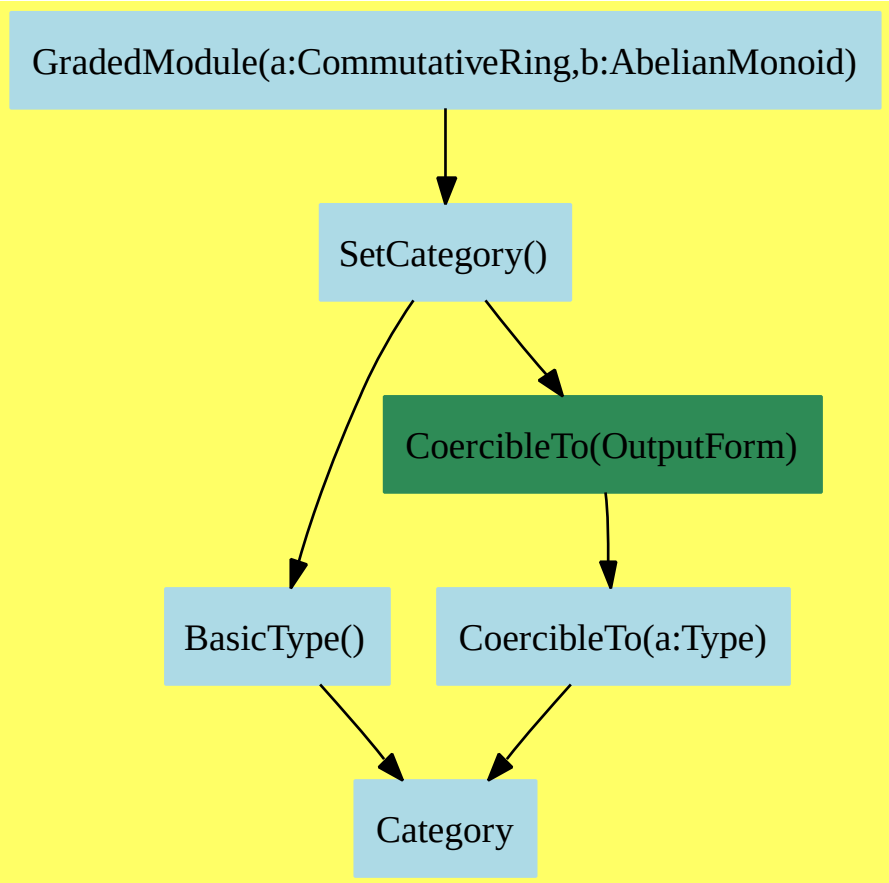
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.11 GradedModule (GRMOD)



See:

⇒ "GradedAlgebra" (GRALG) 5.7 on page 242
⇐ "SetCategory" (SETCAT) 3.13 on page 91

Exports:

0	coerce	degree	hash	latex
?~=?	?*?	?+?	?-?	-?
?=?				

These are directly exported but not implemented:

```
0 : () -> %
degree : % -> E
?*? : (%,R) -> %
?*? : (R,%) -> %
-? : % -> %
```

```
?+? : (%,%) -> %
```

These are implemented by this category:

```
?-? : (%,%) -> %
```

These exports come from (p91) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
```

⟨*category GRMOD GradedModule*⟩≡

```
)abbrev category GRMOD GradedModule
++ Author: Stephen M. Watt
++ Date Created: May 20, 1991
++ Date Last Updated: May 20, 1991
++ Basic Operations: +, *, degree
++ Related Domains: CartesianTensor(n,dim,R)
++ Also See:
++ AMS Classifications:
++ Keywords: graded module, tensor, multi-linear algebra
++ Examples:
++ References: Algebra 2d Edition, MacLane and Birkhoff, MacMillan 1979
++ Description:
++  GradedModule(R,E) denotes ‘‘E-graded R-module’’, i.e. collection of
++  R-modules indexed by an abelian monoid E.
++  An element \spad{g} of \spad{G[s]} for some specific \spad{s} in \spad{E}
++  is said to be an element of \spad{G} with {\em degree} \spad{s}.
++  Sums are defined in each module \spad{G[s]} so two elements of \spad{G}
++  have a sum if they have the same degree.
++
++  Morphisms can be defined and composed by degree to give the
++  mathematical category of graded modules.

GradedModule(R: CommutativeRing, E: AbelianMonoid): Category ==
    SetCategory with
        degree: % -> E
            ++ degree(g) names the degree of g.  The set of all elements
            ++ of a given degree form an R-module.
        0: constant -> %
            ++ 0 denotes the zero of degree 0.
        _*: (R, %) -> %
            ++ r*g is left module multiplication.
        _*: (%, R) -> %
```

```
            ++ g*r is right module multiplication.

        _-: % -> %
            ++ -g is the additive inverse of g in the module of elements
            ++ of the same grade as g.
        _+: (%, %) -> %
            ++ g+h is the sum of g and h in the module of elements of
            ++ the same degree as g and h.  Error: if g and h
            ++ have different degrees.
        _-: (%, %) -> %
            ++ g-h is the difference of g and h in the module of elements of
            ++ the same degree as g and h.  Error: if g and h
            ++ have different degrees.
    add
        (x: %) - (y: %) == x+(-y)
```

⟨*GRMOD.dotabb*⟩≡

```
  "GRMOD"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=GRMOD"];
  "GRMOD" -> "SETCAT"
```

⟨*GRMOD.dotfull*⟩≡

```
  "GradedModule(a:CommutativeRing,b:AbelianMonoid)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=GRMOD"];
  "GradedModule(a:CommutativeRing,b:AbelianMonoid)" -> "SetCategory()"
```

⟨*GRMOD.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"GradedModule(a:CommutativeRing,b:AbelianMonoid)" [color=lightblue];
"GradedModule(a:CommutativeRing,b:AbelianMonoid)" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

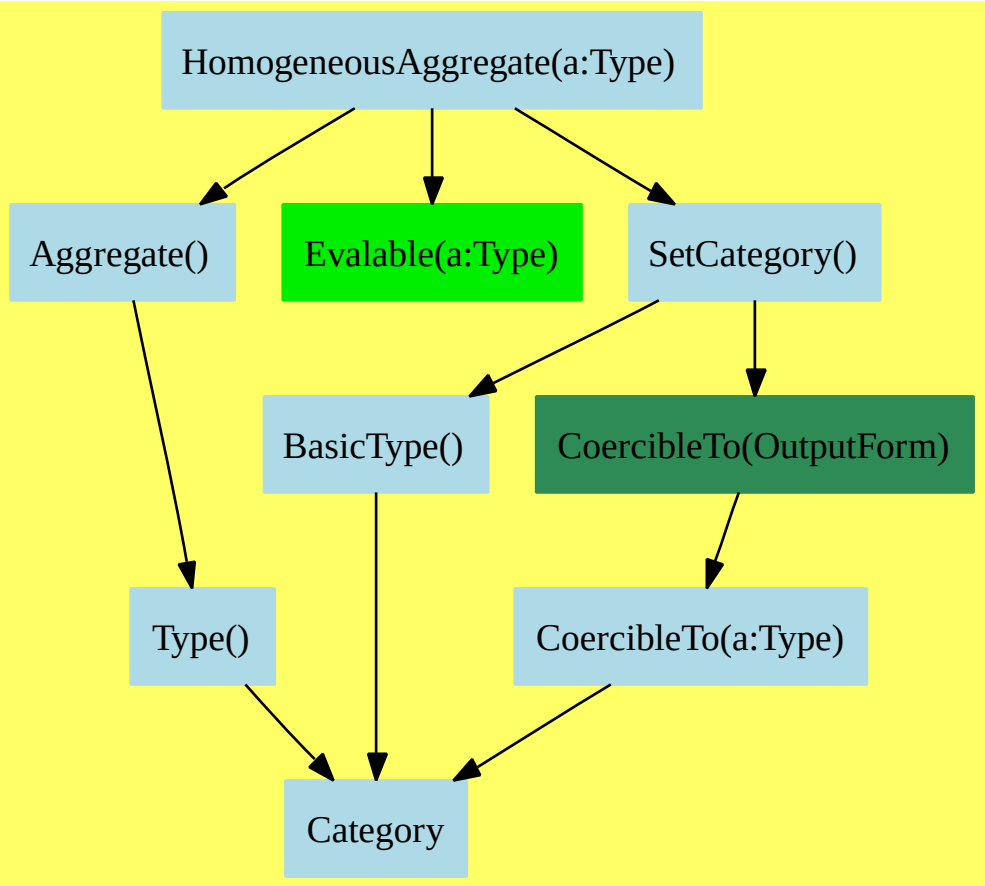
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.12 HomogeneousAggregate (HOAGG)



See:

⇒ “BagAggregate” (BGAGG) 5.2 on page 211
⇒ “Collection” (CLAGG) 5.4 on page 218
⇒ “IndexedAggregate” (IXAGG) 5.8 on page 246
⇒ “RectangularMatrixCategory” (RMATCAT) 10.14 on page 732
⇒ “RecursiveAggregate” (RCAGG) 5.12 on page 263
⇒ “TwoDimensionalArrayCategory” (ARR2CAT) 5.13 on page 268
⇐ “Aggregate” (AGG) 3.1 on page 55
⇐ “Evalable” (EVALAB) 3.4 on page 65
⇐ “SetCategory” (SETCAT) 3.13 on page 91

Attributes exported:

- **nil**

Exports:

any?	coerce	copy	count	empty
empty?	eq?	eval	every?	hash
latex	less?	map	map!	members
member?	more?	parts	sample	size?
#?	?=?	?~=?		

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
parts : % -> List S if $ has finiteAggregate
```

These are implemented by this category:

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p55) Aggregate:

```
copy : % -> %
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
less? : (%,NonNegativeInteger) -> Boolean
more? : (%,NonNegativeInteger) -> Boolean
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
```

These exports come from (p65) Evalable(a:Type):

```
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
```

These exports come from (p91) SetCategory():

```
hash : % -> SingleInteger if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
latex : % -> String if S has SETCAT
```

⟨*category HOAGG HomogeneousAggregate*⟩≡

```
)abbrev category HOAGG HomogeneousAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991, May 1995
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A homogeneous aggregate is an aggregate of elements all of the
++ same type.
++ In the current system, all aggregates are homogeneous.
++ Two attributes characterize classes of aggregates.
++ Aggregates from domains with attribute \spadatt{finiteAggregate}
++ have a finite number of members.
++ Those with attribute \spadatt{shallowlyMutable} allow an element
++ to be modified or updated without changing its overall value.
HomogeneousAggregate(S:Type): Category == Aggregate with
   if S has SetCategory then SetCategory
   if S has SetCategory then
      if S has Evalable S then Evalable S
   map     : (S->S,%) -> %
     ++ map(f,u) returns a copy of u with each element x replaced by f(x).
     ++ For collections, \axiom{map(f,u) = [f(x) for x in u]}.
   if % has shallowlyMutable then
     map_!: (S->S,%) -> %
        ++ map!(f,u) destructively replaces each element x of u
        ++ by \axiom{f(x)}.
   if % has finiteAggregate then
```

```
    any?: (S->Boolean,%) -> Boolean
      ++ any?(p,u) tests if \axiom{p(x)} is true for any element x of u.
      ++ Note: for collections,
      ++ \axiom{any?(p,u) = reduce(or,map(f,u),false,true)}.
    every?: (S->Boolean,%) -> Boolean
      ++ every?(f,u) tests if p(x) is true for all elements x of u.
      ++ Note: for collections,
      ++ \axiom{every?(p,u) = reduce(and,map(f,u),true,false)}.
    count: (S->Boolean,%) -> NonNegativeInteger
      ++ count(p,u) returns the number of elements x in u
      ++ such that \axiom{p(x)} is true. For collections,
      ++ \axiom{count(p,u) = reduce(+,[1 for x in u | p(x)],0)}.
    parts: % -> List S
      ++ parts(u) returns a list of the consecutive elements of u.
      ++ For collections, \axiom{parts([x,y,...,z]) = (x,y,...,z)}.
    members: % -> List S
      ++ members(u) returns a list of the consecutive elements of u.
      ++ For collections, \axiom{parts([x,y,...,z]) = (x,y,...,z)}.
    if S has SetCategory then
      count: (S,%) -> NonNegativeInteger
        ++ count(x,u) returns the number of occurrences of x in u. For
        ++ collections, \axiom{count(x,u) = reduce(+,[x=y for y in u],0)}.
      member?: (S,%) -> Boolean
        ++ member?(x,u) tests if x is a member of u.
        ++ For collections,
        ++ \axiom{member?(x,u) = reduce(or,[x=y for y in u],false)}.
add
 if S has Evalable S then
   eval(u:%,l:List Equation S):% == map(x +-> eval(x,l),u)
 if % has finiteAggregate then
   #c                   == # parts c
   any?(f, c)           == _or/[f x for x in parts c]
   every?(f, c)         == _and/[f x for x in parts c]
   count(f:S -> Boolean, c:%) == _+/[1 for x in parts c | f x]
   members x            == parts x
   if S has SetCategory then
     count(s:S, x:%) == count(y +-> s = y, x)
     member?(e, c)   == any?(x +-> e = x,c)
     x = y ==
        size?(x, #y) and _and/[a = b for a in parts x for b in parts y]
     coerce(x:%):OutputForm ==
       bracket
          commaSeparate [a::OutputForm for a in parts x]$List(OutputForm)
```

⟨*HOAGG.dotabb*⟩≡
```
"HOAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=HOAGG"];
"HOAGG" -> "AGG"
```

⟨*HOAGG.dotfull*⟩≡
```
"HomogeneousAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=HOAGG"];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"HomogeneousAggregate(Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=HOAGG"];
"HomogeneousAggregate(Ring)"
  -> "HomogeneousAggregate(a:Type)"
```

⟨*HOAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"
"HomogeneousAggregate(a:Type)" -> "Evalable(a:Type)"
"HomogeneousAggregate(a:Type)" -> "SetCategory()"

"Evalable(a:Type)" [color="#00EE00"];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

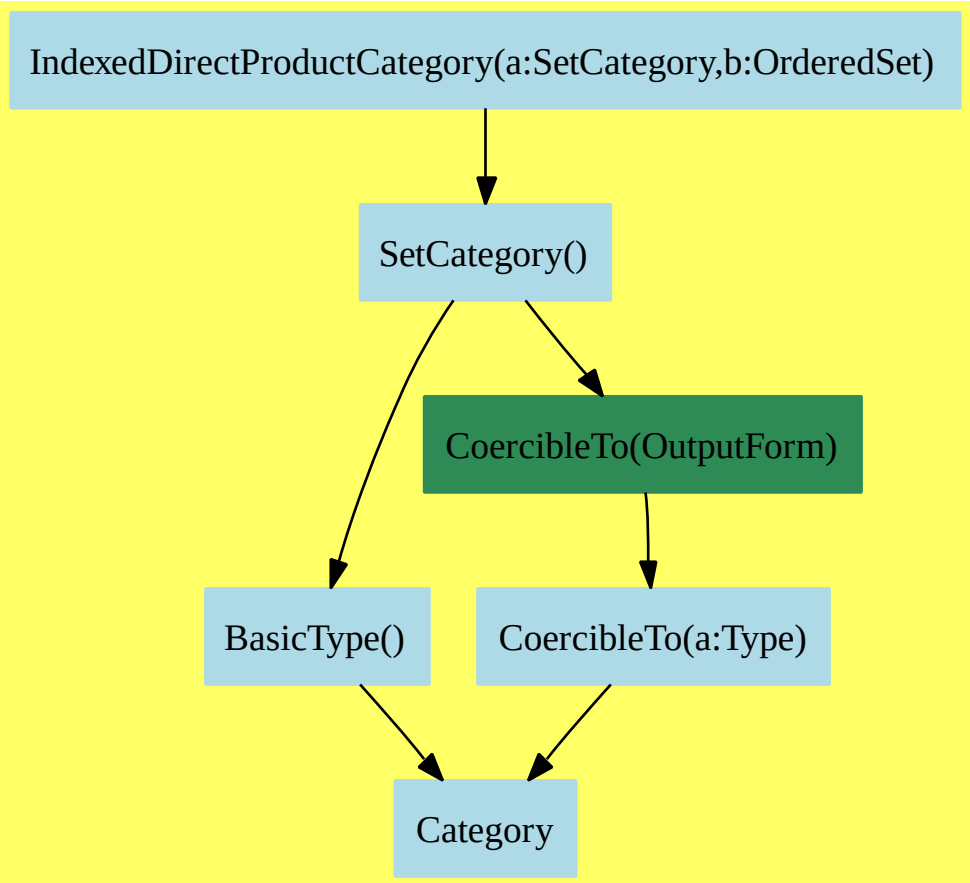
"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];

}
```

4.13 IndexedDirectProductCategory (IDPC)



See:

⇐ "SetCategory" (SETCAT) 3.13 on page 91

Exports:

coerce	hash	latex	leadingCoefficient	leadingSupport
map	monomial	reductum	?=?	?~=?

These are directly exported but not implemented:

```
leadingCoefficient : % -> A
leadingSupport : % -> S
map : ((A -> A),%) -> %
monomial : (A,S) -> %
reductum : % -> %
```

These exports come from (p91) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category IDPC IndexedDirectProductCategory*⟩≡

```
)abbrev category IDPC IndexedDirectProductCategory
++ Author: James Davenport
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This category represents the direct product of some set with
++ respect to an ordered indexing set.

IndexedDirectProductCategory(A:SetCategory,S:OrderedSet): Category ==
  SetCategory with
    map:           (A -> A, %) -> %
       ++ map(f,z) returns the new element created by applying the
       ++ function f to each component of the direct product element z.
    monomial:         (A, S) -> %
       ++ monomial(a,s) constructs a direct product element with the s
       ++ component set to \spad{a}
    leadingCoefficient:   % -> A
       ++ leadingCoefficient(z) returns the coefficient of the leading
       ++ (with respect to the ordering on the indexing set)
       ++ monomial of z.
       ++ Error: if z has no support.
    leadingSupport:   % -> S
       ++ leadingSupport(z) returns the index of leading
       ++ (with respect to the ordering on the indexing set) monomial of z.
       ++ Error: if z has no support.
    reductum:      % -> %
       ++ reductum(z) returns a new element created by removing the
       ++ leading coefficient/support pair from the element z.
       ++ Error: if z has no support.
```

⟨*IDPC.dotabb*⟩≡
```
"IDPC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=IDPC"];
"IDPC" -> "SETCAT"
```

⟨*IDPC.dotfull*⟩≡
```
"IndexedDirectProductCategory(a:SetCategory,b:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=IDPC"];
"IndexedDirectProductCategory(a:SetCategory,b:OrderedSet)" ->
   "SetCategory()"
```

⟨*IDPC.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"IndexedDirectProductCategory(a:SetCategory,b:OrderedSet)" [color=lightblue];
"IndexedDirectProductCategory(a:SetCategory,b:OrderedSet)" ->
   "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

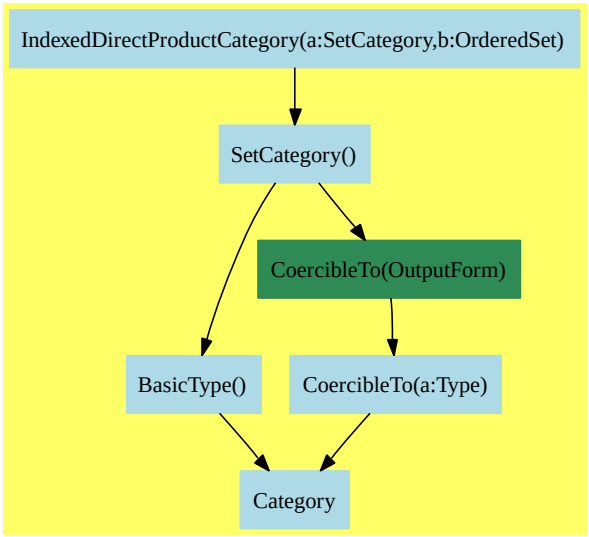
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.14 LiouvillianFunctionCategory (LFCAT)



See:

⇐ "PrimitiveFunctionCategory" (PRIMCAT) 2.16 on page 40
⇐ "TranscendentalFunctionCategory" (TRANFUN) 3.14 on page 94

Exports:

Ci	Ei	Si	acos	acosh
acot	acoth	acsc	acsch	asec
asech	asin	asinh	atan	atanh
cos	cosh	cot	coth	csc
csch	dilog	erf	exp	integral
li	log	pi	sec	sech
sin	sinh	tan	tanh	?**?

These are directly exported but not implemented:

```
Ci : % -> %
dilog : % -> %
Ei : % -> %
erf : % -> %
li : % -> %
Si : % -> %
```

These exports come from (p40) PrimitiveFunctionCategory()

```
integral : (%,Symbol) -> %
integral : (%,SegmentBinding %) -> %
```

These exports come from (p94) TranscendentalFunctionCategory():

```
?**? : (%,%) -> %
acos : % -> %
acosh : % -> %
acot : % -> %
acoth : % -> %
acsc : % -> %
acsch : % -> %
asec : % -> %
asech : % -> %
asin : % -> %
asinh : % -> %
atan : % -> %
atanh : % -> %
cos : % -> %
cosh : % -> %
cot : % -> %
coth : % -> %
csc : % -> %
csch : % -> %
exp : % -> %
log : % -> %
pi : () -> %
sec : % -> %
sech : % -> %
sin : % -> %
sinh : % -> %
tan : % -> %
tanh : % -> %
```

⟨*category LFCAT LiouvillianFunctionCategory*⟩≡

```
)abbrev category LFCAT LiouvillianFunctionCategory
++ Category for the transcendental Liouvillian functions
++ Author: Manuel Bronstein
++ Date Created: ???
++ Date Last Updated: 14 May 1991
++ Description: Category for the transcendental Liouvillian functions;
LiouvillianFunctionCategory(): Category ==
  Join(PrimitiveFunctionCategory, TranscendentalFunctionCategory) with
    Ei      : $  -> $
      ++ Ei(x) returns the exponential integral of x, i.e.
      ++ the integral of \spad{exp(x)/x dx}.
    Si      : $  -> $
      ++ Si(x) returns the sine integral of x, i.e.
      ++ the integral of \spad{sin(x) / x dx}.
    Ci      : $  -> $
      ++ Ci(x) returns the cosine integral of x, i.e.
      ++ the integral of \spad{cos(x) / x dx}.
    li      : $  -> $
      ++ li(x) returns the logarithmic integral of x, i.e.
```

```
      ++ the integral of \spad{dx / log(x)}.
    dilog   : $  -> $
      ++ dilog(x) returns the dilogarithm of x, i.e.
      ++ the integral of \spad{log(x) / (1 - x) dx}.
    erf     : $  -> $
      ++ erf(x) returns the error function of x, i.e.
      ++ \spad{2 / sqrt(%pi)} times the integral of \spad{exp(-x**2) dx}.
```

⟨*LFCAT.dotabb*⟩≡

```
"LFCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LFCAT"];
"LFCAT" -> "PRIMCAT"
"LFCAT" -> "TRANFUN"
```

⟨*LFCAT.dotfull*⟩≡

```
"LiouvillianFunctionCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LFCAT"];
"LiouvillianFunctionCategory()" -> "PrimitiveFunctionCategory()"
"LiouvillianFunctionCategory()" -> "TranscendentalFunctionCategory()"
```

⟨*LFCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"LiouvillianFunctionCategory()" [color=lightblue];
"LiouvillianFunctionCategory()" -> "PrimitiveFunctionCategory()"
"LiouvillianFunctionCategory()" -> "TranscendentalFunctionCategory()"

"PrimitiveFunctionCategory()" [color=lightblue];
"PrimitiveFunctionCategory()" -> "Category"

"TranscendentalFunctionCategory()" [color=lightblue];
"TranscendentalFunctionCategory()" ->
    "TRIGCAT"
"TranscendentalFunctionCategory()" ->
    "ATRIG"
"TranscendentalFunctionCategory()" ->
    "HYPCAT"
"TranscendentalFunctionCategory()" ->
    "AHYP"
"TranscendentalFunctionCategory()" ->
    "ELEMFUN"

"TRIGCAT" [color=lightblue];
"TRIGCAT" -> "Category"

"ATRIG" [color=lightblue];
"ATRIG" -> "Category"

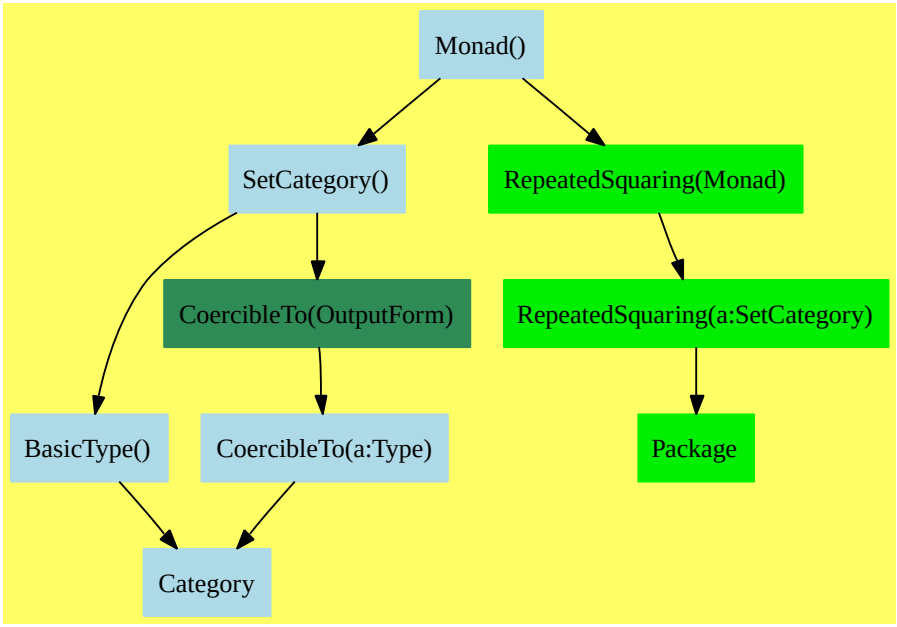
"HYPCAT" [color=lightblue];
"HYPCAT" -> "Category"

"AHYP" [color=lightblue];
"AHYP" -> "Category"

"ELEMFUN" [color=lightblue];
"ELEMFUN" -> "Category"

"Category" [color=lightblue];
}
```

4.15 Monad (MONAD)



See:

⇒ "MonadWithUnit" (MONADWU) 5.9 on page 252
⇒ "NonAssociativeRng" (NARNG) 8.8 on page 552

Exports:

coerce	hash	latex	leftPower	rightPower
?**?	?*?	?=?	?~=?	

These are directly exported but not implemented:

```
?*? : (%,%) -> %
```

These are implemented by this category:

```
leftPower : (%,PositiveInteger) -> %
rightPower : (%,PositiveInteger) -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p91) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category MONAD Monad*⟩≡

```
)abbrev category MONAD Monad
++ Authors: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 11 June 1991
++ Basic Operations: *, **
++ Related Constructors: SemiGroup, Monoid, MonadWithUnit
++ Also See:
++ AMS Classifications:
++ Keywords: Monad,  binary operation
++ Reference:
++  N. Jacobson: Structure and Representations of Jordan Algebras
++  AMS, Providence, 1968
++ Description:
++  Monad is the class of all multiplicative monads, i.e. sets
++  with a binary operation.
Monad(): Category == SetCategory with
      "*": (%,%) -> %
        ++ a*b is the product of \spad{a} and b in a set with
        ++ a binary operation.
      rightPower: (%,PositiveInteger) -> %
        ++ rightPower(a,n) returns the \spad{n}-th right power of \spad{a},
        ++ i.e. \spad{rightPower(a,n) := rightPower(a,n-1) * a} and
        ++ \spad{rightPower(a,1) := a}.
      leftPower: (%,PositiveInteger) -> %
        ++ leftPower(a,n) returns the \spad{n}-th left power of \spad{a},
        ++ i.e. \spad{leftPower(a,n) := a * leftPower(a,n-1)} and
        ++ \spad{leftPower(a,1) := a}.
      "**": (%,PositiveInteger) -> %
        ++ a**n returns the \spad{n}-th power of \spad{a},
        ++ defined by repeated squaring.
    add
      import RepeatedSquaring(%)
      x:% ** n:PositiveInteger == expt(x,n)
      rightPower(a,n) ==
--        one? n => a
        (n = 1) => a
        res := a
        for i in 1..(n-1) repeat res := res * a
        res
      leftPower(a,n) ==
--        one? n => a
        (n = 1) => a
        res := a
        for i in 1..(n-1) repeat res := a * res
        res
```

⟨*MONAD.dotabb*⟩≡

```
"MONAD"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONAD"];
"MONAD" -> "SETCAT"
```

⟨*MONAD.dotfull*⟩≡

```
"Monad()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONAD"];
"Monad()" -> "SetCategory()"
"Monad()" -> "RepeatedSquaring(Monad)"
```

⟨*MONAD.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Monad()" [color=lightblue];
"Monad()" -> "SetCategory()"
"Monad()" -> "RepeatedSquaring(Monad)"

"RepeatedSquaring(Monad)" [color="#00EE00"];
"RepeatedSquaring(Monad)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

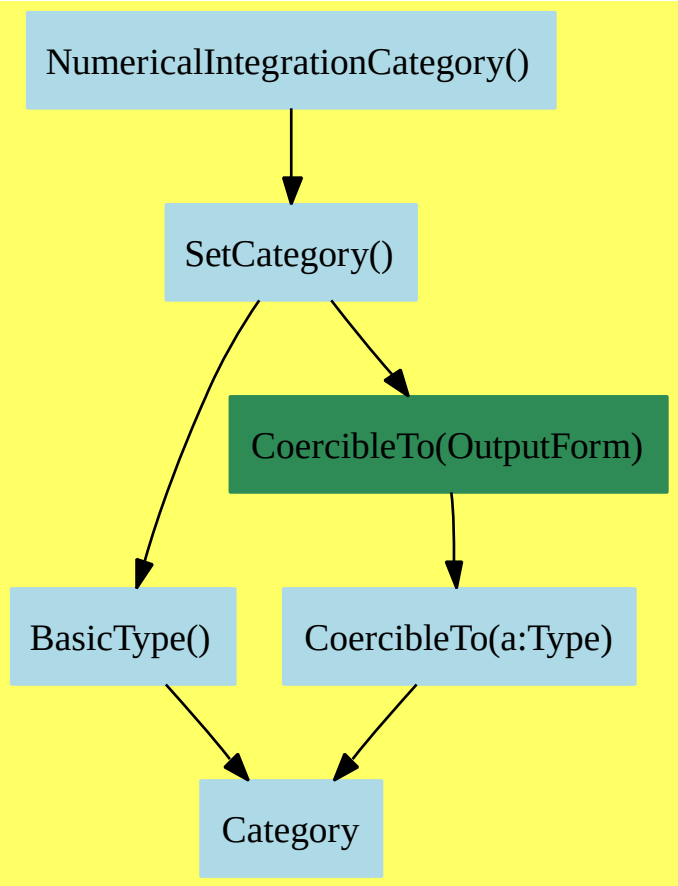
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.16 NumericalIntegrationCategory (NUMINT)



See:

⇐ "SetCategory" (SETCAT) 3.13 on page 91

Exports:

coerce	hash	latex	measure
numericalIntegration	?=?	?~=?	

These are directly exported but not implemented:

```
measure :
 (RoutinesTable,
  Record(fn: Expression DoubleFloat,
         range: List Segment OrderedCompletion DoubleFloat,
         abserr: DoubleFloat,
         relerr: DoubleFloat)) ->
    Record(measure: Float,explanations: String,extra: Result)
```

```
measure :
 (RoutinesTable,
  Record(var: Symbol,
         fn: Expression DoubleFloat,
         range: Segment OrderedCompletion DoubleFloat,
         abserr: DoubleFloat,
         relerr: DoubleFloat)) ->
   Record(measure: Float,explanations: String,extra: Result)
numericalIntegration :
 (Record(fn: Expression DoubleFloat,
         range: List Segment OrderedCompletion DoubleFloat,
         abserr: DoubleFloat,relerr: DoubleFloat),
  Result) ->
 Result
numericalIntegration :
 (Record(var: Symbol,
         fn: Expression DoubleFloat,
         range: Segment OrderedCompletion DoubleFloat,
         abserr: DoubleFloat,
         relerr: DoubleFloat),
  Result) ->
 Result
```

These exports come from (p91) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category NUMINT NumericalIntegrationCategory*⟩≡

```
)abbrev category NUMINT NumericalIntegrationCategory
++ Author: Brian Dupee
++ Date Created: February 1994
++ Date Last Updated: March 1996
++ Description:
++ \axiomType{NumericalIntegrationCategory} is the \axiom{category} for
++ describing the set of Numerical Integration \axiom{domains} with
++ \axiomFun{measure} and \axiomFun{numericalIntegration}.

EDFE   ==> Expression DoubleFloat
SOCDFE ==> Segment OrderedCompletion DoubleFloat
DFE    ==> DoubleFloat
NIAE   ==> Record(var:Symbol,fn:EDFE,range:SOCDFE,abserr:DFE,relerr:DFE)
MDNIAE ==> Record(fn:EDFE,range:List SOCDFE,abserr:DFE,relerr:DFE)
NumericalIntegrationCategory(): Category == SetCategory with

  measure:(RoutinesTable,NIAE) -> _
```

```
     Record(measure:Float,explanations:String,extra:Result)
      ++ measure(R,args) calculates an estimate of the ability of a particular
      ++ method to solve a problem.
      ++
      ++ This method may be either a specific NAG routine or a strategy (such
      ++ as transforming the function from one which is difficult to one which
      ++ is easier to solve).
      ++
      ++ It will call whichever agents are needed to perform analysis on the
      ++ problem in order to calculate the measure. There is a parameter,
      ++ labelled \axiom{sofar}, which would contain the best compatibility
      ++ found so far.

    numericalIntegration: (NIAE, Result) -> Result
      ++ numericalIntegration(args,hints) performs the integration of the
      ++ function given the strategy or method returned by \axiomFun{measure}.

    measure:(RoutinesTable,MDNIAE) -> _
     Record(measure:Float,explanations:String,extra:Result)
      ++ measure(R,args) calculates an estimate of the ability of a particular
      ++ method to solve a problem.
      ++
      ++ This method may be either a specific NAG routine or a strategy (such
      ++ as transforming the function from one which is difficult to one which
      ++ is easier to solve).
      ++
      ++ It will call whichever agents are needed to perform analysis on the
      ++ problem in order to calculate the measure. There is a parameter,
      ++ labelled \axiom{sofar}, which would contain the best compatibility
      ++ found so far.

    numericalIntegration: (MDNIAE, Result) -> Result
      ++ numericalIntegration(args,hints) performs the integration of the
      ++ function given the strategy or method returned by \axiomFun{measure}.
```

⟨*NUMINT.dotabb*⟩≡

```
  "NUMINT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=NUMINT"];
  "NUMINT" -> "SETCAT"
```

⟨*NUMINT.dotfull*⟩≡
```
"NumericalIntegrationCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NUMINT"];
"NumericalIntegrationCategory()" -> "SetCategory()"
```

⟨*NUMINT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"NumericalIntegrationCategory()" [color=lightblue];
"NumericalIntegrationCategory()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

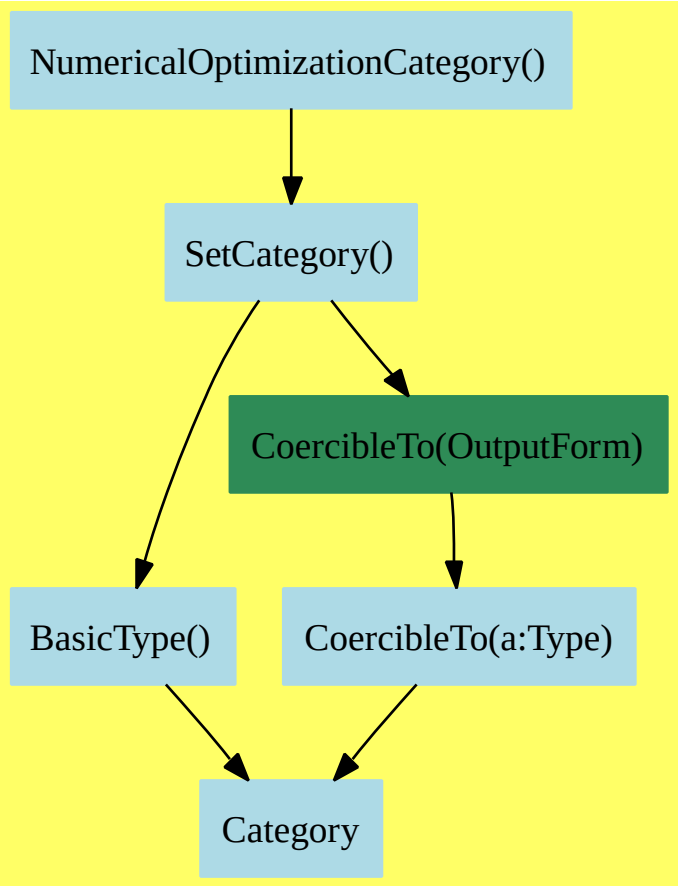
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.17 NumericalOptimizationCategory (OPTCAT)



See:

⇐ "SetCategory" (SETCAT) 3.13 on page 91

Exports:

coerce	hash	latex	measure
numericalOptimization	?=?	?~=?	

These are directly exported but not implemented:

```
measure :
 (RoutinesTable,
  Record(lfn: List Expression DoubleFloat,
         init: List DoubleFloat)) ->
   Record(measure: Float,explanations: String)
measure :
 (RoutinesTable,
```

```
   Record(fn: Expression DoubleFloat,
          init: List DoubleFloat,
          lb: List OrderedCompletion DoubleFloat,
          cf: List Expression DoubleFloat,
          ub: List OrderedCompletion DoubleFloat)) ->
    Record(measure: Float,explanations: String)
 numericalOptimization :
  Record(fn: Expression DoubleFloat,
         init: List DoubleFloat,
         lb: List OrderedCompletion DoubleFloat,
         cf: List Expression DoubleFloat,
         ub: List OrderedCompletion DoubleFloat) ->
   Result
 numericalOptimization :
  Record(lfn: List Expression DoubleFloat,
         init: List DoubleFloat) ->
   Result
```

These exports come from (p91) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category OPTCAT NumericalOptimizationCategory*⟩≡

```
 )abbrev category OPTCAT NumericalOptimizationCategory
 ++ Author: Brian Dupee
 ++ Date Created: January 1996
 ++ Date Last Updated: March 1996
 ++ Description:
 ++ \axiomType{NumericalOptimizationCategory} is the \axiom{category} for
 ++ describing the set of Numerical Optimization \axiom{domains} with
 ++ \axiomFun{measure} and \axiomFun{optimize}.

 LDFH   ==> List DoubleFloat
 LEDFH  ==> List Expression DoubleFloat
 LSAH   ==> Record(lfn:LEDFH, init:LDFH)
 EDFH   ==> Expression DoubleFloat
 LOCDFH ==> List OrderedCompletion DoubleFloat
 NOAH   ==> Record(fn:EDFH, init:LDFH, lb:LOCDFH, cf:LEDFH, ub:LOCDFH)
 NumericalOptimizationCategory(): Category == SetCategory with
   measure:(RoutinesTable,NOAH)->Record(measure:Float,explanations:String)
     ++ measure(R,args) calculates an estimate of the ability of a particular
     ++ method to solve an optimization problem.
     ++
     ++ This method may be either a specific NAG routine or a strategy (such
```

```
      ++ as transforming the function from one which is difficult to one which
      ++ is easier to solve).
      ++
      ++ It will call whichever agents are needed to perform analysis on the
      ++ problem in order to calculate the measure. There is a parameter,
      ++ labelled \axiom{sofar}, which would contain the best compatibility
      ++ found so far.

    measure:(RoutinesTable,LSAH)->Record(measure:Float,explanations:String)
      ++ measure(R,args) calculates an estimate of the ability of a particular
      ++ method to solve an optimization problem.
      ++
      ++ This method may be either a specific NAG routine or a strategy (such
      ++ as transforming the function from one which is difficult to one which
      ++ is easier to solve).
      ++
      ++ It will call whichever agents are needed to perform analysis on the
      ++ problem in order to calculate the measure. There is a parameter,
      ++ labelled \axiom{sofar}, which would contain the best compatibility
      ++ found so far.

    numericalOptimization:LSAH -> Result
      ++ numericalOptimization(args) performs the optimization of the
      ++ function given the strategy or method returned by \axiomFun{measure}.

    numericalOptimization:NOAH -> Result
      ++ numericalOptimization(args) performs the optimization of the
      ++ function given the strategy or method returned by \axiomFun{measure}.
```

⟨*OPTCAT.dotabb*⟩≡

```
  "OPTCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=OPTCAT"];
  "OPTCAT" -> "SETCAT"
```

⟨*OPTCAT.dotfull*⟩≡

```
  "NumericalOptimizationCategory()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=OPTCAT"];
  "NumericalOptimizationCategory()" -> "SetCategory()"
```

⟨*OPTCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"NumericalOptimizationCategory()" [color=lightblue];
"NumericalOptimizationCategory()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

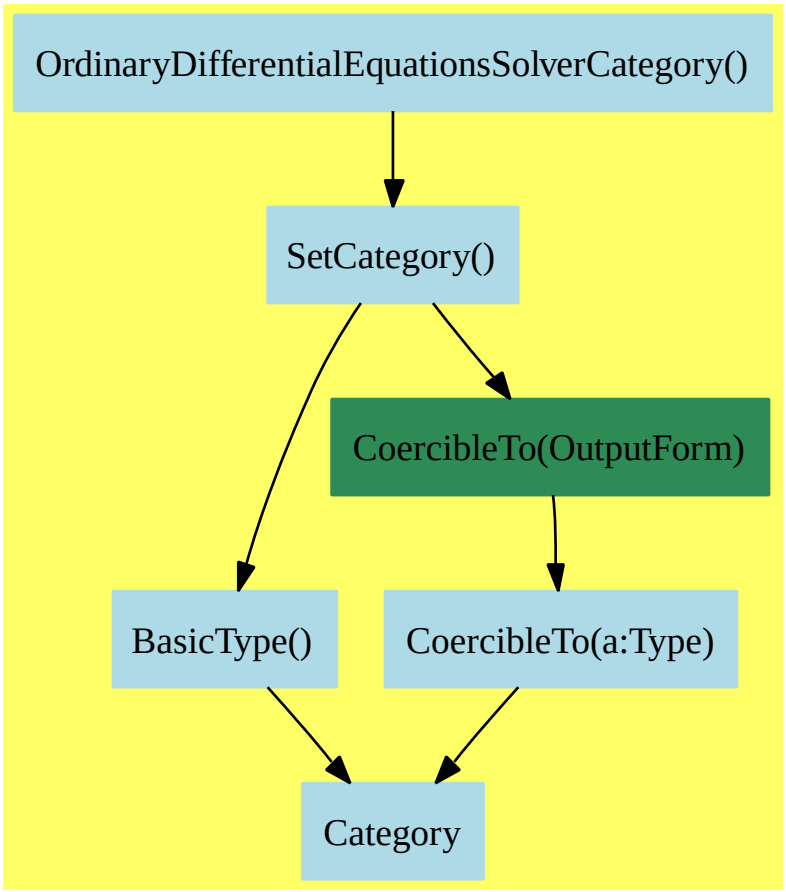
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.18 OrdinaryDifferentialEquationsSolverCategory (ODECAT)



See:

⇐ "SetCategory" (SETCAT) 3.13 on page 91

Exports:

coerce hash latex measure ODESolve
?=? ?~=?

These are directly exported but not implemented:

```
ODESolve :
 Record(xinit: DoubleFloat,
        xend: DoubleFloat,
        fn: Vector Expression DoubleFloat,
        yinit: List DoubleFloat,
```

```
        intvals: List DoubleFloat,
        g: Expression DoubleFloat,
        abserr: DoubleFloat,
        relerr: DoubleFloat) ->
  Result
measure :
 (RoutinesTable,
  Record(xinit: DoubleFloat,
         xend: DoubleFloat,
         fn: Vector Expression DoubleFloat,
         yinit: List DoubleFloat,
         intvals: List DoubleFloat,
         g: Expression DoubleFloat,
         abserr: DoubleFloat,
         relerr: DoubleFloat)) ->
   Record(measure: Float,explanations: String)
```

These exports come from (p91) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category ODECAT OrdinaryDifferentialEquationsSolverCategory*⟩≡

```
 )abbrev category ODECAT OrdinaryDifferentialEquationsSolverCategory
 ++ Author: Brian Dupee
 ++ Date Created: February 1995
 ++ Date Last Updated: June 1995
 ++ Basic Operations:
 ++ Description:
 ++ \axiomType{OrdinaryDifferentialEquationsSolverCategory} is the
 ++ \axiom{category} for describing the set of ODE solver \axiom{domains}
 ++ with \axiomFun{measure} and \axiomFun{ODEsolve}.

 DFF   ==> DoubleFloat
 VEDFF ==> Vector Expression DoubleFloat
 LDFF  ==> List DoubleFloat
 EDFF  ==> Expression DoubleFloat
 ODEAF ==> Record(xinit:DFF,xend:DFF,fn:VEDFF,yinit:LDFF,intvals:LDFF,g:EDFF,abserr:DFF,relerr
 OrdinaryDifferentialEquationsSolverCategory(): Category == SetCategory with

   measure:(RoutinesTable,ODEAF) -> Record(measure:Float,explanations:String)
     ++ measure(R,args) calculates an estimate of the ability of a particular
     ++ method to solve a problem.
     ++
     ++ This method may be either a specific NAG routine or a strategy (such
```

```
      ++ as transforming the function from one which is difficult to one which
      ++ is easier to solve).
      ++
      ++ It will call whichever agents are needed to perform analysis on the
      ++ problem in order to calculate the measure. There is a parameter,
      ++ labelled \axiom{sofar}, which would contain the best compatibility
      ++ found so far.

    ODESolve: ODEAF -> Result
      ++ ODESolve(args) performs the integration of the
      ++ function given the strategy or method returned by \axiomFun{measure}.
```

⟨*ODECAT.dotabb*⟩≡

```
  "ODECAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ODECAT"];
  "ODECAT" -> "SETCAT"
```

⟨*ODECAT.dotfull*⟩≡

```
  "OrdinaryDifferentialEquationsSolverCategory()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ODECAT"];
  "OrdinaryDifferentialEquationsSolverCategory()" -> "SetCategory()"
```

⟨*ODECAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrdinaryDifferentialEquationsSolverCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ODECAT"];
"OrdinaryDifferentialEquationsSolverCategory()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

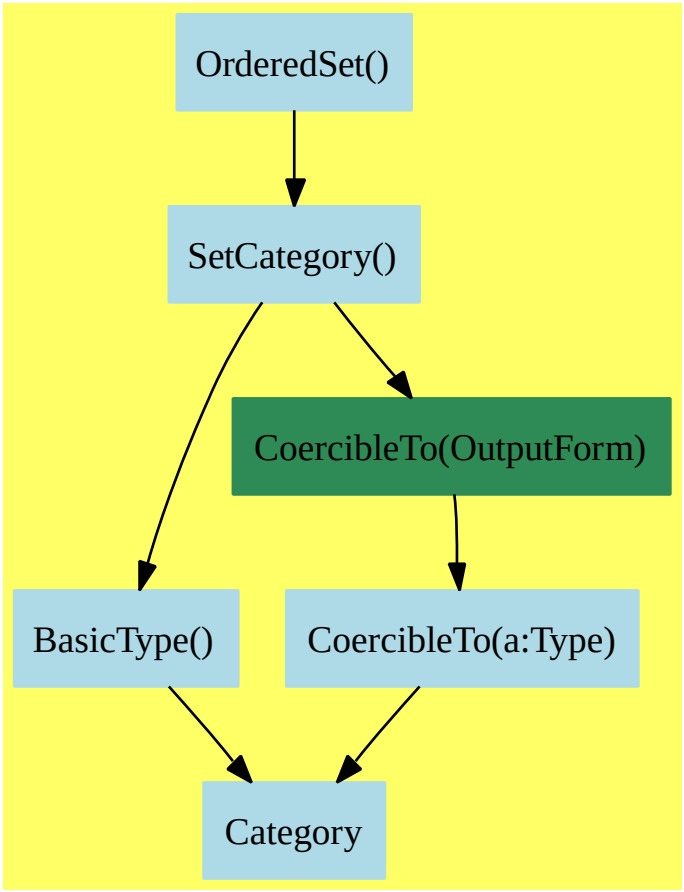
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.19 OrderedSet (ORDSET)



See:

⇒ “BitAggregate” (BTAGG) 9.2 on page 596
⇒ “CachableSet” (CACHSET) 5.3 on page 215
⇒ “DifferentialVariableCategory” (DVARCAT) 5.5 on page 224
⇒ “ExpressionSpace” (ES) 5.6 on page 230
⇒ “FortranMachineTypeCategory” (FMTC) 13.2 on page 913
⇒ “IntervalCategory” (INTCAT) 14.2 on page 950
⇒ “OrderedAbelianSemiGroup” (OASGP) 6.8 on page 364
⇒ “OrderedFinite” (ORDFIN) 5.11 on page 260
⇒ “OrderedMonoid” (ORDMON) 6.9 on page 368
⇒ “PolynomialCategory” (POLYCAT) 16.4 on page 1027
⇒ “QuotientFieldCategory” (QFCAT) 17.6 on page 1121
⇐ “SetCategory” (SETCAT) 3.13 on page 91

Exports:

coerce	hash	latex	max	min
?<?	?<=?	?=?	?>?	?>=?
?~=?				

These are directly exported but not implemented:

```
?<? : (%,%) -> Boolean
```

These are implemented by this category:

```
max : (%,%) -> %
min : (%,%) -> %
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
```

These exports come from (p91) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category ORDSET OrderedSet*⟩≡

```
)abbrev category ORDSET OrderedSet
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The class of totally ordered sets, that is, sets such that for each
++ pair of elements \spad{(a,b)}
++ exactly one of the following relations holds \spad{a<b or a=b or b<a}
++ and the relation is transitive, i.e.  \spad{a<b and b<c => a<c}.
OrderedSet(): Category == SetCategory with
    "<": (%,%) -> Boolean
      ++ x < y is a strict total ordering on the elements of the set.
    ">":          (%, %) -> Boolean
      ++ x > y is a greater than test.
    ">=":         (%, %) -> Boolean
      ++ x >= y is a greater than or equal test.
```

```
    "<=":        (%, %) -> Boolean
      ++ x <= y is a less than or equal test.
    max: (%,%) -> %
      ++ max(x,y) returns the maximum of x and y relative to "<".
    min: (%,%) -> %
      ++ min(x,y) returns the minimum of x and y relative to "<".
  add
    x,y: %
  -- These really ought to become some sort of macro
    max(x,y) ==
      x > y => x
      y
    min(x,y) ==
      x > y => y
      x
    ((x: %) >  (y: %)) : Boolean == y < x
    ((x: %) >= (y: %)) : Boolean == not (x < y)
    ((x: %) <= (y: %)) : Boolean == not (y < x)
```

⟨*ORDSET.dotabb*⟩≡

```
"ORDSET"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDSET"];
"ORDSET" -> "SETCAT"
```

⟨*ORDSET.dotfull*⟩≡

```
"OrderedSet()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDSET"];
"OrderedSet()" -> "SetCategory()"
```

⟨*ORDSET.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

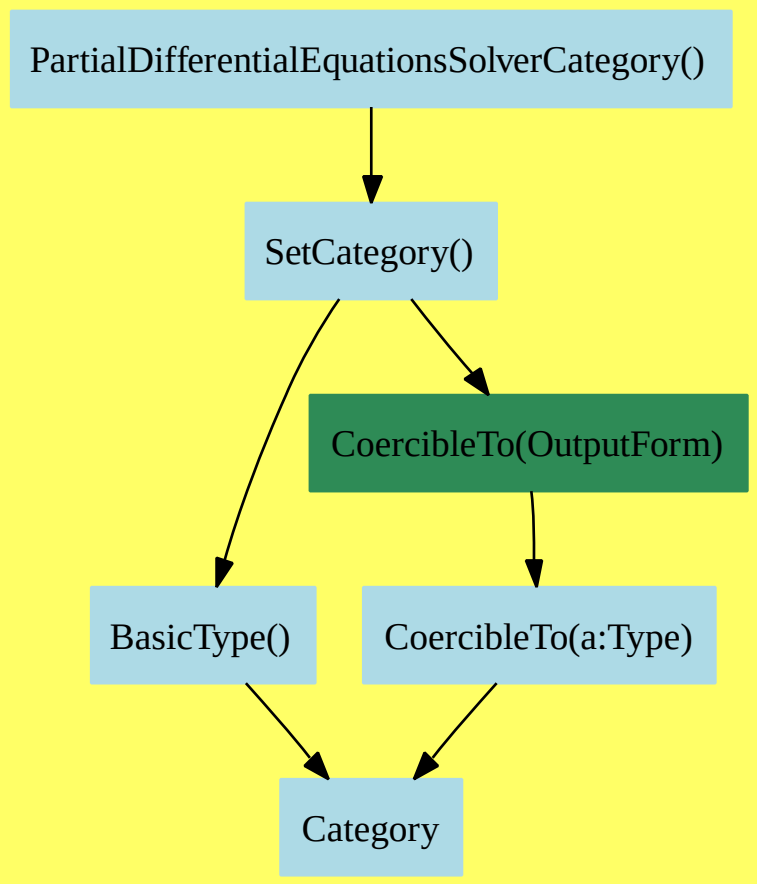
"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

4.20 PartialDifferentialEquationsSolverCategory (PDECAT)



See:

⇐ "SetCategory" (SETCAT) 3.13 on page 91

Exports:

coerce hash latex measure PDESolve
?=? ?~=?

These are directly exported but not implemented:

```
measure :
(RoutinesTable,
 Record(pde: List Expression DoubleFloat,
        constraints:
         List Record(start: DoubleFloat,
```

```
                    finish: DoubleFloat,
                    grid: NonNegativeInteger,
                    boundaryType: Integer,
                    dStart: Matrix DoubleFloat,
                    dFinish: Matrix DoubleFloat),
        f: List List Expression DoubleFloat,
        st: String,
        tol: DoubleFloat)) ->
   Record(measure: Float,explanations: String)
PDESolve :
 Record(pde: List Expression DoubleFloat,
        constraints:
         List Record(start: DoubleFloat,
                    finish: DoubleFloat,
                    grid: NonNegativeInteger,
                    boundaryType: Integer,
                    dStart: Matrix DoubleFloat,
                    dFinish: Matrix DoubleFloat),
        f: List List Expression DoubleFloat,
        st: String,
        tol: DoubleFloat) ->
   Result
```

These exports come from (p428) Dictionary(S:SetCategory):

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category PDECAT PartialDifferentialEquationsSolverCategory*⟩≡

```
 )abbrev category PDECAT PartialDifferentialEquationsSolverCategory
 ++ Author: Brian Dupee
 ++ Date Created: February 1995
 ++ Date Last Updated: June 1995
 ++ Basic Operations:
 ++ Description:
 ++ \axiomType{PartialDifferentialEquationsSolverCategory} is the
 ++ \axiom{category} for describing the set of PDE solver \axiom{domains}
 ++ with \axiomFun{measure} and \axiomFun{PDEsolve}.

 -- PDEA ==> Record(xmin:F,xmax:F,ymin:F,ymax:F,ngx:NNI,ngy:NNI,_
 --             pde:List Expression Float, bounds:List List Expression Float,_
 --             st:String, tol:DF)

 --  measure:(RoutinesTable,PDEA) -> Record(measure:F,explanations:String)
 --    ++ measure(R,args) calculates an estimate of the ability of a particular
 --    ++ method to solve a problem.
```

```
--    ++
--    ++ This method may be either a specific NAG routine or a strategy (such
--    ++ as transforming the function from one which is difficult to one which
--    ++ is easier to solve).
--    ++
--    ++ It will call whichever agents are needed to perform analysis on the
--    ++ problem in order to calculate the measure. There is a parameter,
--    ++ labelled \axiom{sofar}, which would contain the best compatibility
--    ++ found so far.

--  PDESolve: PDEA -> Result
--    ++ PDESolve(args) performs the integration of the
--    ++ function given the strategy or method returned by \axiomFun{measure}.

DFG   ==> DoubleFloat
NNIG  ==> NonNegativeInteger
INTG  ==> Integer
MDFG  ==> Matrix DoubleFloat
PDECG ==> Record(start:DFG, finish:DFG, grid:NNIG, boundaryType:INTG,
                 dStart:MDFG, dFinish:MDFG)
LEDFG ==> List Expression DoubleFloat
PDEBG ==> Record(pde:LEDFG, constraints:List PDECG, f:List LEDFG,
                   st:String, tol:DFG)
PartialDifferentialEquationsSolverCategory(): Category == SetCategory with

  measure:(RoutinesTable,PDEBG) -> Record(measure:Float,explanations:String)
    ++ measure(R,args) calculates an estimate of the ability of a particular
    ++ method to solve a problem.
    ++
    ++ This method may be either a specific NAG routine or a strategy (such
    ++ as transforming the function from one which is difficult to one which
    ++ is easier to solve).
    ++
    ++ It will call whichever agents are needed to perform analysis on the
    ++ problem in order to calculate the measure. There is a parameter,
    ++ labelled \axiom{sofar}, which would contain the best compatibility
    ++ found so far.

  PDESolve: PDEBG -> Result
    ++ PDESolve(args) performs the integration of the
    ++ function given the strategy or method returned by \axiomFun{measure}.
```

⟨*PDECAT.dotabb*⟩≡
```
"PDECAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PDECAT"];
"PDECAT" -> "SETCAT"
```

⟨*PDECAT.dotfull*⟩≡
```
"PartialDifferentialEquationsSolverCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PDECAT"];
"PartialDifferentialEquationsSolverCategory()" -> "SetCategory()"
```

⟨*PDECAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PartialDifferentialEquationsSolverCategory()" [color=lightblue];
"PartialDifferentialEquationsSolverCategory()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

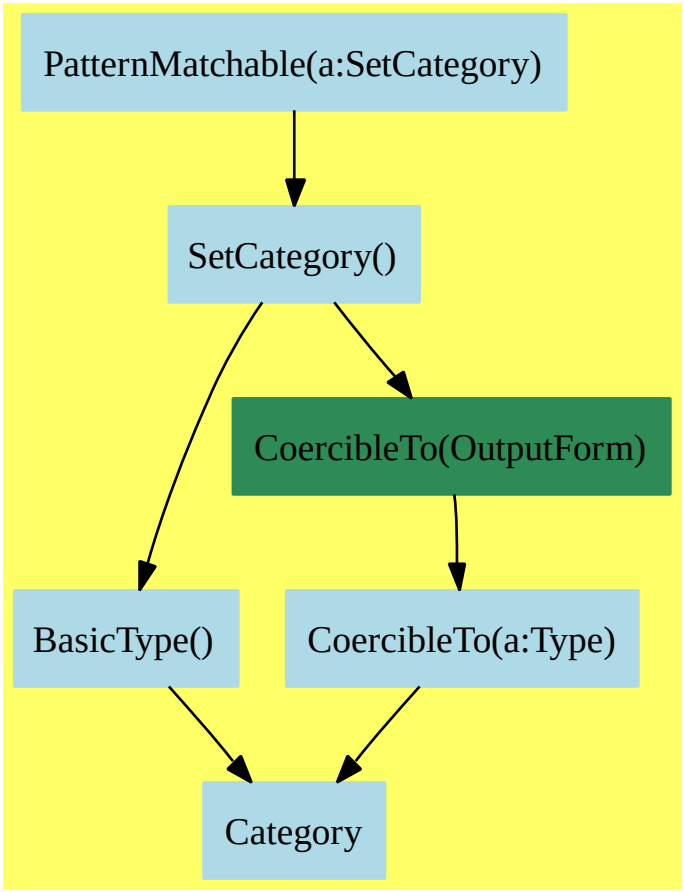
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.21 PatternMatchable (PATMAB)



See:

⇒ "RealNumberSystem" (RNS) 17.8 on page 1141
⇐ "SetCategory" (SETCAT) 3.13 on page 91

Exports:
coerce hash latex patternMatch ?=?
?~=?

These are directly exported but not implemented:

```
patternMatch :
  (%,Pattern S,PatternMatchResult(S,%))
      -> PatternMatchResult(S,%)
```

These exports come from (p91) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category PATMAB PatternMatchable*⟩≡

```
)abbrev category PATMAB PatternMatchable
++ Category of sets that can be pattern-matched on
++ Author: Manuel Bronstein
++ Date Created: 28 Nov 1989
++ Date Last Updated: 15 Mar 1990
++ Description:
++   A set R is PatternMatchable over S if elements of R can
++   be matched to patterns over S.
++ Keywords: pattern, matching.
PatternMatchable(S:SetCategory): Category == SetCategory with
  patternMatch: (%, Pattern S, PatternMatchResult(S, %)) ->
                                              PatternMatchResult(S, %)
    ++ patternMatch(expr, pat, res) matches the pattern pat to the
    ++ expression expr. res contains the variables of pat which
    ++ are already matched and their matches (necessary for recursion).
    ++ Initially, res is just the result of \spadfun{new}
    ++ which is an empty list of matches.
```

⟨*PATMAB.dotabb*⟩≡

```
"PATMAB"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PATMAB"];
"PATMAB" -> "SETCAT"
```

⟨*PATMAB.dotfull*⟩≡

```
"PatternMatchable(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PATMAB"];
"PatternMatchable(a:SetCategory)" -> "SetCategory()"

"PatternMatchable(Integer)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PATMAB"];
"PatternMatchable(Integer)" -> "PatternMatchable(a:SetCategory)"

"PatternMatchable(Float)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PATMAB"];
"PatternMatchable(Float)" -> "PatternMatchable(a:SetCategory)"
```

⟨*PATMAB.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PatternMatchable(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PATMAB"];
"PatternMatchable(a:SetCategory)" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

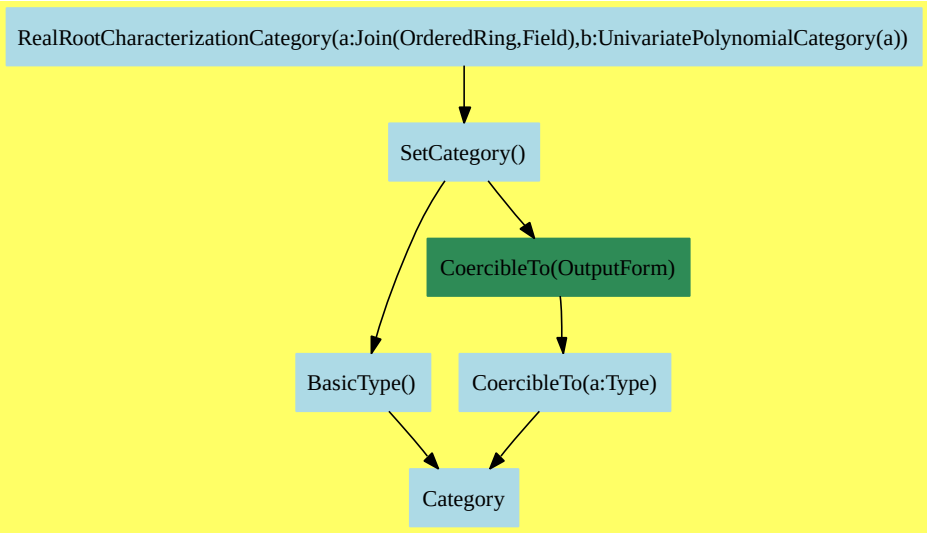
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.22 RealRootCharacterizationCategory (RRCC)



See:

⇐ "SetCategory" (SETCAT) 3.13 on page 91

Exports:

allRootsOf	approximate	coerce	definingPolynomial
hash	latex	negative?	positive?
recip	relativeApprox	rootOf	sign
zero?	?=?	? =?	

These are directly exported but not implemented:

```
approximate : (ThePols,%,TheField) -> TheField
allRootsOf : ThePols -> List %
definingPolynomial : % -> ThePols
relativeApprox : (ThePols,%,TheField) -> TheField
sign : (ThePols,%) -> Integer
```

These are implemented by this category:

```
negative? : (ThePols,%) -> Boolean
positive? : (ThePols,%) -> Boolean
recip : (ThePols,%) -> Union(ThePols,"failed")
rootOf : (ThePols,PositiveInteger) -> Union(%,"failed")
zero? : (ThePols,%) -> Boolean
```

These exports come from (p91) SetCategory():

```
coerce : % -> OutputForm
```

```
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category RRCC RealRootCharacterizationCategory*⟩≡

```
)abbrev category RRCC RealRootCharacterizationCategory
++ Author: Renaud Rioboo
++ Date Created: summer 1992
++ Date Last Updated: January 2004
++ Basic Functions: provides operations with generic real roots of
++                  polynomials
++ Related Constructors: RealClosure, RightOpenIntervalRootCharacterization
++ Also See:
++ AMS Classifications:
++ Keywords: Real Algebraic Numbers
++ References:
++ Description:
++ \axiomType{RealRootCharacterizationCategory} provides common acces
++ functions for all real root codings.
RealRootCharacterizationCategory(TheField, ThePols ) : Category == PUB where

   TheField : Join(OrderedRing, Field)
   ThePols : UnivariatePolynomialCategory(TheField)

   Z ==> Integer
   N ==> PositiveInteger

   PUB ==>
     SetCategory with

         sign:                ( ThePols, $ )   ->            Z
               ++ \axiom{sign(pol,aRoot)} gives the sign of \axiom{pol}
               ++ interpreted as \axiom{aRoot}
         zero? :              ( ThePols, $ )   ->         Boolean
               ++ \axiom{zero?(pol,aRoot)} answers if \axiom{pol}
               ++ interpreted as \axiom{aRoot} is \axiom{0}
         negative?:           ( ThePols, $ )   ->         Boolean
               ++ \axiom{negative?(pol,aRoot)} answers if \axiom{pol}
               ++ interpreted as \axiom{aRoot} is negative
         positive?:           ( ThePols, $ )   ->         Boolean
               ++ \axiom{positive?(pol,aRoot)} answers if \axiom{pol}
               ++ interpreted as \axiom{aRoot} is positive
         recip:               ( ThePols, $ )   ->   Union(ThePols,"failed")
               ++ \axiom{recip(pol,aRoot)} tries to inverse \axiom{pol}
               ++ interpreted as \axiom{aRoot}
         definingPolynomial:         $         ->         ThePols
```

```
        ++ \axiom{definingPolynomial(aRoot)} gives a polynomial
        ++ such that \axiom{definingPolynomial(aRoot).aRoot = 0}
    allRootsOf:              ThePols      ->          List $
        ++ \axiom{allRootsOf(pol)} creates all the roots of \axiom{pol}
        ++ in the Real Closure, assumed in order.
    rootOf:              ( ThePols, N )   ->      Union($,"failed")
        ++ \axiom{rootOf(pol,n)} gives the nth root for the order of the
        ++ Real Closure
    approximate :  (ThePols,$,TheField)   ->    TheField
        ++ \axiom{approximate(term,root,prec)} gives an approximation
        ++ of \axiom{term} over \axiom{root} with precision \axiom{prec}

    relativeApprox :  (ThePols,$,TheField)   ->    TheField
        ++ \axiom{approximate(term,root,prec)} gives an approximation
        ++ of \axiom{term} over \axiom{root} with precision \axiom{prec}

  add

    zero?(toTest, rootChar) ==
      sign(toTest, rootChar) = 0

    negative?(toTest, rootChar) ==
      sign(toTest, rootChar) < 0

    positive?(toTest, rootChar) ==
      sign(toTest, rootChar) > 0

    rootOf(pol,n) ==
      liste:List($):= allRootsOf(pol)
      # liste > n => "failed"
      liste.n

    recip(toInv,rootChar) ==
      degree(toInv) = 0 =>
        res := recip(leadingCoefficient(toInv))
        if (res case "failed") then "failed" else (res::TheField::ThePols)
      defPol := definingPolynomial(rootChar)
      d := principalIdeal([defPol,toInv])
      zero?(d.generator,rootChar) => "failed"
      if (degree(d.generator) ^= 0 )
      then
        defPol := (defPol exquo (d.generator))::ThePols
        d := principalIdeal([defPol,toInv])
      d.coef.2
```

⟨*RRCC.dotabb*⟩≡
```
"RRCC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RRCC"];
"RRCC" -> "SETCAT"
```

⟨*RRCC.dotfull*⟩≡
```
"RealRootCharacterizationCategory(a:Join(OrderedRing,Field),b:UnivariatePolynomia
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RRCC"];
"RealRootCharacterizationCategory(a:Join(OrderedRing,Field),b:UnivariatePolynomia
  -> "SetCategory()"
```

⟨*RRCC.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"RealRootCharacterizationCategory(a:Join(OrderedRing,Field),b:UnivariatePolynomia
 [color=lightblue];
"RealRootCharacterizationCategory(a:Join(OrderedRing,Field),b:UnivariatePolynomia
  -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

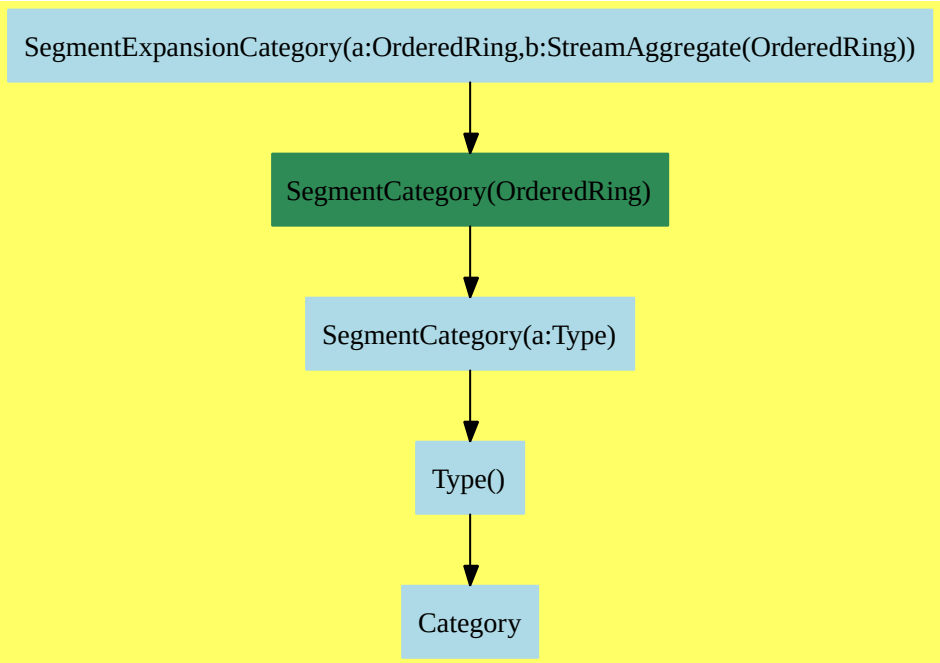
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.23 SegmentExpansionCategory (SEGXCAT)



See:

⇐ "SegmentCategory" (SEGCAT) 3.12 on page 88

Exports:

BY	convert	expand	hi	high
incr	lo	low	map	segment
?..?				

Attributes exported:

- **nil**

These are directly exported but not implemented:

```
expand : % -> L
expand : List % -> L
map : ((S -> S),%) -> L
```

These exports come from (p88) SegmentCategory(OrderedRing):

```
BY : (%,Integer) -> %
convert : S -> %
hi : % -> S
```

```
high : % -> S
incr : % -> Integer
lo : % -> S
low : % -> S
segment : (S,S) -> %
?..? : (S,S) -> %
```

⟨*category SEGXCAT SegmentExpansionCategory*⟩≡

```
)abbrev category SEGXCAT SegmentExpansionCategory
++ Author:  Stephen M. Watt
++ Date Created: June 5, 1991
++ Date Last Updated:
++ Basic Operations:
++ Related Domains: Segment, UniversalSegment
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++   This category provides an interface for expanding segments to
++   a stream of elements.
SegmentExpansionCategory(S: OrderedRing, L: StreamAggregate(S)): Category ==
    SegmentCategory(S) with
      expand: List % -> L
        ++ expand(l) creates a new value of type L in which each segment
        ++ \spad{l..h by k} is replaced with \spad{l, l+k, ... lN},
        ++ where \spad{lN <= h < lN+k}.
        ++ For example, \spad{expand [1..4, 7..9] = [1,2,3,4,7,8,9]}.
      expand: % -> L
        ++ expand(l..h by k) creates value of type L with elements
        ++ \spad{l, l+k, ... lN} where \spad{lN <= h < lN+k}.
        ++ For example, \spad{expand(1..5 by 2) = [1,3,5]}.
      map: (S -> S, %) -> L
        ++ map(f,l..h by k) produces a value of type L by applying f
        ++ to each of the succesive elements of the segment, that is,
        ++ \spad{[f(l), f(l+k), ..., f(lN)]}, where \spad{lN <= h < lN+k}.
```

⟨*SEGXCAT.dotabb*⟩≡

```
"SEGXCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEGXCAT"];
"SEGXCAT" -> "SEGCAT"
```

⟨*SEGXCAT.dotfull*⟩≡

```
"SegmentExpansionCategory(a:OrderedRing,b:StreamAggregate(OrderedRing))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEGXCAT"];
"SegmentExpansionCategory(a:OrderedRing,b:StreamAggregate(OrderedRing))"
   -> "SegmentCategory(OrderedRing)"
```

⟨*SEGXCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"SegmentExpansionCategory(a:OrderedRing,b:StreamAggregate(OrderedRing))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEGXCAT"];
"SegmentExpansionCategory(a:OrderedRing,b:StreamAggregate(OrderedRing))"
   -> "SegmentCategory(OrderedRing)"

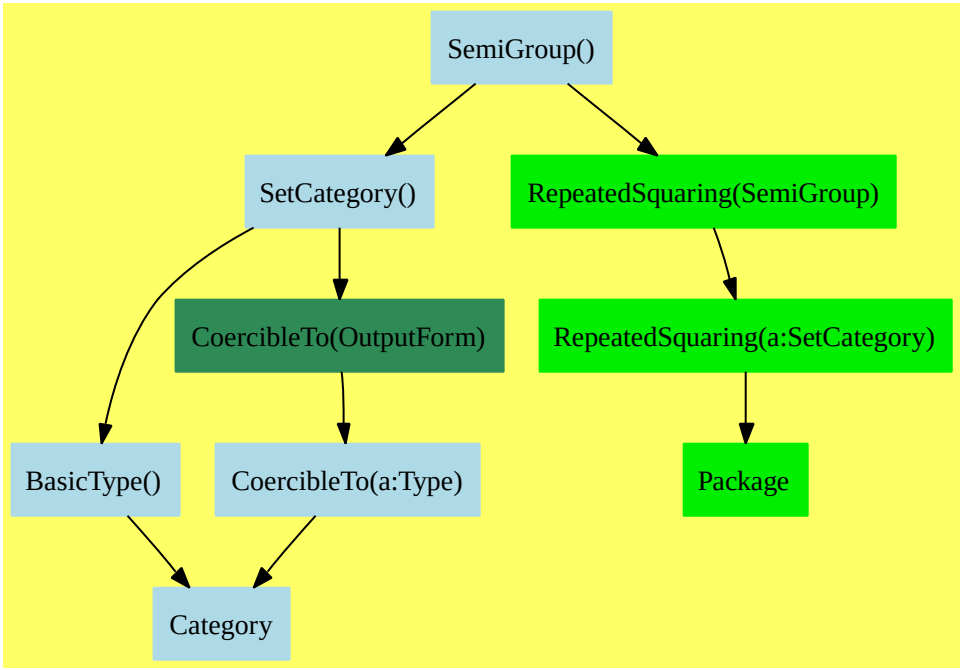
"SegmentCategory(OrderedRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=SEGCAT"];
"SegmentCategory(OrderedRing)" -> "SegmentCategory(a:Type)"

"SegmentCategory(a:Type)" [color=lightblue];
"SegmentCategory(a:Type)" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

4.24 SemiGroup (SGROUP)



A Semigroup is defined as a set S with a binary multiplicative operator "*". A Semigroup $G(S, *)$ is:

- a set S which can be null
- a binary multiplicative operator "*"
- associative. $\forall a, b, c \in S, a * (b * c) = (a * b) * c$

See:

⇒ "FunctionSpace" (FS) 17.5 on page 1095
⇒ "Monoid" (MONOID) 5.10 on page 256
⇒ "Rng" (RNG) 8.13 on page 587
⇐ "SetCategory" (SETCAT) 3.13 on page 91

Exports:

coerce	hash	latex	?*?	?**?
?=?	?^?	?~=?		

These are directly exported but not implemented:

```
?*? : (%,%) -> %
```

These are implemented by this category:

```
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p91) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category SGROUP SemiGroup*⟩≡

```
)abbrev category SGROUP SemiGroup
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ the class of all multiplicative semigroups, i.e. a set
++ with an associative operation \spadop{*}.
++
++ Axioms:
++    \spad{associative("*":(%,%)->%)}\tab{30}\spad{ (x*y)*z = x*(y*z)}
++
++ Conditional attributes:
++    \spad{commutative("*":(%,%)->%)}\tab{30}\spad{ x*y = y*x }
SemiGroup(): Category == SetCategory with
      "*": (%,%) -> %                  ++ x*y returns the product of x and y.
      "**": (%,PositiveInteger) -> %   ++ x**n returns the repeated product
                                       ++ of x n times, i.e. exponentiation.
      "^": (%,PositiveInteger) -> %    ++ x^n returns the repeated product
                                       ++ of x n times, i.e. exponentiation.
    add
      import RepeatedSquaring(%)
      x:% ** n:PositiveInteger == expt(x,n)
      _^(x:%, n:PositiveInteger):% == x ** n
```

⟨*SGROUP.dotabb*⟩≡

```
"SGROUP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SGROUP"];
"SGROUP" -> "SETCAT"
```

⟨*SGROUP.dotfull*⟩≡
```
"SemiGroup()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SGROUP"];
"SemiGroup()" -> "SetCategory()"
"SemiGroup()" -> "RepeatedSquaring(a:SemiGroup)"
```

⟨*SGROUP.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SetCategory()"
"SemiGroup()" -> "RepeatedSquaring(a:SemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedSquaring(a:SemiGroup)" [color="#00EE00"];
"RepeatedSquaring(a:SemiGroup)" -> "RepeatedSquaring(a:SetCategory)"

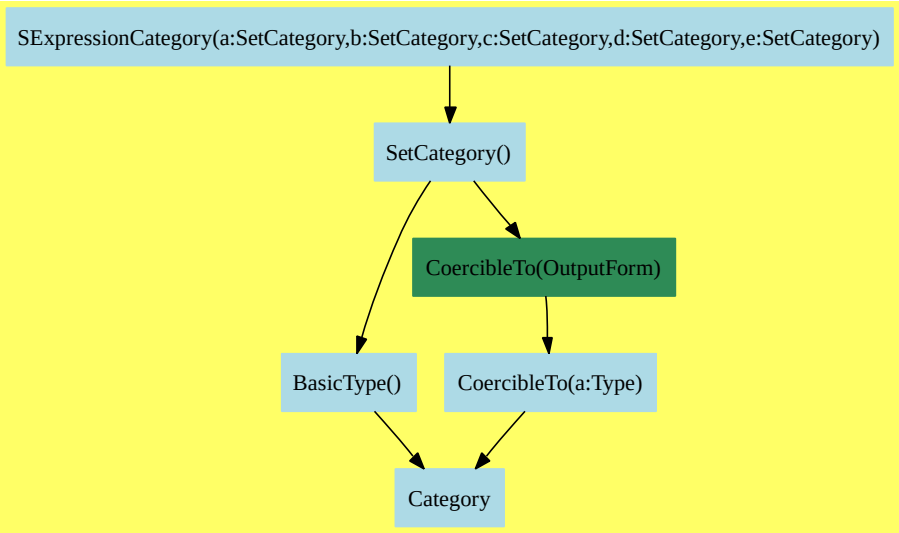
"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

4.25 SExpressionCategory (SEXCAT)



See:

⇐ "SetCategory" (SETCAT) 3.13 on page 91

Exports:

atom?	car	cdr	coerce	convert
destruct	eq	expr	float	float?
hash	integer	integer?	latex	list?
null?	pair?	string	string?	symbol
symbol?	#?	?=?	?~=?	?.?

These are directly exported but not implemented:

```
atom? : % -> Boolean
car : % -> %
cdr : % -> %
convert : Expr -> %
convert : Flt -> %
convert : Int -> %
convert : Sym -> %
convert : Str -> %
convert : List % -> %
destruct : % -> List %
eq : (%,%) -> Boolean
expr : % -> Expr
float : % -> Flt
float? : % -> Boolean
integer : % -> Int
integer? : % -> Boolean
```

```
list? : % -> Boolean
null? : % -> Boolean
pair? : % -> Boolean
string : % -> Str
string? : % -> Boolean
symbol : % -> Sym
symbol? : % -> Boolean
#? : % -> Integer
?.? : (%,List Integer) -> %
?.? : (%,Integer) -> %
```

These exports come from (p91) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category SEXCAT SExpressionCategory*⟩≡

```
)abbrev category SEXCAT SExpressionCategory
++ Category for Lisp values
++ Author: S.M.Watt
++ Date Created: July 1987
++ Date Last Modified: 23 May 1991
++ Description:
++  This category allows the manipulation of Lisp values while keeping
++  the grunge fairly localized.
--  The coerce to expression lets the
--  values be displayed in the usual parenthesized way (displaying
--  them as type Expression can cause the formatter to die, since
--  certain magic cookies are in unexpected places).
--  SMW July 87
SExpressionCategory(Str, Sym, Int, Flt, Expr): Category == Decl where
    Str, Sym, Int, Flt, Expr: SetCategory

    Decl ==> SetCategory with
        eq:        (%,%) -> Boolean
          ++ eq(s, t) is true if EQ(s,t) is true in Lisp.
        null?:     % -> Boolean
          ++ null?(s) is true if s is the S-expression ().
        atom?:     % -> Boolean
          ++ atom?(s) is true if s is a Lisp atom.
        pair?:     % -> Boolean
          ++ pair?(s) is true if s has is a non-null Lisp list.
        list?:     % -> Boolean
          ++ list?(s) is true if s is a Lisp list, possibly ().
        string?:   % -> Boolean
```

```
    ++ string?(s) is true if s is an atom and belong to Str.
  symbol?:   % -> Boolean
    ++ symbol?(s) is true if s is an atom and belong to Sym.
  integer?:  % -> Boolean
    ++ integer?(s) is true if s is an atom and belong to Int.
  float?:    % -> Boolean
    ++ float?(s) is true if s is an atom and belong to Flt.
  destruct:  % -> List %
    ++ destruct((a1,...,an)) returns the list [a1,...,an].
  string:    % -> Str
    ++ string(s) returns s as an element of Str.
    ++ Error: if s is not an atom that also belongs to Str.
  symbol:    % -> Sym
    ++ symbol(s) returns s as an element of Sym.
    ++ Error: if s is not an atom that also belongs to Sym.
  integer:   % -> Int
    ++ integer(s) returns s as an element of Int.
    ++ Error: if s is not an atom that also belongs to Int.
  float:     % -> Flt
    ++ float(s) returns s as an element of Flt;
    ++ Error: if s is not an atom that also belongs to Flt.
  expr:      % -> Expr
    ++ expr(s) returns s as an element of Expr;
    ++ Error: if s is not an atom that also belongs to Expr.
  convert:   List % -> %
    ++ convert([a1,...,an]) returns an S-expression \spad{(a1,...,an)}.
  convert:   Str    -> %
    ++ convert(x) returns the Lisp atom x;
  convert:   Sym    -> %
    ++ convert(x) returns the Lisp atom x.
  convert:   Int    -> %
    ++ convert(x) returns the Lisp atom x.
  convert:   Flt    -> %
    ++ convert(x) returns the Lisp atom x.
  convert:   Expr   -> %
    ++ convert(x) returns the Lisp atom x.
  car:       % -> %
    ++ car((a1,...,an)) returns a1.
  cdr:       % -> %
    ++ cdr((a1,...,an)) returns \spad{(a2,...,an)}.
  "#":       % -> Integer
    ++ #((a1,...,an)) returns n.
  elt:       (%, Integer)      -> %
    ++ elt((a1,...,an), i) returns \spad{ai}.
  elt:       (%, List Integer) -> %
    ++ elt((a1,...,an), [i1,...,im]) returns \spad{(a_i1,...,a_im)}.
```

⟨*SEXCAT.dotabb*⟩≡
```
"SEXCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEXCAT"];
"SEXCAT" -> "SETCAT"
```

⟨*SEXCAT.dotfull*⟩≡
```
"SExpressionCategory(a:SetCategory,b:SetCategory,c:SetCategory,d:SetCategory,e:Se
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SEXCAT"];
"SExpressionCategory(a:SetCategory,b:SetCategory,c:SetCategory,d:SetCategory,e:Se
   "SetCategory()"
```

⟨*SEXCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"SExpressionCategory(a:SetCategory,b:SetCategory,c:SetCategory,d:SetCategory,e:Se
 [color=lightblue];
"SExpressionCategory(a:SetCategory,b:SetCategory,c:SetCategory,d:SetCategory,e:Se
   "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

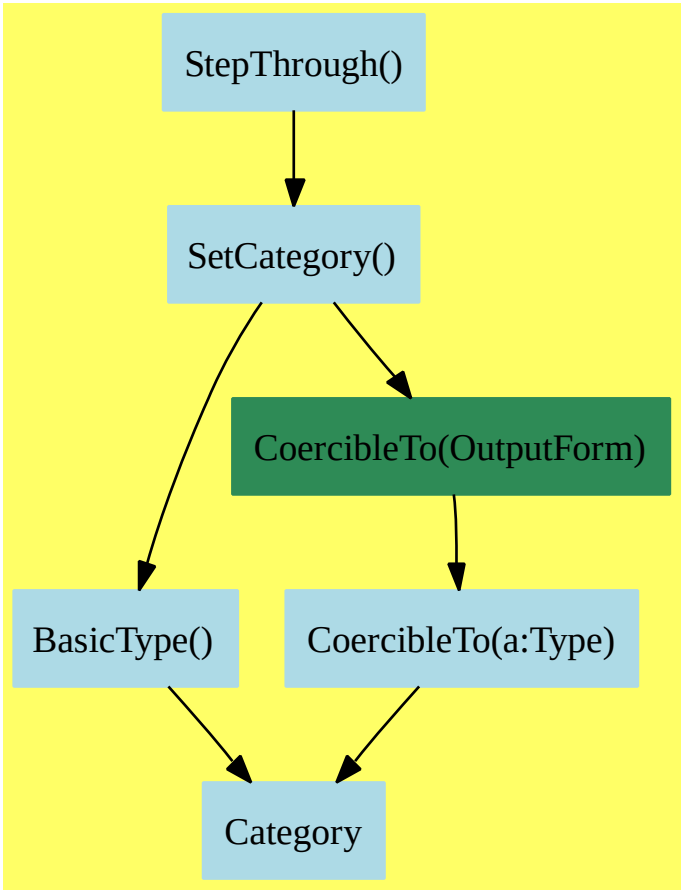
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.26 StepThrough (STEP)



See:

⇒ "FiniteFieldCategory" (FFIELDC) 18.3 on page 1244
⇒ "IntegerNumberSystem" (INS) 16.2 on page 1011
⇒ "QuotientFieldCategory" (QFCAT) 17.6 on page 1121
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.12 on page 1204
⇐ "SetCategory" (SETCAT) 3.13 on page 91

Exports:

coerce	hash	init	latex	?=?
?~=?	nextItem			

These are directly exported but not implemented:

```
init : () -> %
nextItem : % -> Union(%,"failed")
```

These exports come from (p91) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category STEP StepThrough*⟩≡

```
)abbrev category STEP StepThrough
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A class of objects which can be 'stepped through'.
++ Repeated applications of \spadfun{nextItem} is guaranteed never to
++ return duplicate items and only return "failed" after exhausting
++ all elements of the domain.
++ This assumes that the sequence starts with \spad{init()}.
++ For infinite domains, repeated application
++ of \spadfun{nextItem} is not required to reach all possible domain elements
++ starting from any initial element.
++
++ Conditional attributes:
++   infinite\tab{15}repeated \spad{nextItem}'s are never "failed".
StepThrough(): Category == SetCategory with
      init: constant -> %
        ++ init() chooses an initial object for stepping.
      nextItem: % -> Union(%,"failed")
        ++ nextItem(x) returns the next item, or "failed"
        ++ if domain is exhausted.
```

⟨*STEP.dotabb*⟩≡

```
"STEP" [color=lightblue,href="bookvol10.2.pdf#nameddest=STEP"];
"STEP" -> "SETCAT"
```

⟨*STEP.dotfull*⟩≡
```
"StepThrough()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=STEP"];
"StepThrough()" -> "SetCategory()"
```

⟨*STEP.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"StepThrough()" [color=lightblue];
"StepThrough()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

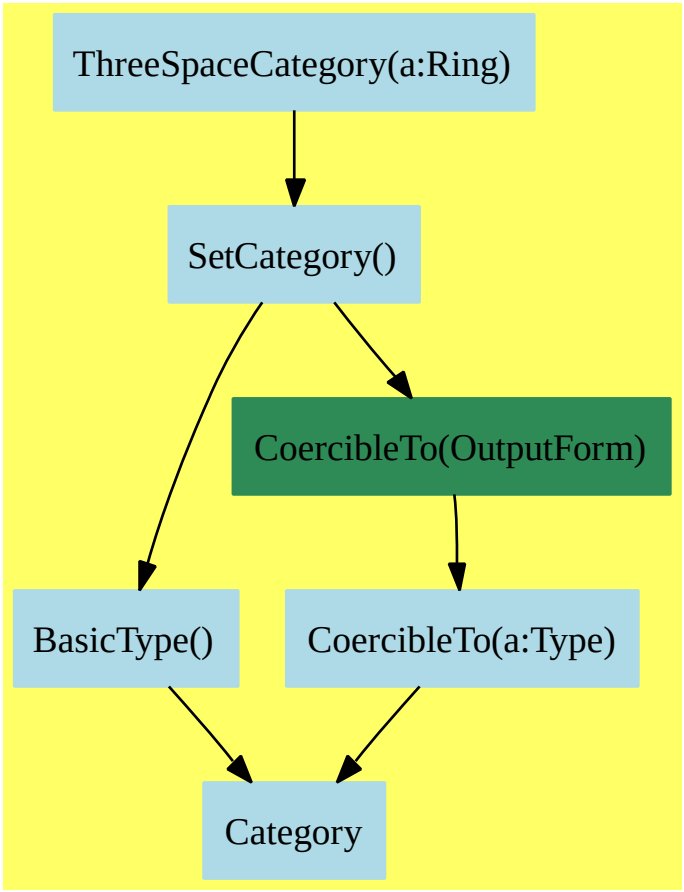
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

4.27 ThreeSpaceCategory (SPACEC)



See:

⇐ "SetCategory" (SETCAT) 3.13 on page 91

Exports:

check	closedCurve	closedCurve?	coerce
components	composite	composites	copy
create3Space	curve	curve?	enterPointData
hash	latex	lllip	lllp
llprop	lp	lprop	merge
mesh	mesh?	modifyPointData	numberOfComponents
numberOfComposites	objects	point	point?
polygon	polygon?	subspace	?=?
?~=?			

These are directly exported but not implemented:

```
check : % -> %
closedCurve : (%,List List R) -> %
closedCurve : (%,List Point R) -> %
closedCurve : List Point R -> %
closedCurve : % -> List Point R
closedCurve? : % -> Boolean
coerce : % -> OutputForm
components : % -> List %
composite : List % -> %
composites : % -> List %
copy : % -> %
create3Space : () -> %
create3Space : SubSpace(3,R) -> %
curve : (%,List List R) -> %
curve : (%,List Point R) -> %
curve : List Point R -> %
curve : % -> List Point R
curve? : % -> Boolean
enterPointData : (%,List Point R) -> NonNegativeInteger
lllip : % -> List List List NonNegativeInteger
lllp : % -> List List List Point R
llprop : % -> List List SubSpaceComponentProperty
lp : % -> List Point R
lprop : % -> List SubSpaceComponentProperty
merge : List % -> %
merge : (%,%) -> %
mesh : % -> List List Point R
mesh : List List Point R -> %
mesh : (List List Point R,Boolean,Boolean) -> %
mesh : (%,List List List R,Boolean,Boolean) -> %
mesh : (%,List List Point R,Boolean,Boolean) -> %
mesh : (%,List List List R,
        List SubSpaceComponentProperty,
        SubSpaceComponentProperty) -> %
mesh : (%,List List Point R,
        List SubSpaceComponentProperty,
        SubSpaceComponentProperty) -> %
mesh? : % -> Boolean
modifyPointData : (%,NonNegativeInteger,Point R) -> %
numberOfComponents : % -> NonNegativeInteger
numberOfComposites : % -> NonNegativeInteger
objects : % ->
  Record(points: NonNegativeInteger,
         curves: NonNegativeInteger,
         polygons: NonNegativeInteger,
         constructs: NonNegativeInteger)
point : (%,Point R) -> %
point : (%,List R) -> %
point : (%,NonNegativeInteger) -> %
point : Point R -> %
```

```
point : % -> Point R
point? : % -> Boolean
polygon : (%,List Point R) -> %
polygon : (%,List List R) -> %
polygon : List Point R -> %
polygon : % -> List Point R
polygon? : % -> Boolean
subspace : % -> SubSpace(3,R)
```

These exports come from (p91) SetCategory():

```
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category SPACEC ThreeSpaceCategory*⟩≡

```
)abbrev category SPACEC ThreeSpaceCategory
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Operations: create3Space, numberOfComponents, numberOfComposites,
++ merge, composite, components, copy, enterPointData, modifyPointData,
++ point, point?, curve, curve?, closedCurve, closedCurve?, polygon,
++ polygon? mesh, mesh?, lp, lllip, lllp, llprop, lprop, objects,
++ check, subspace, coerce
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description: The category ThreeSpaceCategory is used for creating
++ three dimensional objects using functions for defining points, curves,
++ polygons, constructs and the subspaces containing them.

ThreeSpaceCategory(R:Ring): Exports == Implementation where
 I    ==> Integer
 PI   ==> PositiveInteger
 NNI  ==> NonNegativeInteger
 L    ==> List
 B    ==> Boolean
 O    ==> OutputForm
 SUBSPACE ==> SubSpace(3,R)
 POINT   ==> Point(R)
 PROP    ==> SubSpaceComponentProperty()
 REP3D ==> Record(lp:L POINT,lliPt:L L L NNI, llProp:L L PROP, lProp:L PROP)
 OBJ3D   ==> Record(points:NNI, curves:NNI, polygons:NNI, constructs:NNI)
```

```
Exports ==> Category
Implementation ==>
 SetCategory with
   create3Space : () -> %
     ++ create3Space() creates a \spadtype{ThreeSpace} object capable of
     ++ holding point, curve, mesh components and any combination.
   create3Space : SUBSPACE -> %
     ++ create3Space(s) creates a \spadtype{ThreeSpace} object containing
     ++ objects pre-defined within some \spadtype{SubSpace} s.
   numberOfComponents : % -> NNI
     ++ numberOfComponents(s) returns the number of distinct
     ++ object components in the indicated \spadtype{ThreeSpace}, s, such
     ++ as points, curves, polygons, and constructs.
   numberOfComposites : % -> NNI
     ++ numberOfComposites(s) returns the number of supercomponents,
     ++ or composites, in the \spadtype{ThreeSpace}, s; Composites are
     ++ arbitrary groupings of otherwise distinct and unrelated components;
     ++ A \spadtype{ThreeSpace} need not have any composites defined at all
     ++ and, outside of the requirement that no component can belong
     ++ to more than one composite at a time, the definition and
     ++ interpretation of composites are unrestricted.
   merge : L % -> %
     ++ merge([s1,s2,...,sn]) will create a new \spadtype{ThreeSpace} that
     ++ has the components of all the ones in the list; Groupings of
     ++ components into composites are maintained.
   merge : (%,%) -> %
     ++ merge(s1,s2) will create a new \spadtype{ThreeSpace} that has the
     ++ components of \spad{s1} and \spad{s2}; Groupings of components
     ++ into composites are maintained.
   composite : L % -> %
     ++ composite([s1,s2,...,sn]) will create a new \spadtype{ThreeSpace}
     ++ that is a union of all the components from each
     ++ \spadtype{ThreeSpace} in the parameter list, grouped as a composite.
   components : % -> L %
     ++ components(s) takes the \spadtype{ThreeSpace} s, and creates a list
     ++ containing a unique \spadtype{ThreeSpace} for each single component
     ++ of s. If s has no components defined, the list returned is empty.
   composites : % -> L %
     ++ composites(s) takes the \spadtype{ThreeSpace} s, and creates a list
     ++ containing a unique \spadtype{ThreeSpace} for each single composite
     ++ of s. If s has no composites defined (composites need to be
     ++ explicitly created), the list returned is empty. Note that not all
     ++ the components need to be part of a composite.
   copy : % -> %
     ++ copy(s) returns a new \spadtype{ThreeSpace} that is an exact copy
     ++ of s.
```

```
enterPointData  : (%,L POINT) -> NNI
  ++ enterPointData(s,[p0,p1,...,pn]) adds a list of points from p0
  ++ through pn to the \spadtype{ThreeSpace}, s, and returns the index,
  ++ to the starting point of the list;
modifyPointData : (%,NNI,POINT) -> %
  ++ modifyPointData(s,i,p) changes the point at the indexed
  ++ location i in the \spadtype{ThreeSpace}, s, to that of point p.
  ++ This is useful for making changes to a point which has been
  ++ transformed.

-- 3D primitives
point        : (%,POINT) -> %
  ++ point(s,p) adds a point component defined by the point, p,
  ++ specified as a list from \spad{List(R)}, to the
  ++ \spadtype{ThreeSpace}, s, where R is the \spadtype{Ring} over
  ++ which the point is defined.
point        : (%,L R) -> %
  ++ point(s,[x,y,z]) adds a point component defined by a list of
  ++ elements which are from the \spad{PointDomain(R)} to the
  ++ \spadtype{ThreeSpace}, s, where R is the \spadtype{Ring} over
  ++ which the point elements are defined.
point        : (%,NNI) -> %
  ++ point(s,i) adds a point component which is placed into a component
  ++ list of the \spadtype{ThreeSpace}, s, at the index given by i.
point        : POINT -> %
  ++ point(p) returns a \spadtype{ThreeSpace} object which is composed
  ++ of one component, the point p.
point        : % -> POINT
  ++ point(s) checks to see if the \spadtype{ThreeSpace}, s, is
  ++ composed of only a single point and if so, returns the point.
  ++ An error is signaled otherwise.
point?       : % -> B
  ++ point?(s) queries whether the \spadtype{ThreeSpace}, s, is
  ++ composed of a single component which is a point and returns the
  ++ boolean result.
curve        : (%,L POINT) -> %
  ++ curve(s,[p0,p1,...,pn]) adds a space curve component defined by a
  ++ list of points \spad{p0} through \spad{pn}, to the
  ++ \spadtype{ThreeSpace} s.
curve        : (%,L L R) -> %
  ++ curve(s,[[p0],[p1],...,[pn]]) adds a space curve which is a list of
  ++ points p0 through pn defined by lists of elements from the domain
  ++ \spad{PointDomain(m,R)}, where R is the \spadtype{Ring} over which
  ++ the point elements are defined and m is the dimension of the
  ++ points, to the \spadtype{ThreeSpace} s.
curve         : L POINT -> %
```

```
    ++ curve([p0,p1,p2,...,pn]) creates a space curve defined
    ++ by the list of points \spad{p0} through \spad{pn}, and returns the
    ++ \spadtype{ThreeSpace} whose component is the curve.
  curve         : % -> L POINT
    ++ curve(s) checks to see if the \spadtype{ThreeSpace}, s, is
    ++ composed of a single curve defined by a list of points and if so,
    ++ returns the curve, i.e., list of points. An error is signaled
    ++ otherwise.
  curve?        : % -> B
    ++ curve?(s) queries whether the \spadtype{ThreeSpace}, s, is a curve,
    ++ i.e., has one component, a list of list of points, and returns
    ++ true if it is, or false otherwise.
  closedCurve   : (%,L POINT) -> %
    ++ closedCurve(s,[p0,p1,...,pn,p0]) adds a closed curve component
    ++ which is a list of points defined by the first element p0 through
    ++ the last element pn and back to the first element p0 again, to the
    ++ \spadtype{ThreeSpace} s.
  closedCurve   : (%,L L R) -> %
    ++ closedCurve(s,[[lr0],[lr1],...,[lrn],[lr0]]) adds a closed curve
    ++ component defined by a list of points \spad{lr0} through
    ++ \spad{lrn}, which are lists of elements from the domain
    ++ \spad{PointDomain(m,R)}, where R is the \spadtype{Ring} over which
    ++ the point elements are defined and m is the dimension of the
    ++ points, in which the last element of the list of points contains
    ++ a copy of the first element list, lr0.
    ++ The closed curve is added to the \spadtype{ThreeSpace}, s.
  closedCurve         : L POINT -> %
    ++ closedCurve(lp) sets a list of points defined by the first element
    ++ of lp through the last element of lp and back to the first elelment
    ++ again and returns a \spadtype{ThreeSpace} whose component is the
    ++ closed curve defined by lp.
  closedCurve         : % -> L POINT
    ++ closedCurve(s) checks to see if the \spadtype{ThreeSpace}, s, is
    ++ composed of a single closed curve component defined by a list of
    ++ points in which the first point is also the last point, all of
    ++ which are from the domain \spad{PointDomain(m,R)} and if so,
    ++ returns the list of points.  An error is signaled otherwise.
  closedCurve?        : % -> B
    ++ closedCurve?(s) returns true if the \spadtype{ThreeSpace} s
    ++ contains a single closed curve component, i.e., the first element
    ++ of the curve is also the last element, or false otherwise.
  polygon       : (%,L POINT) -> %
    ++ polygon(s,[p0,p1,...,pn]) adds a polygon component defined by a
    ++ list of points, p0 throught pn, to the \spadtype{ThreeSpace} s.
  polygon       : (%,L L R) -> %
    ++ polygon(s,[[r0],[r1],...,[rn]]) adds a polygon component defined
```

```
  ++ by a list of points \spad{r0} through \spad{rn}, which are lists of
  ++ elements from the domain \spad{PointDomain(m,R)} to the
  ++ \spadtype{ThreeSpace} s, where m is the dimension of the points
  ++ and R is the \spadtype{Ring} over which the points are defined.
polygon         : L POINT -> %
  ++ polygon([p0,p1,...,pn]) creates a polygon defined by a list of
  ++ points, p0 through pn, and returns a \spadtype{ThreeSpace} whose
  ++ component is the polygon.
polygon         : % -> L POINT
  ++ polygon(s) checks to see if the \spadtype{ThreeSpace}, s, is
  ++ composed of a single polygon component defined by a list of
  ++ points, and if so, returns the list of points;  An error is
  ++ signaled otherwise.
polygon?        : % -> B
  ++ polygon?(s) returns true if the \spadtype{ThreeSpace} s contains
  ++ a single polygon component, or false otherwise.
mesh         : (%,L L POINT,L PROP,PROP) -> %
  ++ mesh(s,[[p0],[p1],...,[pn]],[props],prop) adds a surface component,
  ++ defined over a list curves which contains lists of points, to the
  ++ \spadtype{ThreeSpace} s; props is a list which contains the
  ++ subspace component properties for each surface parameter, and
  ++ prop is the subspace component property by which the points are
  ++ defined.
mesh         : (%,L L L R,L PROP,PROP) -> %
  ++ mesh(s,[ [[r10]...,[r1m]],[[r20]...,[r2m]],...,[[rn0]...,[rnm]] ],
  ++        [props], prop)
  ++ adds a surface component to the \spadtype{ThreeSpace} s, which is
  ++ defined over a rectangular domain of size WxH where W is the number
  ++ of lists of points from the domain \spad{PointDomain(R)} and H is
  ++ the number of elements in each of those lists; lprops is the list
  ++ of the subspace component properties for each curve list, and
  ++ prop is the subspace component property by which the points are
  ++ defined.
mesh         : (%,L L POINT,B,B) -> %
  ++ mesh(s,[[p0],[p1],...,[pn]], close1, close2) adds a surface
  ++ component to the \spadtype{ThreeSpace}, which is defined over a
  ++ list of curves, in which each of these curves is a list of points.
  ++ The boolean arguments close1 and close2 indicate how the surface
  ++ is to be closed. Argument close1 equal true
  ++ means that each individual list (a curve) is to be closed, i.e. the
  ++ last point of the list is to be connected to the first point.
  ++ Argument close2 equal true
  ++ means that the boundary at one end of the surface is to be
  ++ connected to the boundary at the other end, i.e. the boundaries
  ++ are defined as the first list of points (curve) and
  ++ the last list of points (curve).
```

```
mesh         : (%,L L L R,B,B) -> %
  ++ mesh(s,[ [[r10]...,[r1m]],[[r20]...,[r2m]],...,[[rn0]...,[rnm]] ],
  ++      close1, close2)
  ++ adds a surface component to the \spadtype{ThreeSpace} s, which is
  ++ defined over a rectangular domain of size WxH where W is the number
  ++ of lists of points from the domain \spad{PointDomain(R)} and H is
  ++ the number of elements in each of those lists; the booleans close1
  ++ and close2 indicate how the surface is to be closed: if close1 is
  ++ true this means that each individual list (a curve) is to be
  ++ closed (i.e.,
  ++ the last point of the list is to be connected to the first point);
  ++ if close2 is true, this means that the boundary at one end of the
  ++ surface is to be connected to the boundary at the other end
  ++ (the boundaries are defined as the first list of points (curve)
  ++ and the last list of points (curve)).
mesh         : L L POINT -> %
  ++ mesh([[p0],[p1],...,[pn]]) creates a surface defined by a list of
  ++ curves which are lists, p0 through pn, of points, and returns a
  ++ \spadtype{ThreeSpace} whose component is the surface.
mesh         : (L L POINT,B,B) -> %
  ++ mesh([[p0],[p1],...,[pn]], close1, close2) creates a surface
  ++ defined over a list of curves, p0 through pn, which are lists of
  ++ points; the booleans close1 and close2 indicate how the surface is
  ++ to be closed: close1 set to true means that each individual list
  ++ (a curve) is to be closed (that is, the last point of the list is
  ++ to be connected to the first point); close2 set to true means
  ++ that the boundary at one end of the surface is to be connected to
  ++ the boundary at the other end (the boundaries are defined as the
  ++ first list of points (curve) and the last list of points (curve));
  ++ the \spadtype{ThreeSpace} containing this surface is returned.
mesh         : % -> L L POINT
  ++ mesh(s) checks to see if the \spadtype{ThreeSpace}, s, is
  ++ composed of a single surface component defined by a list curves
  ++ which contain lists of points, and if so, returns the list of
  ++ lists of points;  An error is signaled otherwise.
mesh?        : % -> B
  ++ mesh?(s) returns true if the \spadtype{ThreeSpace} s is composed
  ++ of one component, a mesh comprising a list of curves which are lists
  ++ of points, or returns false if otherwise
lp           : % -> L POINT
  ++ lp(s) returns the list of points component which the
  ++ \spadtype{ThreeSpace}, s, contains; these points are used by
  ++ reference, i.e., the component holds indices referring to the
  ++ points rather than the points themselves. This allows for sharing
  ++ of the points.
lllip        : % -> L L L NNI
```

```
      ++ lllip(s) checks to see if the \spadtype{ThreeSpace}, s, is
      ++ composed of a list of components, which are lists of curves,
      ++ which are lists of indices to points, and if so, returns the list
      ++ of lists of lists;  An error is signaled otherwise.
    lllp          : % -> L L L POINT   -- used by view3D
      ++ lllp(s) checks to see if the \spadtype{ThreeSpace}, s, is
      ++ composed of a list of components, which are lists of curves,
      ++ which are lists of points, and if so, returns the list of
      ++ lists of lists;  An error is signaled otherwise.
    llprop        : % -> L L PROP      -- used by view3D
      ++ llprop(s) checks to see if the \spadtype{ThreeSpace}, s, is
      ++ composed of a list of curves which are lists of the
      ++ subspace component properties of the curves, and if so, returns the
      ++ list of lists;  An error is signaled otherwise.
    lprop         : % -> L PROP        -- used by view3D
      ++ lprop(s) checks to see if the \spadtype{ThreeSpace}, s, is
      ++ composed of a list of subspace component properties, and if so,
      ++ returns the list;  An error is signaled otherwise.
    objects       : % -> OBJ3D
      ++ objects(s) returns the \spadtype{ThreeSpace}, s, in the form of a
      ++ 3D object record containing information on the number of points,
      ++ curves, polygons and constructs comprising the
      ++ \spadtype{ThreeSpace}..
    check         : % -> %             -- used by mesh
      ++ check(s) returns lllpt, list of lists of lists of point information
      ++ about the \spadtype{ThreeSpace} s.
    subspace      : % -> SUBSPACE
      ++ subspace(s) returns the \spadtype{SubSpace} which holds all the
      ++ point information in the \spadtype{ThreeSpace}, s.
    coerce        : % -> O
      ++ coerce(s) returns the \spadtype{ThreeSpace} s to Output format.
```

⟨*SPACEC.dotabb*⟩≡

```
"SPACEC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SPACEC"];
"SPACEC" -> "SETCAT"
```

⟨*SPACEC.dotfull*⟩≡

```
"ThreeSpaceCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SPACEC"];
"ThreeSpaceCategory(a:Ring)" -> "SetCategory()"
```

⟨*SPACEC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ThreeSpaceCategory(a:Ring)" [color=lightblue];
"ThreeSpaceCategory(a:Ring)" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

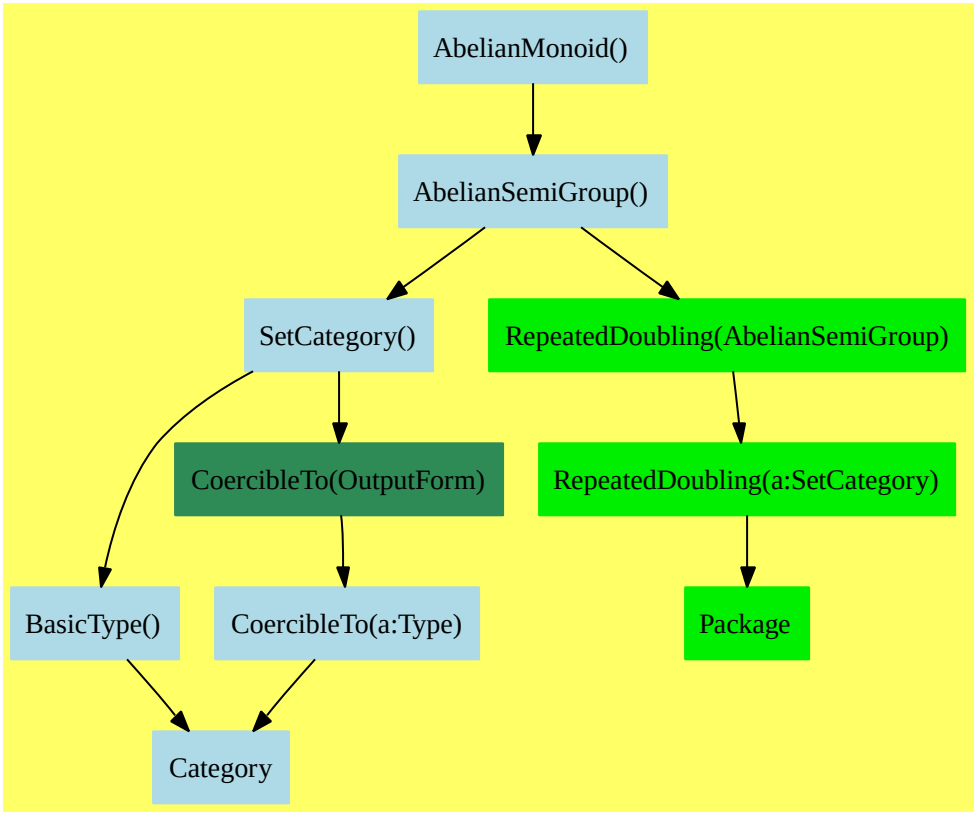
"Category" [color=lightblue];

}
```

Chapter 5

Category Layer 4

5.1 AbelianMonoid (ABELMON)



See:

⇒ “CancellationAbelianMonoid” (CABMON) 6.2 on page 289
⇒ “FunctionSpace” (FS) 17.5 on page 1095
⇒ “OrderedAbelianMonoid” (OAMON) 7.9 on page 461
⇒ “OrderedAbelianSemiGroup” (OASGP) 6.8 on page 364
⇐ “AbelianSemiGroup” (ABELSG) 4.1 on page 99

Exports:

0	coerce	hash	latex	sample
zero?	?*?	?+?	?=?	?~=?

These are directly exported but not implemented:

```
0 : () -> %
```

These are implemented by this category:

```
sample : () -> %
zero? : % -> Boolean
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
```

These exports come from (p99) AbelianSemiGroup():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?+? : (%,%) -> %
```

⟨*category ABELMON AbelianMonoid*⟩≡

```
)abbrev category ABELMON AbelianMonoid
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The class of multiplicative monoids, i.e. semigroups with an
++ additive identity element.
++
++ Axioms:
++   \spad{leftIdentity("+":(%,%)->%,0)}\tab{30}\spad{ 0+x=x }
++   \spad{rightIdentity("+":(%,%)->%,0)}\tab{30}\spad{ x+0=x }
```

```
-- following domain must be compiled with subsumption disabled
-- define SourceLevelSubset to be EQUAL
AbelianMonoid(): Category == AbelianSemiGroup with
      0: constant -> %
        ++ 0 is the additive identity element.
      sample: constant -> %
        ++ sample yields a value of type %
      zero?: % -> Boolean
        ++ zero?(x) tests if x is equal to 0.
      "*": (NonNegativeInteger,%) -> %
        ++ n * x is left-multiplication by a non negative integer
    add
      import RepeatedDoubling(%)
      zero? x == x = 0
      n:PositiveInteger * x:% == (n::NonNegativeInteger) * x
      sample() == 0
      if not (% has Ring) then
        n:NonNegativeInteger * x:% ==
          zero? n => 0
          double(n pretend PositiveInteger,x)
```

⟨*ABELMON.dotabb*⟩≡

```
"ABELMON"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ABELMON"];
"ABELMON" -> "ABELSG"
```

⟨*ABELMON.dotfull*⟩≡

```
"AbelianMonoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ABELMON"];
"AbelianMonoid()" -> "AbelianSemiGroup()"
```

⟨*ABELMON.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SetCategory()"
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

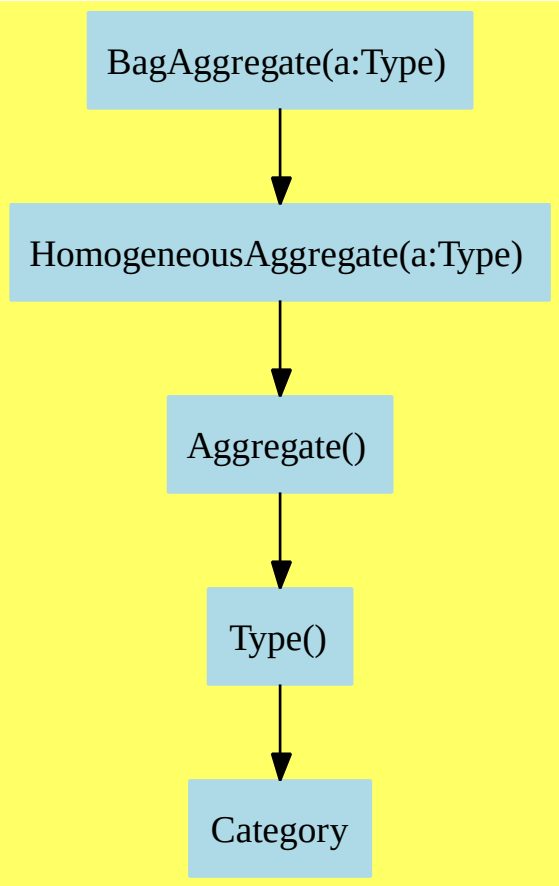
"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];
}
```

5.2 BagAggregate (BGAGG)



See:

⇒ “DictionaryOperations” (DIOPS) 6.3 on page 293
⇒ “PriorityQueueAggregate” (PRQAGG) 6.11 on page 386
⇒ “QueueAggregate” (QUAGG) 6.12 on page 390
⇒ “StackAggregate” (SKAGG) 6.14 on page 402
⇐ “HomogeneousAggregate” (HOAGG) 4.12 on page 139

Exports:

any?	bag	coerce	copy	count
empty	empty?	eq?	eval	every?
extract!	hash	insert!	inspect	latex
less?	map	map!	member?	members
more?	parts	sample	size?	#?
?=?	?~=?			

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
extract! : % -> S
insert! : (S,%) -> %
inspect : % -> S
```

These are implemented by this category:

```
bag : List S -> %
```

These exports come from (p139) HomogeneousAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
members : % -> List S if $ has finiteAggregate
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
```

```
parts : % -> List S if $ has finiteAggregate
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
\#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category BGAGG BagAggregate*⟩≡

```
)abbrev category BGAGG BagAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A bag aggregate is an aggregate for which one can insert and extract
++ objects, and where the order in which objects are inserted determines
++ the order of extraction.
++ Examples of bags are stacks, queues, and dequeues.
BagAggregate(S:Type): Category == HomogeneousAggregate S with
   shallowlyMutable
     ++ shallowlyMutable means that elements of bags may be
     ++ destructively changed.
   bag: List S -> %
     ++ bag([x,y,...,z]) creates a bag with elements x,y,...,z.
   extract_!: % -> S
     ++ extract!(u) destructively removes a (random) item from bag u.
   insert_!: (S,%) -> %
     ++ insert!(x,u) inserts item x into bag u.
   inspect: % -> S
     ++ inspect(u) returns an (random) element from a bag.
 add
   bag(l) ==
     x:=empty()
     for s in l repeat x:=insert_!(s,x)
     x
```

⟨*BGAGG.dotabb*⟩≡

```
"BGAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=BGAGG"];
"BGAGG" -> "HOAGG"
```

⟨*BGAGG.dotfull*⟩≡

```
"BagAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BGAGG"];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"BagAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=BGAGG"];
"BagAggregate(a:SetCategory)" -> "BagAggregate(a:Type)"
```

⟨*BGAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

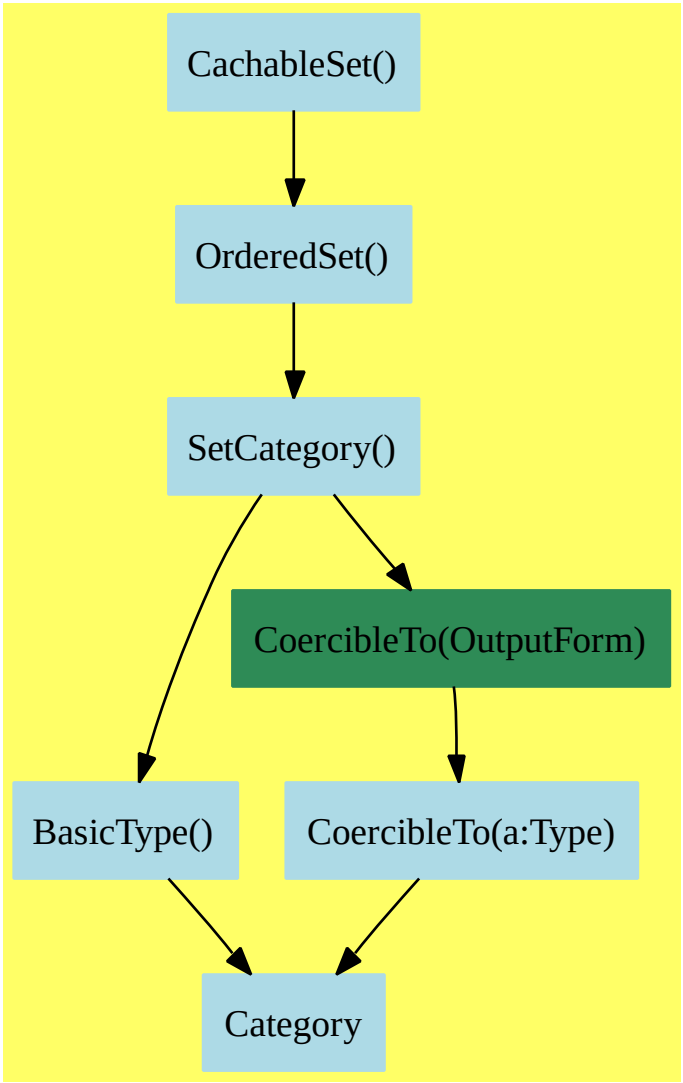
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

5.3 CachableSet (CACHSET)



See:

⇐ "OrderedSet" (ORDSET) 4.19 on page 168

Exports:

coerce	hash	latex	max	min
position	setPosition	?~=?	?<?	?<=?
?=?	?>?	?>=?		

These are directly exported but not implemented:

```
position : % -> NonNegativeInteger
setPosition : (%,NonNegativeInteger) -> Void
```

These exports come from (p168) OrderedSet():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?~=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

⟨*category CACHSET CachableSet*⟩≡

```
)abbrev category CACHSET CachableSet
++ Sets whose elements can cache an integer
++ Author: Manuel Bronstein
++ Date Created: 31 Oct 1988
++ Date Last Updated: 14 May 1991
++ Description:
++   A cachable set is a set whose elements keep an integer as part
++   of their structure.
CachableSet: Category == OrderedSet with
  position   : % -> NonNegativeInteger
    ++ position(x) returns the integer n associated to x.
  setPosition: (%, NonNegativeInteger) -> Void
    ++ setPosition(x, n) associates the integer n to x.
```

⟨*CACHSET.dotabb*⟩≡

```
"CACHSET"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CACHSET"];
"CACHSET" -> "ORDSET"
```

⟨*CACHSET.dotfull*⟩≡

```
"CachableSet()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CACHSET"];
"CachableSet()" -> "OrderedSet()"
```

⟨*CACHSET.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"CachableSet()" [color=lightblue];
"CachableSet()" -> "OrderedSet()"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

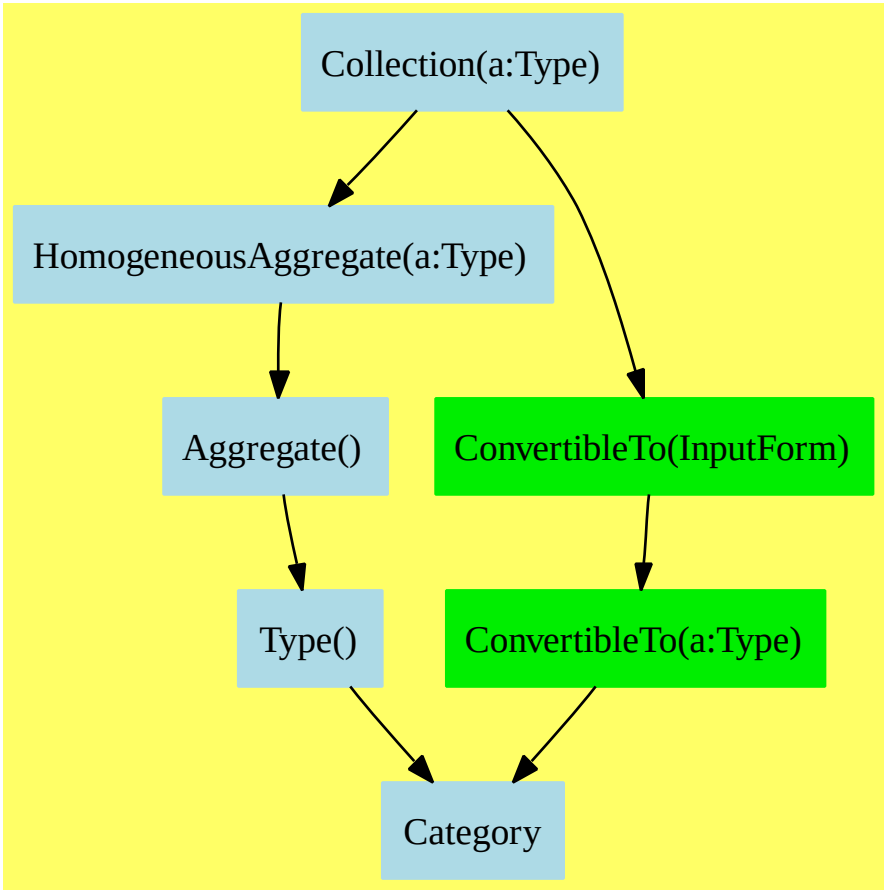
"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

5.4 Collection (CLAGG)



See:

⇒ “DictionaryOperations” (DIOPS) 6.3 on page 293
⇒ “LinearAggregate” (LNAGG) 6.6 on page 308
⇒ “PolynomialSetCategory” (PSETCAT) 6.10 on page 373
⇒ “SetAggregate” (SETAGG) 6.13 on page 395
⇐ “ConvertibleTo” (KONVERT) 2.8 on page 19
⇐ “HomogeneousAggregate” (HOAGG) 4.12 on page 139

Exports:

any?	coerce	construct	copy	convert
count	empty	empty?	eq?	eval
every?	find	hash	latex	less?
map	map!	member?	members	more?
parts	reduce	remove	removeDuplicates	sample
select	size?	#?	?=?	?~=?

Attributes exported:

- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are directly exported but not implemented:

```
construct: List S -> %
```

These are implemented by this category:

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
find : ((S -> Boolean),%) -> Union(S,"failed")
reduce : (((S,S) -> S),%,S,S) -> S
         if S has SETCAT and $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
removeDuplicates : % -> %
         if S has SETCAT and $ has finiteAggregate
select : ((S -> Boolean),%) -> % if $ has finiteAggregate
#? : % -> NonNegativeInteger if $ has finiteAggregate
```

These exports come from (p139) HomogeneousAggregate(S:Type):

```
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
            if S has SETCAT and $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
```

```
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p19) ConvertibleTo(S:Type):

```
convert : % -> InputForm if S has KONVERT INFORM
```

⟨*category CLAGG Collection*⟩≡

```
)abbrev category CLAGG Collection
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A collection is a homogeneous aggregate which can built from
++ list of members. The operation used to build the aggregate is
++ generically named \spadfun{construct}. However, each collection
++ provides its own special function with the same name as the
++ data type, except with an initial lower case letter, e.g.
++ \spadfun{list} for \spadtype{List},
++ \spadfun{flexibleArray} for \spadtype{FlexibleArray}, and so on.
Collection(S:Type): Category == HomogeneousAggregate(S) with
   construct: List S -> %
     ++ \axiom{construct(x,y,...,z)} returns the collection of elements
     ++ \axiom{x,y,...,z} ordered as given. Equivalently written as
     ++ \axiom{[x,y,...,z]$D}, where
     ++ D is the domain. D may be omitted for those of type List.
   find: (S->Boolean, %) -> Union(S, "failed")
     ++ find(p,u) returns the first x in u such that \axiom{p(x)} is true,
     ++ and "failed" otherwise.
```

```
    if % has finiteAggregate then
       reduce: ((S,S)->S,%) -> S
         ++ reduce(f,u) reduces the binary operation f across u. For example,
         ++ if u is \axiom{[x,y,...,z]} then \axiom{reduce(f,u)}
         ++ returns \axiom{f(..f(f(x,y),...),z)}.
         ++ Note: if u has one element x, \axiom{reduce(f,u)} returns x.
         ++ Error: if u is empty.
         ++
         ++C )clear all
         ++X reduce(+,[C[i]*x**i for i in 1..5])

       reduce: ((S,S)->S,%,S) -> S
         ++ reduce(f,u,x) reduces the binary operation f across u, where x is
         ++ the identity operation of f.
         ++ Same as \axiom{reduce(f,u)} if u has 2 or more elements.
         ++ Returns \axiom{f(x,y)} if u has one element y,
         ++ x if u is empty.
         ++ For example, \axiom{reduce(+,u,0)} returns the
         ++ sum of the elements of u.
       remove: (S->Boolean,%) -> %
         ++ remove(p,u) returns a copy of u removing all elements x such that
         ++ \axiom{p(x)} is true.
         ++ Note: \axiom{remove(p,u) == [x for x in u | not p(x)]}.
       select: (S->Boolean,%) -> %
         ++ select(p,u) returns a copy of u containing only those elements
         ++ such \axiom{p(x)} is true.
         ++ Note: \axiom{select(p,u) == [x for x in u | p(x)]}.
       if S has SetCategory then
         reduce: ((S,S)->S,%,S,S) -> S
           ++ reduce(f,u,x,z) reduces the binary operation f across u,
           ++ stopping when an "absorbing element" z is encountered.
           ++ As for \axiom{reduce(f,u,x)}, x is the identity operation of f.
           ++ Same as \axiom{reduce(f,u,x)} when u contains no element z.
           ++ Thus the third argument x is returned when u is empty.
         remove: (S,%) -> %
           ++ remove(x,u) returns a copy of u with all
           ++ elements \axiom{y = x} removed.
           ++ Note: \axiom{remove(y,c) == [x for x in c | x ^= y]}.
         removeDuplicates: % -> %
           ++ removeDuplicates(u) returns a copy of u with all duplicates
           ++ removed.
    if S has ConvertibleTo InputForm then ConvertibleTo InputForm
  add
    if % has finiteAggregate then
      #c                    == # parts c
      count(f:S -> Boolean, c:%) == _+/[1 for x in parts c | f x]
```

```
    any?(f, c)           == _or/[f x for x in parts c]
    every?(f, c)         == _and/[f x for x in parts c]
    find(f:S -> Boolean, c:%) == find(f, parts c)
    reduce(f:(S,S)->S, x:%) == reduce(f, parts x)
    reduce(f:(S,S)->S, x:%, s:S) == reduce(f, parts x, s)
    remove(f:S->Boolean, x:%) ==
      construct remove(f, parts x)
    select(f:S->Boolean, x:%) ==
      construct select(f, parts x)

    if S has SetCategory then
      remove(s:S, x:%) == remove(y +-> y = s, x)
      reduce(f:(S,S)->S, x:%, s1:S, s2:S) == reduce(f, parts x, s1, s2)
      removeDuplicates(x) == construct removeDuplicates parts x
```

⟨*CLAGG.dotabb*⟩≡

```
"CLAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=CLAGG"];
"CLAGG" -> "HOAGG"
```

⟨*CLAGG.dotfull*⟩≡

```
"Collection(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CLAGG"];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"Collection(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=CLAGG"];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet
 [color=seagreen,href="bookvol10.2.pdf#nameddest=CLAGG"];
"Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet
  -> "Collection(a:Type)"
```

⟨*CLAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"
"Collection(a:Type)" -> "ConvertibleTo(InputForm)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

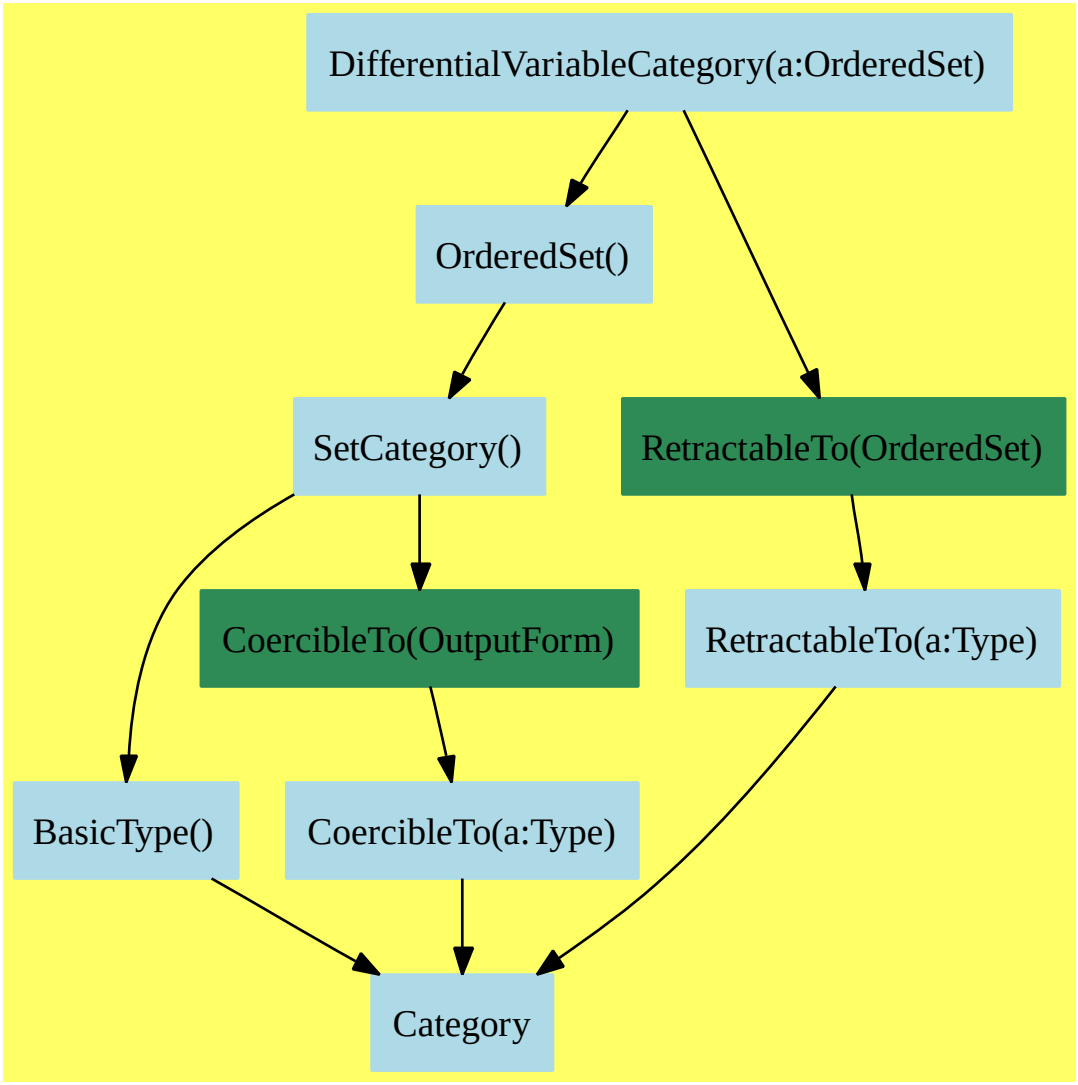
"Type()" [color=lightblue];
"Type()" -> "Category"

"ConvertibleTo(InputForm)" [color="#00EE00"];
"ConvertibleTo(InputForm)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(a:Type)" [color="#00EE00"];
"ConvertibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

5.5 DifferentialVariableCategory (DVARCAT)



See:

⇐ "OrderedSet" (ORDSET) 4.19 on page 168
⇐ "RetractableTo" (RETRACT) 2.18 on page 44

Exports:

coerce	differentiate	hash	latex	makeVariable
max	min	order	retract	retractIfCan
variable	weight	?~=?	?<?	?<=?
?=?	?>?	?>=?		

These are directly exported but not implemented:

```
makeVariable : (S,NonNegativeInteger) -> %
order : % -> NonNegativeInteger
variable : % -> S
```

These are implemented by this category:

```
coerce : S -> %
coerce : % -> OutputForm
differentiate : % -> %
differentiate : (%,NonNegativeInteger) -> %
retract : % -> S
retractIfCan : % -> Union(S,"failed")
weight : % -> NonNegativeInteger
?<? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
```

These exports come from (p168) OrderedSet():

```
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

These exports come from (p44) RetractableTo(S:OrderedSet):

⟨*category DVARCAT DifferentialVariableCategory*⟩≡

```
)abbrev category DVARCAT DifferentialVariableCategory
++ Author:  William Sit
++ Date Created: 19 July 1990
++ Date Last Updated: 13 September 1991
++ Basic Operations:
++ Related Constructors:DifferentialPolynomialCategory
++ See Also:OrderedDifferentialVariable,
++          SequentialDifferentialVariable,
++          DifferentialSparseMultivariatePolynomial.
++ AMS Classifications:12H05
++ Keywords: differential indeterminates, ranking, order, weight
++ References:Ritt, J.F. "Differential Algebra" (Dover, 1950).
++ Description:
++   \spadtype{DifferentialVariableCategory} constructs the
```

```
++   set of derivatives of a given set of
++   (ordinary) differential indeterminates.
++   If x,...,y is an ordered set of differential indeterminates,
++   and the prime notation is used for differentiation, then
++   the set of derivatives (including
++   zero-th order) of the differential indeterminates is
++   x,\spad{x'},\spad{x''},..., y,\spad{y'},\spad{y''},...
++   (Note: in the interpreter, the n-th derivative of y is displayed as
++   y with a subscript n.)  This set is
++   viewed as a set of algebraic indeterminates, totally ordered in a
++   way compatible with differentiation and the given order on the
++   differential indeterminates.  Such a total order is called a
++   ranking of the differential indeterminates.
++
++   A domain in this category is needed to construct a differential
++   polynomial domain.  Differential polynomials are ordered
++   by a ranking on the derivatives,  and by an order (extending the
++   ranking) on
++   on the set of differential monomials.  One may thus associate
++   a domain in this category with a ranking of the differential
++   indeterminates, just as one associates a domain in the category
++   \spadtype{OrderedAbelianMonoidSup} with an ordering of the set of
++   monomials in a set of algebraic indeterminates.  The ranking
++   is specified through the binary relation \spadfun{<}.
++   For example, one may define
++   one derivative to be less than another by lexicographically comparing
++   first the \spadfun{order}, then the given order of the differential
++   indeterminates appearing in the derivatives.  This is the default
++   implementation.
++
++   The notion of weight generalizes that of degree.  A
++   polynomial domain may be made into a graded ring
++   if a weight function is given on the set of indeterminates,
++   Very often, a grading is the first step in ordering the set of
++   monomials.  For differential polynomial domains, this
++   constructor provides a function \spadfun{weight}, which
++   allows the assignment of a non-negative number to each derivative of a
++   differential indeterminate.  For example, one may define
++   the weight of a derivative to be simply its \spadfun{order}
++   (this is the default assignment).
++   This weight function can then be extended to the set of
++   all differential polynomials, providing a graded ring
++   structure.
DifferentialVariableCategory(S:OrderedSet): Category ==
  Join(OrderedSet, RetractableTo S) with
    -- Examples:
```

```
      -- v:=makeVariable('s,5)
      makeVariable  : (S, NonNegativeInteger) -> $
         ++ makeVariable(s, n) returns the n-th derivative of a
         ++ differential indeterminate s as an algebraic indeterminate.
         -- Example: makeVariable('s, 5)
      order         : $ -> NonNegativeInteger
         ++ order(v) returns n if v is the n-th derivative of any
         ++ differential indeterminate.
         -- Example: order(v)
      variable      : $ -> S
         ++ variable(v) returns s if v is any derivative of the differential
         ++ indeterminate s.
         -- Example: variable(v)
            --  default implementation using above primitives --

      weight        : $ -> NonNegativeInteger
         ++ weight(v) returns the weight of the derivative v.
         -- Example: weight(v)
      differentiate : $ -> $
         ++ differentiate(v) returns the derivative of v.
         -- Example: differentiate(v)
      differentiate : ($, NonNegativeInteger) -> $
         ++ differentiate(v, n) returns the n-th derivative of v.
         -- Example: differentiate(v,2)
      coerce        : S  -> $
         ++ coerce(s) returns s, viewed as the zero-th order derivative of s.
         -- Example: coerce('s); differentiate(%,5)
   add
      import NumberFormats

      coerce (s:S):$ == makeVariable(s, 0)

      differentiate v     == differentiate(v, 1)

      differentiate(v, n) == makeVariable(variable v, n + order v)

      retractIfCan v == (zero?(order v) => variable v; "failed")

      v = u      == (variable v = variable u) and (order v = order u)

      coerce(v:$):OutputForm ==
        a := variable(v)::OutputForm
        zero?(nn := order v) => a
        sub(a, outputForm nn)

      retract v ==
```

```
      zero?(order v) => variable v
      error "Not retractable"

    v < u ==
      -- the ranking below is orderly, and is the default --
      order v = order u => variable v < variable u
      order v < order u

    weight v == order v
      --  the default weight is just the order
```

⟨*DVARCAT.dotabb*⟩≡

```
  "DVARCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=DVARCAT"];
  "DVARCAT" -> "ORDSET"
  "DVARCAT" -> "RETRACT"
```

⟨*DVARCAT.dotfull*⟩≡

```
  "DifferentialVariableCategory(a:OrderedSet)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=DVARCAT"];
  "DifferentialVariableCategory(a:OrderedSet)" -> "OrderedSet()"
  "DifferentialVariableCategory(a:OrderedSet)" -> "RetractableTo(OrderedSet)"
```

⟨*DVARCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DifferentialVariableCategory(a:OrderedSet)" [color=lightblue];
"DifferentialVariableCategory(a:OrderedSet)" -> "OrderedSet()"
"DifferentialVariableCategory(a:OrderedSet)" -> "RetractableTo(OrderedSet)"

"RetractableTo(OrderedSet)" [color=seagreen];
"RetractableTo(OrderedSet)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

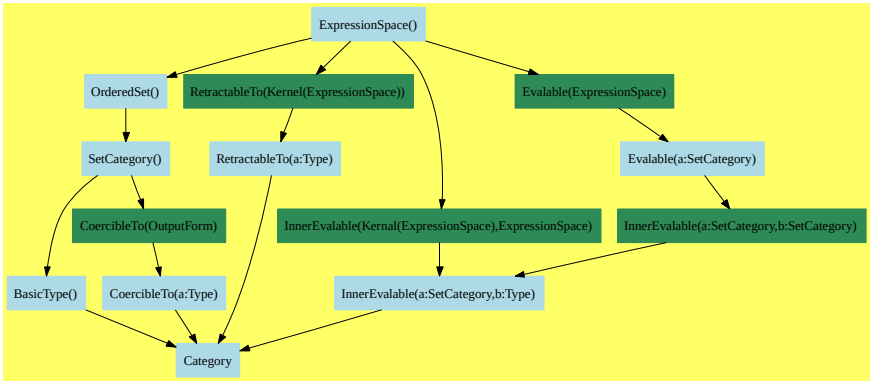
"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

5.6 ExpressionSpace (ES)



See:

⇒ “FunctionSpace” (FS) 17.5 on page 1095
⇐ “Evalable” (EVALAB) 3.4 on page 65
⇐ “InnerEvalable” (IEVALAB) 2.12 on page 29
⇐ “OrderedSet” (ORDSET) 4.19 on page 168
⇐ “RetractableTo” (RETRACT) 2.18 on page 44

Exports:

belong?	box	coerce	definingPolynomial
distribute	elt	eval	even?
freeOf?	hash	height	is?
kernel	kernels	latex	mainKernel
map	max	min	minPoly
odd?	operator	operators	paren
retract	retractIfCan	subst	tower
?<?	?<=?	?=?	?>?
?>=?	?~=?		

These are directly exported but not implemented:

```
definingPolynomial : % -> % if $ has RING
eval : (%,List Symbol,List (List % -> %)) -> %
eval : (%,List Kernel %,List %) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
eval : (%,Equation %) -> %
eval : (%,Kernel %,%) -> %
kernels : % -> List Kernel %
minPoly : Kernel % -> SparseUnivariatePolynomial % if $ has RING
subst : (%,List Kernel %,List %) -> %
```

These are implemented by this category:

```
belong? : BasicOperator -> Boolean
box : List % -> %
box : % -> %
distribute : % -> %
distribute : (%,%) -> %
elt : (BasicOperator,%,%,%,%) -> %
elt : (BasicOperator,%,%,%) -> %
elt : (BasicOperator,%,%) -> %
elt : (BasicOperator,%) -> %
elt : (BasicOperator,List %) -> %
eval : (%,List Equation %) -> %
eval : (%,Symbol,(% -> %)) -> %
eval : (%,Symbol,(List % -> %)) -> %
eval : (%,BasicOperator,(% -> %)) -> %
eval : (%,BasicOperator,(List % -> %)) -> %
eval : (%,List BasicOperator,List (% -> %)) -> %
eval : (%,List Symbol,List (% -> %)) -> %
eval : (%,List BasicOperator,List (List % -> %)) -> %
even? : % -> Boolean if $ has RETRACT INT
freeOf? : (%,Symbol) -> Boolean
freeOf? : (%,%) -> Boolean
height : % -> NonNegativeInteger
is? : (%,BasicOperator) -> Boolean
is? : (%,Symbol) -> Boolean
kernel : (BasicOperator,%) -> %
kernel : (BasicOperator,List %) -> %
mainKernel : % -> Union(Kernel %,"failed")
map : ((% -> %),Kernel %) -> %
odd? : % -> Boolean if $ has RETRACT INT
operator : BasicOperator -> BasicOperator
operators : % -> List BasicOperator
paren : % -> %
paren : List % -> %
retract : % -> Kernel %
retractIfCan : % -> Union(Kernel %,"failed")
subst : (%,Equation %) -> %
subst : (%,List Equation %) -> %
tower : % -> List Kernel %
```

These exports come from (p168) OrderedSet():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
```

```
?>=? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
```

These exports come from (p44) RetractableTo(a:Type):

```
coerce : Kernel % -> %
```

These exports come from (p29) InnerEvalable(a:SetCategory,b:Type):

These exports come from (p65) Evalable(a:SetCategory):

⟨*category ES ExpressionSpace*⟩≡

```
)abbrev category ES ExpressionSpace
++ Category for domains on which operators can be applied
++ Author: Manuel Bronstein
++ Date Created: 22 March 1988
++ Date Last Updated: 27 May 1994
++ Description:
++ An expression space is a set which is closed under certain operators;
++ Keywords: operator, kernel, expression, space.
ExpressionSpace(): Category == Defn where
  N   ==> NonNegativeInteger
  K   ==> Kernel %
  OP  ==> BasicOperator
  SY  ==> Symbol
  PAREN  ==> "%paren"::SY
  BOX    ==> "%box"::SY
  DUMMYVAR ==> "%dummyVar"

  Defn ==> Join(OrderedSet, RetractableTo K,
                InnerEvalable(K, %), Evalable %) with
    elt          : (OP, %) -> %
      ++ elt(op,x) or op(x) applies the unary operator op to x.
    elt          : (OP, %, %) -> %
      ++ elt(op,x,y) or op(x, y) applies the binary operator op to x and y.
    elt          : (OP, %, %, %) -> %
      ++ elt(op,x,y,z) or op(x, y, z) applies the ternary operator op
      ++ to x, y and z.
    elt          : (OP, %, %, %, %) -> %
      ++ elt(op,x,y,z,t) or op(x, y, z, t) applies the 4-ary operator op
      ++ to x, y, z and t.
    elt          : (OP, List %) -> %
      ++ elt(op,[x1,...,xn]) or op([x1,...,xn]) applies the n-ary operator
```

```
  ++ op to x1,...,xn.
subst        : (%, Equation %) -> %
  ++ subst(f, k = g) replaces the kernel k by g formally in f.
subst        : (%, List Equation %) -> %
  ++ subst(f, [k1 = g1,...,kn = gn]) replaces the kernels k1,...,kn
  ++ by g1,...,gn formally in f.
subst        : (%, List K, List %) -> %
  ++ subst(f, [k1...,kn], [g1,...,gn]) replaces the kernels k1,...,kn
  ++ by g1,...,gn formally in f.
box          : % -> %
  ++ box(f) returns f with a 'box' around it that prevents f from
  ++ being evaluated when operators are applied to it. For example,
  ++ \spad{log(1)} returns 0, but \spad{log(box 1)}
  ++ returns the formal kernel log(1).
box          : List % -> %
  ++ box([f1,...,fn]) returns \spad{(f1,...,fn)} with a 'box'
  ++ around them that
  ++ prevents the fi from being evaluated when operators are applied to
  ++ them, and makes them applicable to a unary operator. For example,
  ++ \spad{atan(box [x, 2])} returns the formal kernel \spad{atan(x, 2)}.
paren        : % -> %
  ++ paren(f) returns (f). This prevents f from
  ++ being evaluated when operators are applied to it. For example,
  ++ \spad{log(1)} returns 0, but \spad{log(paren 1)} returns the
  ++ formal kernel log((1)).
paren        : List % -> %
  ++ paren([f1,...,fn]) returns \spad{(f1,...,fn)}. This
  ++ prevents the fi from being evaluated when operators are applied to
  ++ them, and makes them applicable to a unary operator. For example,
  ++ \spad{atan(paren [x, 2])} returns the formal
  ++ kernel \spad{atan((x, 2))}.
distribute   : % -> %
  ++ distribute(f) expands all the kernels in f that are
  ++ formally enclosed by a \spadfunFrom{box}{ExpressionSpace}
  ++ or \spadfunFrom{paren}{ExpressionSpace} expression.
distribute   : (%, %) -> %
  ++ distribute(f, g) expands all the kernels in f that contain g in their
  ++ arguments and that are formally
  ++ enclosed by a \spadfunFrom{box}{ExpressionSpace}
  ++ or a \spadfunFrom{paren}{ExpressionSpace} expression.
height       : %  -> N
  ++ height(f) returns the highest nesting level appearing in f.
  ++ Constants have height 0. Symbols have height 1. For any
  ++ operator op and expressions f1,...,fn, \spad{op(f1,...,fn)} has
  ++ height equal to \spad{1 + max(height(f1),...,height(fn))}.
mainKernel   : %  -> Union(K, "failed")
```

```
  ++ mainKernel(f) returns a kernel of f with maximum nesting level, or
  ++ if f has no kernels (i.e. f is a constant).
kernels      : %  -> List K
  ++ kernels(f) returns the list of all the top-level kernels
  ++ appearing in f, but not the ones appearing in the arguments
  ++ of the top-level kernels.
tower        : %  -> List K
  ++ tower(f) returns all the kernels appearing in f, no matter
  ++ what their levels are.
operators    : %  -> List OP
  ++ operators(f) returns all the basic operators appearing in f,
  ++ no matter what their levels are.
operator     : OP -> OP
  ++ operator(op) returns a copy of op with the domain-dependent
  ++ properties appropriate for %.
belong?      : OP -> Boolean
  ++ belong?(op) tests if % accepts op as applicable to its
  ++ elements.
is?          : (%, OP)     -> Boolean
  ++ is?(x, op) tests if x is a kernel and is its operator is op.
is?          : (%, SY) -> Boolean
  ++ is?(x, s) tests if x is a kernel and is the name of its
  ++ operator is s.
kernel       : (OP, %) -> %
  ++ kernel(op, x) constructs op(x) without evaluating it.
kernel       : (OP, List %) -> %
  ++ kernel(op, [f1,...,fn]) constructs \spad{op(f1,...,fn)} without
  ++ evaluating it.
map          : (% -> %, K) -> %
  ++ map(f, k) returns \spad{op(f(x1),...,f(xn))} where
  ++ \spad{k = op(x1,...,xn)}.
freeOf?      : (%, %)  -> Boolean
  ++ freeOf?(x, y) tests if x does not contain any occurrence of y,
  ++ where y is a single kernel.
freeOf?      : (%, SY) -> Boolean
  ++ freeOf?(x, s) tests if x does not contain any operator
  ++ whose name is s.
eval         : (%, List SY, List(% -> %)) -> %
  ++ eval(x, [s1,...,sm], [f1,...,fm]) replaces
  ++ every \spad{si(a)} in x by \spad{fi(a)} for any \spad{a}.
eval         : (%, List SY, List(List % -> %)) -> %
  ++ eval(x, [s1,...,sm], [f1,...,fm]) replaces
  ++ every \spad{si(a1,...,an)} in x by
  ++ \spad{fi(a1,...,an)} for any \spad{a1},...,\spad{an}.
eval         : (%, SY, List % -> %) -> %
  ++ eval(x, s, f) replaces every \spad{s(a1,..,am)} in x
```

```
      ++ by \spad{f(a1,..,am)} for any \spad{a1},...,\spad{am}.
    eval         : (%, SY, % -> %) -> %
      ++ eval(x, s, f) replaces every \spad{s(a)} in x by \spad{f(a)}
      ++ for any \spad{a}.
    eval         : (%, List OP, List(% -> %)) -> %
      ++ eval(x, [s1,...,sm], [f1,...,fm]) replaces
      ++ every \spad{si(a)} in x by \spad{fi(a)} for any \spad{a}.
    eval         : (%, List OP, List(List % -> %)) -> %
      ++ eval(x, [s1,...,sm], [f1,...,fm]) replaces
      ++ every \spad{si(a1,...,an)} in x by
      ++ \spad{fi(a1,...,an)} for any \spad{a1},...,\spad{an}.
    eval         : (%, OP, List % -> %) -> %
      ++ eval(x, s, f) replaces every \spad{s(a1,..,am)} in x
      ++ by \spad{f(a1,..,am)} for any \spad{a1},...,\spad{am}.
    eval         : (%, OP, % -> %) -> %
      ++ eval(x, s, f) replaces every \spad{s(a)} in x by \spad{f(a)}
      ++ for any \spad{a}.
    if % has Ring then
      minPoly: K -> SparseUnivariatePolynomial %
        ++ minPoly(k) returns p such that \spad{p(k) = 0}.
      definingPolynomial: % -> %
        ++ definingPolynomial(x) returns an expression p such that
        ++ \spad{p(x) = 0}.
    if % has RetractableTo Integer then
      even?: % -> Boolean
        ++ even? x is true if x is an even integer.
      odd? : % -> Boolean
        ++ odd? x is true if x is an odd integer.

  add

-- the 7 functions not provided are:
--        kernels   minPoly   definingPolynomial
--        coerce:K -> %  eval:(%, List K, List %) -> %
--        subst:(%, List K, List %) -> %
--        eval:(%, List Symbol, List(List % -> %)) -> %

    allKernels: %      -> Set K
    listk     : %      -> List K
    allk      : List % -> Set K
    unwrap    : (List K, %) -> %
    okkernel  : (OP, List %) -> %
    mkKerLists: List Equation % -> Record(lstk: List K, lstv:List %)

    oppren := operator(PAREN)$CommonOperators()
    opbox  := operator(BOX)$CommonOperators()
```

```
box(x:%)     == box [x]
paren(x:%)   == paren [x]
belong? op   == op = oppren or op = opbox
listk f      == parts allKernels f
tower f      == sort_! listk f
allk l       == reduce("union", [allKernels f for f in l], {})
operators f  == [operator k for k in listk f]
height f     == reduce("max", [height k for k in kernels f], 0)
freeOf?(x:%, s:SY)       == not member?(s, [name k for k in listk x])
distribute x == unwrap([k for k in listk x | is?(k, oppren)], x)
box(l:List %)                  == opbox l
paren(l:List %)                == oppren l
freeOf?(x:%, k:%)              == not member?(retract k, listk x)
kernel(op:OP, arg:%)           == kernel(op, [arg])
elt(op:OP, x:%)                == op [x]
elt(op:OP, x:%, y:%)           == op [x, y]
elt(op:OP, x:%, y:%, z:%)      == op [x, y, z]
elt(op:OP, x:%, y:%, z:%, t:%) == op [x, y, z, t]
eval(x:%, s:SY, f:List % -> %) == eval(x, [s], [f])
eval(x:%, s:OP, f:List % -> %) == eval(x, [name s], [f])
eval(x:%, s:SY, f:% -> %)      ==
  eval(x, [s], [(y:List %):% +-> f(first y)])
eval(x:%, s:OP, f:% -> %)      ==
  eval(x, [s], [(y:List %):% +-> f(first y)])
subst(x:%, e:Equation %)       == subst(x, [e])

eval(x:%, ls:List OP, lf:List(% -> %)) ==
  eval(x, ls, [y +-> f(first y) for f in lf]$List(List % -> %))

eval(x:%, ls:List SY, lf:List(% -> %)) ==
  eval(x, ls, [y +-> f(first y) for f in lf]$List(List % -> %))

eval(x:%, ls:List OP, lf:List(List % -> %)) ==
  eval(x, [name s for s in ls]$List(SY), lf)

map(fn, k) ==
  (l := [fn x for x in argument k]$List(%)) = argument k => k::%
  (operator k) l

operator op ==
  is?(op, PAREN) => oppren
  is?(op, BOX) => opbox
  error "Unknown operator"

mainKernel x ==
```

```
        empty?(l := kernels x) => "failed"
        n := height(k := first l)
        for kk in rest l repeat
          if height(kk) > n then
            n := height kk
            k := kk
        k

  -- takes all the kernels except for the dummy variables, which are second
  -- arguments of rootOf's, integrals, sums and products which appear only in
  -- their first arguments
      allKernels f ==
        s := brace(l := kernels f)
        for k in l repeat
            t :=
                (u := property(operator k, DUMMYVAR)) case None =>
                    arg := argument k
                    s0  := remove_!(retract(second arg)@K, allKernels first arg)
                    arg := rest rest arg
                    n   := (u::None) pretend N
                    if n > 1 then arg := rest arg
                    union(s0, allk arg)
                allk argument k
            s := union(s, t)
        s

      kernel(op:OP, args:List %) ==
        not belong? op => error "Unknown operator"
        okkernel(op, args)

      okkernel(op, l) ==
        kernel(op, l, 1 + reduce("max", [height f for f in l], 0))$K :: %

      elt(op:OP, args:List %) ==
        not belong? op => error "Unknown operator"
        ((u := arity op) case N) and (#args ^= u::N)
                                      => error "Wrong number of arguments"
        (v := evaluate(op,args)$BasicOperatorFunctions1(%)) case % => v::%
        okkernel(op, args)

      retract f ==
        (k := mainKernel f) case "failed" => error "not a kernel"
        k::K::% ^= f => error "not a kernel"
        k::K

      retractIfCan f ==
```

```
    (k := mainKernel f) case "failed" => "failed"
    k::K::% ^= f => "failed"
    k

  is?(f:%, s:SY) ==
    (k := retractIfCan f) case "failed" => false
    is?(k::K, s)

  is?(f:%, op:OP) ==
    (k := retractIfCan f) case "failed" => false
    is?(k::K, op)

  unwrap(l, x) ==
    for k in reverse_! l repeat
      x := eval(x, k, first argument k)
    x

  distribute(x, y) ==
    ky := retract y
    unwrap([k for k in listk x |
            is?(k, "%paren"::SY) and member?(ky, listk(k::%))], x)

  -- in case of conflicting substitutions e.g. [x = a, x = b],
  -- the first one prevails.
  -- this is not part of the semantics of the function, but just
  -- a feature of this implementation.
  eval(f:%, leq:List Equation %) ==
    rec := mkKerLists leq
    eval(f, rec.lstk, rec.lstv)

  subst(f:%, leq:List Equation %) ==
    rec := mkKerLists leq
    subst(f, rec.lstk, rec.lstv)

  mkKerLists leq ==
    lk := empty()$List(K)
    lv := empty()$List(%)
    for eq in leq repeat
      (k := retractIfCan(lhs eq)@Union(K, "failed")) case "failed" =>
                        error "left hand side must be a single kernel"
      if not member?(k::K, lk) then
        lk := concat(k::K, lk)
        lv := concat(rhs eq, lv)
    [lk, lv]

  if % has RetractableTo Integer then
```

```
        intpred?: (%, Integer -> Boolean) -> Boolean

        even? x == intpred?(x, even?)
        odd? x  == intpred?(x, odd?)

        intpred?(x, pred?) ==
            (u := retractIfCan(x)@Union(Integer, "failed")) case Integer
                   and pred?(u::Integer)
```

⟨*ES.dotabb*⟩≡

```
  "ES"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ES"];
  "ES" -> "ORDSET"
  "ES" -> "RETRACT"
  "ES" -> "IEVALAB"
  "ES" -> "EVALAB"
```

⟨*ES.dotfull*⟩≡

```
  "ExpressionSpace()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ES"];
  "ExpressionSpace()" -> "OrderedSet()"
  "ExpressionSpace()" -> "RetractableTo(Kernel(ExpressionSpace))"
  "ExpressionSpace()" ->
      "InnerEvalable(Kernal(ExpressionSpace),ExpressionSpace)"
  "ExpressionSpace()" -> "Evalable(ExpressionSpace)"
```

⟨*ES.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ExpressionSpace()" [color=lightblue];
"ExpressionSpace()" -> "OrderedSet()"
"ExpressionSpace()" -> "RetractableTo(Kernel(ExpressionSpace))"
"ExpressionSpace()" ->
    "InnerEvalable(Kernal(ExpressionSpace),ExpressionSpace)"
"ExpressionSpace()" -> "Evalable(ExpressionSpace)"

"Evalable(ExpressionSpace)" [color=seagreen];
"Evalable(ExpressionSpace)" -> "Evalable(a:SetCategory)"

"Evalable(a:SetCategory)" [color=lightblue];
"Evalable(a:SetCategory)" -> "InnerEvalable(a:SetCategory,b:SetCategory)"

"InnerEvalable(Kernal(ExpressionSpace),ExpressionSpace)" [color=seagreen];
"InnerEvalable(Kernal(ExpressionSpace),ExpressionSpace)" ->
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:SetCategory,b:SetCategory)" [color=seagreen];
"InnerEvalable(a:SetCategory,b:SetCategory)" ->
    "InnerEvalable(a:SetCategory,b:Type)"

"InnerEvalable(a:SetCategory,b:Type)" [color=lightblue];
"InnerEvalable(a:SetCategory,b:Type)" -> "Category"

"RetractableTo(Kernel(ExpressionSpace))" [color=seagreen];
"RetractableTo(Kernel(ExpressionSpace))" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

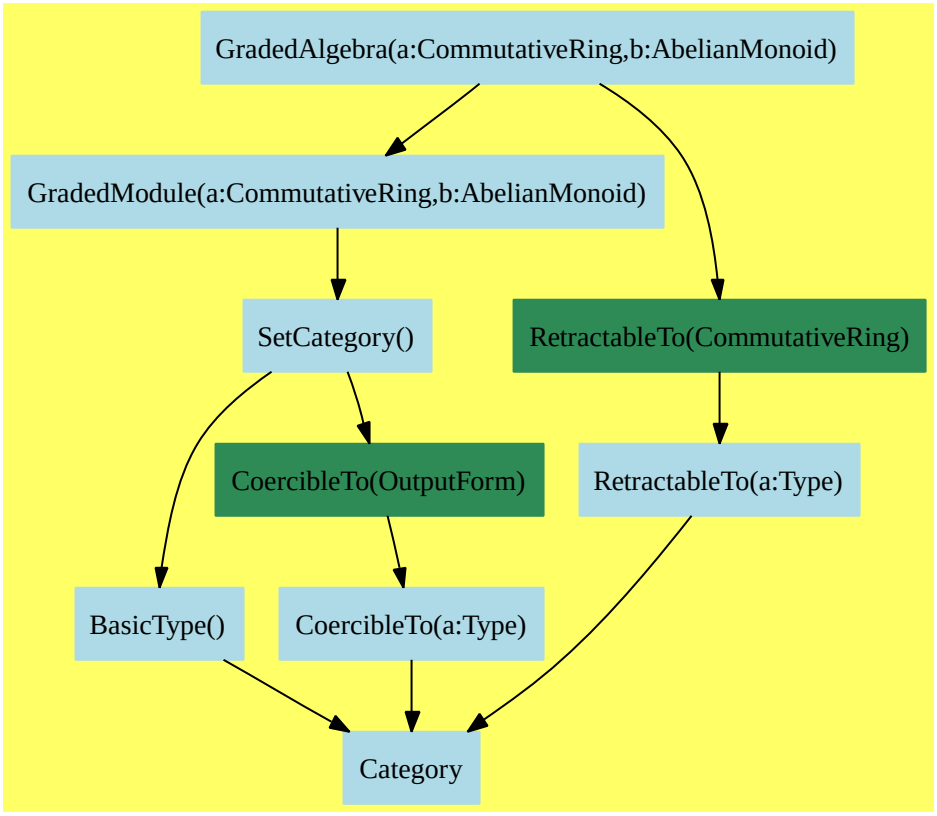
"BasicType()" [color=lightblue];
"BasicType()" -> "Category"
```

```
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

5.7 GradedAlgebra (GRALG)



See:

⇐ "GradedModule" (GRMOD) 4.11 on page 135
⇐ "RetractableTo" (RETRACT) 2.18 on page 44

Exports:

0	1	coerce	degree	hash
latex	product	retract	retractIfCan	?~=?
?*?	?+?	?-?	-?	?=?

These are directly exported but not implemented:

```
product : (%,%) -> %
```

These are implemented by this category:

```
0 : () -> %
1 : () -> %
?*? : (%,R) -> %
?*? : (R,%) -> %
```

These exports come from (p135) GradedModule(R, E)
where R:CommutativeRing and E:AbelianMonoid:

```
coerce : % -> OutputForm
degree : % -> E
hash : % -> SingleInteger
latex : % -> String
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?-? : (%,%) -> %
-? : % -> %
?+? : (%,%) -> %
```

These exports come from (p44) RetractableTo(R:CommutativeRing):

```
coerce : R -> %
retract : % -> R
retractIfCan : % -> Union(R,"failed")
```

⟨*category GRALG GradedAlgebra*⟩≡

```
)abbrev category GRALG GradedAlgebra
++ Author: Stephen M. Watt
++ Date Created: May 20, 1991
++ Date Last Updated: May 20, 1991
++ Basic Operations: +, *, degree
++ Related Domains: CartesianTensor(n,dim,R)
++ Also See:
++ AMS Classifications:
++ Keywords: graded module, tensor, multi-linear algebra
++ Examples:
++ References: Encyclopedic Dictionary of Mathematics, MIT Press, 1977
++ Description:
++  GradedAlgebra(R,E) denotes ‘‘E-graded R-algebra’’.
++  A graded algebra is a graded module together with a degree preserving
++  R-linear map, called the {\em product}.
++
++  The name ‘‘product’’ is written out in full so inner and outer products
++  with the same mapping type can be distinguished by name.

GradedAlgebra(R: CommutativeRing, E: AbelianMonoid): Category ==
    Join(GradedModule(R, E),RetractableTo(R)) with
        1: constant -> %
            ++ 1 is the identity for \spad{product}.
        product: (%, %) -> %
            ++ product(a,b) is the degree-preserving R-linear product:
            ++
            ++   \spad{degree product(a,b) = degree a + degree b}
            ++   \spad{product(a1+a2,b) = product(a1,b) + product(a2,b)}
```

```
            ++   \spad{product(a,b1+b2) = product(a,b1) + product(a,b2)}
            ++   \spad{product(r*a,b) = product(a,r*b) = r*product(a,b)}
            ++   \spad{product(a,product(b,c)) = product(product(a,b),c)}
    add
          if not (R is %) then
              0: % == (0$R)::%
              1: % == 1$R::%
              (r: R)*(x: %) == product(r::%, x)
              (x: %)*(r: R) == product(x, r::%)
```

⟨*GRALG.dotabb*⟩≡

```
  "GRALG"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=GRALG"];
  "GRALG" -> "GRMOD"
  "GRALG" -> "RETRACT"
```

⟨*GRALG.dotfull*⟩≡

```
  "GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=GRALG"];
  "GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)" ->
      "GradedModule(a:CommutativeRing,b:AbelianMonoid)"
  "GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)" ->
      "RetractableTo(CommutativeRing)"
```

⟨*GRALG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)" [color=lightblue];
"GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)" ->
    "GradedModule(a:CommutativeRing,b:AbelianMonoid)"
"GradedAlgebra(a:CommutativeRing,b:AbelianMonoid)" ->
    "RetractableTo(CommutativeRing)"

"RetractableTo(CommutativeRing)" [color=seagreen];
"RetractableTo(CommutativeRing)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"GradedModule(a:CommutativeRing,b:AbelianMonoid)" [color=lightblue];
"GradedModule(a:CommutativeRing,b:AbelianMonoid)" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

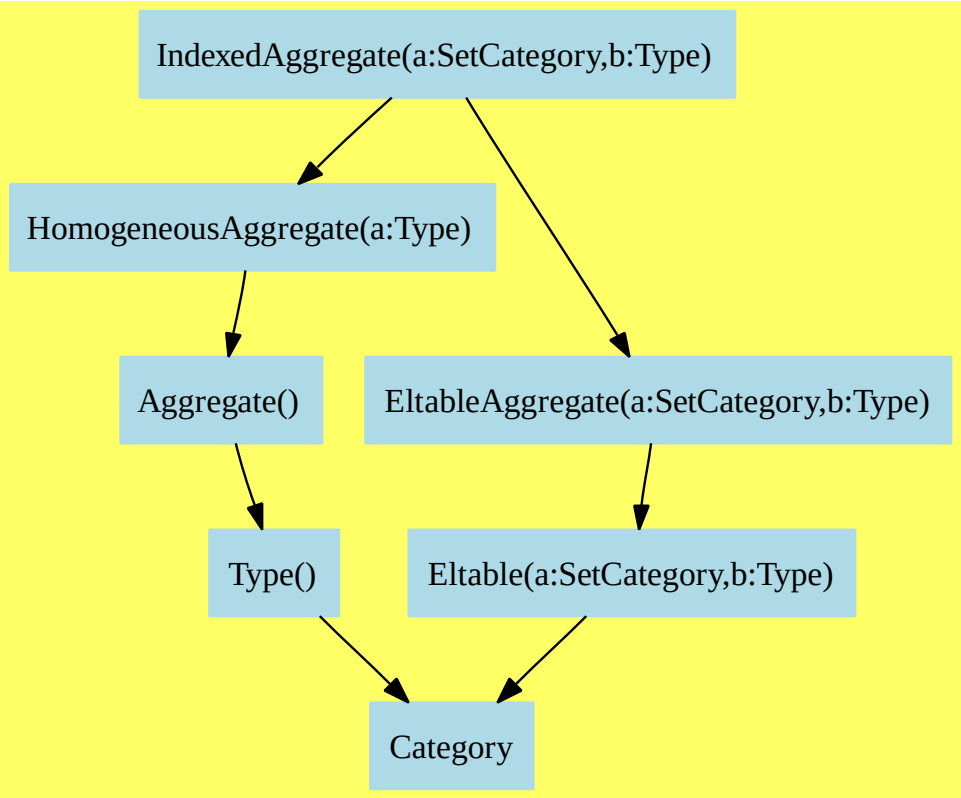
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

5.8 IndexedAggregate (IXAGG)



See:

⇒ "DirectProductCategory" (DIRPCAT) 12.1 on page 815
⇒ "LinearAggregate" (LNAGG) 6.6 on page 308
⇒ "TableAggregate" (TBAGG) 9.11 on page 649
⇐ "EltableAggregate" (ELTAGG) 3.3 on page 62
⇐ "HomogeneousAggregate" (HOAGG) 4.12 on page 139

Exports:

any?	coerce	copy	count	elt
empty	empty?	entries	entry?	eq?
eval	every?	fill!	first	hash
index?	indices	latex	less?	map
map!	maxIndex	member?	members	minIndex
more?	parts	qelt	qsetelt!	sample
setelt	size?	swap!	?.?	?~=?
#?	?=?			

Attributes exported:

- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
index? : (Index,%) -> Boolean
indices : % -> List Index
```

These are implemented by this category:

```
elt : (%,Index,Entry) -> Entry
entries : % -> List Entry
entry? : (Entry,%) -> Boolean
           if $ has finiteAggregate and Entry has SETCAT
fill! : (%,Entry) -> % if $ has shallowlyMutable
first : % -> Entry if Index has ORDSET
map : ((Entry -> Entry),%) -> %
map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
maxIndex : % -> Index if Index has ORDSET
minIndex : % -> Index if Index has ORDSET
swap! : (%,Index,Index) -> Void if $ has shallowlyMutable
```

These exports come from (p139) HomogeneousAggregate(Entry:Type):

```
any? : ((Entry -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if Entry has SETCAT
copy : % -> %
count : (Entry,%) -> NonNegativeInteger
         if Entry has SETCAT and $ has finiteAggregate
count : ((Entry -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Entry,List Entry) -> %
      if Entry has EVALAB Entry and Entry has SETCAT
eval : (%,Entry,Entry) -> %
      if Entry has EVALAB Entry and Entry has SETCAT
eval : (%,Equation Entry) -> %
      if Entry has EVALAB Entry and Entry has SETCAT
```

```
eval : (%,List Equation Entry) -> %
      if Entry has EVALAB Entry and Entry has SETCAT
every? : ((Entry -> Boolean),%) -> Boolean
      if $ has finiteAggregate
hash : % -> SingleInteger if Entry has SETCAT
latex : % -> String if Entry has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
member? : (Entry,%) -> Boolean
          if Entry has SETCAT and $ has finiteAggregate
members : % -> List Entry if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List Entry if $ has finiteAggregate
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
?~=? : (%,%) -> Boolean if Entry has SETCAT
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if Entry has SETCAT
```

These exports come from (p62) EltableAggregate(Index:SetCategory,Entry:Type):

```
qelt : (%,Index) -> Entry
qsetelt! : (%,Index,Entry) -> Entry if $ has shallowlyMutable
setelt : (%,Index,Entry) -> Entry if $ has shallowlyMutable
?.? : (%,Index) -> Entry
```

⟨*category IXAGG IndexedAggregate*⟩≡

```
)abbrev category IXAGG IndexedAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ An indexed aggregate is a many-to-one mapping of indices to entries.
++ For example, a one-dimensional-array is an indexed aggregate where
++ the index is an integer.  Also, a table is an indexed aggregate
++ where the indices and entries may have any type.
IndexedAggregate(Index: SetCategory, Entry: Type): Category ==
  Join(HomogeneousAggregate(Entry), EltableAggregate(Index, Entry)) with
   entries: % -> List Entry
      ++ entries(u) returns a list of all the entries of aggregate u
      ++ in no assumed order.
      -- to become entries: % -> Entry* and
      -- entries: % -> Iterator(Entry,Entry)
```

```
    index?: (Index,%) -> Boolean
       ++ index?(i,u) tests if i is an index of aggregate u.
    indices: % -> List Index
       ++ indices(u) returns a list of indices of aggregate u in no
       ++ particular order. to become indices:
       --  % -> Index* and indices: % -> Iterator(Index,Index).
  -- map: ((Entry,Entry)->Entry,%,%,Entry) -> %
  --    ++ exists c = map(f,a,b,x), i:Index where
  --    ++    c.i = f(a(i,x),b(i,x)) | index?(i,a) or index?(i,b)
    if Entry has SetCategory and % has finiteAggregate then
       entry?: (Entry,%) -> Boolean
         ++ entry?(x,u) tests if x equals \axiom{u . i} for some index i.
    if Index has OrderedSet then
       maxIndex: % -> Index
         ++ maxIndex(u) returns the maximum index i of aggregate u.
         ++ Note: in general,
         ++ \axiom{maxIndex(u) = reduce(max,[i for i in indices u])};
         ++ if u is a list, \axiom{maxIndex(u) = #u}.
       minIndex: % -> Index
         ++ minIndex(u) returns the minimum index i of aggregate u.
         ++ Note: in general,
         ++ \axiom{minIndex(a) = reduce(min,[i for i in indices a])};
         ++ for lists, \axiom{minIndex(a) = 1}.
       first   : % -> Entry
         ++ first(u) returns the first element x of u.
         ++ Note: for collections, \axiom{first([x,y,...,z]) = x}.
         ++ Error: if u is empty.

    if % has shallowlyMutable then
       fill_!: (%,Entry) -> %
         ++ fill!(u,x) replaces each entry in aggregate u by x.
         ++ The modified u is returned as value.
       swap_!: (%,Index,Index) -> Void
         ++ swap!(u,i,j) interchanges elements i and j of aggregate u.
         ++ No meaningful value is returned.
  add
   elt(a, i, x) == (index?(i, a) => qelt(a, i); x)

   if % has finiteAggregate then
     entries x == parts x
     if Entry has SetCategory then
       entry?(x, a) == member?(x, a)

   if Index has OrderedSet then
     maxIndex a == "max"/indices(a)
     minIndex a == "min"/indices(a)
```

```
      first a    == a minIndex a

    if % has shallowlyMutable then
      map(f, a) == map_!(f, copy a)

      map_!(f, a) ==
        for i in indices a repeat qsetelt_!(a, i, f qelt(a, i))
        a

      fill_!(a, x) ==
        for i in indices a repeat qsetelt_!(a, i, x)
        a

      swap_!(a, i, j) ==
        t := a.i
        qsetelt_!(a, i, a.j)
        qsetelt_!(a, j, t)
        void
```

⟨*IXAGG.dotabb*⟩≡

```
  "IXAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=IXAGG"];
  "IXAGG" -> "HOAGG"
  "IXAGG" -> "ELTAGG"
```

⟨*IXAGG.dotfull*⟩≡

```
  "IndexedAggregate(a:SetCategory,b:Type)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=IXAGG"];
  "IndexedAggregate(a:SetCategory,b:Type)" ->
      "HomogeneousAggregate(a:Type)"
  "IndexedAggregate(a:SetCategory,b:Type)" ->
      "EltableAggregate(a:SetCategory,b:Type)"

  "IndexedAggregate(a:SetCategory,b:SetCategory)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=IXAGG"];
  "IndexedAggregate(a:SetCategory,b:SetCategory)" ->
      "IndexedAggregate(a:SetCategory,b:Type)"

  "IndexedAggregate(b:Integer,a:Type)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=IXAGG"];
  "IndexedAggregate(b:Integer,a:Type)" ->
      "IndexedAggregate(a:SetCategory,b:Type)"
```

⟨*IXAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "HomogeneousAggregate(a:Type)"
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "EltableAggregate(a:SetCategory,b:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"EltableAggregate(a:SetCategory,b:Type)" [color=lightblue];
"EltableAggregate(a:SetCategory,b:Type)" -> "Eltable(a:SetCategory,b:Type)"

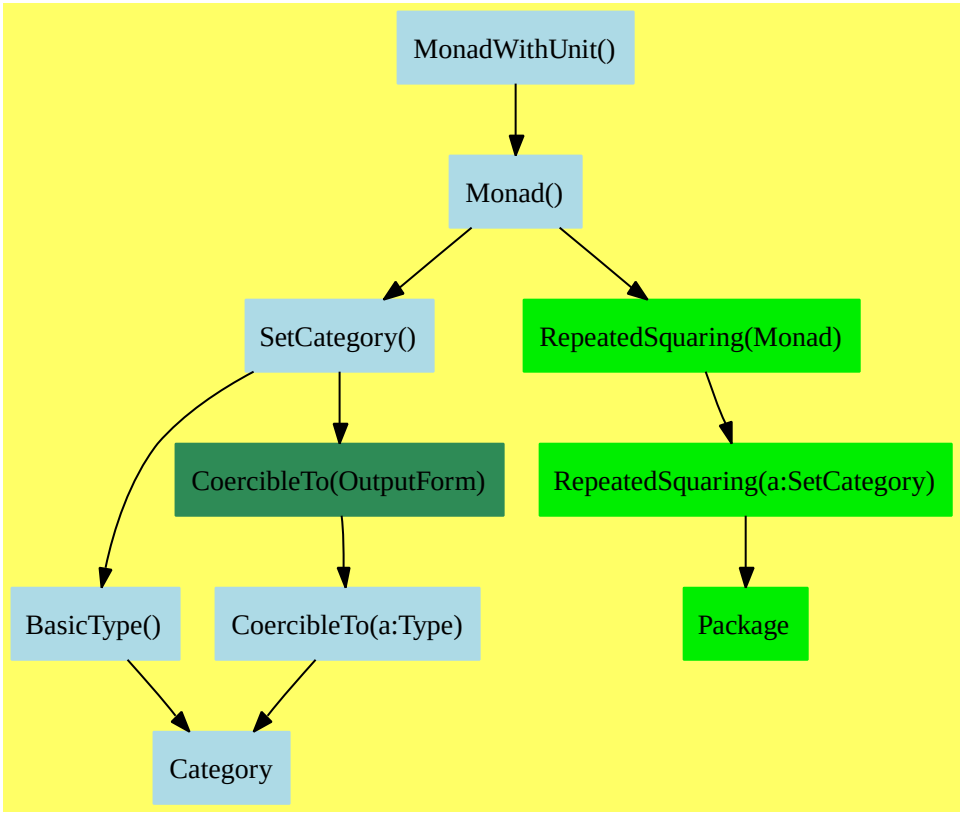
"Eltable(a:SetCategory,b:Type)" [color=lightblue];
"Eltable(a:SetCategory,b:Type)" -> "Category"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

5.9 MonadWithUnit (MONADWU)



See:

⇒ "NonAssociativeRing" (NASRING) 9.3 on page 604
⇐ "Monad" (MONAD) 4.15 on page 152

Exports:

1	coerce	hash	latex	one?
recip	leftPower	leftRecip	rightPower	rightRecip
?*?	?~=?	?**?	?=?	

These are directly exported but not implemented:

```
1 : () -> %
leftRecip : % -> Union(%,"failed")
recip : % -> Union(%,"failed")
rightRecip : % -> Union(%,"failed")
```

These are implemented by this category:

```
leftPower : (%,NonNegativeInteger) -> %
```

```
one? : % -> Boolean
rightPower : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
```

These exports come from (p152) Monad():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
leftPower : (%,PositiveInteger) -> %
rightPower : (%,PositiveInteger) -> %
?**? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
```

⟨*category MONADWU MonadWithUnit*⟩≡

```
)abbrev category MONADWU MonadWithUnit
++ Authors: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 11 June 1991
++ Basic Operations: *, **, 1
++ Related Constructors: SemiGroup, Monoid, Monad
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Keywords: Monad with unit, binary operation
++ Reference:
++  N. Jacobson: Structure and Representations of Jordan Algebras
++  AMS, Providence, 1968
++ Description:
++  MonadWithUnit is the class of multiplicative monads with unit,
++  i.e. sets with a binary operation and a unit element.
++ Axioms
++    leftIdentity("*":(%,%)->%,1)   \tab{30} 1*x=x
++    rightIdentity("*":(%,%)->%,1)  \tab{30} x*1=x
++ Common Additional Axioms
++    unitsKnown---if "recip" says "failed", that PROVES input wasn't a unit
MonadWithUnit(): Category == Monad with
      1: constant ->  %
        ++ 1 returns the unit element, denoted by 1.
      one?: % -> Boolean
        ++ one?(a) tests whether \spad{a} is the unit 1.
      rightPower: (%,NonNegativeInteger) -> %
        ++ rightPower(a,n) returns the \spad{n}-th right power of \spad{a},
        ++ i.e. \spad{rightPower(a,n) := rightPower(a,n-1) * a} and
        ++ \spad{rightPower(a,0) := 1}.
      leftPower: (%,NonNegativeInteger) -> %
```

```
      ++ leftPower(a,n) returns the \spad{n}-th left power of \spad{a},
      ++ i.e. \spad{leftPower(a,n) := a * leftPower(a,n-1)} and
      ++ \spad{leftPower(a,0) := 1}.
    "**": (%,NonNegativeInteger) -> %
      ++ \spad{a**n} returns the \spad{n}-th power of \spad{a},
      ++ defined by repeated squaring.
    recip: % -> Union(%,"failed")
      ++ recip(a) returns an element, which is both a left and a right
      ++ inverse of \spad{a},
      ++ or \spad{"failed"} if such an element doesn't exist or cannot
      ++ be determined (see unitsKnown).
    leftRecip: % -> Union(%,"failed")
      ++ leftRecip(a) returns an element, which is a left inverse of
      ++ \spad{a}, or \spad{"failed"} if such an element doesn't exist
      ++ or cannot be determined (see unitsKnown).
    rightRecip: % -> Union(%,"failed")
      ++ rightRecip(a) returns an element, which is a right inverse of
      ++ \spad{a}, or \spad{"failed"} if such an element doesn't exist
      ++ or cannot be determined (see unitsKnown).
  add
    import RepeatedSquaring(%)
    one? x == x = 1
    x:% ** n:NonNegativeInteger ==
       zero? n => 1
       expt(x,n pretend PositiveInteger)
    rightPower(a,n) ==
      zero? n => 1
      res := 1
      for i in 1..n repeat res := res * a
      res
    leftPower(a,n) ==
      zero? n => 1
      res := 1
      for i in 1..n repeat res := a * res
      res
```

⟨*MONADWU.dotabb*⟩≡

```
"MONADWU"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONADWU"];
"MONADWU" -> "MONAD"
```

⟨*MONADWU.dotfull*⟩≡
```
"MonadWithUnit()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONADWU"];
"MonadWithUnit()" -> "Monad()"
```

⟨*MONADWU.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"MonadWithUnit()" [color=lightblue];
"MonadWithUnit()" -> "Monad()"

"Monad()" [color=lightblue];
"Monad()" -> "SetCategory()"
"Monad()" -> "RepeatedSquaring(Monad)"

"RepeatedSquaring(Monad)" [color="#00EE00"];
"RepeatedSquaring(Monad)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

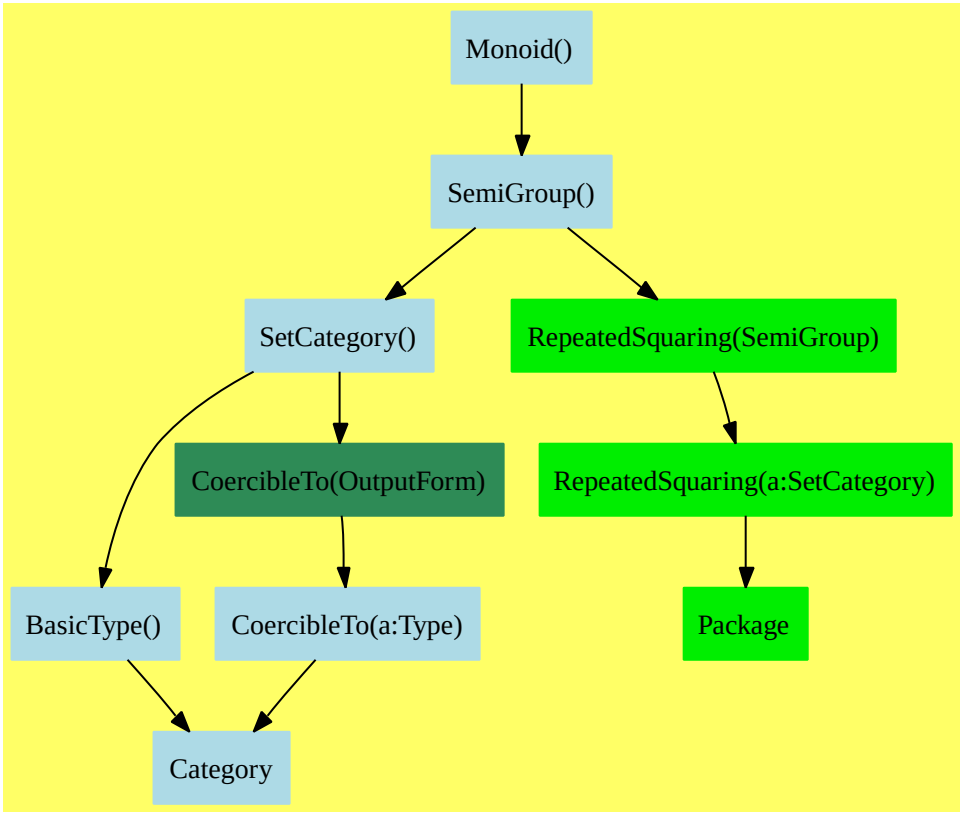
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

5.10 Monoid (MONOID)



See:

⇒ “FunctionSpace” (FS) 17.5 on page 1095
⇒ “Group” (GROUP) 6.5 on page 304
⇒ “OrderedMonoid” (ORDMON) 6.9 on page 368
⇒ “OrderedRing” (ORDRING) 10.11 on page 715
⇒ “Ring” (RING) 9.8 on page 628
⇐ “SemiGroup” (SGROUP) 4.24 on page 186

Exports:

1	coerce	hash	latex	one?
recip	sample	?*?	?=?	?~=?
?**?	?^?			

These are directly exported but not implemented:

```
1 : () -> %
```

These are implemented by this category:

```
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
```

These exports come from (p186) SemiGroup():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?*? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category MONOID Monoid*⟩≡

```
)abbrev category MONOID Monoid
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The class of multiplicative monoids, i.e. semigroups with a
++ multiplicative identity element.
++
++ Axioms:
++    \spad{leftIdentity("*":(%,%)->%,1)}\tab{30}\spad{1*x=x}
++    \spad{rightIdentity("*":(%,%)->%,1)}\tab{30}\spad{x*1=x}
++
++ Conditional attributes:
++    unitsKnown\tab{15}\spadfun{recip} only returns "failed" on non-units
Monoid(): Category == SemiGroup with
      1: constant ->  %
        ++ 1 is the multiplicative identity.
      sample: constant -> %
        ++ sample yields a value of type %
      one?: % -> Boolean
        ++ one?(x) tests if x is equal to 1.
      "**": (%,NonNegativeInteger) -> %
        ++ x**n returns the repeated product
        ++ of x n times, i.e. exponentiation.
```

```
    "^" : (%,NonNegativeInteger) -> %
      ++ x^n returns the repeated product
      ++ of x n times, i.e. exponentiation.
    recip: % -> Union(%,"failed")
      ++ recip(x) tries to compute the multiplicative inverse for x
      ++ or "failed" if it cannot find the inverse (see unitsKnown).
  add
    import RepeatedSquaring(%)
    _^(x:%, n:NonNegativeInteger):% == x ** n
    one? x == x = 1
    sample() == 1
    recip x ==
      (x = 1) => x
      "failed"
    x:% ** n:NonNegativeInteger ==
       zero? n => 1
       expt(x,n pretend PositiveInteger)
```

⟨*MONOID.dotabb*⟩≡

```
"MONOID"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONOID"];
"MONOID" -> "SGROUP"
```

⟨*MONOID.dotfull*⟩≡

```
"Monoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONOID"];
"Monoid()" -> "SemiGroup()"
```

⟨*MONOID.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SetCategory()"
"SemiGroup()" -> "RepeatedSquaring(a:SemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedSquaring(a:SemiGroup)" [color="#00EE00"];
"RepeatedSquaring(a:SemiGroup)" -> "RepeatedSquaring(a:SetCategory)"

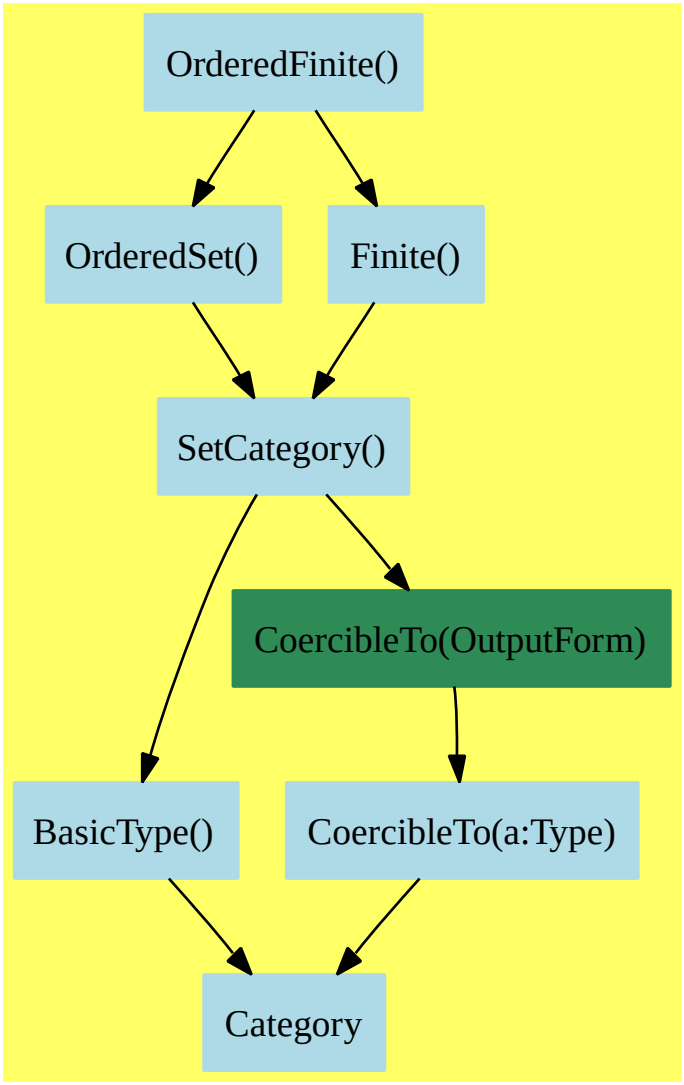
"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

5.11 OrderedFinite (ORDFIN)



See:

⇐ "Finite" (FINITE) 4.9 on page 129
⇐ "OrderedSet" (ORDSET) 4.19 on page 168

Exports:

coerce	hash	index	latex	lookup
max	min	random	size	?~=?
?<?	?<=?	?=?	?>?	?>=?

These exports come from (p168) OrderedSet():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

These exports come from (p129) Finite():

```
index : PositiveInteger -> %
lookup : % -> PositiveInteger
random : () -> %
size : () -> NonNegativeInteger
```

⟨*category ORDFIN OrderedFinite*⟩≡

```
)abbrev category ORDFIN OrderedFinite
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Ordered finite sets.

OrderedFinite(): Category == Join(OrderedSet, Finite)
```

⟨*ORDFIN.dotabb*⟩≡

```
"ORDFIN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDFIN"];
"ORDFIN" -> "ORDSET"
"ORDFIN" -> "FINITE"
```

⟨*ORDFIN.dotfull*⟩≡
```
"OrderedFinite()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDFIN"];
"OrderedFinite()" -> "OrderedSet()"
"OrderedFinite()" -> "Finite()"
```

⟨*ORDFIN.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedFinite()" [color=lightblue];
"OrderedFinite()" -> "OrderedSet()"
"OrderedFinite()" -> "Finite()"

"Finite()" [color=lightblue];
"Finite()" -> "SetCategory()"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

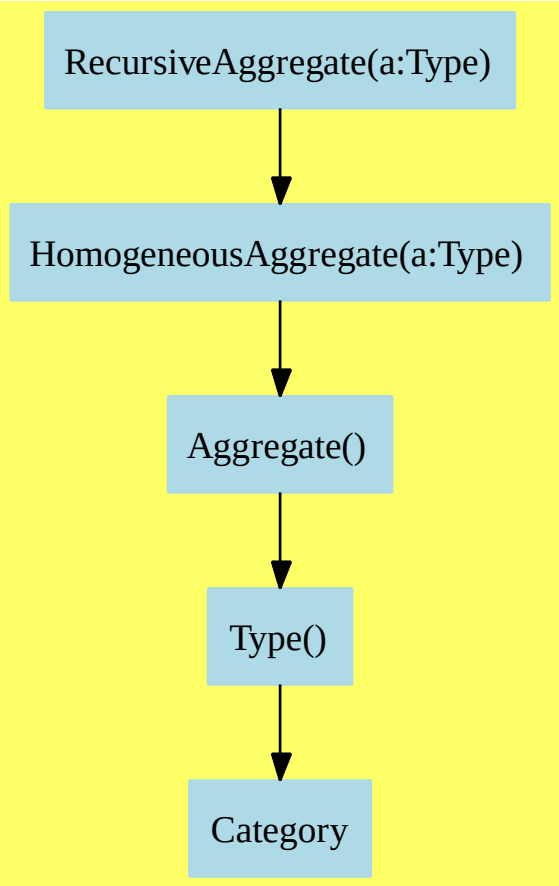
"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];

}
```

5.12 RecursiveAggregate (RCAGG)



See:

⇒ “BinaryRecursiveAggregate” (BRAGG) 6.1 on page 282
⇒ “DoublyLinkedAggregate” (DLAGG) 6.4 on page 299
⇒ “UnaryRecursiveAggregate” (URAGG) 6.15 on page 407
⇐ “HomogeneousAggregate” (HOAGG) 4.12 on page 139

Exports:

any?	child?	children	coerce	copy
count	cyclic?	distance	empty	empty?
eq?	eval	every?	hash	latex
leaf?	leaves	less?	map	map!
member?	members	more?	nodes	node?
parts	sample	setchildren!	setelt	setvalue!
size?	value	?.value	?~=?	#?
?=?				

Attributes exported:

- **nil**

Attributes Used:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
children : % -> List %
cyclic? : % -> Boolean
distance : (%,%) -> Integer
nodes : % -> List %
leaf? : % -> Boolean
leaves : % -> List S
node? : (%,%) -> Boolean if S has SETCAT
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
value : % -> S
```

These are implemented by this category:

```
child? : (%,%) -> Boolean if S has SETCAT
setelt : (%,value,S) -> S if $ has shallowlyMutable
?.value : (%,value) -> S
```

These exports come from (p139) HomogeneousAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
      if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
      if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
      if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
```

```
      if S has EVALAB S and S has SETCAT
 every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
 hash : % -> SingleInteger if S has SETCAT
 latex : % -> String if S has SETCAT
 less? : (%,NonNegativeInteger) -> Boolean
 map : ((S -> S),%) -> %
 map! : ((S -> S),%) -> % if $ has shallowlyMutable
 member? : (S,%) -> Boolean
           if S has SETCAT and $ has finiteAggregate
 members : % -> List S if $ has finiteAggregate
 more? : (%,NonNegativeInteger) -> Boolean
 parts : % -> List S if $ has finiteAggregate
 sample : () -> %
 size? : (%,NonNegativeInteger) -> Boolean
 ?~=? : (%,%) -> Boolean if S has SETCAT
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category RCAGG RecursiveAggregate*⟩≡

```
 )abbrev category RCAGG RecursiveAggregate
 ++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
 ++ Date Created: August 87 through August 88
 ++ Date Last Updated: April 1991
 ++ Basic Operations:
 ++ Related Constructors:
 ++ Also See:
 ++ AMS Classifications:
 ++ Keywords:
 ++ References:
 ++ Description:
 ++ A recursive aggregate over a type S is a model for a
 ++ a directed graph containing values of type S.
 ++ Recursively, a recursive aggregate is a {\em node}
 ++ consisting of a \spadfun{value} from S and 0 or more \spadfun{children}
 ++ which are recursive aggregates.
 ++ A node with no children is called a \spadfun{leaf} node.
 ++ A recursive aggregate may be cyclic for which some operations as noted
 ++ may go into an infinite loop.
 RecursiveAggregate(S:Type): Category == HomogeneousAggregate(S) with
    children: % -> List %
      ++ children(u) returns a list of the children of aggregate u.
    -- should be % -> %* and also needs children: % -> Iterator(S,S)
    nodes: % -> List %
      ++ nodes(u) returns a list of all of the nodes of aggregate u.
    -- to become % -> %* and also nodes: % -> Iterator(S,S)
    leaf?: % -> Boolean
      ++ leaf?(u) tests if u is a terminal node.
    value: % -> S
```

```
      ++ value(u) returns the value of the node u.
    elt: (%,"value") -> S
      ++ elt(u,"value") (also written: \axiom{a. value}) is
      ++ equivalent to \axiom{value(a)}.
    cyclic?: % -> Boolean
      ++ cyclic?(u) tests if u has a cycle.
    leaves: % -> List S
      ++ leaves(t) returns the list of values in obtained by visiting the
      ++ nodes of tree \axiom{t} in left-to-right order.
    distance: (%,%) -> Integer
      ++ distance(u,v) returns the path length (an integer) from node u to v.
    if S has SetCategory then
       child?: (%,%) -> Boolean
         ++ child?(u,v) tests if node u is a child of node v.
       node?: (%,%) -> Boolean
         ++ node?(u,v) tests if node u is contained in node v
         ++ (either as a child, a child of a child, etc.).
    if % has shallowlyMutable then
       setchildren_!: (%,List %)->%
         ++ setchildren!(u,v) replaces the current children of node u
         ++ with the members of v in left-to-right order.
       setelt: (%,"value",S) -> S
         ++ setelt(a,"value",x) (also written \axiom{a . value := x})
         ++ is equivalent to \axiom{setvalue!(a,x)}
       setvalue_!: (%,S) -> S
         ++ setvalue!(u,x) sets the value of node u to x.
  add
    elt(x,"value") == value x
    if % has shallowlyMutable then
      setelt(x,"value",y) == setvalue_!(x,y)
    if S has SetCategory then
      child?(x,l) == member?(x,children(l))
```

⟨*RCAGG.dotabb*⟩≡

```
"RCAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=RCAGG"];
"RCAGG" -> "HOAGG"
```

⟨*RCAGG.dotfull*⟩≡

```
"RecursiveAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RCAGG"];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"
```

⟨*RCAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

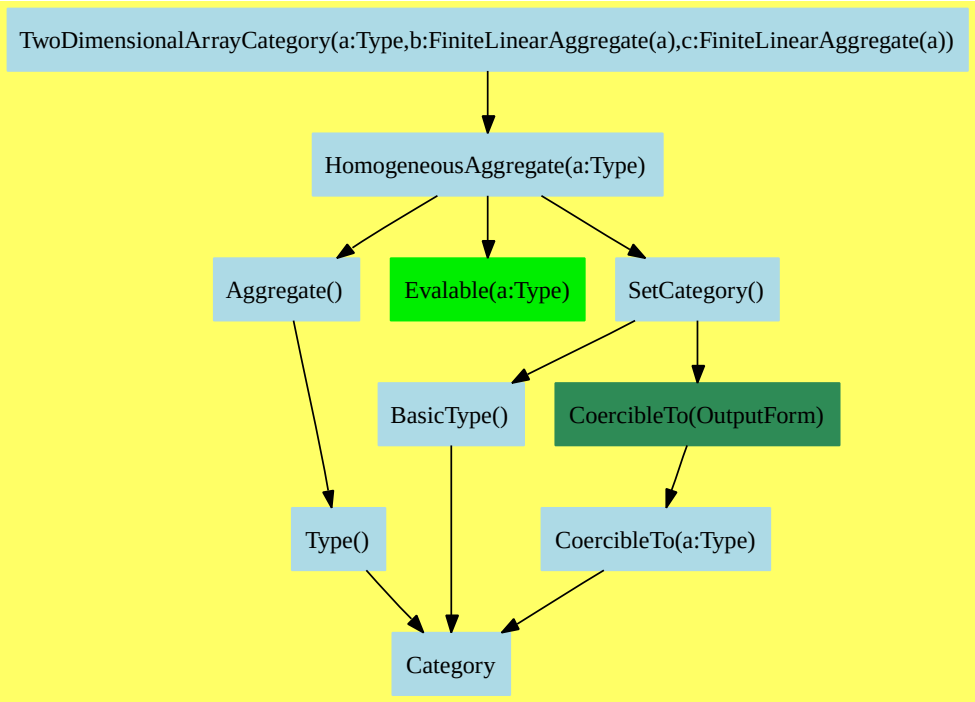
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

5.13 TwoDimensionalArrayCategory (ARR2CAT)



TwoDimensionalArrayCategory is a general array category which allows different representations and indexing schemes. Rows and columns may be extracted with rows returned as objects of type Row and columns returned as objects of type Col. The index of the 'first' row may be obtained by calling the function 'minRowIndex'. The index of the 'first' column may be obtained by calling the function 'minColIndex'. The index of the first element of a 'Row' is the same as the index of the first column in an array and vice versa.

See:

⇒ "MatrixCategory" (MATCAT) 6.7 on page 315
⇐ "HomogeneousAggregate" (HOAGG) 4.12 on page 139

Exports:

any?	column	coerce	copy	count
elt	empty	empty?	eq?	eval
every?	fill!	hash	latex	less?
map	map!	maxColIndex	maxRowIndex	member?
members	minColIndex	minRowIndex	more?	ncols
new	nrows	parts	qelt	qsetelt!
row	sample	setColumn!	setRow!	setelt
size?	#?	?=?	?~=?	

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

Attributes Used:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
elt : (%,Integer,Integer) -> R
maxColIndex : % -> Integer
maxRowIndex : % -> Integer
minColIndex : % -> Integer
minRowIndex : % -> Integer
new : (NonNegativeInteger,NonNegativeInteger,R) -> %
ncols : % -> NonNegativeInteger
nrows : % -> NonNegativeInteger
qelt : (%,Integer,Integer) -> R
qsetelt! : (%,Integer,Integer,R) -> R
setelt : (%,Integer,Integer,R) -> R
```

These are implemented by this category:

```
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
coerce : % -> OutputForm if R has SETCAT
column : (%,Integer) -> Col
copy : % -> %
count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
elt : (%,Integer,Integer,R) -> R
every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
fill! : (%,R) -> %
less? : (%,NonNegativeInteger) -> Boolean
map : ((R -> R),%) -> %
map : (((R,R) -> R),%,%) -> %
map : (((R,R) -> R),%,%,R) -> %
map! : ((R -> R),%) -> %
member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
```

```
parts : % -> List R
row : (%,Integer) -> Row
setColumn! : (%,Integer,Col) -> %
setRow! : (%,Integer,Row) -> %
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if R has SETCAT
```

These exports come from (p139) HomogeneousAggregate(R:Type)

```
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
hash : % -> SingleInteger if R has SETCAT
latex : % -> String if R has SETCAT
members : % -> List R if $ has finiteAggregate
sample : () -> %
?~=? : (%,%) -> Boolean if R has SETCAT
```

⟨*category ARR2CAT TwoDimensionalArrayCategory*⟩≡

```
)abbrev category ARR2CAT TwoDimensionalArrayCategory
++ Two dimensional array categories and domains
++ Author:
++ Date Created: 27 October 1989
++ Date Last Updated: 27 June 1990
++ Keywords: array, data structure
++ Examples:
++ References:
TwoDimensionalArrayCategory(R,Row,Col): Category == Definition where
 R   : Type
 Row : FiniteLinearAggregate R
 Col : FiniteLinearAggregate R

 Definition == HomogeneousAggregate(R) with

   shallowlyMutable
    ++ one may destructively alter arrays

   finiteAggregate
    ++ two-dimensional arrays are finite

 --% Array creation

   new: (NonNegativeInteger,NonNegativeInteger,R) -> %
```

```
     ++ new(m,n,r) is an m-by-n array all of whose entries are r
     ++
     ++X arr : ARRAY2 INT := new(5,4,0)

    fill_!: (%,R) -> %
     ++ fill!(m,r) fills m with r's
     ++
     ++X arr : ARRAY2 INT := new(5,4,0)
     ++X fill!(arr,10)

  --% Size inquiries

    minRowIndex : % -> Integer
     ++ minRowIndex(m) returns the index of the 'first' row of the array m
     ++
     ++X arr : ARRAY2 INT := new(5,4,10)
     ++X minRowIndex(arr)

    maxRowIndex : % -> Integer
     ++ maxRowIndex(m) returns the index of the 'last' row of the array m
     ++
     ++X arr : ARRAY2 INT := new(5,4,10)
     ++X maxRowIndex(arr)

    minColIndex : % -> Integer
     ++ minColIndex(m) returns the index of the 'first' column of the array m
     ++
     ++X arr : ARRAY2 INT := new(5,4,10)
     ++X minColIndex(arr)

    maxColIndex : % -> Integer
     ++ maxColIndex(m) returns the index of the 'last' column of the array m
     ++
     ++X arr : ARRAY2 INT := new(5,4,10)
     ++X maxColIndex(arr)

    nrows : % -> NonNegativeInteger
     ++ nrows(m) returns the number of rows in the array m
     ++
     ++X arr : ARRAY2 INT := new(5,4,10)
     ++X nrows(arr)

    ncols : % -> NonNegativeInteger
     ++ ncols(m) returns the number of columns in the array m
     ++
     ++X arr : ARRAY2 INT := new(5,4,10)
```

```
     ++X ncols(arr)

  --% Part extractions

     elt: (%,Integer,Integer) -> R
      ++ elt(m,i,j) returns the element in the ith row and jth
      ++ column of the array m
      ++ error check to determine if indices are in proper ranges
      ++
      ++X arr : ARRAY2 INT := new(5,4,10)
      ++X elt(arr,1,1)

     qelt: (%,Integer,Integer) -> R
      ++ qelt(m,i,j) returns the element in the ith row and jth
      ++ column of the array m
      ++ NO error check to determine if indices are in proper ranges
      ++
      ++X arr : ARRAY2 INT := new(5,4,10)
      ++X qelt(arr,1,1)

     elt: (%,Integer,Integer,R) -> R
      ++ elt(m,i,j,r) returns the element in the ith row and jth
      ++ column of the array m, if m has an ith row and a jth column,
      ++ and returns r otherwise
      ++
      ++X arr : ARRAY2 INT := new(5,4,10)
      ++X elt(arr,1,1,6)
      ++X elt(arr,1,10,6)

     row: (%,Integer) -> Row
      ++ row(m,i) returns the ith row of m
      ++ error check to determine if index is in proper ranges
      ++
      ++X arr : ARRAY2 INT := new(5,4,10)
      ++X row(arr,1)

     column: (%,Integer) -> Col
      ++ column(m,j) returns the jth column of m
      ++ error check to determine if index is in proper ranges
      ++
      ++X arr : ARRAY2 INT := new(5,4,10)
      ++X column(arr,1)

     parts: % -> List R
      ++ parts(m) returns a list of the elements of m in row major order
      ++
```

```
     ++X arr : ARRAY2 INT := new(5,4,10)
     ++X parts(arr)

  --% Part assignments

    setelt: (%,Integer,Integer,R) -> R
     -- will become setelt_!
     ++ setelt(m,i,j,r) sets the element in the ith row and jth
     ++ column of m to r
     ++ error check to determine if indices are in proper ranges
     ++
     ++X arr : ARRAY2 INT := new(5,4,0)
     ++X setelt(arr,1,1,17)

    qsetelt_!: (%,Integer,Integer,R) -> R
     ++ qsetelt!(m,i,j,r) sets the element in the ith row and jth
     ++ column of m to r
     ++ NO error check to determine if indices are in proper ranges
     ++
     ++X arr : ARRAY2 INT := new(5,4,0)
     ++X qsetelt!(arr,1,1,17)

    setRow_!: (%,Integer,Row) -> %
     ++ setRow!(m,i,v) sets to ith row of m to v
     ++
     ++X T1:=TwoDimensionalArray Integer
     ++X arr:T1:= new(5,4,0)
     ++X T2:=OneDimensionalArray Integer
     ++X arow:=construct([1,2,3,4]::List(INT))$T2
     ++X setRow!(arr,1,arow)$T1

    setColumn_!: (%,Integer,Col) -> %
     ++ setColumn!(m,j,v) sets to jth column of m to v
     ++
     ++X T1:=TwoDimensionalArray Integer
     ++X arr:T1:= new(5,4,0)
     ++X T2:=OneDimensionalArray Integer
     ++X acol:=construct([1,2,3,4,5]::List(INT))$T2
     ++X setColumn!(arr,1,acol)$T1

  --% Map and Zip

    map: (R -> R,%) -> %
     ++ map(f,a) returns \spad{b}, where \spad{b(i,j) = f(a(i,j))}
     ++ for all \spad{i, j}
     ++
```

```
      ++X arr : ARRAY2 INT := new(5,4,10)
      ++X map(-,arr)
      ++X map((x +-> x + x),arr)

     map_!: (R -> R,%) -> %
      ++ map!(f,a)  assign \spad{a(i,j)} to \spad{f(a(i,j))}
      ++ for all \spad{i, j}
      ++X arr : ARRAY2 INT := new(5,4,10)
      ++X map!(-,arr)

     map:((R,R) -> R,%,%) -> %
      ++ map(f,a,b) returns \spad{c}, where \spad{c(i,j) = f(a(i,j),b(i,j))}
      ++ for all \spad{i, j}
      ++
      ++X adder(a:Integer,b:Integer):Integer == a+b
      ++X arr : ARRAY2 INT := new(5,4,10)
      ++X map(adder,arr,arr)

     map:((R,R) -> R,%,%,R) -> %
      ++ map(f,a,b,r) returns \spad{c}, where \spad{c(i,j) = f(a(i,j),b(i,j))}
      ++ when both \spad{a(i,j)} and \spad{b(i,j)} exist;
      ++ else \spad{c(i,j) = f(r, b(i,j))} when \spad{a(i,j)} does not exist;
      ++ else \spad{c(i,j) = f(a(i,j),r)} when \spad{b(i,j)} does not exist;
      ++ otherwise \spad{c(i,j) = f(r,r)}.
      ++
      ++X adder(a:Integer,b:Integer):Integer == a+b
      ++X arr1 : ARRAY2 INT := new(5,4,10)
      ++X arr2 : ARRAY2 INT := new(3,3,10)
      ++X map(adder,arr1,arr2,17)

    add

  --% Predicates

      any?(f,m) ==
        for i in minRowIndex(m)..maxRowIndex(m) repeat
          for j in minColIndex(m)..maxColIndex(m) repeat
            f(qelt(m,i,j)) => return true
        false

      every?(f,m) ==
        for i in minRowIndex(m)..maxRowIndex(m) repeat
          for j in minColIndex(m)..maxColIndex(m) repeat
            not f(qelt(m,i,j)) => return false
        true
```

```
    size?(m,n) == nrows(m) * ncols(m) = n
    less?(m,n) == nrows(m) * ncols(m) < n
    more?(m,n) == nrows(m) * ncols(m) > n

--% Size inquiries

    # m == nrows(m) * ncols(m)

--% Part extractions

    elt(m,i,j,r) ==
      i < minRowIndex(m) or i > maxRowIndex(m) => r
      j < minColIndex(m) or j > maxColIndex(m) => r
      qelt(m,i,j)

    count(f:R -> Boolean,m:%) ==
      num : NonNegativeInteger := 0
      for i in minRowIndex(m)..maxRowIndex(m) repeat
        for j in minColIndex(m)..maxColIndex(m) repeat
          if f(qelt(m,i,j)) then num := num + 1
      num

    parts m ==
      entryList : List R := nil()
      for i in maxRowIndex(m)..minRowIndex(m) by -1 repeat
        for j in maxColIndex(m)..minColIndex(m) by -1 repeat
          entryList := concat(qelt(m,i,j),entryList)
      entryList

--% Creation

    copy m ==
      ans := new(nrows m,ncols m,NIL$Lisp)
      for i in minRowIndex(m)..maxRowIndex(m) repeat
        for j in minColIndex(m)..maxColIndex(m) repeat
          qsetelt_!(ans,i,j,qelt(m,i,j))
      ans

    fill_!(m,r) ==
      for i in minRowIndex(m)..maxRowIndex(m) repeat
        for j in minColIndex(m)..maxColIndex(m) repeat
          qsetelt_!(m,i,j,r)
      m

    map(f,m) ==
      ans := new(nrows m,ncols m,NIL$Lisp)
```

```
  for i in minRowIndex(m)..maxRowIndex(m) repeat
    for j in minColIndex(m)..maxColIndex(m) repeat
      qsetelt_!(ans,i,j,f(qelt(m,i,j)))
  ans

map_!(f,m) ==
  for i in minRowIndex(m)..maxRowIndex(m) repeat
    for j in minColIndex(m)..maxColIndex(m) repeat
      qsetelt_!(m,i,j,f(qelt(m,i,j)))
  m

map(f,m,n) ==
  (nrows(m) ^= nrows(n)) or (ncols(m) ^= ncols(n)) =>
    error "map: arguments must have same dimensions"
  ans := new(nrows m,ncols m,NIL$Lisp)
  for i in minRowIndex(m)..maxRowIndex(m) repeat
    for j in minColIndex(m)..maxColIndex(m) repeat
      qsetelt_!(ans,i,j,f(qelt(m,i,j),qelt(n,i,j)))
  ans

map(f,m,n,r) ==
  maxRow := max(maxRowIndex m,maxRowIndex n)
  maxCol := max(maxColIndex m,maxColIndex n)
  ans := new(max(nrows m,nrows n),max(ncols m,ncols n),NIL$Lisp)
  for i in minRowIndex(m)..maxRow repeat
    for j in minColIndex(m)..maxCol repeat
      qsetelt_!(ans,i,j,f(elt(m,i,j,r),elt(n,i,j,r)))
  ans

setRow_!(m,i,v) ==
  i < minRowIndex(m) or i > maxRowIndex(m) =>
    error "setRow!: index out of range"
  for j in minColIndex(m)..maxColIndex(m) _
    for k in minIndex(v)..maxIndex(v) repeat
      qsetelt_!(m,i,j,v.k)
  m

setColumn_!(m,j,v) ==
  j < minColIndex(m) or j > maxColIndex(m) =>
    error "setColumn!: index out of range"
  for i in minRowIndex(m)..maxRowIndex(m) _
    for k in minIndex(v)..maxIndex(v) repeat
      qsetelt_!(m,i,j,v.k)
  m

if R has _= : (R,R) -> Boolean then
```

```
  m = n ==
    eq?(m,n) => true
    (nrows(m) ^= nrows(n)) or (ncols(m) ^= ncols(n)) => false
    for i in minRowIndex(m)..maxRowIndex(m) repeat
      for j in minColIndex(m)..maxColIndex(m) repeat
        not (qelt(m,i,j) = qelt(n,i,j)) => return false
    true

  member?(r,m) ==
    for i in minRowIndex(m)..maxRowIndex(m) repeat
      for j in minColIndex(m)..maxColIndex(m) repeat
        qelt(m,i,j) = r => return true
    false

  count(r:R,m:%) == count(x +-> x = r,m)

if Row has shallowlyMutable then

  row(m,i) ==
    i < minRowIndex(m) or i > maxRowIndex(m) =>
      error "row: index out of range"
    v : Row := new(ncols m,NIL$Lisp)
    for j in minColIndex(m)..maxColIndex(m) _
      for k in minIndex(v)..maxIndex(v) repeat
        qsetelt_!(v,k,qelt(m,i,j))
    v

if Col has shallowlyMutable then

  column(m,j) ==
    j < minColIndex(m) or j > maxColIndex(m) =>
      error "column: index out of range"
    v : Col := new(nrows m,NIL$Lisp)
    for i in minRowIndex(m)..maxRowIndex(m) _
      for k in minIndex(v)..maxIndex(v) repeat
        qsetelt_!(v,k,qelt(m,i,j))
    v

if R has CoercibleTo(OutputForm) then

  coerce(m:%) ==
    l : List List OutputForm
    l := [[qelt(m,i,j) :: OutputForm _
              for j in minColIndex(m)..maxColIndex(m)] _
              for i in minRowIndex(m)..maxRowIndex(m)]
```

```
        matrix l
```

⟨*ARR2CAT.dotabb*⟩≡

```
  "ARR2CAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ARR2CAT"];
  "ARR2CAT" -> "HOAGG"
```

⟨*ARR2CAT.dotfull*⟩≡

```
  "TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggr
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ARR2CAT"];
  "TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggr
      -> "HomogeneousAggregate(a:Type)"

  "TwoDimensionalArrayCategory(a:Type,d:IndexedOneDimensionalArray(a,b),e:IndexedOn
   [color=seagreen,href="bookvol10.2.pdf#nameddest=ARR2CAT"];
  "TwoDimensionalArrayCategory(a:Type,d:IndexedOneDimensionalArray(a,b),e:IndexedOn
  -> "TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearA
```

⟨*ARR2CAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))"
 [color=lightblue];
"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))"
    -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"
"HomogeneousAggregate(a:Type)" -> "Evalable(a:Type)"
"HomogeneousAggregate(a:Type)" -> "SetCategory()"

"Evalable(a:Type)" [color="#00EE00"];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

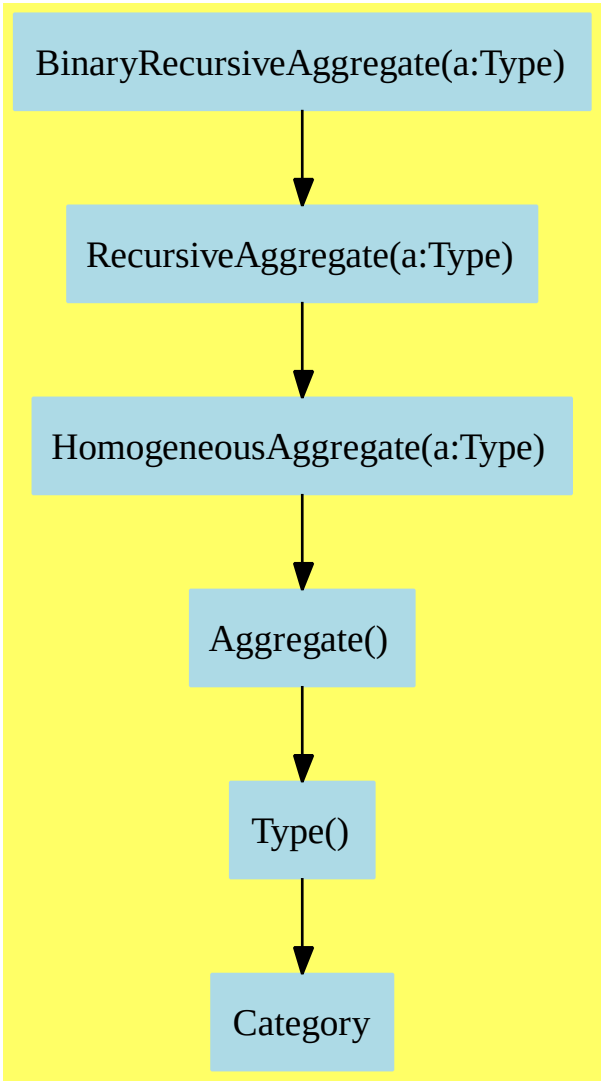
"Category" [color=lightblue];

}
```

Chapter 6

Category Layer 5

6.1 BinaryRecursiveAggregate (BRAGG)



See:

⇒ "BinaryTreeCategory" (BTCAT) 7.2 on page 423
⇐ "RecursiveAggregate" (RCAGG) 5.12 on page 263

Exports:

any?	children	child?	coerce	copy
count	cyclic?	distance	empty	empty?
eq?	eval	every?	hash	latex
leaf?	leaves	left	less?	map
map!	member?	members	more?	nodes
node?	parts	right	sample	setchildren!
setelt	setleft!	setright!	setvalue!	size?
value	#?	?=?	?~=?	?.right
?.left	?.value			

Attributes exported:

- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
left : % -> %
right : % -> %
setelt : (%,right,%) -> % if $ has shallowlyMutable
setelt : (%,left,%) -> % if $ has shallowlyMutable
setleft! : (%,%) -> % if $ has shallowlyMutable
setright! : (%,%) -> % if $ has shallowlyMutable
```

These are implemented by this category:

```
children : % -> List %
coerce : % -> OutputForm if S has SETCAT
cyclic? : % -> Boolean
leaf? : % -> Boolean
leaves : % -> List S
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
nodes : % -> List %
node? : (%,%) -> Boolean if S has SETCAT
```

```
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?.right : (%,right) -> %
?.left : (%,left) -> %
```

These exports come from (p263) RecursiveAggregate(S:Type)

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
child? : (%,%) -> Boolean if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
distance : (%,%) -> Integer
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
sample : () -> %
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,value,S) -> S if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
value : % -> S
?~=? : (%,%) -> Boolean if S has SETCAT
?.value : (%,value) -> S
```

⟨*category BRAGG BinaryRecursiveAggregate*⟩≡

```
)abbrev category BRAGG BinaryRecursiveAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
```

```
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A binary-recursive aggregate has 0, 1 or 2 children and serves
++ as a model for a binary tree or a doubly-linked aggregate structure
BinaryRecursiveAggregate(S:Type):Category == RecursiveAggregate S with
   -- needs preorder, inorder and postorder iterators
   left: % -> %
     ++ left(u) returns the left child.
   elt: (%,"left") -> %
     ++ elt(u,"left") (also written: \axiom{a . left}) is
     ++ equivalent to \axiom{left(a)}.
   right: % -> %
     ++ right(a) returns the right child.
   elt: (%,"right") -> %
     ++ elt(a,"right") (also written: \axiom{a . right})
     ++ is equivalent to \axiom{right(a)}.
   if % has shallowlyMutable then
      setelt: (%,"left",%) -> %
        ++ setelt(a,"left",b) (also written \axiom{a . left := b}) is
        ++ equivalent to \axiom{setleft!(a,b)}.
      setleft_!: (%,%) -> %
         ++ setleft!(a,b) sets the left child of \axiom{a} to be b.
      setelt: (%,"right",%) -> %
         ++ setelt(a,"right",b) (also written \axiom{b . right := b})
         ++ is equivalent to \axiom{setright!(a,b)}.
      setright_!: (%,%) -> %
         ++ setright!(a,x) sets the right child of t to be x.
 add
   cycleMax ==> 1000

   elt(x,"left")  == left x
   elt(x,"right") == right x
   leaf? x == empty? x or empty? left x and empty? right x
   leaves t ==
     empty? t => empty()$List(S)
     leaf? t => [value t]
     concat(leaves left t,leaves right t)
   nodes x ==
     l := empty()$List(%)
     empty? x => l
     concat(nodes left x,concat([x],nodes right x))
```

```
children x ==
  l := empty()$List(%)
  empty? x => l
  empty? left x  => [right x]
  empty? right x => [left x]
  [left x, right x]
if % has SetAggregate(S) and S has SetCategory then
  node?(u,v) ==
    empty? v => false
    u = v => true
    for y in children v repeat node?(u,y) => return true
    false
  x = y ==
    empty?(x) => empty?(y)
    empty?(y) => false
    value x = value y and left x = left y and right x = right y
  if % has finiteAggregate then
    member?(x,u) ==
      empty? u => false
      x = value u => true
      member?(x,left u) or member?(x,right u)

if S has SetCategory then
  coerce(t:%): OutputForm ==
    empty? t =>  "[]"::OutputForm
    v := value(t):: OutputForm
    empty? left t =>
      empty? right t => v
      r := coerce(right t)@OutputForm
      bracket [".":: OutputForm, v, r]
    l := coerce(left t)@OutputForm
    r :=
      empty? right t => ".":: OutputForm
      coerce(right t)@OutputForm
    bracket [l, v, r]

if % has finiteAggregate then
  aggCount: (%,NonNegativeInteger) -> NonNegativeInteger
  #x == aggCount(x,0)
  aggCount(x,k) ==
    empty? x => 0
    k := k + 1
    k = cycleMax and cyclic? x => error "cyclic tree"
    for y in children x repeat k := aggCount(y,k)
    k
```

```
    isCycle?:  (%, List %) -> Boolean
    eqMember?: (%, List %) -> Boolean
    cyclic? x     == not empty? x and isCycle?(x,empty()$(List %))
    isCycle?(x,acc) ==
      empty? x => false
      eqMember?(x,acc) => true
      for y in children x | not empty? y repeat
        isCycle?(y,acc) => return true
      false
    eqMember?(y,l) ==
      for x in l repeat eq?(x,y) => return true
      false
    if % has shallowlyMutable then
      setelt(x,"left",b)  == setleft_!(x,b)
      setelt(x,"right",b) == setright_!(x,b)
```

⟨*BRAGG.dotabb*⟩≡

```
  "BRAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=BRAGG"];
  "BRAGG" -> "RCAGG"
```

⟨*BRAGG.dotfull*⟩≡

```
  "BinaryRecursiveAggregate(a:Type)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=BRAGG"];
  "BinaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

  "BinaryRecursiveAggregate(a:SetCategory)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=BRAGG"];
  "BinaryRecursiveAggregate(a:SetCategory)" ->
     "BinaryRecursiveAggregate(a:Type)"
```

⟨*BRAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"BinaryRecursiveAggregate(a:Type)" [color=lightblue];
"BinaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

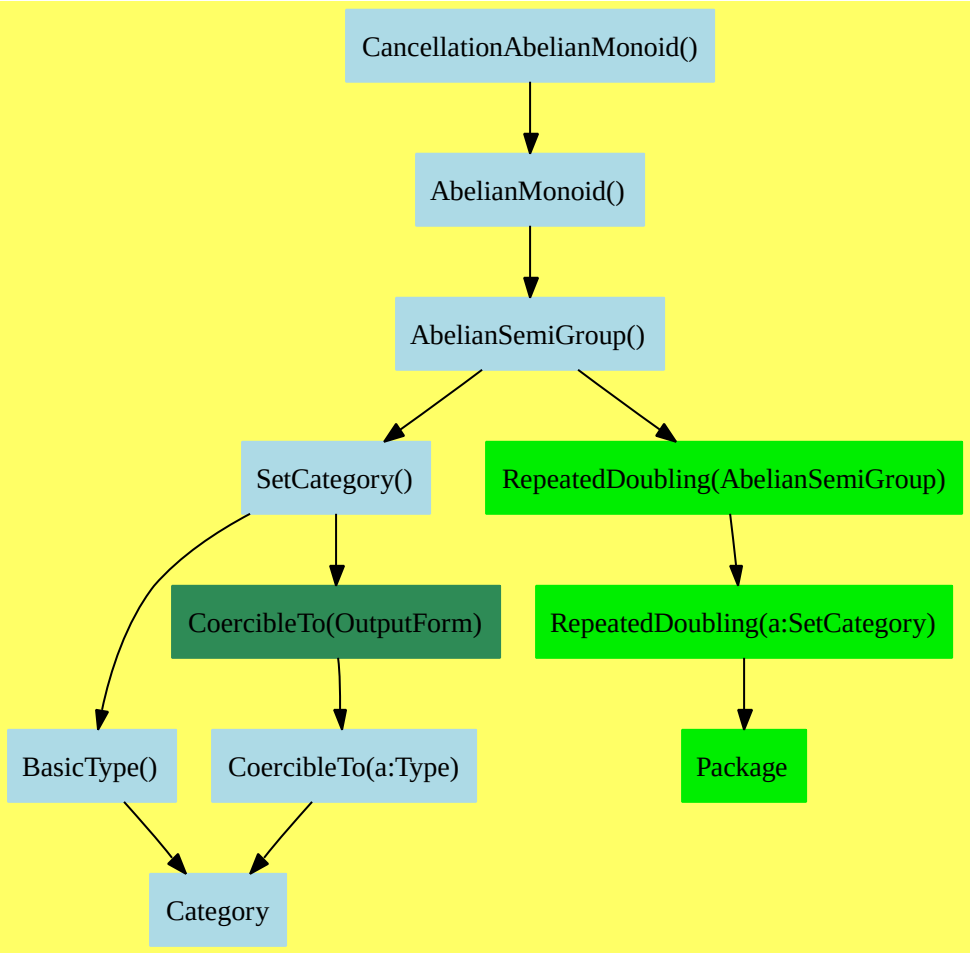
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

6.2 CancellationAbelianMonoid (CABMON)



See:

⇒ "AbelianGroup" (ABELGRP) 7.1 on page 418
⇒ "FreeAbelianMonoidCategory" (FAMONC) 7.7 on page 451
⇒ "OrderedCancellationAbelianMonoid" (OCAMON) 8.10 on page 567
⇐ "AbelianMonoid" (ABELMON) 5.1 on page 207

Exports:

0	coerce	hash	latex	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?=?				

These are directly exported but not implemented:

```
subtractIfCan : (%,%) -> Union(%,"failed")
```

These exports come from (p207) AbelianMonoid():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
zero? : % -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?~=? : (%,%) -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
```

⟨*category CABMON CancellationAbelianMonoid*⟩≡

```
)abbrev category CABMON CancellationAbelianMonoid
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References: Davenport & Trager I
++ Description:
++ This is an \spadtype{AbelianMonoid} with the cancellation property, i.e.
++ \spad{ a+b = a+c => b=c }.
++ This is formalised by the partial subtraction operator,
++ which satisfies the axioms listed below:
++
++ Axioms:
++   \spad{c = a+b <=> c-b = a}
CancellationAbelianMonoid(): Category == AbelianMonoid with
      subtractIfCan: (%,%) -> Union(%,"failed")
         ++ subtractIfCan(x, y) returns an element z such that \spad{z+y=x}
         ++ or "failed" if no such element exists.
```

⟨*CABMON.dotabb*⟩≡

```
"CABMON"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CABMON"];
"CABMON" -> "ABELMON"
```

⟨*CABMON.dotfull*⟩≡

```
"CancellationAbelianMonoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CABMON"];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"
```

⟨*CABMON.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SetCategory()"
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

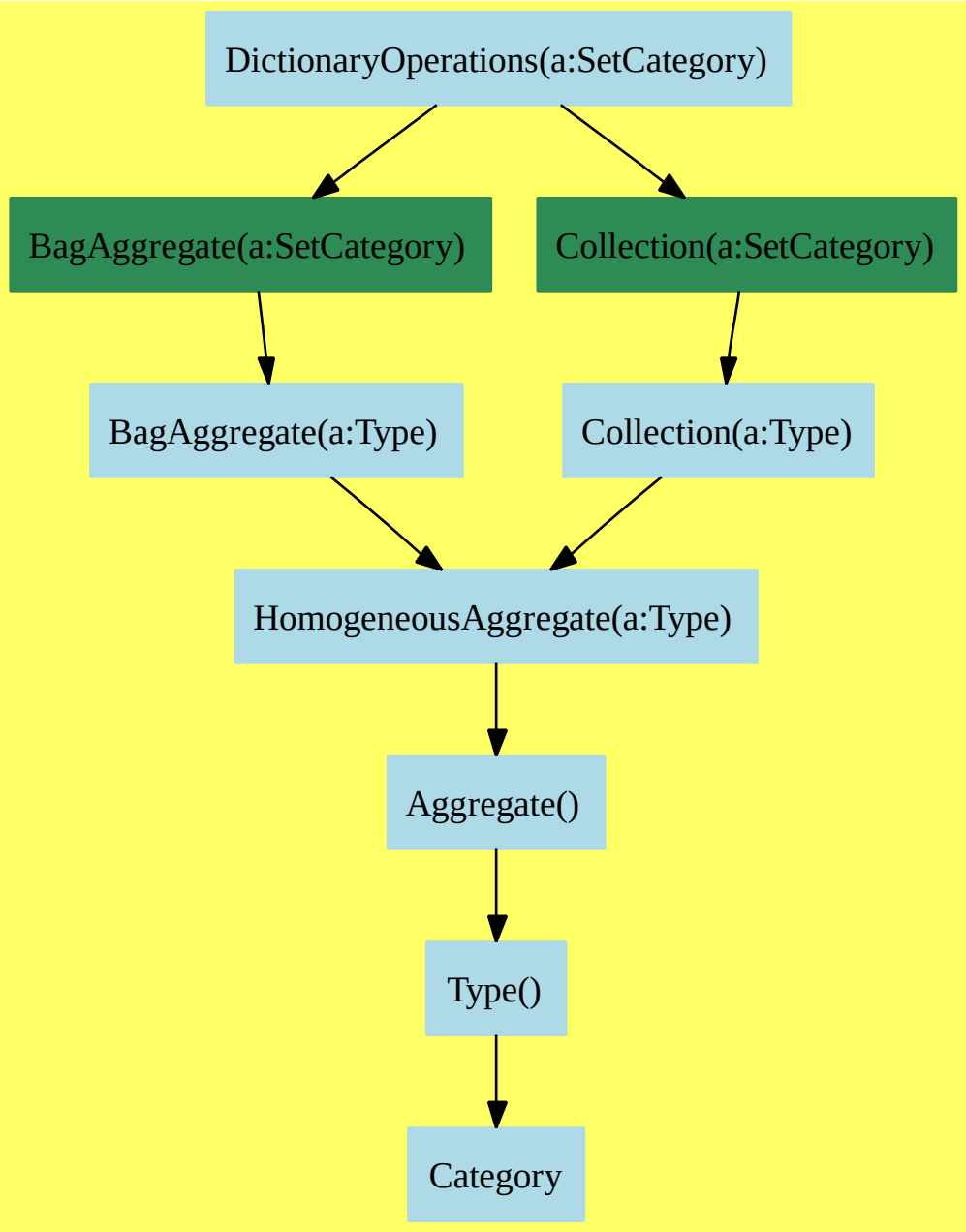
"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];
}
```

6.3 DictionaryOperations (DIOPS)



See:

⇒ “Dictionary” (DIAGG) 7.3 on page 428

⇒ "MultiDictionary" (MDAGG) 7.8 on page 456
⇐ "BagAggregate" (BGAGG) 5.2 on page 211
⇐ "Collection" (CLAGG) 5.4 on page 218

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	empty	empty?
eq?	eval	every?	extract!	find
hash	insert!	inspect	latex	less?
map	map!	member?	members	more?
parts	reduce	remove	remove!	removeDuplicates
sample	select	select!	size?	#?
?=?	?~=?			

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are directly exported but not implemented:

```
dictionary : List S -> %
remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
remove! : (S,%) -> % if $ has finiteAggregate
select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

These are implemented by this category:

```
coerce : % -> OutputForm if S has SETCAT
construct : List S -> %
copy : % -> %
dictionary : () -> %
```

These exports come from (p211) BagAggregate(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
```

```
          if $ has finiteAggregate
 empty : () -> %
 empty? : % -> Boolean
 eq? : (%,%) -> Boolean
 eval : (%,List S,List S) -> %
          if S has EVALAB S and S has SETCAT
 eval : (%,S,S) -> %
          if S has EVALAB S and S has SETCAT
 eval : (%,Equation S) -> %
          if S has EVALAB S and S has SETCAT
 eval : (%,List Equation S) -> %
          if S has EVALAB S and S has SETCAT
 every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
 extract! : % -> S
 hash : % -> SingleInteger if S has SETCAT
 insert! : (S,%) -> %
 inspect : % -> S
 latex : % -> String if S has SETCAT
 less? : (%,NonNegativeInteger) -> Boolean
 map : ((S -> S),%) -> %
 map! : ((S -> S),%) -> % if $ has shallowlyMutable
 member? : (S,%) -> Boolean
          if S has SETCAT and $ has finiteAggregate
 members : % -> List S if $ has finiteAggregate
 more? : (%,NonNegativeInteger) -> Boolean
 parts : % -> List S if $ has finiteAggregate
 sample : () -> %
 size? : (%,NonNegativeInteger) -> Boolean
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?=? : (%,%) -> Boolean if S has SETCAT
 ?~=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p218) Collection(S:SetCategory)

```
 convert : % -> InputForm if S has KONVERT INFORM
 find : ((S -> Boolean),%) -> Union(S,"failed")
 reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
 reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
 reduce : (((S,S) -> S),%,S,S) -> S
                 if S has SETCAT and $ has finiteAggregate
 remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
 remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
 removeDuplicates : % -> %
          if S has SETCAT and $ has finiteAggregate
 select : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

⟨*category DIOPS DictionaryOperations*⟩≡

```
 )abbrev category DIOPS DictionaryOperations
 ++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
 ++ Date Created: August 87 through August 88
```

```
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This category is a collection of operations common to both
++ categories \spadtype{Dictionary} and \spadtype{MultiDictionary}
DictionaryOperations(S:SetCategory): Category ==
  Join(BagAggregate S, Collection(S)) with
   dictionary: () -> %
     ++ dictionary()$D creates an empty dictionary of type D.
   dictionary: List S -> %
     ++ dictionary([x,y,...,z]) creates a dictionary consisting of
     ++ entries \axiom{x,y,...,z}.
-- insert: (S,%) -> S                 ++ insert an entry
-- member?: (S,%) -> Boolean                  ++ search for an entry
-- remove_!: (S,%,NonNegativeInteger) -> %
--   ++ remove!(x,d,n) destructively changes dictionary d by removing
--   ++ up to n entries y such that \axiom{y = x}.
-- remove_!: (S->Boolean,%,NonNegativeInteger) -> %
--   ++ remove!(p,d,n) destructively changes dictionary d by removing
--   ++ up to n entries x such that \axiom{p(x)} is true.
   if % has finiteAggregate then
     remove_!: (S,%) -> %
       ++ remove!(x,d) destructively changes dictionary d by removing
       ++ all entries y such that \axiom{y = x}.
     remove_!: (S->Boolean,%) -> %
       ++ remove!(p,d) destructively changes dictionary d by removeing
       ++ all entries x such that \axiom{p(x)} is true.
     select_!: (S->Boolean,%) -> %
       ++ select!(p,d) destructively changes dictionary d by removing
       ++ all entries x such that \axiom{p(x)} is not true.
 add
   construct l == dictionary l
   dictionary() == empty()
   if % has finiteAggregate then
     copy d == dictionary parts d
     coerce(s:%):OutputForm ==
       prefix("dictionary"@String :: OutputForm,
                                      [x::OutputForm for x in parts s])
```

⟨*DIOPS.dotabb*⟩≡
```
"DIOPS" [color=lightblue,href="bookvol10.2.pdf#nameddest=DIOPS"];
"DIOPS" -> "BGAGG"
"DIOPS" -> "CLAGG"
```

⟨*DIOPS.dotfull*⟩≡
```
"DictionaryOperations(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIOPS"];
"DictionaryOperations(a:SetCategory)" -> "BagAggregate(a:SetCategory)"
"DictionaryOperations(a:SetCategory)" -> "Collection(a:SetCategory)"
```

⟨*DIOPS.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BagAggregate(a:SetCategory)"
"DictionaryOperations(a:SetCategory)" -> "Collection(a:SetCategory)"

"BagAggregate(a:SetCategory)" [color=seagreen];
"BagAggregate(a:SetCategory)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"Collection(a:SetCategory)" [color=seagreen];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

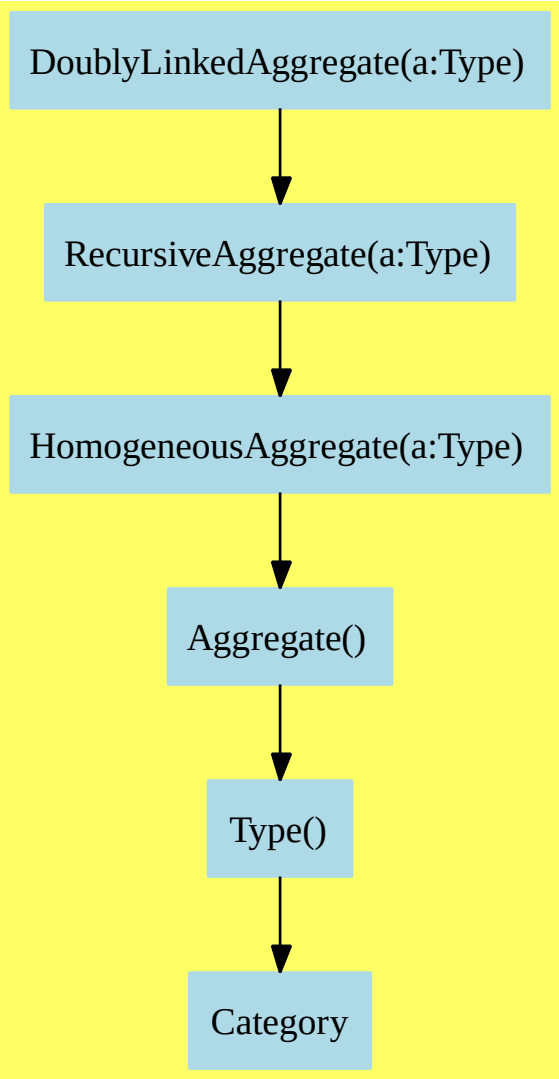
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

6.4 DoublyLinkedAggregate (DLAGG)



See:

⇐ “RecursiveAggregate” (RCAGG) 5.12 on page 263

Exports:

any?	children	child?	coerce	concat!
copy	count	cyclic?	distance	empty
empty?	eq?	eval	every?	hash
head	last	latex	leaf?	leaves
less?	map	map!	member?	members
more?	next	nodes	node?	parts
previous	sample	setchildren!	setelt	setnext!
setprevious!	setvalue!	size?	tail	value
#?	?=?	?~=?	?.value	

Attributes exported:

- **nil**

Attributes Used:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
concat! : (%,%) -> % if $ has shallowlyMutable
head : % -> %
last : % -> S
next : % -> %
previous : % -> %
setnext! : (%,%) -> % if $ has shallowlyMutable
setprevious! : (%,%) -> % if $ has shallowlyMutable
tail : % -> %
```

These exports come from (p263) RecursiveAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
children : % -> List %
child? : (%,%) -> Boolean if S has SETCAT
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
cyclic? : % -> Boolean
distance : (%,%) -> Integer
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
```

```
          if S has EVALAB S and S has SETCAT
 eval : (%,S,S) -> %
          if S has EVALAB S and S has SETCAT
 eval : (%,Equation S) -> %
          if S has EVALAB S and S has SETCAT
 eval : (%,List Equation S) -> %
          if S has EVALAB S and S has SETCAT
 every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
 hash : % -> SingleInteger if S has SETCAT
 latex : % -> String if S has SETCAT
 leaf? : % -> Boolean
 leaves : % -> List S
 less? : (%,NonNegativeInteger) -> Boolean
 map : ((S -> S),%) -> %
 map! : ((S -> S),%) -> % if $ has shallowlyMutable
 member? : (S,%) -> Boolean
          if S has SETCAT and $ has finiteAggregate
 members : % -> List S if $ has finiteAggregate
 more? : (%,NonNegativeInteger) -> Boolean
 nodes : % -> List %
 node? : (%,%) -> Boolean if S has SETCAT
 parts : % -> List S if $ has finiteAggregate
 sample : () -> %
 setchildren! : (%,List %) -> % if $ has shallowlyMutable
 setelt : (%,value,S) -> S if $ has shallowlyMutable
 setvalue! : (%,S) -> S if $ has shallowlyMutable
 size? : (%,NonNegativeInteger) -> Boolean
 value : % -> S
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?=? : (%,%) -> Boolean if S has SETCAT
 ?~=? : (%,%) -> Boolean if S has SETCAT
 ?.value : (%,value) -> S
```

⟨*category DLAGG DoublyLinkedAggregate*⟩≡

```
 )abbrev category DLAGG DoublyLinkedAggregate
 ++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
 ++ Date Created: August 87 through August 88
 ++ Date Last Updated: April 1991
 ++ Basic Operations:
 ++ Related Constructors:
 ++ Also See:
 ++ AMS Classifications:
 ++ Keywords:
 ++ References:
 ++ Description:
 ++ A doubly-linked aggregate serves as a model for a doubly-linked
 ++ list, that is, a list which can has links to both next and previous
 ++ nodes and thus can be efficiently traversed in both directions.
 DoublyLinkedAggregate(S:Type): Category == RecursiveAggregate S with
```

```
    last: % -> S
      ++ last(l) returns the last element of a doubly-linked aggregate l.
      ++ Error: if l is empty.
    head: % -> %
      ++ head(l) returns the first element of a doubly-linked aggregate l.
      ++ Error: if l is empty.
    tail: % -> %
      ++ tail(l) returns the doubly-linked aggregate l starting at
      ++ its second element.
      ++ Error: if l is empty.
    previous: % -> %
      ++ previous(l) returns the doubly-link list beginning with its previous
      ++ element.
      ++ Error: if l has no previous element.
      ++ Note: \axiom{next(previous(l)) = l}.
    next: % -> %
      ++ next(l) returns the doubly-linked aggregate beginning with its next
      ++ element.
      ++ Error: if l has no next element.
      ++ Note: \axiom{next(l) = rest(l)} and \axiom{previous(next(l)) = l}.
    if % has shallowlyMutable then
       concat_!: (%,%) -> %
         ++ concat!(u,v) destructively concatenates doubly-linked aggregate v
         ++ to the end of doubly-linked aggregate u.
       setprevious_!: (%,%) -> %
         ++ setprevious!(u,v) destructively sets the previous node of
         ++ doubly-linked aggregate u to v, returning v.
       setnext_!: (%,%) -> %
         ++ setnext!(u,v) destructively sets the next node of doubly-linked
         ++ aggregate u to v, returning v.
```

⟨*DLAGG.dotabb*⟩≡

```
"DLAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=DLAGG"];
"DLAGG" -> "RCAGG"
```

⟨*DLAGG.dotfull*⟩≡

```
"DoublyLinkedAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DLAGG"];
"DoublyLinkedAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"
```

⟨*DLAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DoublyLinkedAggregate(a:Type)" [color=lightblue];
"DoublyLinkedAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

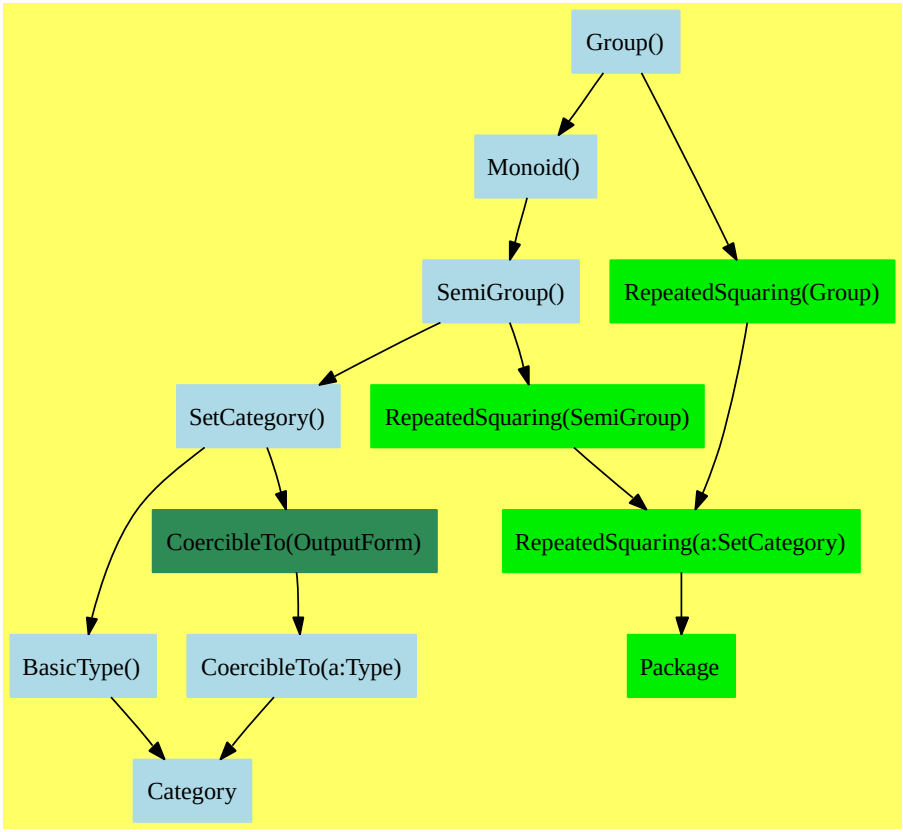
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

6.5 Group (GROUP)



See:

⇒ “FunctionSpace” (FS) 17.5 on page 1095
⇒ “PermutationCategory” (PERMCAT) 7.10 on page 464
⇐ “Monoid” (MONOID) 5.10 on page 256

Exports:

1	coerce	commutator	conjugate	hash
inv	latex	one?	recip	sample
?~=?	?*?	?**?	?/?	?=?
?^?				

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return “failed” if its argument is not a unit.

These are directly exported but not implemented:

```
inv : % -> %
```

These are implemented by this category:

```
commutator : (%,%) -> %
conjugate : (%,%) -> %
recip : % -> Union(%,"failed")
?/? : (%,%) -> %
?^? : (%,Integer) -> %
?**? : (%,Integer) -> %
```

These exports come from (p256) Monoid():

```
1 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
sample : () -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category GROUP Group*⟩≡

```
)abbrev category GROUP Group
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The class of multiplicative groups, i.e. monoids with
++ multiplicative inverses.
++
++ Axioms:
++   \spad{leftInverse("*":(%,%)->%,inv)}\tab{30}\spad{ inv(x)*x = 1 }
++   \spad{rightInverse("*":(%,%)->%,inv)}\tab{30}\spad{ x*inv(x) = 1 }
Group(): Category == Monoid with
      inv: % -> %
        ++ inv(x) returns the inverse of x.
```

```
    "/": (%,%) -> %
      ++ x/y is the same as x times the inverse of y.
    "**": (%,Integer) -> %
      ++ x**n returns x raised to the integer power n.
    "^": (%,Integer) -> %
      ++ x^n returns x raised to the integer power n.
    unitsKnown
      ++ unitsKnown asserts that recip only returns
      ++ "failed" for non-units.
    conjugate: (%,%) -> %
      ++ conjugate(p,q) computes \spad{inv(q) * p * q}; this is
      ++ 'right action by conjugation'.
    commutator: (%,%) -> %
      ++ commutator(p,q) computes \spad{inv(p) * inv(q) * p * q}.
  add
    import RepeatedSquaring(%)
    x:% / y:% == x*inv(y)
    recip(x:%) == inv(x)
    _^(x:%, n:Integer):% == x ** n
    x:% ** n:Integer ==
       zero? n => 1
       n<0 => expt(inv(x),(-n) pretend PositiveInteger)
       expt(x,n pretend PositiveInteger)
    conjugate(p,q) == inv(q) * p * q
    commutator(p,q) == inv(p) * inv(q) * p * q
```

⟨*GROUP.dotabb*⟩≡

```
"GROUP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=GROUP"];
"GROUP" -> "MONOID"
```

⟨*GROUP.dotfull*⟩≡

```
"Group()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=GROUP"];
"Group()" -> "Monoid()"
```

⟨*GROUP.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Group()" [color=lightblue];
"Group()" -> "Monoid()"
"Group()" -> "RepeatedSquaring(Group)"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SetCategory()"
"SemiGroup()" -> "RepeatedSquaring(SemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedSquaring(Group)" [color="#00EE00"];
"RepeatedSquaring(Group)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(SemiGroup)" [color="#00EE00"];
"RepeatedSquaring(SemiGroup)" -> "RepeatedSquaring(a:SetCategory)"

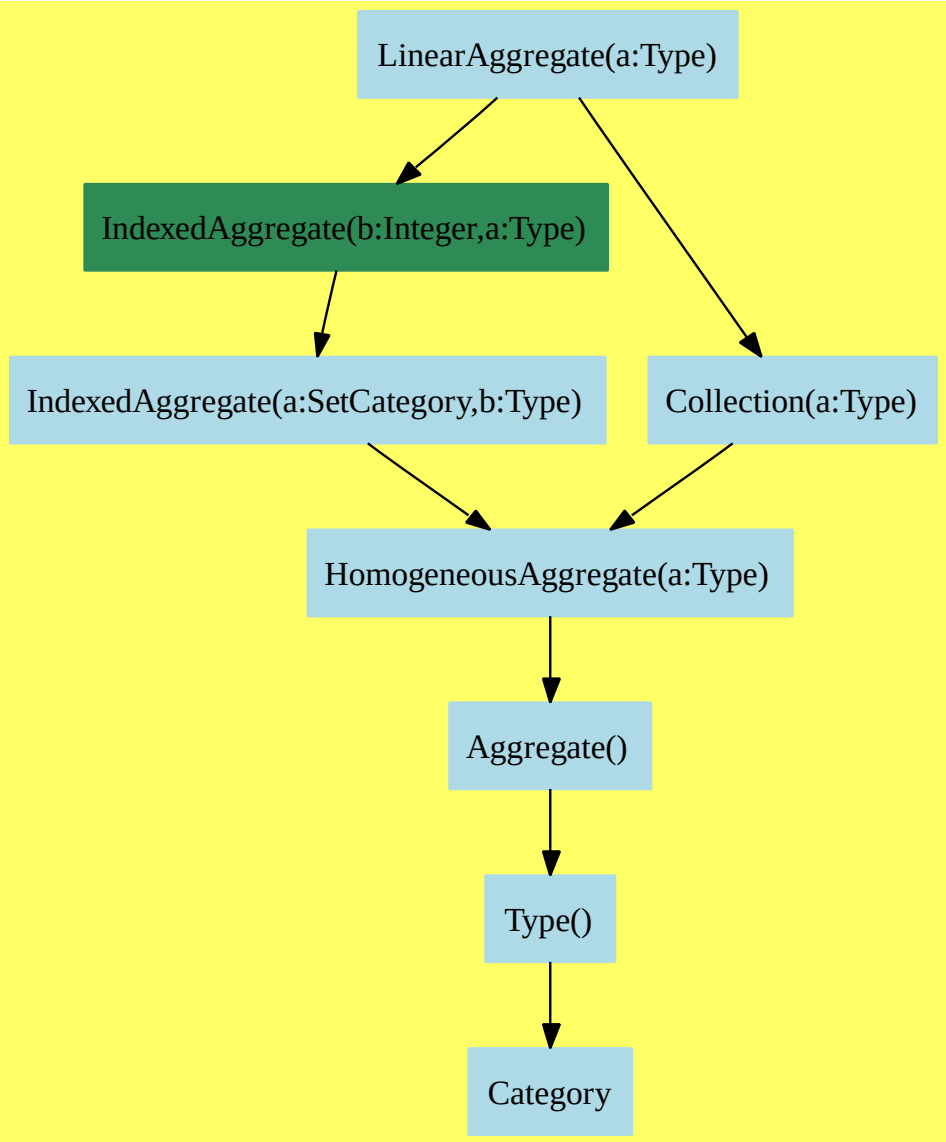
"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

6.6 LinearAggregate (LNAGG)



See:

⇒ “ExtensibleLinearAggregate” (ELAGG) 7.5 on page 438
⇒ “FiniteLinearAggregate” (FLAGG) 7.6 on page 444
⇒ “StreamAggregate” (STAGG) 7.11 on page 470
⇐ “Collection” (CLAGG) 5.4 on page 218
⇐ “IndexedAggregate” (IXAGG) 5.8 on page 246

Exports:

any?	coerce	concat	construct	convert
copy	count	delete	elt	empty
empty?	entries	entry?	eq?	eval
every?	fill!	find	first	hash
index?	indices	insert	latex	less?
map	map!	maxIndex	member?	members
minIndex	more?	new	parts	qelt
qsetelt!	reduce	remove	removeDuplicates	sample
setelt	size?	swap!	?~=?	# ?
?=?	?.?			

Attributes exported:

- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
concat : (%,%) -> %
concat : List % -> %
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
insert : (%,%,Integer) -> %
map : (((S,S) -> S),%,%) -> %
new : (NonNegativeInteger,S) -> %
setelt : (%,UniversalSegment Integer,S) -> S
         if $ has shallowlyMutable
?.? : (%,UniversalSegment Integer) -> %
```

These are implemented by this category:

```
concat : (%,S) -> %
concat : (S,%) -> %
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (S,%,Integer) -> %
maxIndex : % -> Integer if Integer has ORDSET
```

These exports come from (p246) IndexedAggregate(Integer,S:Type)

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
elt : (%,Integer,S) -> S
empty : () -> %
empty? : % -> Boolean
entries : % -> List S
entry? : (S,%) -> Boolean
         if $ has finiteAggregate and S has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
fill! : (%,S) -> % if $ has shallowlyMutable
first : % -> S if Integer has ORDSET
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
sample : () -> %
setelt : (%,Integer,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.? : (%,Integer) -> S
?~=? : (%,%) -> Boolean if S has SETCAT
?=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p218) Collection(S:Type):

```
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
find : ((S -> Boolean),%) -> Union(S,"failed")
reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
               if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
         if S has SETCAT and $ has finiteAggregate
select : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

⟨*category LNAGG LinearAggregate*⟩≡

```
)abbrev category LNAGG LinearAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A linear aggregate is an aggregate whose elements are indexed by integers.
++ Examples of linear aggregates are strings, lists, and
++ arrays.
++ Most of the exported operations for linear aggregates are non-destructive
++ but are not always efficient for a particular aggregate.
++ For example, \spadfun{concat} of two lists needs only to copy its first
++ argument, whereas \spadfun{concat} of two arrays needs to copy both
++ arguments. Most of the operations exported here apply to infinite
++ objects (e.g. streams) as well to finite ones.
++ For finite linear aggregates, see \spadtype{FiniteLinearAggregate}.
LinearAggregate(S:Type): Category ==
  Join(IndexedAggregate(Integer, S), Collection(S)) with
   new   : (NonNegativeInteger,S) -> %
     ++ new(n,x) returns \axiom{fill!(new n,x)}.
   concat: (%,S) -> %
     ++ concat(u,x) returns aggregate u with additional element x at the end.
     ++ Note: for lists, \axiom{concat(u,x) == concat(u,[x])}
   concat: (S,%) -> %
     ++ concat(x,u) returns aggregate u with additional element at the front.
     ++ Note: for lists: \axiom{concat(x,u) == concat([x],u)}.
   concat: (%,%) -> %
      ++ concat(u,v) returns an aggregate consisting of the elements of u
```

```
      ++ followed by the elements of v.
      ++ Note: if \axiom{w = concat(u,v)} then
      ++ \axiom{w.i = u.i for i in indices u}
      ++ and \axiom{w.(j + maxIndex u) = v.j for j in indices v}.
    concat: List % -> %
      ++ concat(u), where u is a lists of aggregates \axiom{[a,b,...,c]},
      ++ returns a single aggregate consisting of the elements of \axiom{a}
      ++ followed by those
      ++ of b followed ... by the elements of c.
      ++ Note: \axiom{concat(a,b,...,c) = concat(a,concat(b,...,c))}.
    map: ((S,S)->S,%,%) -> %
      ++ map(f,u,v) returns a new collection w with elements
      ++ \axiom{z = f(x,y)} for corresponding elements x and y from u and v.
      ++ Note: for linear aggregates, \axiom{w.i = f(u.i,v.i)}.
    elt: (%,UniversalSegment(Integer)) -> %
      ++ elt(u,i..j) (also written: \axiom{a(i..j)}) returns the aggregate of
      ++ elements \axiom{u} for k from i to j in that order.
      ++ Note: in general, \axiom{a.s = [a.k for i in s]}.
    delete: (%,Integer) -> %
      ++ delete(u,i) returns a copy of u with the \axiom{i}th
      ++ element deleted. Note: for lists,
      ++ \axiom{delete(a,i) == concat(a(0..i - 1),a(i + 1,..))}.
    delete: (%,UniversalSegment(Integer)) -> %
      ++ delete(u,i..j) returns a copy of u with the \axiom{i}th through
      ++ \axiom{j}th element deleted.
      ++ Note: \axiom{delete(a,i..j) = concat(a(0..i-1),a(j+1..))}.
    insert: (S,%,Integer) -> %
      ++ insert(x,u,i) returns a copy of u having x as its
      ++ \axiom{i}th element.
      ++ Note: \axiom{insert(x,a,k) = concat(concat(a(0..k-1),x),a(k..))}.
    insert: (%,%,Integer) -> %
      ++ insert(v,u,k) returns a copy of u having v inserted beginning at the
      ++ \axiom{i}th element.
      ++ Note: \axiom{insert(v,u,k) = concat( u(0..k-1), v, u(k..) )}.
    if % has shallowlyMutable then
     setelt: (%,UniversalSegment(Integer),S) -> S
      ++ setelt(u,i..j,x) (also written: \axiom{u(i..j) := x}) destructively
      ++ replaces each element in the segment \axiom{u(i..j)} by x.
      ++ The value x is returned.
      ++ Note: u is destructively change so
      ++ that \axiom{u.k := x for k in i..j};
      ++ its length remains unchanged.
  add
   indices a      == [i for i in minIndex a .. maxIndex a]
   index?(i, a)   == i >= minIndex a and i <= maxIndex a
   concat(a:%, x:S)      == concat(a, new(1, x))
```

```
    concat(x:S, y:%)      == concat(new(1, x), y)
    insert(x:S, a:%, i:Integer) == insert(new(1, x), a, i)
    if % has finiteAggregate then
      maxIndex l == #l - 1 + minIndex l

  --if % has shallowlyMutable then new(n, s)  == fill_!(new n, s)
```

⟨*LNAGG.dotabb*⟩≡

```
"LNAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=LNAGG"];
"LNAGG" -> "IXAGG"
"LNAGG" -> "CLAGG"
```

⟨*LNAGG.dotfull*⟩≡

```
"LinearAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LNAGG"];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "Collection(a:Type)"
```

⟨*LNAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "Collection(a:Type)"

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "HomogeneousAggregate(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

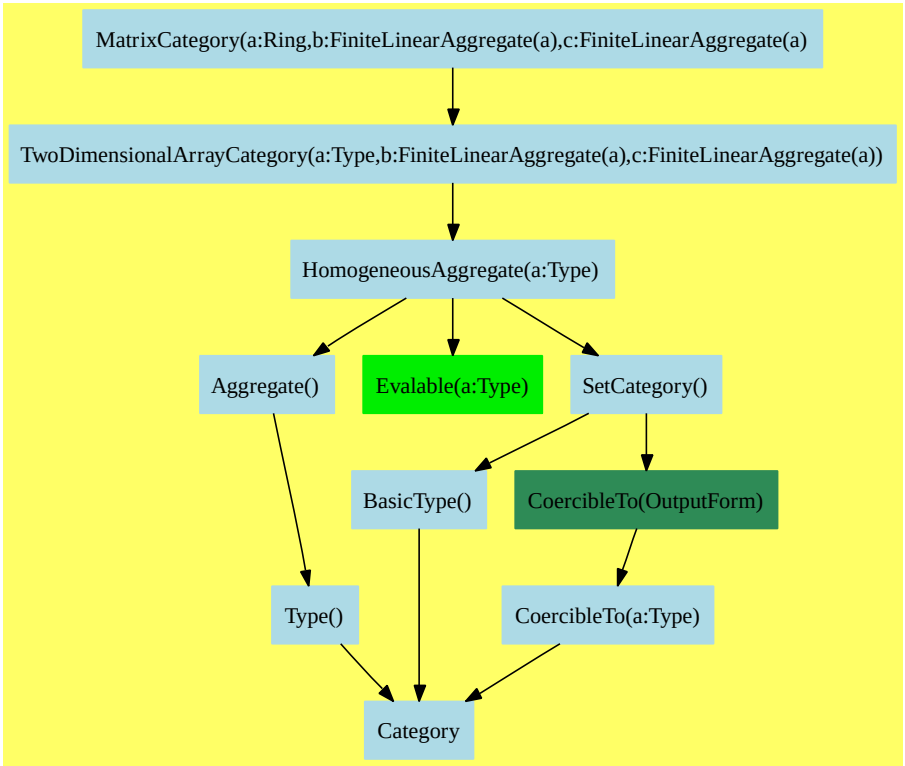
"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];

}
```

6.7 MatrixCategory (MATCAT)



⟨*MatrixCategory.input*⟩≡

```
)set break resume
)sys rm -f MatrixCategory.output
)spool MatrixCategory.output
)set message test on
)set message auto off
)clear all


--S 1 of 59
square? matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R   (1)  true
--R                                                    Type: Boolean
--E 1

--S 2 of 59
diagonal? matrix [[j**i for i in 0..4] for j in 1..5]
```

```
--R
--R
--R   (2)  false
--R                                                          Type: Boolean
--E 2

--S 3 of 59
symmetric? matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R   (3)  false
--R                                                          Type: Boolean
--E 3

--S 4 of 59
antisymmetric? matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R   (4)  false
--R                                                          Type: Boolean
--E 4

--S 5 of 59
z:Matrix(INT):=zero(3,3)
--R
--R
--R        +0  0  0+
--R        |       |
--R   (5)  |0  0  0|
--R        |       |
--R        +0  0  0+
--R                                                 Type: Matrix Integer
--E 5

--S 6 of 59
matrix [[1,2,3],[4,5,6],[7,8,9],[1,1,1]]
--R
--R
--R        +1  2  3+
--R        |       |
--R        |4  5  6|
--R   (6)  |       |
--R        |7  8  9|
--R        |       |
--R        +1  1  1+
--R                                                 Type: Matrix Integer
```

```
--E 6

--S 7 of 59
z:Matrix(INT):=scalarMatrix(3,5)
--R
--R
--R        +5  0  0+
--R        |       |
--R   (7)  |0  5  0|
--R        |       |
--R        +0  0  5+
--R                                                    Type: Matrix Integer
--E 7

--S 8 of 59
diagonalMatrix [1,2,3]
--R
--R
--R        +1  0  0+
--R        |       |
--R   (8)  |0  2  0|
--R        |       |
--R        +0  0  3+
--R                                                    Type: Matrix Integer
--E 8

--S 9 of 59
diagonalMatrix [matrix [[1,2],[3,4]], matrix [[4,5],[6,7]]]
--R
--R
--R        +1  2  0  0+
--R        |          |
--R        |3  4  0  0|
--R   (9)  |          |
--R        |0  0  4  5|
--R        |          |
--R        +0  0  6  7+
--R                                                    Type: Matrix Integer
--E 9

--S 10 of 59
coerce([1,2,3])@Matrix(INT)
--R
--R
--R         +1+
--R         | |
```

```
--R   (10)  |2|
--R         | |
--R         +3+
--R                                                    Type: Matrix Integer
--E 10

--S 11 of 59
transpose([1,2,3])@Matrix(INT)
--R
--R
--R   (11)  [1  2  3]
--R                                                    Type: Matrix Integer
--E 11

--S 12 of 59
transpose matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1   1    1    1 +
--R         |                   |
--R         |1  2   3    4    5 |
--R         |                   |
--R   (12)  |1  4   9   16   25 |
--R         |                   |
--R         |1  8   27  64   125|
--R         |                   |
--R         +1  16  81  256  625+
--R                                                    Type: Matrix Integer
--E 12

--S 13 of 59
squareTop matrix [[j**i for i in 0..2] for j in 1..5]
--R
--R
--R         +1  1  1+
--R         |      |
--R   (13)  |1  2  4|
--R         |      |
--R         +1  3  9+
--R                                                    Type: Matrix Integer
--E 13

--S 14 of 59
t1:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
```

```
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (14)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                    Type: Matrix Integer
--E 14

--S 15 of 59
horizConcat(t1,t1)
--R
--R
--R         +1  1  1    1    1   1  1  1    1    1 +
--R         |                                     |
--R         |1  2  4    8   16   1  2  4    8   16 |
--R         |                                     |
--R   (15)  |1  3  9   27   81   1  3  9   27   81 |
--R         |                                     |
--R         |1  4  16  64   256  1  4  16  64   256|
--R         |                                     |
--R         +1  5  25  125  625  1  5  25  125  625+
--R                                                    Type: Matrix Integer
--E 15

--S 16 of 59
t2:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (16)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                    Type: Matrix Integer
--E 16

--S 17 of 59
vertConcat(t2,t2)
```

```
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R         |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         |1  5  25  125  625|
--R   (17)  |                  |
--R         |1  1  1    1    1 |
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R         |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                    Type: Matrix Integer
--E 17

--S 18 of 59
t3:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (18)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                    Type: Matrix Integer
--E 18

--S 19 of 59
listOfLists t3
--R
--R
--R   (19)
--R   [[1,1,1,1,1],[1,2,4,8,16],[1,3,9,27,81],[1,4,16,64,256],[1,5,25,125,625]]
```

```
--R                                                          Type: List List Integer
--E 19

--S 20 of 59
t4:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (20)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                    Type: Matrix Integer
--E 20

--S 21 of 59
elt(t4,3,3)
--R
--R
--R   (21)  9
--R                                                  Type: PositiveInteger
--E 21

--S 22 of 59
t5:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (22)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                    Type: Matrix Integer
--E 22

--S 23 of 59
setelt(t5,3,3,10)
--R
```

```
--R
--R   (23)  10
--R                                                    Type: PositiveInteger
--E 23

--S 24 of 59
t6:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (24)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                     Type: Matrix Integer
--E 24

--S 25 of 59
swapRows!(t6,2,4)
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R   (25)  |1  3  9   27   81 |
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R         +1  5  25  125  625+
--R                                                     Type: Matrix Integer
--E 25

--S 26 of 59
t7:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (26)  |1  3  9   27   81 |
```

```
--R         |                    |
--R         |1  4  16  64   256|
--R         |                    |
--R         +1  5  25  125  625+
--R                                                    Type: Matrix Integer
--E 26

--S 27 of 59
swapColumns!(t7,2,4)
--R
--R
--R         +1   1   1   1   1 +
--R         |                  |
--R         |1   8   4   2  16 |
--R         |                  |
--R   (27)  |1  27   9   3  81 |
--R         |                  |
--R         |1  64   16  4  256|
--R         |                  |
--R         +1  125  25  5  625+
--R                                                    Type: Matrix Integer
--E 27

--S 28 of 59
t8:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (28)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                    Type: Matrix Integer
--E 28

--S 29 of 59
subMatrix(t8,1,3,2,4)
--R
--R
--R         +1  1  1 +
--R         |        |
--R   (29)  |2  4  8 |
```

```
--R         |        |
--R         +3  9  27+
--R                                                    Type: Matrix Integer
--E 29

--S 30 of 59
t9:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (30)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                    Type: Matrix Integer
--E 30

--S 31 of 59
setsubMatrix!(t9,2,2,matrix [[3,3],[3,3]])
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  3  3    8   16 |
--R         |                  |
--R   (31)  |1  3  3   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                    Type: Matrix Integer
--E 31

--S 32 of 59
t0:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (32)  |1  3  9   27   81 |
```

```
--R         |                    |
--R         |1  4  16  64   256|
--R         |                    |
--R         +1  5  25  125  625+
--R                                                    Type: Matrix Integer
--E 32

--S 33 of 59
t0+t0
--R
--R
--R         +2  2   2    2    2  +
--R         |                    |
--R         |2  4   8   16    32 |
--R         |                    |
--R   (33)  |2  6   18  54   162 |
--R         |                    |
--R         |2  8   32  128  512 |
--R         |                    |
--R         +2  10  50  250  1250+
--R                                                    Type: Matrix Integer
--E 33

--S 34 of 59
t0-t0
--R
--R
--R         +0  0  0  0  0+
--R         |             |
--R         |0  0  0  0  0|
--R         |             |
--R   (34)  |0  0  0  0  0|
--R         |             |
--R         |0  0  0  0  0|
--R         |             |
--R         +0  0  0  0  0+
--R                                                    Type: Matrix Integer
--E 34

--S 35 of 59
-t0
--R
--R
--R         +- 1  - 1  - 1    - 1    - 1 +
--R         |                            |
--R         |- 1  - 2  - 4    - 8   - 16 |
```

```
--R         |                                |
--R   (35)  |- 1  - 3  - 9   - 27   - 81 |
--R         |                                |
--R         |- 1  - 4  - 16  - 64   - 256|
--R         |                                |
--R         +- 1  - 5  - 25  - 125  - 625+
--R                                                    Type: Matrix Integer
--E 35

--S 36 of 59
t0*t0
--R
--R
--R         + 5    15    55     225    979  +
--R         |                                |
--R         |31   129    573   2637   12405 |
--R         |                                |
--R   (36)  |121  547   2551   12121  58315 |
--R         |                                |
--R         |341  1593  7585   36561  177745|
--R         |                                |
--R         +781  3711  17871  86841  424731+
--R                                                    Type: Matrix Integer
--E 36

--S 37 of 59
1/3*t0
--R
--R
--R         +1  1  1    1    1 +
--R         |-  -  -    -    - |
--R         |3  3  3    3    3 |
--R         |                   |
--R         |1  2  4    8   16 |
--R         |-  -  -    -   -- |
--R         |3  3  3    3    3 |
--R         |                   |
--R         |1                  |
--R   (37)  |-  1  3    9   27 |
--R         |3                  |
--R         |                   |
--R         |1  4  16  64   256|
--R         |-  -  --  --   ---|
--R         |3  3   3   3    3 |
--R         |                   |
--R         |1  5  25  125  625|
```

```
--R         |-  -  --  ---  ---|
--R         +3  3   3   3    3 +
--R                                                  Type: Matrix Fraction Integer
--E 37

--S 38 of 59
m:=matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  1  1    1    1 +
--R         |                  |
--R         |1  2  4    8   16 |
--R         |                  |
--R   (38)  |1  3  9   27   81 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |                  |
--R         +1  5  25  125  625+
--R                                                Type: Matrix Integer
--E 38

--S 39 of 59
t0*1/3
--R
--R
--R         +1  1  1    1    1 +
--R         |-  -  -    -    - |
--R         |3  3  3    3    3 |
--R         |                  |
--R         |1  2  4    8   16 |
--R         |-  -  -    -   -- |
--R         |3  3  3    3    3 |
--R         |                  |
--R         |1                 |
--R   (39)  |-  1  3    9   27 |
--R         |3                 |
--R         |                  |
--R         |1  4  16  64   256|
--R         |-  -  --  --   ---|
--R         |3  3   3   3    3 |
--R         |                  |
--R         |1  5  25  125  625|
--R         |-  -  --  ---  ---|
--R         +3  3   3   3    3 +
--R                                                  Type: Matrix Fraction Integer
--E 39
```

```
--S 40 of 59
3*t0
--R
--R
--R         +3  3   3    3    3  +
--R         |                    |
--R         |3  6   12  24    48 |
--R         |                    |
--R   (40)  |3  9   27  81   243 |
--R         |                    |
--R         |3  12  48  192  768 |
--R         |                    |
--R         +3  15  75  375  1875+
--R                                                    Type: Matrix Integer
--E 40

--S 41 of 59
c:=coerce([1,2,3,4,5])@Matrix(INT)
--R
--R
--R         +1+
--R         | |
--R         |2|
--R         | |
--R   (41)  |3|
--R         | |
--R         |4|
--R         | |
--R         +5+
--R                                                    Type: Matrix Integer
--E 41

--S 42 of 59
t0*c
--R
--R
--R         + 15 +
--R         |    |
--R         |129 |
--R         |    |
--R   (42)  |547 |
--R         |    |
--R         |1593|
--R         |    |
--R         +3711+
```

```
--R                                                          Type: Matrix Integer
--E 42

--S 43 of 59
r:=transpose([1,2,3,4,5])@Matrix(INT)
--R
--R
--R   (43)  [1  2  3  4  5]
--R                                                          Type: Matrix Integer
--E 43

--S 44 of 59
r*t0
--R
--R
--R   (44)  [15  55  225  979  4425]
--R                                                          Type: Matrix Integer
--E 44

--S 45 of 59
t0**3
--R
--R
--R         + 1279    5995     28635     138385    674175  +
--R         |                                              |
--R         |15775    74581    358021   1735927    8476705 |
--R         |                                              |
--R   (45)  |73655   348927   1677079   8138493   39765355 |
--R         |                                              |
--R         |223825  1061251  5103579   24775909  121090455|
--R         |                                              |
--R         +533935  2532835  12184195  59162185  289195879+
--R                                                          Type: Matrix Integer
--E 45

--S 46 of 59
t10:=matrix [[2**i for i in 2..4] for j in 1..5]
--R
--R
--R         +4  8  16+
--R         |        |
--R         |4  8  16|
--R         |        |
--R   (46)  |4  8  16|
--R         |        |
--R         |4  8  16|
```

```
--R         |        |
--R         +4  8  16+
--R                                                    Type: Matrix Integer
--E 46

--S 47 of 59
exquo(t10,2)
--R
--R
--R         +2  4  8+
--R         |       |
--R         |2  4  8|
--R         |       |
--R   (47)  |2  4  8|
--R         |       |
--R         |2  4  8|
--R         |       |
--R         +2  4  8+
--R                                          Type: Union(Matrix Integer,...)
--E 47

--S 48 of 59
t10/4
--R
--R
--R         +1  2  4+
--R         |       |
--R         |1  2  4|
--R         |       |
--R   (48)  |1  2  4|
--R         |       |
--R         |1  2  4|
--R         |       |
--R         +1  2  4+
--R                                            Type: Matrix Fraction Integer
--E 48

--S 49 of 59
rowEchelon matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         +1  0  0  0  0 +
--R         |              |
--R         |0  1  1  1  1 |
--R         |              |
--R   (49)  |0  0  2  0  2 |
```

```
--R         |              |
--R         |0  0  0  6  12|
--R         |              |
--R         +0  0  0  0  24+
--R                                                    Type: Matrix Integer
--E 49

--S 50 of 59
columnSpace matrix [[1,2,3],[4,5,6],[7,8,9],[1,1,1]]
--R
--R
--R   (50)  [[1,4,7,1],[2,5,8,1]]
--R                                                 Type: List Vector Integer
--E 50

--S 51 of 59
rank matrix [[1,2,3],[4,5,6],[7,8,9]]
--R
--R
--R   (51)  2
--R                                                     Type: PositiveInteger
--E 51

--S 52 of 59
nullity matrix [[1,2,3],[4,5,6],[7,8,9]]
--R
--R
--R   (52)  1
--R                                                     Type: PositiveInteger
--E 52

--S 53 of 59
nullSpace matrix [[1,2,3],[4,5,6],[7,8,9]]
--R
--R
--R   (53)  [[1,- 2,1]]
--R                                                 Type: List Vector Integer
--E 53

--S 54 of 59
determinant matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R   (54)  288
--R                                                     Type: PositiveInteger
--E 54
```

```
--S 55 of 59
minordet matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R   (55)  288
--R                                                    Type: PositiveInteger
--E 55

--S 56 of 59
pfaffian [[0,1,0,0],[-1,0,0,0],[0,0,0,1],[0,0,-1,0]]
--R
--R
--R   (56)  1
--R                                                    Type: PositiveInteger
--E 56

--S 57 of 59
inverse matrix [[j**i for i in 0..4] for j in 1..5]
--R
--R
--R         + 5    - 10   10   - 5    1  +
--R         |                            |
--R         |  77  107     39   61     25|
--R         |- --  ---   - --   --   - --|
--R         |  12   6       2    6     12|
--R         |                            |
--R         | 71     59   49     41   35 |
--R         | --   - --   --   - --   -- |
--R   (57)  | 24      6    4      6   24 |
--R         |                            |
--R         |   7   13          11      5|
--R         |- --   --   - 3    --   - --|
--R         |  12    6           6     12|
--R         |                            |
--R         |  1     1    1      1     1 |
--R         | --   - -    -    - -    -- |
--R         + 24     6    4      6    24 +
--R                                  Type: Union(Matrix Fraction Integer,...)
--E 57

--S 58 of 59
(matrix [[j**i for i in 0..4] for j in 1..5]) ** 2
--R
--R
--R         + 5    15    55     225    979  +
```

```
--R         |                                    |
--R         |31   129    573   2637   12405 |
--R         |                                    |
--R   (58)  |121  547   2551   12121  58315 |
--R         |                                    |
--R         |341  1593  7585   36561  177745|
--R         |                                    |
--R         +781  3711  17871  86841  424731+
--R                                                    Type: Matrix Integer
--E 58

--S 59 of 59
)show MatrixCategory
--R
--R MatrixCategory(R: Ring,Row: FiniteLinearAggregate t#1,Col: FiniteLinearAggregate t#1)  i
--R Abbreviation for MatrixCategory is MATCAT
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.2.spad.pamphlet to see algebra source code for MATCAT
--R
--R------------------------------ Operations --------------------------------
--R ?*? : (Row,%) -> Row                  ?*? : (%,Col) -> Col
--R ?*? : (Integer,%) -> %                ?*? : (%,R) -> %
--R ?*? : (R,%) -> %                      ?*? : (%,%) -> %
--R ?+? : (%,%) -> %                      -? : % -> %
--R ?-? : (%,%) -> %                      antisymmetric? : % -> Boolean
--R coerce : Col -> %                     column : (%,Integer) -> Col
--R copy : % -> %                         diagonal? : % -> Boolean
--R diagonalMatrix : List % -> %          diagonalMatrix : List R -> %
--R elt : (%,Integer,Integer,R) -> R      elt : (%,Integer,Integer) -> R
--R empty : () -> %                       empty? : % -> Boolean
--R eq? : (%,%) -> Boolean                fill! : (%,R) -> %
--R horizConcat : (%,%) -> %              listOfLists : % -> List List R
--R map : (((R,R) -> R),%,%,R) -> %       map : (((R,R) -> R),%,%) -> %
--R map : ((R -> R),%) -> %               map! : ((R -> R),%) -> %
--R matrix : List List R -> %             maxColIndex : % -> Integer
--R maxRowIndex : % -> Integer            minColIndex : % -> Integer
--R minRowIndex : % -> Integer            ncols : % -> NonNegativeInteger
--R nrows : % -> NonNegativeInteger       parts : % -> List R
--R qelt : (%,Integer,Integer) -> R       row : (%,Integer) -> Row
--R sample : () -> %                      setRow! : (%,Integer,Row) -> %
--R square? : % -> Boolean                squareTop : % -> %
--R symmetric? : % -> Boolean             transpose : % -> %
--R transpose : Row -> %                  vertConcat : (%,%) -> %
--R #? : % -> NonNegativeInteger if $ has finiteAggregate
--R ?**? : (%,Integer) -> % if R has FIELD
--R ?**? : (%,NonNegativeInteger) -> %
```

```
--R ?/? : (%,R) -> % if R has FIELD
--R ?=? : (%,%) -> Boolean if R has SETCAT
--R any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R coerce : % -> OutputForm if R has SETCAT
--R columnSpace : % -> List Col if R has EUCDOM
--R count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
--R count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
--R determinant : % -> R if R has commutative *
--R elt : (%,List Integer,List Integer) -> %
--R eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
--R eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
--R every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
--R exquo : (%,R) -> Union(%,"failed") if R has INTDOM
--R hash : % -> SingleInteger if R has SETCAT
--R inverse : % -> Union(%,"failed") if R has FIELD
--R latex : % -> String if R has SETCAT
--R less? : (%,NonNegativeInteger) -> Boolean
--R member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
--R members : % -> List R if $ has finiteAggregate
--R minordet : % -> R if R has commutative *
--R more? : (%,NonNegativeInteger) -> Boolean
--R new : (NonNegativeInteger,NonNegativeInteger,R) -> %
--R nullSpace : % -> List Col if R has INTDOM
--R nullity : % -> NonNegativeInteger if R has INTDOM
--R pfaffian : % -> R if R has COMRING
--R qsetelt! : (%,Integer,Integer,R) -> R
--R rank : % -> NonNegativeInteger if R has INTDOM
--R rowEchelon : % -> % if R has EUCDOM
--R scalarMatrix : (NonNegativeInteger,R) -> %
--R setColumn! : (%,Integer,Col) -> %
--R setelt : (%,List Integer,List Integer,%) -> %
--R setelt : (%,Integer,Integer,R) -> R
--R setsubMatrix! : (%,Integer,Integer,%) -> %
--R size? : (%,NonNegativeInteger) -> Boolean
--R subMatrix : (%,Integer,Integer,Integer,Integer) -> %
--R swapColumns! : (%,Integer,Integer) -> %
--R swapRows! : (%,Integer,Integer) -> %
--R zero : (NonNegativeInteger,NonNegativeInteger) -> %
--R ?~=? : (%,%) -> Boolean if R has SETCAT
--R
--E 59

)spool
)lisp (bye)
```

⟨*MatrixCategory.help*⟩≡

```
==========================================================================
MatrixCategory examples
==========================================================================

Predicates:

square?(m) returns true if m is a square matrix
(if m has the same number of rows as columns) and false otherwise.

  square matrix [[j**i for i in 0..4] for j in 1..5]

diagonal?(m) returns true if the matrix m is square and
diagonal (i.e. all entries of m not on the diagonal are zero) and
false otherwise.

  diagonal? matrix [[j**i for i in 0..4] for j in 1..5]

symmetric?(m) returns true if the matrix m is square and
symmetric (i.e. \spad{m[i,j] = m[j,i]} for all i and j) and false
otherwise.

  symmetric? matrix [[j**i for i in 0..4] for j in 1..5]

antisymmetric?(m) returns true if the matrix m is square and
antisymmetric (i.e. m[i,j] = -m[j,i] for all i and j)
and false otherwise.

  antisymmetric? matrix [[j**i for i in 0..4] for j in 1..5]


Creation

zero(m,n) returns an m-by-n zero matrix.

  z:Matrix(INT):=zero(3,3)

matrix(l) converts the list of lists l to a matrix, where the
list of lists is viewed as a list of the rows of the matrix.

  matrix [[1,2,3],[4,5,6],[7,8,9],[1,1,1]]

scalarMatrix(n,r) returns an n-by-n matrix with r's on the
diagonal and zeroes elsewhere.

  z:Matrix(INT):=scalarMatrix(3,5)
```

```
diagonalMatrix(l) returns a diagonal matrix with the elements
of l on the diagonal.

  diagonalMatrix [1,2,3]

diagonalMatrix([m1,...,mk]) creates a block diagonal matrix
M with block matrices m1,...,mk down the diagonal,
with 0 block matrices elsewhere.

More precisly: if ri := nrows mi, ci := ncols mi,
then m is an (r1+..+rk) by (c1+..+ck) - matrix  with entries
m.i.j = ml.(i-r1-..-r(l-1)).(j-n1-..-n(l-1)),
if (r1+..+r(l-1)) < i <= r1+..+rl and
   (c1+..+c(l-1)) < i <= c1+..+cl,
    m.i.j = 0  otherwise.

  diagonalMatrix [matrix [[1,2],[3,4]], matrix [[4,5],[6,7]]]

coerce(col) converts the column col to a column matrix.

  coerce([1,2,3])@Matrix(INT)

transpose(r) converts the row r to a row matrix.

  transpose([1,2,3])@Matrix(INT)

Creation of new matrices from old

transpose(m) returns the transpose of the matrix m.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  transpose m

squareTop(m) returns an n-by-n matrix consisting of the first
n rows of the m-by-n matrix m. Error: if m < n.

  m:=matrix [[j**i for i in 0..2] for j in 1..5]
  squareTop m

horizConcat(x,y) horizontally concatenates two matrices with
an equal number of rows. The entries of y appear to the right
of the entries of x.  Error: if the matrices
do not have the same number of rows.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
```

```
  horizConcat(m,m)

vertConcat(x,y) vertically concatenates two matrices with an
equal number of columns. The entries of y appear below
of the entries of x.  Error: if the matrices
do not have the same number of columns.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  vertConcat(m,m)

Part extractions/assignments

listOfLists(m) returns the rows of the matrix m as a list of lists

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  listOfLists m

elt(x,rowList,colList) returns an m-by-n matrix consisting
of elements of x, where m = # rowList and n = # colList
If rowList = [i<1>,i<2>,...,i<m>] and
   colList = [j<1>,j<2>,...,j<n>],
 then the (k,l)-th entry of elt(x,rowList,colList) is x(i<k>,j<l>).

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  elt(m,3,3)

setelt(x,rowList,colList,y) destructively alters the matrix x.
If y is m-by-n,
   rowList = [i<1>,i<2>,...,i<m>] and
   colList = [j<1>,j<2>,...,j<n>],
 then x(i<k>,j<l>)
 is set to y(k,l) for k = 1,...,m and l = 1,...,n

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  setelt(m,3,3,10)

swapRows!(m,i,j) interchanges the i-th and j-th
rows of m. This destructively alters the matrix.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  swapRows!(m,2,4)

swapColumns!(m,i,j) interchanges the i-th and j-th
columns of m. This destructively alters the matrix.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
```

```
  swapColumns!(m,2,4)

subMatrix(x,i1,i2,j1,j2) extracts the submatrix [x(i,j)]
where the index i ranges from i1 to i2
and   the index j ranges from j1 to j2.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  subMatrix(m,1,3,2,4)

setsubMatrix(x,i1,j1,y) destructively alters the matrix x.
Here x(i,j) is set to y(i-i1+1,j-j1+1) for
i = i1,...,i1-1+nrows y and j = j1,...,j1-1+ncols y.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  setsubMatrix!(m,2,2,matrix [[3,3],[3,3]])


Arithmetic

x + y is the sum of the matrices x and y.
It is an error if the dimensions are incompatible.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  m+m

x - y is the difference of the matrices x and y.
It is an error if the dimensions are incompatible.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  m-m

-x returns the negative of the matrix x.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  -m

x * y is the product of the matrices x and y.
It is an error if the dimensions are incompatible.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  m*m

r*x is the left scalar multiple of the scalar r and the matrix x.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  1/3*m
```

```
x * r is the right scalar multiple of the scalar r and the matrix x.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  m*1/3

n * x is an integer multiple.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  3*m

x * c is the product of the matrix x and the column vector c.
It is an error if the dimensions are incompatible.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  c:=coerce([1,2,3,4,5])@Matrix(INT)
  m *c

r * x is the product of the row vector r and the matrix x.
It is an error if the dimensions are incompatible.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  r:=transpose([1,2,3,4,5])@Matrix(INT)
  r*m

x ** n computes a non-negative integral power of the matrix x.
It is an error if the matrix is not square.

  m:=matrix [[j**i for i in 0..4] for j in 1..5]
  m**3

exquo(m,r) computes the exact quotient of the elements
of m by r, returning "failed" if this is not possible.

  m:=matrix [[2**i for i in 2..4] for j in 1..5]
  exquo(m,2)

m/r divides the elements of m by r, r must be non-zero.

  m:=matrix [[2**i for i in 2..4] for j in 1..5]
  m/4

Linear algebra

rowEchelon(m) returns the row echelon form of the matrix m.
```

```
  rowEchelon matrix [[j**i for i in 0..4] for j in 1..5]

columnSpace(m) returns a sublist of columns of the matrix m

  columnSpace matrix [[1,2,3],[4,5,6],[7,8,9],[1,1,1]]

rank(m) returns the rank of the matrix m.

  rank matrix [[1,2,3],[4,5,6],[7,8,9]]

nullity(m) returns the nullity of the matrix m. This is
the dimension of the null space of the matrix m.

  nullity matrix [[1,2,3],[4,5,6],[7,8,9]]

nullSpace(m) returns a basis for the null space of the matrix m.

  nullSpace matrix [[1,2,3],[4,5,6],[7,8,9]]

determinant(m) returns the determinant of the matrix m.
It is an error if the matrix is not square.

  determinant matrix [[j**i for i in 0..4] for j in 1..5]

minordet(m) computes the determinant of the matrix m using minors.
It is an error if the matrix is not square.

  minordet matrix [[j**i for i in 0..4] for j in 1..5]

pfaffian(m) returns the Pfaffian of the matrix m.
It is an error if the matrix is not antisymmetric

  pfaffian [[0,1,0,0],[-1,0,0,0],[0,0,0,1],[0,0,-1,0]]

inverse(m) returns the inverse of the matrix m.
If the matrix is not invertible, "failed" is returned.
It is an error if the matrix is not square.

  inverse matrix [[j**i for i in 0..4] for j in 1..5]

m**n computes an integral power of the matrix m.
It is an error if matrix is not square or
if the matrix is square but not invertible.

  (matrix [[j**i for i in 0..4] for j in 1..5]) ** 2
```

We define three categories for matrices

- MatrixCategory is the category of all matrices
- RectangularMatrixCategory is the category of all matrices of a given dimension
- SquareMatrixCategory inherits from RectangularMatrixCategory

The Matrix domain is the domain of all matrices.

All three domains share the same representation, inherited from Matrix. Most algorithms are only implemented for Matrix but implemented in separate packages.

- MatrixLinearAlgebraFunctions is the top-level package that calls the other packages
- InnerMatrixLinearAlgebraFunctions contains implementations that work over a Field
- InnerMatrixQuotientFieldFunctions contain implementations that work over a quotient field

Implementations that rely on the representation of matrices used in Matrix should be put into these packages.

See:

⇐ "TwoDimensionalArrayCategory" (ARR2CAT) 5.13 on page 268
⇒ "RectangularMatrixCategory" (RMATCAT) 10.14 on page 732
⇒ "SquareMatrixCategory" (SMATCAT) 12.9 on page 887

Exports:

antisymmetric?	any?	coerce	column	columnSpace
copy	count	determinant	diagonal?	diagonalMatrix
elt	empty	empty?	eq?	eval
every?	exquo	fill!	hash	horizConcat
inverse	latex	less?	listOfLists	map
map!	matrix	maxColIndex	maxRowIndex	member?
members	minColIndex	minordet	minRowIndex	more?
ncols	new	nrows	nullSpace	nullity
parts	pfaffian	qelt	qsetelt!	rank
row	rowEchelon	sample	scalarMatrix	setColumn!
setelt	setRow!	setsubMatrix!	size?	square?
squareTop	subMatrix	swapColumns!	swapRows!	symmetric?
transpose	vertConcat	zero	#?	?**?
?/?	?=?	?~=?	?*?	?+?
-?	?-?			

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
determinant : % -> R if R has commutative *
inverse : % -> Union(%,"failed") if R has FIELD
minordet : % -> R if R has commutative *
nullity : % -> NonNegativeInteger if R has INTDOM
nullSpace : % -> List Col if R has INTDOM
rowEchelon : % -> % if R has EUCDOM
rank : % -> NonNegativeInteger if R has INTDOM
```

These are implemented by this category:

```
antisymmetric? : % -> Boolean
coerce : Col -> %
columnSpace: % -> List Col if R has EUCDOM
diagonal? : % -> Boolean
diagonalMatrix : List % -> %
diagonalMatrix : List R -> %
elt : (%,List Integer,List Integer) -> %
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
horizConcat : (%,%) -> %
listOfLists : % -> List List R
pfaffian: % -> R if R has COMRING
matrix : List List R -> %
scalarMatrix : (NonNegativeInteger,R) -> %
setelt : (%,List Integer,List Integer,%) -> %
setsubMatrix! : (%,Integer,Integer,%) -> %
square? : % -> Boolean
squareTop : % -> %
subMatrix : (%,Integer,Integer,Integer,Integer) -> %
swapColumns! : (%,Integer,Integer) -> %
swapRows! : (%,Integer,Integer) -> %
symmetric? : % -> Boolean
transpose : Row -> %
transpose : % -> %
vertConcat : (%,%) -> %
zero : (NonNegativeInteger,NonNegativeInteger) -> %
?/? : (%,R) -> % if R has FIELD
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

```
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,Integer) -> % if R has FIELD
?*? : (%,Col) -> Col
?*? : (Row,%) -> Row
?*? : (%,%) -> %
```

These exports come from (p268) TwoDimensionalArrayCategory(R,Row,Col) where R:Ring, Row:FiniteLinearAggregate(R), Col:FiniteLinearAggregate(R):

```
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
coerce : % -> OutputForm if R has SETCAT
column : (%,Integer) -> Col
copy : % -> %
count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
elt : (%,Integer,Integer,R) -> R
elt : (%,Integer,Integer) -> R
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
fill! : (%,R) -> %
hash : % -> SingleInteger if R has SETCAT
latex : % -> String if R has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : (((R,R) -> R),%,%,R) -> %
map : (((R,R) -> R),%,%) -> %
map : ((R -> R),%) -> %
map! : ((R -> R),%) -> %
maxColIndex : % -> Integer
maxRowIndex : % -> Integer
member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
members : % -> List R if $ has finiteAggregate
minColIndex : % -> Integer
minRowIndex : % -> Integer
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,NonNegativeInteger,R) -> %
ncols : % -> NonNegativeInteger
nrows : % -> NonNegativeInteger
parts : % -> List R
qelt : (%,Integer,Integer) -> R
qsetelt! : (%,Integer,Integer,R) -> R
```

```
row : (%,Integer) -> Row
sample : () -> %
setColumn! : (%,Integer,Col) -> %
setelt : (%,Integer,Integer,R) -> R
setRow! : (%,Integer,Row) -> %
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean if R has SETCAT
?=? : (%,%) -> Boolean if R has SETCAT
```

⟨*category MATCAT MatrixCategory*⟩≡

```
)abbrev category MATCAT MatrixCategory
++ Authors: Grabmeier, Gschnitzer, Williamson
++ Date Created: 1987
++ Date Last Updated: July 1990
++ Basic Operations:
++ Related Domains: Matrix(R)
++ Also See:
++ AMS Classifications:
++ Keywords: matrix, linear algebra
++ Examples:
++ References:
++ Description:
++   \spadtype{MatrixCategory} is a general matrix category which allows
++   different representations and indexing schemes.  Rows and
++   columns may be extracted with rows returned as objects of
++   type Row and colums returned as objects of type Col.
++   A domain belonging to this category will be shallowly mutable.
++   The index of the 'first' row may be obtained by calling the
++   function \spadfun{minRowIndex}.  The index of the 'first' column may
++   be obtained by calling the function \spadfun{minColIndex}.  The index of
++   the first element of a Row is the same as the index of the
++   first column in a matrix and vice versa.
MatrixCategory(R,Row,Col): Category == Definition where
  R   : Ring
  Row : FiniteLinearAggregate R
  Col : FiniteLinearAggregate R

  Definition ==> TwoDimensionalArrayCategory(R,Row,Col) with
     shallowlyMutable
       ++ One may destructively alter matrices

     finiteAggregate
       ++ matrices are finite

--% Predicates
```

```
      square?  : % -> Boolean
        ++ \spad{square?(m)} returns true if m is a square matrix
        ++ (if m has the same number of rows as columns) and false otherwise.
        ++
        ++X square matrix [[j**i for i in 0..4] for j in 1..5]

      diagonal?: % -> Boolean
        ++ \spad{diagonal?(m)} returns true if the matrix m is square and
        ++ diagonal (i.e. all entries of m not on the diagonal are zero) and
        ++ false otherwise.
        ++
        ++X diagonal? matrix [[j**i for i in 0..4] for j in 1..5]

      symmetric?: % -> Boolean
        ++ \spad{symmetric?(m)} returns true if the matrix m is square and
        ++ symmetric (i.e. \spad{m[i,j] = m[j,i]} for all i and j) and false
        ++ otherwise.
        ++
        ++X symmetric? matrix [[j**i for i in 0..4] for j in 1..5]

      antisymmetric?: % -> Boolean
        ++ \spad{antisymmetric?(m)} returns true if the matrix m is square and
        ++ antisymmetric (i.e. \spad{m[i,j] = -m[j,i]} for all i and j)
        ++ and false otherwise.
        ++
        ++X antisymmetric? matrix [[j**i for i in 0..4] for j in 1..5]


    --% Creation

      zero: (NonNegativeInteger,NonNegativeInteger) -> %
        ++ \spad{zero(m,n)} returns an m-by-n zero matrix.
        ++
        ++X z:Matrix(INT):=zero(3,3)

      matrix: List List R -> %
        ++ \spad{matrix(l)} converts the list of lists l to a matrix, where the
        ++ list of lists is viewed as a list of the rows of the matrix.
        ++
        ++X matrix [[1,2,3],[4,5,6],[7,8,9],[1,1,1]]

      scalarMatrix: (NonNegativeInteger,R) -> %
        ++ \spad{scalarMatrix(n,r)} returns an n-by-n matrix with r's on the
        ++ diagonal and zeroes elsewhere.
        ++
        ++X z:Matrix(INT):=scalarMatrix(3,5)
```

```
      diagonalMatrix: List R -> %
        ++ \spad{diagonalMatrix(l)} returns a diagonal matrix with the elements
        ++ of l on the diagonal.
        ++
        ++X diagonalMatrix [1,2,3]

      diagonalMatrix: List % -> %
        ++ \spad{diagonalMatrix([m1,...,mk])} creates a block diagonal matrix
        ++ M with block matrices {\em m1},...,{\em mk} down the diagonal,
        ++ with 0 block matrices elsewhere.
        ++ More precisly: if \spad{ri := nrows mi}, \spad{ci := ncols mi},
        ++ then m is an (r1+..+rk) by (c1+..+ck) - matrix  with entries
        ++ \spad{m.i.j = ml.(i-r1-..-r(l-1)).(j-n1-..-n(l-1))}, if
        ++ \spad{(r1+..+r(l-1)) < i <= r1+..+rl} and
        ++ \spad{(c1+..+c(l-1)) < i <= c1+..+cl},
        ++ \spad{m.i.j} = 0  otherwise.
        ++
        ++X diagonalMatrix [matrix [[1,2],[3,4]], matrix [[4,5],[6,7]]]

      coerce: Col -> %
        ++ \spad{coerce(col)} converts the column col to a column matrix.
        ++
        ++X coerce([1,2,3])@Matrix(INT)

      transpose: Row -> %
        ++ \spad{transpose(r)} converts the row r to a row matrix.
        ++
        ++X transpose([1,2,3])@Matrix(INT)

    --% Creation of new matrices from old

      transpose: % -> %
        ++ \spad{transpose(m)} returns the transpose of the matrix m.
        ++
        ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
        ++X transpose m

      squareTop: % -> %
        ++ \spad{squareTop(m)} returns an n-by-n matrix consisting of the first
        ++ n rows of the m-by-n matrix m. Error: if
        ++ \spad{m < n}.
        ++
        ++X m:=matrix [[j**i for i in 0..2] for j in 1..5]
        ++X squareTop m
```

```
      horizConcat: (%,%) -> %
        ++ \spad{horizConcat(x,y)} horizontally concatenates two matrices with
        ++ an equal number of rows. The entries of y appear to the right
        ++ of the entries of x.  Error: if the matrices
        ++ do not have the same number of rows.
        ++
        ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
        ++X horizConcat(m,m)

      vertConcat: (%,%) -> %
        ++ \spad{vertConcat(x,y)} vertically concatenates two matrices with an
        ++ equal number of columns. The entries of y appear below
        ++ of the entries of x.  Error: if the matrices
        ++ do not have the same number of columns.
        ++
        ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
        ++X vertConcat(m,m)

    --% Part extractions/assignments

      listOfLists: % -> List List R
        ++ \spad{listOfLists(m)} returns the rows of the matrix m as a list
        ++ of lists.
        ++
        ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
        ++X listOfLists m

      elt: (%,List Integer,List Integer) -> %
        ++ \spad{elt(x,rowList,colList)} returns an m-by-n matrix consisting
        ++ of elements of x, where \spad{m = # rowList} and \spad{n = # colList}
        ++ If \spad{rowList = [i<1>,i<2>,...,i<m>]} and \spad{colList =
        ++ [j<1>,j<2>,...,j<n>]}, then the \spad{(k,l)}th entry of
        ++ \spad{elt(x,rowList,colList)} is \spad{x(i<k>,j<l>)}.
        ++
        ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
        ++X elt(m,3,3)

      setelt: (%,List Integer,List Integer, %) -> %
        ++ \spad{setelt(x,rowList,colList,y)} destructively alters the matrix x.
        ++ If y is \spad{m}-by-\spad{n}, \spad{rowList = [i<1>,i<2>,...,i<m>]}
        ++ and \spad{colList = [j<1>,j<2>,...,j<n>]}, then \spad{x(i<k>,j<l>)}
        ++ is set to \spad{y(k,l)} for \spad{k = 1,...,m} and \spad{l = 1,...,n}
        ++
        ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
        ++X setelt(m,3,3,10)
```

```
      swapRows_!: (%,Integer,Integer) -> %
        ++ \spad{swapRows!(m,i,j)} interchanges the \spad{i}th and \spad{j}th
        ++ rows of m. This destructively alters the matrix.
        ++
        ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
        ++X swapRows!(m,2,4)

      swapColumns_!: (%,Integer,Integer) -> %
        ++ \spad{swapColumns!(m,i,j)} interchanges the \spad{i}th and \spad{j}th
        ++ columns of m. This destructively alters the matrix.
        ++
        ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
        ++X swapColumns!(m,2,4)

      subMatrix: (%,Integer,Integer,Integer,Integer) -> %
        ++ \spad{subMatrix(x,i1,i2,j1,j2)} extracts the submatrix
        ++ \spad{[x(i,j)]} where the index i ranges from \spad{i1} to \spad{i2}
        ++ and the index j ranges from \spad{j1} to \spad{j2}.
        ++
        ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
        ++X subMatrix(m,1,3,2,4)

      setsubMatrix_!: (%,Integer,Integer,%) -> %
        ++ \spad{setsubMatrix(x,i1,j1,y)} destructively alters the
        ++ matrix x. Here \spad{x(i,j)} is set to \spad{y(i-i1+1,j-j1+1)} for
        ++ \spad{i = i1,...,i1-1+nrows y} and \spad{j = j1,...,j1-1+ncols y}.
        ++
        ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
        ++X setsubMatrix!(m,2,2,matrix [[3,3],[3,3]])


    --% Arithmetic

      "+": (%,%) -> %
        ++ \spad{x + y} is the sum of the matrices x and y.
        ++ Error: if the dimensions are incompatible.
        ++
        ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
        ++X m+m

      "-": (%,%) -> %
        ++ \spad{x - y} is the difference of the matrices x and y.
        ++ Error: if the dimensions are incompatible.
        ++
        ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
        ++X m-m
```

```
"-":  %    -> %
  ++ \spad{-x} returns the negative of the matrix x.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X -m

"*": (%,%) -> %
  ++ \spad{x * y} is the product of the matrices x and y.
  ++ Error: if the dimensions are incompatible.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X m*m

"*": (R,%) -> %
  ++ \spad{r*x} is the left scalar multiple of the scalar r and the
  ++ matrix x.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X 1/3*m

"*": (%,R) -> %
  ++ \spad{x * r} is the right scalar multiple of the scalar r and the
  ++ matrix x.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X m*1/3

"*": (Integer,%) -> %
  ++ \spad{n * x} is an integer multiple.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X 3*m

"*": (%,Col) -> Col
  ++ \spad{x * c} is the product of the matrix x and the column vector c.
  ++ Error: if the dimensions are incompatible.
  ++
  ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
  ++X c:=coerce([1,2,3,4,5])@Matrix(INT)
  ++X m*c

"*": (Row,%) -> Row
  ++ \spad{r * x} is the product of the row vector r and the matrix x.
  ++ Error: if the dimensions are incompatible.
  ++
```

```
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X r:=transpose([1,2,3,4,5])@Matrix(INT)
      ++X r*m

    "**": (%,NonNegativeInteger) -> %
      ++ \spad{x ** n} computes a non-negative integral power of the matrix x.
      ++ Error: if the matrix is not square.
      ++
      ++X m:=matrix [[j**i for i in 0..4] for j in 1..5]
      ++X m**3

    if R has IntegralDomain then
      "exquo": (%,R) -> Union(%,"failed")
        ++ \spad{exquo(m,r)} computes the exact quotient of the elements
        ++ of m by r, returning \axiom{"failed"} if this is not possible.
        ++
        ++X m:=matrix [[2**i for i in 2..4] for j in 1..5]
        ++X exquo(m,2)

    if R has Field then
      "/": (%,R) -> %
        ++ \spad{m/r} divides the elements of m by r. Error: if \spad{r = 0}.
        ++
        ++X m:=matrix [[2**i for i in 2..4] for j in 1..5]
        ++X m/4

  --% Linear algebra

    if R has EuclideanDomain then
      rowEchelon: % -> %
        ++ \spad{rowEchelon(m)} returns the row echelon form of the matrix m.
        ++
        ++X rowEchelon matrix [[j**i for i in 0..4] for j in 1..5]

      columnSpace: % -> List Col
        ++ \spad{columnSpace(m)} returns a sublist of columns of the matrix m
        ++ forming a basis of its column space
        ++
        ++X columnSpace matrix [[1,2,3],[4,5,6],[7,8,9],[1,1,1]]

    if R has IntegralDomain then
      rank: % -> NonNegativeInteger
        ++ \spad{rank(m)} returns the rank of the matrix m.
        ++
        ++X rank matrix [[1,2,3],[4,5,6],[7,8,9]]
```

```
    nullity: % -> NonNegativeInteger
      ++ \spad{nullity(m)} returns the nullity of the matrix m. This is
      ++ the dimension of the null space of the matrix m.
      ++
      ++X nullity matrix [[1,2,3],[4,5,6],[7,8,9]]

    nullSpace: % -> List Col
      ++ \spad{nullSpace(m)} returns a basis for the null space of
      ++ the matrix m.
      ++
      ++X nullSpace matrix [[1,2,3],[4,5,6],[7,8,9]]

  if R has commutative("*") then
    determinant: % -> R
      ++ \spad{determinant(m)} returns the determinant of the matrix m.
      ++ Error: if the matrix is not square.
      ++
      ++X determinant matrix [[j**i for i in 0..4] for j in 1..5]

    minordet: % -> R
      ++ \spad{minordet(m)} computes the determinant of the matrix m using
      ++ minors. Error: if the matrix is not square.
      ++
      ++X minordet matrix [[j**i for i in 0..4] for j in 1..5]

  if R has CommutativeRing then
    pfaffian: % -> R
      ++ \spad{pfaffian(m)} returns the Pfaffian of the matrix m.
      ++ Error if the matrix is not antisymmetric
      ++
      ++X pfaffian [[0,1,0,0],[-1,0,0,0],[0,0,0,1],[0,0,-1,0]]
  if R has Field then
    inverse: % -> Union(%,"failed")
      ++ \spad{inverse(m)} returns the inverse of the matrix m.
      ++ If the matrix is not invertible, "failed" is returned.
      ++ Error: if the matrix is not square.
      ++
      ++X inverse matrix [[j**i for i in 0..4] for j in 1..5]

    "**": (%,Integer) -> %
      ++ \spad{m**n} computes an integral power of the matrix m.
      ++ Error: if matrix is not square or if the matrix
      ++ is square but not invertible.
      ++
      ++X (matrix [[j**i for i in 0..4] for j in 1..5]) ** 2
```

```
    add
      minr ==> minRowIndex
      maxr ==> maxRowIndex
      minc ==> minColIndex
      maxc ==> maxColIndex
      mini ==> minIndex
      maxi ==> maxIndex

  --% Predicates

      square? x == nrows x = ncols x

      diagonal? x ==
        not square? x => false
        for i in minr x .. maxr x repeat
          for j in minc x .. maxc x | (j - minc x) ^= (i - minr x) repeat
            not zero? qelt(x, i, j) => return false
        true

      symmetric? x ==
        (nRows := nrows x) ^= ncols x => false
        mr := minRowIndex x; mc := minColIndex x
        for i in 0..(nRows - 1) repeat
          for j in (i + 1)..(nRows - 1) repeat
            qelt(x,mr + i,mc + j) ^= qelt(x,mr + j,mc + i) => return false
        true

      antisymmetric? x ==
        (nRows := nrows x) ^= ncols x => false
        mr := minRowIndex x; mc := minColIndex x
        for i in 0..(nRows - 1) repeat
          for j in i..(nRows - 1) repeat
            qelt(x,mr + i,mc + j) ^= -qelt(x,mr + j,mc + i) =>
              return false
        true

  --% Creation of matrices

      zero(rows,cols) == new(rows,cols,0)

      matrix(l: List List R) ==
        null l => new(0,0,0)
        -- error check: this is a top level function
        rows : NonNegativeInteger := 1; cols := # first l
        cols = 0 => error "matrices with zero columns are not supported"
        for ll in rest l repeat
```

```
    cols ^= # ll => error "matrix: rows of different lengths"
    rows := rows + 1
  ans := new(rows,cols,0)
  for i in minr(ans)..maxr(ans) for ll in l repeat
    for j in minc(ans)..maxc(ans) for r in ll repeat
      qsetelt_!(ans,i,j,r)
  ans

scalarMatrix(n,r) ==
  ans := zero(n,n)
  for i in minr(ans)..maxr(ans) for j in minc(ans)..maxc(ans) repeat
    qsetelt_!(ans,i,j,r)
  ans

diagonalMatrix(l: List R) ==
  n := #l; ans := zero(n,n)
  for i in minr(ans)..maxr(ans) for j in minc(ans)..maxc(ans) _
      for r in l repeat qsetelt_!(ans,i,j,r)
  ans

diagonalMatrix(list: List %) ==
  rows : NonNegativeInteger := 0
  cols : NonNegativeInteger := 0
  for mat in list repeat
    rows := rows + nrows mat
    cols := cols + ncols mat
  ans := zero(rows,cols)
  loR := minr ans; loC := minc ans
  for mat in list repeat
    hiR := loR + nrows(mat) - 1; hiC := loC + nrows(mat) - 1
    for i in loR..hiR for k in minr(mat)..maxr(mat) repeat
      for j in loC..hiC for l in minc(mat)..maxc(mat) repeat
        qsetelt_!(ans,i,j,qelt(mat,k,l))
    loR := hiR + 1; loC := hiC + 1
  ans

coerce(v:Col) ==
  x := new(#v,1,0)
  one := minc(x)
  for i in minr(x)..maxr(x) for k in mini(v)..maxi(v) repeat
    qsetelt_!(x,i,one,qelt(v,k))
  x

transpose(v:Row) ==
  x := new(1,#v,0)
  one := minr(x)
```

```
      for j in minc(x)..maxc(x) for k in mini(v)..maxi(v) repeat
        qsetelt_!(x,one,j,qelt(v,k))
      x

    transpose(x:%) ==
      ans := new(ncols x,nrows x,0)
      for i in minr(ans)..maxr(ans) repeat
        for j in minc(ans)..maxc(ans) repeat
          qsetelt_!(ans,i,j,qelt(x,j,i))
      ans

    squareTop x ==
      nrows x < (cols := ncols x) =>
        error "squareTop: number of columns exceeds number of rows"
      ans := new(cols,cols,0)
      for i in minr(x)..(minr(x) + cols - 1) repeat
        for j in minc(x)..maxc(x) repeat
          qsetelt_!(ans,i,j,qelt(x,i,j))
      ans

    horizConcat(x,y) ==
      (rows := nrows x) ^= nrows y =>
        error "HConcat: matrices must have same number of rows"
      ans := new(rows,(cols := ncols x) + ncols y,0)
      for i in minr(x)..maxr(x) repeat
        for j in minc(x)..maxc(x) repeat
          qsetelt_!(ans,i,j,qelt(x,i,j))
      for i in minr(y)..maxr(y) repeat
        for j in minc(y)..maxc(y) repeat
          qsetelt_!(ans,i,j + cols,qelt(y,i,j))
      ans

    vertConcat(x,y) ==
      (cols := ncols x) ^= ncols y =>
        error "HConcat: matrices must have same number of columns"
      ans := new((rows := nrows x) + nrows y,cols,0)
      for i in minr(x)..maxr(x) repeat
        for j in minc(x)..maxc(x) repeat
          qsetelt_!(ans,i,j,qelt(x,i,j))
      for i in minr(y)..maxr(y) repeat
        for j in minc(y)..maxc(y) repeat
          qsetelt_!(ans,i + rows,j,qelt(y,i,j))
      ans

  --% Part extraction/assignment
```

```
listOfLists x ==
  ll : List List R := nil()
  for i in maxr(x)..minr(x) by -1 repeat
    l : List R := nil()
    for j in maxc(x)..minc(x) by -1 repeat
      l := cons(qelt(x,i,j),l)
    ll := cons(l,ll)
  ll

swapRows_!(x,i1,i2) ==
  (i1 < minr(x)) or (i1 > maxr(x)) or (i2 < minr(x)) or _
      (i2 > maxr(x)) => error "swapRows!: index out of range"
  i1 = i2 => x
  for j in minc(x)..maxc(x) repeat
    r := qelt(x,i1,j)
    qsetelt_!(x,i1,j,qelt(x,i2,j))
    qsetelt_!(x,i2,j,r)
  x

swapColumns_!(x,j1,j2) ==
  (j1 < minc(x)) or (j1 > maxc(x)) or (j2 < minc(x)) or _
      (j2 > maxc(x)) => error "swapColumns!: index out of range"
  j1 = j2 => x
  for i in minr(x)..maxr(x) repeat
    r := qelt(x,i,j1)
    qsetelt_!(x,i,j1,qelt(x,i,j2))
    qsetelt_!(x,i,j2,r)
  x

elt(x:%,rowList:List Integer,colList:List Integer) ==
  for ei in rowList repeat
    (ei < minr(x)) or (ei > maxr(x)) =>
      error "elt: index out of range"
  for ej in colList repeat
    (ej < minc(x)) or (ej > maxc(x)) =>
      error "elt: index out of range"
  y := new(# rowList,# colList,0)
  for ei in rowList for i in minr(y)..maxr(y) repeat
    for ej in colList for j in minc(y)..maxc(y) repeat
      qsetelt_!(y,i,j,qelt(x,ei,ej))
  y

setelt(x:%,rowList:List Integer,colList:List Integer,y:%) ==
  for ei in rowList repeat
    (ei < minr(x)) or (ei > maxr(x)) =>
      error "setelt: index out of range"
```

```
    for ej in colList repeat
      (ej < minc(x)) or (ej > maxc(x)) =>
        error "setelt: index out of range"
    ((# rowList) ^= (nrows y)) or ((# colList) ^= (ncols y)) =>
      error "setelt: matrix has bad dimensions"
    for ei in rowList for i in minr(y)..maxr(y) repeat
      for ej in colList for j in minc(y)..maxc(y) repeat
        qsetelt_!(x,ei,ej,qelt(y,i,j))
    y

  subMatrix(x,i1,i2,j1,j2) ==
    (i2 < i1) => error "subMatrix: bad row indices"
    (j2 < j1) => error "subMatrix: bad column indices"
    (i1 < minr(x)) or (i2 > maxr(x)) =>
      error "subMatrix: index out of range"
    (j1 < minc(x)) or (j2 > maxc(x)) =>
      error "subMatrix: index out of range"
    rows := (i2 - i1 + 1) pretend NonNegativeInteger
    cols := (j2 - j1 + 1) pretend NonNegativeInteger
    y := new(rows,cols,0)
    for i in minr(y)..maxr(y) for k in i1..i2 repeat
      for j in minc(y)..maxc(y) for l in j1..j2 repeat
        qsetelt_!(y,i,j,qelt(x,k,l))
    y

  setsubMatrix_!(x,i1,j1,y) ==
    i2 := i1 + nrows(y) -1
    j2 := j1 + ncols(y) -1
    (i1 < minr(x)) or (i2 > maxr(x)) =>
     error _
      "setsubMatrix!: inserted matrix too big, use subMatrix to restrict it"
    (j1 < minc(x)) or (j2 > maxc(x)) =>
     error _
      "setsubMatrix!: inserted matrix too big, use subMatrix to restrict it"
    for i in minr(y)..maxr(y) for k in i1..i2 repeat
      for j in minc(y)..maxc(y) for l in j1..j2 repeat
        qsetelt_!(x,k,l,qelt(y,i,j))
    x

--% Arithmetic

  x + y ==
    ((r := nrows x) ^= nrows y) or ((c := ncols x) ^= ncols y) =>
      error "can't add matrices of different dimensions"
    ans := new(r,c,0)
    for i in minr(x)..maxr(x) repeat
```

```
        for j in minc(x)..maxc(x) repeat
          qsetelt_!(ans,i,j,qelt(x,i,j) + qelt(y,i,j))
      ans

    x - y ==
      ((r := nrows x) ^= nrows y) or ((c := ncols x) ^= ncols y) =>
        error "can't subtract matrices of different dimensions"
      ans := new(r,c,0)
      for i in minr(x)..maxr(x) repeat
        for j in minc(x)..maxc(x) repeat
          qsetelt_!(ans,i,j,qelt(x,i,j) - qelt(y,i,j))
      ans

    - x == map((r1:R):R +-> - r1,x)

    a:R * x:% == map((r1:R):R +-> a * r1,x)

    x:% * a:R == map((r1:R):R +-> r1 * a,x)

    m:Integer * x:% == map((r1:R):R +-> m * r1,x)

    x:% * y:% ==
      (ncols x ^= nrows y) =>
        error "can't multiply matrices of incompatible dimensions"
      ans := new(nrows x,ncols y,0)
      for i in minr(x)..maxr(x) repeat
        for j in minc(y)..maxc(y) repeat
          entry :=
            sum : R := 0
            for k in minr(y)..maxr(y) for l in minc(x)..maxc(x) repeat
              sum := sum + qelt(x,i,l) * qelt(y,k,j)
            sum
          qsetelt_!(ans,i,j,entry)
      ans

    positivePower:(%,Integer) -> %
    positivePower(x,n) ==
--      one? n => x
      (n = 1) => x
      odd? n => x * positivePower(x,n - 1)
      y := positivePower(x,n quo 2)
      y * y

    x:% ** n:NonNegativeInteger ==
      not((nn:= nrows x) = ncols x) => error "**: matrix must be square"
      zero? n => scalarMatrix(nn,1)
```

```
    positivePower(x,n)

  --if R has ConvertibleTo InputForm then
    --convert(x:%):InputForm ==
      --convert [convert("matrix"::Symbol)@InputForm,
               --convert listOfLists x]$List(InputForm)

  if Col has shallowlyMutable then

    x:% * v:Col ==
      ncols(x) ^= #v =>
        error "can't multiply matrix A and vector v if #cols A ^= #v"
      w : Col := new(nrows x,0)
      for i in minr(x)..maxr(x) for k in mini(w)..maxi(w) repeat
        w.k :=
          sum : R := 0
          for j in minc(x)..maxc(x) for l in mini(v)..maxi(v) repeat
            sum := sum + qelt(x,i,j) * v(l)
          sum
      w

  if Row has shallowlyMutable then

    v:Row * x:% ==
      nrows(x) ^= #v =>
        error "can't multiply vector v and matrix A if #rows A ^= #v"
      w : Row := new(ncols x,0)
      for j in minc(x)..maxc(x) for k in mini(w)..maxi(w) repeat
        w.k :=
          sum : R := 0
          for i in minr(x)..maxr(x) for l in mini(v)..maxi(v) repeat
            sum := sum + qelt(x,i,j) * v(l)
          sum
      w

  if R has EuclideanDomain then
    columnSpace M ==
      M2 := rowEchelon M
      basis: List Col := []
      n: Integer := ncols M
      m: Integer := nrows M
      indRow: Integer := 1
      for k in 1..n while indRow <= m repeat
        if not zero?(M2.(indRow,k)) then
          basis := cons(column(M,k),basis)
          indRow := indRow + 1
```

```
    reverse! basis

  if R has CommutativeRing then
    skewSymmetricUnitMatrix(n:PositiveInteger):% ==
      matrix [[(if i=j+1 and odd? j
                  then -1
                  else if i=j-1 and odd? i
                         then 1
                         else 0) for j in 1..n] for i in 1..n]

    SUPR ==> SparseUnivariatePolynomial R

    PfChar(A:%):SUPR ==
      n := nrows A
      (n = 2) => monomial(1$R,2)$SUPR + qelt(A,1,2)::SUPR
      M:=subMatrix(A,3,n,3,n)
      r:=subMatrix(A,1,1,3,n)
      s:=subMatrix(A,3,n,2,2)
      p:=PfChar(M)
      d:=degree(p)$SUPR
      B:=skewSymmetricUnitMatrix((n-2)::PositiveInteger)
      C:=r*B
      g:List R := [qelt(C*s,1,1), qelt(A,1,2), 1]
      if d >= 4 then
        B:=M*B
        for i in 4..d by 2 repeat
          C:=C*B
          g:=cons(qelt(C*s,1,1),g)
      g:=reverse! g
      res:SUPR := 0
      for i in 0..d by 2 for j in 2..d+2 repeat
        c:=coefficient(p,i)
        for e in first(g,j) for k in 2..-d by -2 repeat
          res:=res+monomial(c*e,(k+i)::NonNegativeInteger)$SUPR
      res

    pfaffian a ==
      if antisymmetric? a
        then if odd? nrows a
               then 0
               else PfChar(a).0
        else
          error "pfaffian: only defined for antisymmetric square matrices"

  if R has IntegralDomain then
    x exquo a ==
```

```
            ans := new(nrows x,ncols x,0)
            for i in minr(x)..maxr(x) repeat
              for j in minc(x)..maxc(x) repeat
                entry :=
                  (r := (qelt(x,i,j) exquo a)) case "failed" =>
                    return "failed"
                  r :: R
                qsetelt_!(ans,i,j,entry)
            ans

          if R has Field then
            x / r == map((r1:R):R +-> r1 / r,x)

            x:% ** n:Integer ==
              not((nn:= nrows x) = ncols x) => error "**: matrix must be square"
              zero? n => scalarMatrix(nn,1)
              positive? n => positivePower(x,n)
              (xInv := inverse x) case "failed" =>
                error "**: matrix must be invertible"
              positivePower(xInv :: %,-n)
```

⟨*MATCAT.dotabb*⟩≡

```
"MATCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MATCAT"];
"MATCAT" -> "ARR2CAT"
```

⟨*MATCAT.dotfull*⟩≡

```
"MatrixCategory(a:Ring,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MATCAT"];
"MatrixCategory(a:Ring,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a)"
 ->
"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a))"
```

⟨*MATCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"MatrixCategory(a:Ring,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a)"
 [color=lightblue];
"MatrixCategory(a:Ring,b:FiniteLinearAggregate(a),c:FiniteLinearAggregate(a)"
 ->
"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggr

"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggr
 [color=lightblue];
"TwoDimensionalArrayCategory(a:Type,b:FiniteLinearAggregate(a),c:FiniteLinearAggr
    -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"
"HomogeneousAggregate(a:Type)" -> "Evalable(a:Type)"
"HomogeneousAggregate(a:Type)" -> "SetCategory()"

"Evalable(a:Type)" [color="#00EE00"];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

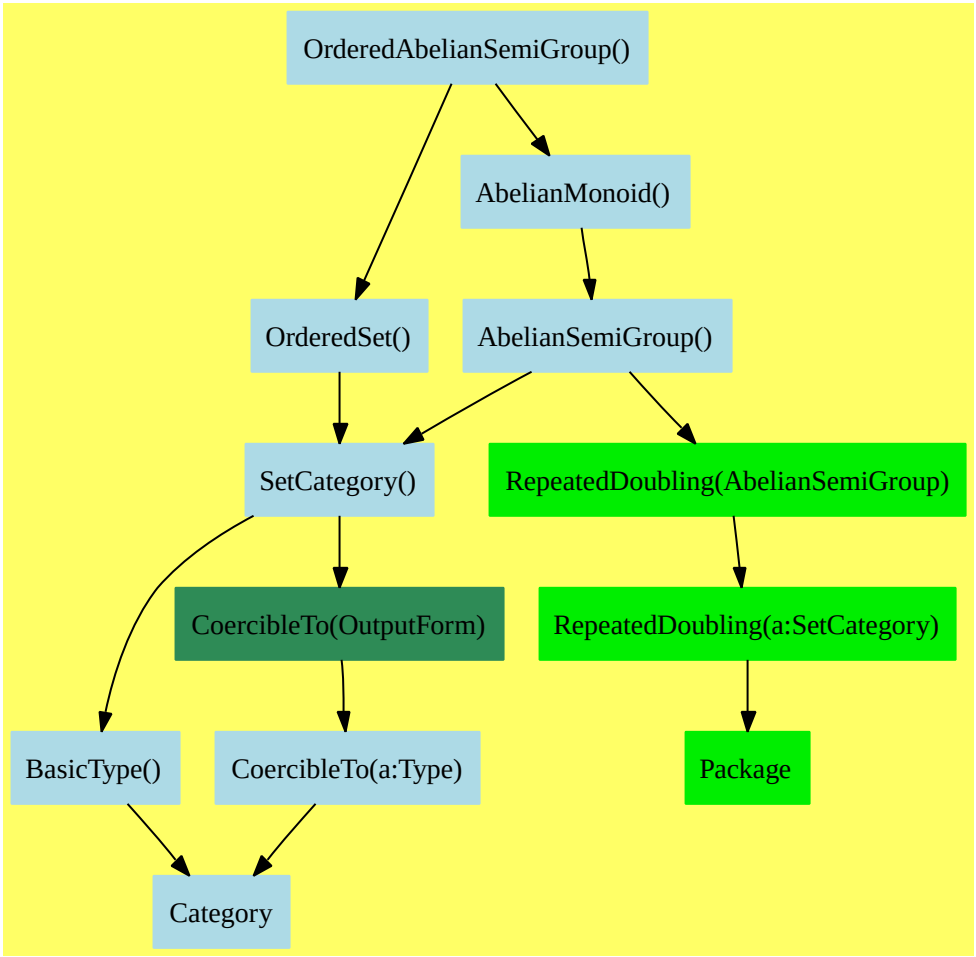
"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];

}
```

6.8 OrderedAbelianSemiGroup (OASGP)



See:

⇒ "OrderedAbelianMonoid" (OAMON) 7.9 on page 461
⇐ "AbelianMonoid" (ABELMON) 5.1 on page 207
⇐ "OrderedSet" (ORDSET) 4.19 on page 168

Exports:

0	coerce	hash	latex	max
min	sample	zero?	?~=?	?*?
?+?	?<?	?<=?	?=?	?>?
?>=?				

These exports come from (p168) OrderedSet():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?<? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

These exports come from (p207) AbelianMonoid():

```
0 : () -> %
sample : () -> %
zero? : % -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
```

⟨*category OASGP OrderedAbelianSemiGroup*⟩≡

```
)abbrev category OASGP OrderedAbelianSemiGroup
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Ordered sets which are also abelian semigroups, such that the addition
++ preserves the ordering.
++   \spad{ x < y => x+z < y+z}
OrderedAbelianSemiGroup(): Category == Join(OrderedSet, AbelianMonoid)
```

⟨*OASGP.dotabb*⟩≡

```
"OASGP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OASGP"];
"OASGP" -> "ORDSET"
"OASGP" -> "ABELMON"
```

⟨*OASGP.dotfull*⟩≡

```
"OrderedAbelianSemiGroup()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OASGP"];
"OrderedAbelianSemiGroup()" -> "OrderedSet()"
"OrderedAbelianSemiGroup()" -> "AbelianMonoid()"
```

⟨*OASGP.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedAbelianSemiGroup()" [color=lightblue];
"OrderedAbelianSemiGroup()" -> "OrderedSet()"
"OrderedAbelianSemiGroup()" -> "AbelianMonoid()"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SetCategory()"
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

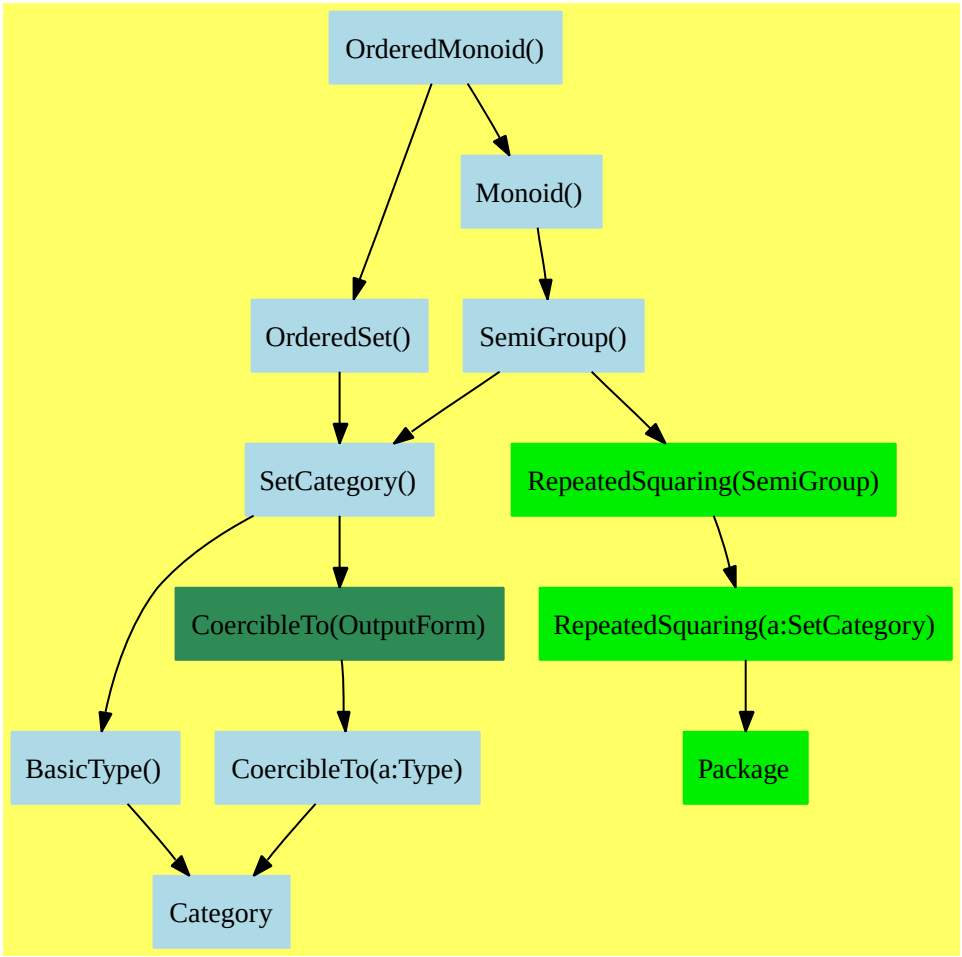
"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];
}
```

6.9 OrderedMonoid (ORDMON)



See:

⇐ "Monoid" (MONOID) 5.10 on page 256
⇐ "OrderedSet" (ORDSET) 4.19 on page 168

Exports:

1	coerce	hash	latex	max
min	one?	recip	sample	?*?
?**?	?<?	?<=?	?=?	?>?
?>=?	?~=?	?^?		

These exports come from (p256) Monoid():

```
1 : () -> %
```

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
?*? : (%,%) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

These exports come from (p168) OrderedSet():

```
max : (%,%) -> %
min : (%,%) -> %
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

⟨*category ORDMON OrderedMonoid*⟩≡

```
)abbrev category ORDMON OrderedMonoid
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Ordered sets which are also monoids, such that multiplication
++ preserves the ordering.
++
++ Axioms:
++   \spad{x < y => x*z < y*z}
++   \spad{x < y => z*x < z*y}

OrderedMonoid(): Category == Join(OrderedSet, Monoid)
```

⟨*ORDMON.dotabb*⟩≡
```
  "ORDMON"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDMON"];
  "ORDMON" -> "ORDSET"
  "ORDMON" -> "MONOID"
```

⟨*ORDMON.dotfull*⟩≡
```
  "OrderedMonoid()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDMON"];
  "OrderedMonoid()" -> "OrderedSet()"
  "OrderedMonoid()" -> "Monoid()"
```

⟨*ORDMON.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedMonoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDMON"];
"OrderedMonoid()" -> "OrderedSet()"
"OrderedMonoid()" -> "Monoid()"

"OrderedSet()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDSET"];
"OrderedSet()" -> "SetCategory()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SetCategory()"
"SemiGroup()" -> "RepeatedSquaring(SemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedSquaring(SemiGroup)" [color="#00EE00"];
"RepeatedSquaring(SemiGroup)" -> "RepeatedSquaring(a:SetCategory)"

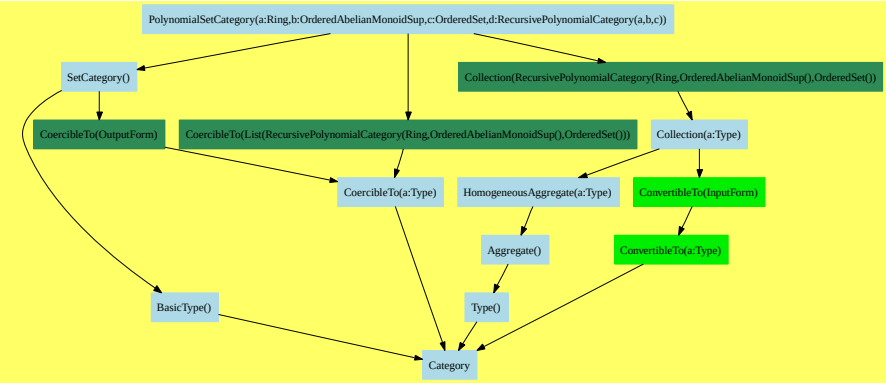
"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];

}
```

6.10 PolynomialSetCategory (PSETCAT)



See:

⇒ "TriangularSetCategory" (TSETCAT) 7.12 on page 478
⇐ "CoercibleTo" (KOERCE) 2.6 on page 15
⇐ "Collection" (CLAGG) 5.4 on page 218
⇐ "SetCategory" (SETCAT) 3.13 on page 91

Exports:

any?	coerce	collect
collectUnder	collectUpper	construct
convert	copy	count
empty	empty?	eq?
eval	every?	find
hash	headRemainder	latex
less?	mainVariables	mainVariable?
map	map!	member?
members	more?	mvar
parts	reduce	remainder
remove	removeDuplicates	retract
retractIfCan	rewriteIdealWithHeadRemainder	rewriteIdealWithRemainder
roughBase?	roughEqualIdeals?	roughSubIdeal?
roughUnitIdeal?	sample	select
size?	sort	triangular?
trivialIdeal?	variables	#?
?~=?	?=?	

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
mvar : % -> VarSet
retract : List P -> %
retractIfCan : List P -> Union(%,"failed")
```

These are implemented by this category:

```
collect : (%,VarSet) -> %
collectUnder : (%,VarSet) -> %
collectUpper : (%,VarSet) -> %
headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
mainVariables : % -> List VarSet
mainVariable? : (VarSet,%) -> Boolean
remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
roughBase? : % -> Boolean if R has INTDOM
roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
roughUnitIdeal? : % -> Boolean if R has INTDOM
sort : (%,VarSet) -> Record(under: %,floor: %,upper: %)
triangular? : % -> Boolean if R has INTDOM
trivialIdeal? : % -> Boolean
variables : % -> List VarSet
?=? : (%,%) -> Boolean
```

These exports come from (p91) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?~=? : (%,%) -> Boolean
```

These exports come from (p218) Collection(P)
where P:RecursivePolynomialCategory(R,E,V)
where R:Ring, E:OrderedAbelianMonoidSup, V:OrderedSet

```
any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
construct : List P -> %
copy : % -> %
count : (P,%) -> NonNegativeInteger if P has SETCAT and $ has finiteAggregate
count : ((P -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
```

```
eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
find : ((P -> Boolean),%) -> Union(P,"failed")
less? : (%,NonNegativeInteger) -> Boolean
map : ((P -> P),%) -> %
map! : ((P -> P),%) -> % if $ has shallowlyMutable
member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
members : % -> List P if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List P if $ has finiteAggregate
reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P,P) -> P if P has SETCAT and $ has finiteAggregate
reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
sample : () -> %
select : ((P -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
```

These exports come from (p15) CoercibleTo(List(P))
where P:RecursivePolynomialCategory(R,E,V)
where R:Ring, E:OrderedAbelianMonoidSup, V:OrderedSet

```
coerce : % -> List P
convert : % -> InputForm if P has KONVERT INFORM
```

These exports come from (p857) IntegralDomain():

⟨*category PSETCAT PolynomialSetCategory*⟩≡

```
)abbrev category PSETCAT PolynomialSetCategory
++ Author: Marc Moreno Maza
++ Date Created: 04/26/1994
++ Date Last Updated: 12/15/1998
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: polynomial, multivariate, ordered variables set
++ References:
++ Description: A category for finite subsets of a polynomial ring.
++ Such a set is only regarded as a set of polynomials and not
++ identified to the ideal it generates. So two distinct sets may
++ generate the same the ideal. Furthermore, for \spad{R} being an
++ integral domain, a set of polynomials may be viewed as a representation
```

```
++ of the ideal it generates in the polynomial ring \spad{(R)^(-1) P},
++ or the set of its zeros (described for instance by the radical of the
++ previous ideal, or a split of the associated affine variety) and so on.
++ So this category provides operations about those different notions.
++ Version: 2

PolynomialSetCategory(R:Ring, E:OrderedAbelianMonoidSup,_
  VarSet:OrderedSet, P:RecursivePolynomialCategory(R,E,VarSet)): Category ==
    Join(SetCategory,Collection(P),CoercibleTo(List(P))) with
     finiteAggregate
     retractIfCan : List(P) -> Union($,"failed")
         ++ \axiom{retractIfCan(lp)} returns an element of the domain
         ++ whose elements are the members of \axiom{lp} if such an element
         ++ exists, otherwise \axiom{"failed"} is returned.
     retract : List(P) -> $
         ++ \axiom{retract(lp)} returns an element of the domain whose elements
         ++ are the members of \axiom{lp} if such an element exists, otherwise
         ++ an error is produced.
     mvar : $ -> VarSet
         ++  \axiom{mvar(ps)} returns the main variable of the non constant
         ++ polynomial with the greatest main variable, if any, else an
         ++ error is returned.
     variables : $ -> List VarSet
         ++  \axiom{variables(ps)} returns the decreasingly sorted list of the
         ++  variables which are variables of some polynomial in \axiom{ps}.
     mainVariables : $  -> List VarSet
         ++ \axiom{mainVariables(ps)} returns the decreasingly sorted list
         ++ of the variables which are main variables of some polynomial
         ++ in \axiom{ps}.
     mainVariable? : (VarSet,$) -> Boolean
         ++ \axiom{mainVariable?(v,ps)} returns true iff \axiom{v} is the
         ++ main variable of some polynomial in \axiom{ps}.
     collectUnder : ($,VarSet) -> $
         ++ \axiom{collectUnder(ps,v)} returns the set consisting of the
         ++ polynomials of \axiom{ps} with main variable less than \axiom{v}.
     collect : ($,VarSet) -> $
         ++ \axiom{collect(ps,v)}  returns the set consisting of the
         ++ polynomials of \axiom{ps} with \axiom{v} as main variable.
     collectUpper : ($,VarSet) -> $
         ++ \axiom{collectUpper(ps,v)} returns the set consisting of the
         ++ polynomials of \axiom{ps} with main variable greater
         ++ than \axiom{v}.
     sort : ($,VarSet) -> Record(under:$,floor:$,upper:$)
         ++ \axiom{sort(v,ps)} returns \axiom{us,vs,ws} such that \axiom{us}
         ++ is \axiom{collectUnder(ps,v)}, \axiom{vs} is \axiom{collect(ps,v)}
         ++ and \axiom{ws} is \axiom{collectUpper(ps,v)}.
```

```
trivialIdeal?: $ -> Boolean
    ++ \axiom{trivialIdeal?(ps)} returns true iff \axiom{ps} does
    ++ not contain non-zero elements.
if R has IntegralDomain
then
  roughBase? : $ -> Boolean
      ++ \axiom{roughBase?(ps)} returns true iff for every pair
      ++ \axiom{{p,q}} of polynomials in \axiom{ps} their leading
      ++ monomials are relatively prime.
  roughSubIdeal?  : ($,$) -> Boolean
      ++  \axiom{roughSubIdeal?(ps1,ps2)} returns true iff it can proved
      ++ that all polynomials in  \axiom{ps1} lie in the ideal generated
      ++ by \axiom{ps2} in \axiom{\axiom{(R)^(-1) P}} without computing
      ++ Groebner bases.
  roughEqualIdeals? : ($,$) -> Boolean
      ++ \axiom{roughEqualIdeals?(ps1,ps2)} returns true iff it can
      ++ proved that \axiom{ps1} and \axiom{ps2} generate the same ideal
      ++ in \axiom{(R)^(-1) P} without computing Groebner bases.
  roughUnitIdeal? : $ -> Boolean
      ++ \axiom{roughUnitIdeal?(ps)} returns true iff \axiom{ps} contains
      ++  some non null element lying in the base ring \axiom{R}.
  headRemainder : (P,$) -> Record(num:P,den:R)
      ++ \axiom{headRemainder(a,ps)} returns \axiom{[b,r]} such that the
      ++ leading monomial of \axiom{b} is reduced in the sense of
      ++ Groebner bases w.r.t. \axiom{ps} and \axiom{r*a - b} lies in
      ++ the ideal generated by \axiom{ps}.
  remainder : (P,$) -> Record(rnum:R,polnum:P,den:R)
      ++ \axiom{remainder(a,ps)} returns \axiom{[c,b,r]} such that
      ++ \axiom{b} is fully reduced in the sense of Groebner bases
      ++ w.r.t. \axiom{ps}, \axiom{r*a - c*b} lies in the ideal
      ++ generated by \axiom{ps}. Furthermore, if \axiom{R} is a
      ++ gcd-domain, \axiom{b} is primitive.
  rewriteIdealWithHeadRemainder : (List(P),$) -> List(P)
      ++ \axiom{rewriteIdealWithHeadRemainder(lp,cs)} returns \axiom{lr}
      ++ such that the leading monomial of every polynomial in \axiom{lr}
      ++ is reduced in the sense of Groebner bases w.r.t. \axiom{cs}
      ++ and \axiom{(lp,cs)} and \axiom{(lr,cs)} generate the same
      ++ ideal in \axiom{(R)^(-1) P}.
  rewriteIdealWithRemainder : (List(P),$) -> List(P)
      ++ \axiom{rewriteIdealWithRemainder(lp,cs)} returns \axiom{lr}
      ++ such that every polynomial in \axiom{lr} is fully reduced in
      ++ the sense of Groebner bases w.r.t. \axiom{cs} and
      ++ \axiom{(lp,cs)} and \axiom{(lr,cs)} generate the same ideal
      ++ in \axiom{(R)^(-1) P}.
  triangular? : $ -> Boolean
      ++ \axiom{triangular?(ps)} returns true iff \axiom{ps} is a
```

```
          ++ triangular set, i.e. two distinct polynomials have distinct
          ++ main variables and no constant lies in \axiom{ps}.

    add

      NNI ==> NonNegativeInteger
      B ==> Boolean

      elements: $ -> List(P)

      elements(ps:$):List(P) ==
        lp : List(P) := members(ps)$$

      variables1(lp:List(P)):(List VarSet) ==
        lvars : List(List(VarSet)) := [variables(p)$P for p in lp]
        sort((z1:VarSet,z2:VarSet):Boolean +-> z1 > z2,
              removeDuplicates(concat(lvars)$List(VarSet)))

      variables2(lp:List(P)):(List VarSet) ==
        lvars : List(VarSet) := [mvar(p)$P for p in lp]
        sort((z1:VarSet,z2:VarSet):Boolean +-> z1 > z2,
              removeDuplicates(lvars)$List(VarSet))

      variables (ps:$) ==
        variables1(elements(ps))

      mainVariables (ps:$) ==
        variables2(remove(ground?,elements(ps)))

      mainVariable? (v,ps) ==
        lp : List(P) := remove(ground?,elements(ps))
        while (not empty? lp) and (not (mvar(first(lp)) = v)) repeat
          lp := rest lp
        (not empty? lp)

      collectUnder (ps,v) ==
        lp : List P := elements(ps)
        lq : List P := []
        while (not empty? lp) repeat
          p := first lp
          lp := rest lp
          if (ground?(p)) or (mvar(p) < v)
            then
              lq := cons(p,lq)
        construct(lq)$$
```

```
collectUpper (ps,v) ==
  lp : List P := elements(ps)
  lq : List P := []
  while (not empty? lp) repeat
    p := first lp
    lp := rest lp
    if (not ground?(p)) and (mvar(p) > v)
      then
        lq := cons(p,lq)
  construct(lq)$$

collect (ps,v) ==
  lp : List P := elements(ps)
  lq : List P := []
  while (not empty? lp) repeat
    p := first lp
    lp := rest lp
    if (not ground?(p)) and (mvar(p) = v)
      then
        lq := cons(p,lq)
  construct(lq)$$

sort (ps,v) ==
  lp : List P := elements(ps)
  us : List P := []
  vs : List P := []
  ws : List P := []
  while (not empty? lp) repeat
    p := first lp
    lp := rest lp
    if (ground?(p)) or (mvar(p) < v)
      then
        us := cons(p,us)
      else
        if (mvar(p) = v)
          then
            vs := cons(p,vs)
          else
            ws := cons(p,ws)
  [construct(us)$$,_
   construct(vs)$$,_
   construct(ws)$$]$Record(under:$,floor:$,upper:$)

ps1 = ps2 ==
  {p for p in elements(ps1)} =$(Set P) {p for p in elements(ps2)}
```

```
exactQuo : (R,R) -> R

localInf? (p:P,q:P):B ==
  degree(p) <$E degree(q)

localTriangular? (lp:List(P)):B ==
  lp := remove(zero?, lp)
  empty? lp => true
  any? (ground?, lp) => false
  lp := sort((z1:P,z2:P):Boolean +-> mvar(z1)$P > mvar(z2)$P, lp)
  p,q : P
  p := first lp
  lp := rest lp
  while (not empty? lp) and (mvar(p) > mvar((q := first(lp)))) repeat
    p := q
    lp := rest lp
  empty? lp

triangular? ps ==
  localTriangular? elements ps

trivialIdeal? ps ==
  empty?(remove(zero?,elements(ps))$(List(P)))$(List(P))

if R has IntegralDomain
then

  roughUnitIdeal? ps ==
    any?(ground?,remove(zero?,elements(ps))$(List(P)))$(List P)

  relativelyPrimeLeadingMonomials? (p:P,q:P):B ==
    dp : E := degree(p)
    dq : E := degree(q)
    (sup(dp,dq)$E =$E dp +$E dq)@B

  roughBase? ps ==
    lp := remove(zero?,elements(ps))$(List(P))
    empty? lp => true
    rB? : B := true
    while (not empty? lp) and rB? repeat
      p := first lp
      lp := rest lp
      copylp := lp
      while (not empty? copylp) and rB? repeat
        rB? := relativelyPrimeLeadingMonomials?(p,first(copylp))
        copylp := rest copylp
```

```
      rB?

    roughSubIdeal?(ps1,ps2) ==
      lp: List(P) := rewriteIdealWithRemainder(elements(ps1),ps2)
      empty? (remove(zero?,lp))

    roughEqualIdeals? (ps1,ps2) ==
      ps1 =$$ ps2 => true
      roughSubIdeal?(ps1,ps2) and roughSubIdeal?(ps2,ps1)

  if (R has GcdDomain) and (VarSet has ConvertibleTo (Symbol))
  then

    LPR ==> List Polynomial R
    LS ==> List Symbol

    if R has EuclideanDomain
      then
        exactQuo(r:R,s:R):R ==
          r quo$R s
      else
        exactQuo(r:R,s:R):R ==
          (r exquo$R s)::R

    headRemainder (a,ps) ==
      lp1 : List(P) := remove(zero?, elements(ps))$(List(P))
      empty? lp1 => [a,1$R]
      any?(ground?,lp1) => [reductum(a),1$R]
      r : R := 1$R
      lp1 := sort(localInf?, reverse elements(ps))
      lp2 := lp1
      e : Union(E, "failed")
      while (not zero? a) and (not empty? lp2) repeat
        p := first lp2
        if ((e:= subtractIfCan(degree(a),degree(p))) case E)
          then
            g := gcd((lca := leadingCoefficient(a)),_
                     (lcp := leadingCoefficient(p)))$R
            (lca,lcp) := (exactQuo(lca,g),exactQuo(lcp,g))
            a := lcp * reductum(a) - monomial(lca, e::E)$P * reductum(p)
            r := r * lcp
            lp2 := lp1
          else
            lp2 := rest lp2
      [a,r]
```

```
        makeIrreducible! (frac:Record(num:P,den:R)):Record(num:P,den:R) ==
          g := gcd(frac.den,frac.num)$P
--          one? g => frac
          (g = 1) => frac
          frac.num := exactQuotient!(frac.num,g)
          frac.den := exactQuo(frac.den,g)
          frac

        remainder (a,ps) ==
          hRa := makeIrreducible! headRemainder (a,ps)
          a := hRa.num
          r : R := hRa.den
          zero? a => [1$R,a,r]
          b : P := monomial(1$R,degree(a))$P
          c : R := leadingCoefficient(a)
          while not zero?(a := reductum a) repeat
            hRa := makeIrreducible!  headRemainder (a,ps)
            a := hRa.num
            r := r * hRa.den
            g := gcd(c,(lca := leadingCoefficient(a)))$R
            b := ((hRa.den) * exactQuo(c,g)) * b + _
                  monomial(exactQuo(lca,g),degree(a))$P
            c := g
          [c,b,r]

        rewriteIdealWithHeadRemainder(ps,cs) ==
          trivialIdeal? cs => ps
          roughUnitIdeal? cs => [0$P]
          ps := remove(zero?,ps)
          empty? ps => ps
          any?(ground?,ps) => [1$P]
          rs : List P := []
          while not empty? ps repeat
            p := first ps
            ps := rest ps
            p := (headRemainder(p,cs)).num
            if not zero? p
              then
                if ground? p
                  then
                    ps := []
                    rs := [1$P]
                  else
                    primitivePart! p
                    rs := cons(p,rs)
          removeDuplicates rs
```

```
        rewriteIdealWithRemainder(ps,cs) ==
          trivialIdeal? cs => ps
          roughUnitIdeal? cs => [0$P]
          ps := remove(zero?,ps)
          empty? ps => ps
          any?(ground?,ps) => [1$P]
          rs : List P := []
          while not empty? ps repeat
            p := first ps
            ps := rest ps
            p := (remainder(p,cs)).polnum
            if not zero? p
              then
                if ground? p
                  then
                    ps := []
                    rs := [1$P]
                  else
                    rs := cons(unitCanonical(p),rs)
          removeDuplicates rs
```

⟨*PSETCAT.dotabb*⟩≡

```
"PSETCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PSETCAT"];
"PSETCAT" -> "KOERCE"
"PSETCAT" -> "CLAGG"
"PSETCAT" -> "SETCAT"
```

⟨*PSETCAT.dotfull*⟩≡

```
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PSETCAT"];
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 -> "SetCategory()"
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 -> "Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet())"
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 -> "CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet(

"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PSETCAT"];
"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
  -> "PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom
```

⟨*PSETCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 [color=lightblue];
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 -> "SetCategory()"
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 -> "Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),Ordere
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 -> "CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),

"CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),Orde
 [color=seagreen];
"CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),Orde
  -> "CoercibleTo(a:Type)"

"Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet
"Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet
  -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"
"Collection(a:Type)" -> "ConvertibleTo(InputForm)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"
```

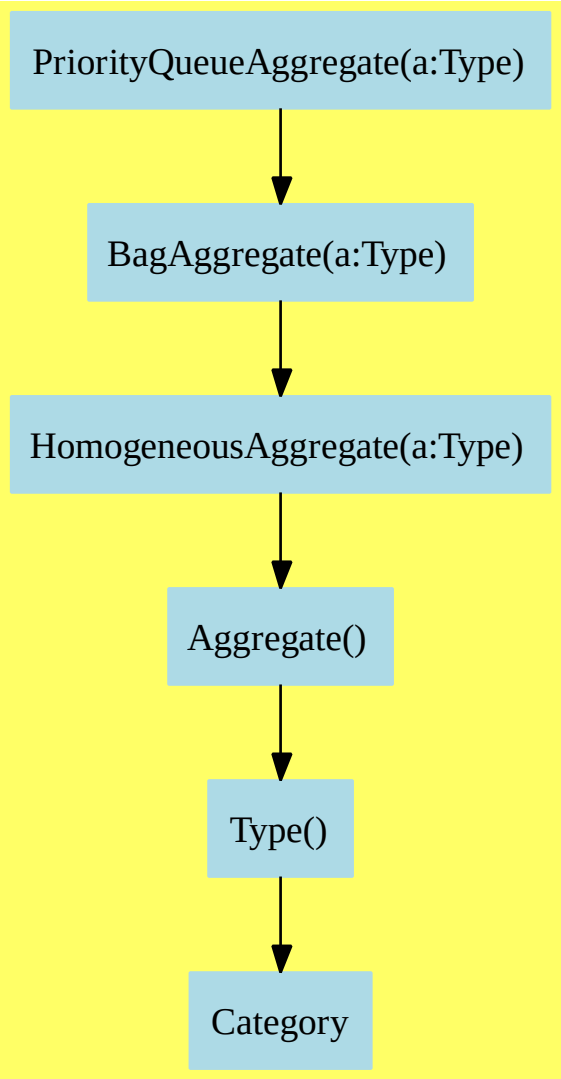
```
"Type()" [color=lightblue];
"Type()" -> "Category"

"ConvertibleTo(InputForm)" [color="#00EE00"];
"ConvertibleTo(InputForm)" -> "ConvertibleTo(a:Type)"

"ConvertibleTo(a:Type)" [color="#00EE00"];
"ConvertibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

6.11 PriorityQueueAggregate (PRQAGG)



See:

⇒ "OrderedMultisetAggregate" (OMSAGG) 9.7 on page 623
⇐ "BagAggregate" (BGAGG) 5.2 on page 211

Exports:

any?	bag	copy	coerce	count
empty	empty?	eq?	eval	every?
extract!	hash	insert!	inspect	latex
less?	map	map!	max	member?
members	merge	merge!	more?	parts
sample	size?	#?	?=?	?~=?

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
max : % -> S
merge : (%,%) -> %
merge! : (%,%) -> %
```

These exports come from (p211) BagAggregate(S:OrderedSet):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
extract! : % -> S
hash : % -> SingleInteger if S has SETCAT
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String if S has SETCAT
```

```
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category PRQAGG PriorityQueueAggregate*⟩≡

```
)abbrev category PRQAGG PriorityQueueAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A priority queue is a bag of items from an ordered set where the item
++ extracted is always the maximum element.
PriorityQueueAggregate(S:OrderedSet): Category == BagAggregate S with
   finiteAggregate
   max: % -> S
     ++ max(q) returns the maximum element of priority queue q.
   merge: (%,%) -> %
     ++ merge(q1,q2) returns combines priority queues q1 and q2 to return
     ++ a single priority queue q.
   merge_!: (%,%) -> %
     ++ merge!(q,q1) destructively changes priority queue q to include the
     ++ values from priority queue q1.
```

⟨*PRQAGG.dotabb*⟩≡

```
"PRQAGG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PRQAGG"];
"PRQAGG" -> "BGAGG"
```

⟨*PRQAGG.dotfull*⟩≡

```
"PriorityQueueAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PRQAGG"];
"PriorityQueueAggregate(a:Type)" -> "BagAggregate(a:Type)"

"PriorityQueueAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PRQAGG"];
"PriorityQueueAggregate(a:SetCategory)" -> "PriorityQueueAggregate(a:Type)"

"PriorityQueueAggregate(a:OrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PRQAGG"];
"PriorityQueueAggregate(a:OrderedSet)" ->
   "PriorityQueueAggregate(a:SetCategory)"
```

⟨*PRQAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PriorityQueueAggregate(a:Type)" [color=lightblue];
"PriorityQueueAggregate(a:Type)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

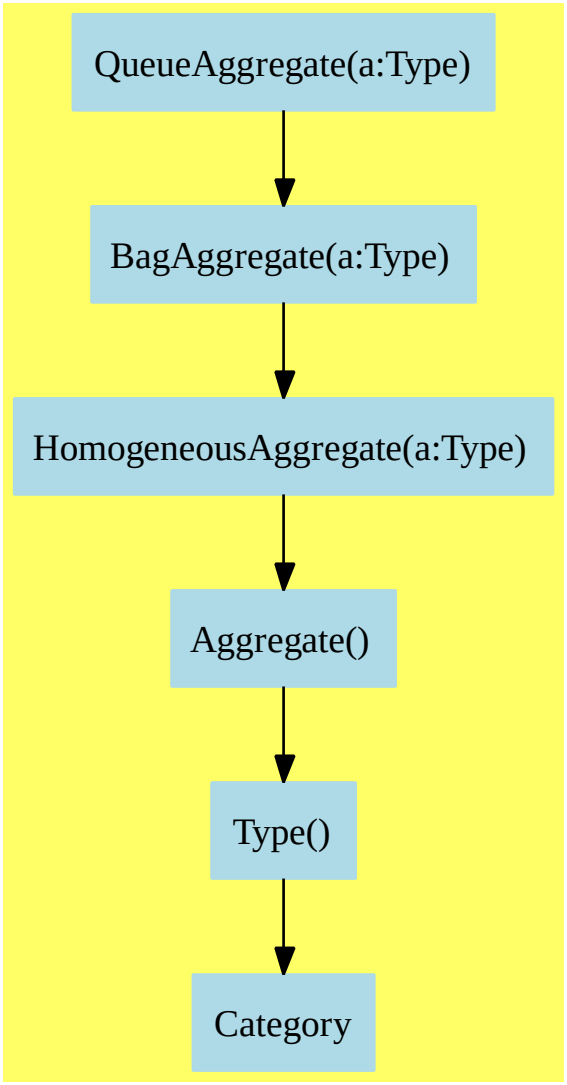
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

6.12 QueueAggregate (QUAGG)



See:

⇒ "DequeueAggregate" (DQAGG) 7.4 on page 433
⇐ "BagAggregate" (BGAGG) 5.2 on page 211

Exports:

any?	bag	back	coerce	copy
count	dequeue!	empty	empty?	enqueue!
eq?	eval	every?	extract!	front
hash	insert!	inspect	latex	length
less?	map	map!	member?	members
more?	parts	rotate!	sample	size?
#?	?=?	?~=?		

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are implemented by this category:

```
back : % -> S
dequeue! : % -> S
enqueue! : (S,%) -> S
front : % -> S
length : % -> NonNegativeInteger
rotate! : % -> %
```

These exports come from (p211) BagAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
```

```
extract! : % -> S
hash : % -> SingleInteger if S has SETCAT
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category QUAGG QueueAggregate*⟩≡

```
)abbrev category QUAGG QueueAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A queue is a bag where the first item inserted is the first
++ item extracted.
QueueAggregate(S:Type): Category == BagAggregate S with
   finiteAggregate
   enqueue_!: (S, %) -> S
     ++ enqueue!(x,q) inserts x into the queue q at the back end.
   dequeue_!: % -> S
     ++ dequeue! s destructively extracts the first (top) element
     ++ from queue q. The element previously second in the queue becomes
     ++ the first element. Error: if q is empty.
   rotate_!: % -> %
     ++ rotate! q rotates queue q so that the element at the front of
     ++ the queue goes to the back of the queue.
     ++ Note: rotate! q is equivalent to enqueue!(dequeue!(q)).
   length: % -> NonNegativeInteger
     ++ length(q) returns the number of elements in the queue.
     ++ Note: \axiom{length(q) = #q}.
```

```
    front: % -> S
      ++ front(q) returns the element at the front of the queue.
      ++ The queue q is unchanged by this operation.
      ++ Error: if q is empty.
    back: % -> S
      ++ back(q) returns the element at the back of the queue.
      ++ The queue q is unchanged by this operation.
      ++ Error: if q is empty.
```

⟨*QUAGG.dotabb*⟩≡

```
"QUAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=QUAGG"];
"QUAGG" -> "BGAGG"
```

⟨*QUAGG.dotfull*⟩≡

```
"QueueAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=QUAGG"];
"QueueAggregate(a:Type)" -> "BagAggregate(a:Type)"

"QueueAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=QUAGG"];
"QueueAggregate(a:SetCategory)" -> "QueueAggregate(a:Type)"
```

⟨*QUAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"QueueAggregate(a:Type)" [color=lightblue];
"QueueAggregate(a:Type)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

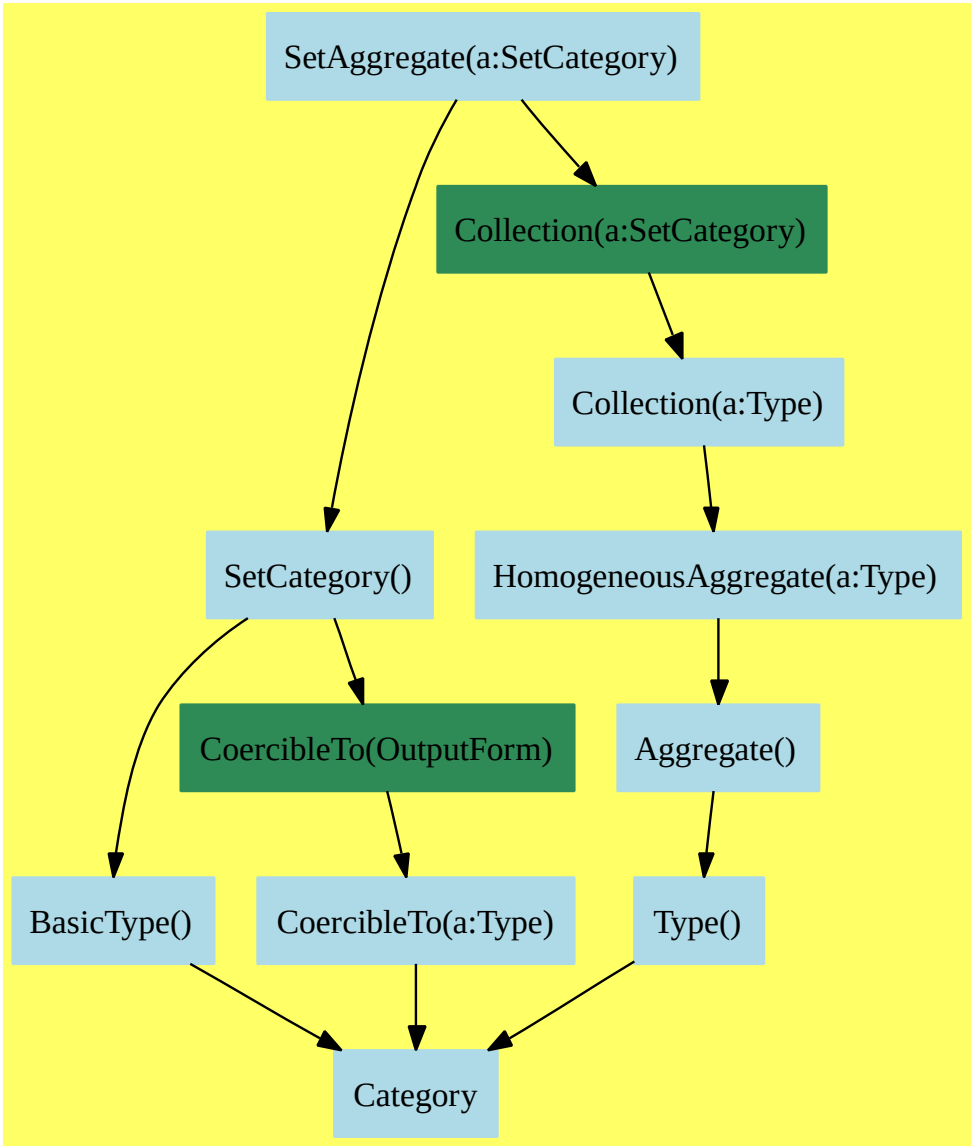
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

6.13 SetAggregate (SETAGG)



See:

⇒ “FiniteSetAggregate” (FSAGG) 8.2 on page 503
⇒ “MultisetAggregate” (MSETAGG) 8.7 on page 548
⇐ “Collection” (CLAGG) 5.4 on page 218
⇐ “SetCategory” (SETCAT) 3.13 on page 91

Exports:

any?	brace	coerce	construct	convert
copy	count	difference	empty	empty?
eq?	eval	every?	find	hash
intersect	latex	less?	map	map!
member?	members	more?	parts	reduce
remove	removeDuplicates	sample	select	set
size?	subset?	symmetricDifference	union	#?
?<?	?=?	?~=?		

Attributes Exported:

- **partiallyOrderedSet** is true if a set with $<$ which is transitive, but not$(a < b$ or $a = b)$ does not necessarily imply $b < a$.
- **nil**

These are directly exported but not implemented:

```
brace : List S -> %
brace : () -> %
difference : (%,%) -> %
intersect : (%,%) -> %
set : List S -> %
set : () -> %
subset? : (%,%) -> Boolean
union : (%,%) -> %
?<? : (%,%) -> Boolean
```

These are implemented by this category:

```
difference : (%,S) -> %
symmetricDifference : (%,%) -> %
union : (S,%) -> %
union : (%,S) -> %
```

These exports come from (p91) SetCategory():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

These exports come from (p218) Collection(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
construct : List S -> %
```

```
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : ((S -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
count : (S,%) -> NonNegativeInteger
    if S has SETCAT
    and $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,Equation S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,S,S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,List S,List S) -> %
    if S has EVALAB S
    and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
find : ((S -> Boolean),%) -> Union(S,"failed")
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
members : % -> List S if $ has finiteAggregate
member? : (S,%) -> Boolean
    if S has SETCAT and $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
    if S has SETCAT
    and $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
    if $ has finiteAggregate
remove : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove : (S,%) -> %
    if S has SETCAT
    and $ has finiteAggregate
removeDuplicates : % -> %
    if S has SETCAT
    and $ has finiteAggregate
sample : () -> %
select : ((S -> Boolean),%) -> %
```

```
    if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger
    if $ has finiteAggregate
```

⟨*category SETAGG SetAggregate*⟩≡

```
)abbrev category SETAGG SetAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: 14 Oct, 1993 by RSS
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A set category lists a collection of set-theoretic operations
++ useful for both finite sets and multisets.
++ Note however that finite sets are distinct from multisets.
++ Although the operations defined for set categories are
++ common to both, the relationship between the two cannot
++ be described by inclusion or inheritance.
SetAggregate(S:SetCategory):
  Category == Join(SetCategory, Collection(S)) with
   partiallyOrderedSet
   "<"         : (%, %) -> Boolean
     ++ s < t returns true if all elements of set aggregate s are also
     ++ elements of set aggregate t.
   brace       : () -> %
     ++ brace()$D (otherwise written {}$D)
     ++ creates an empty set aggregate of type D.
     ++ This form is considered obsolete. Use \axiomFun{set} instead.
   brace       : List S -> %
     ++ brace([x,y,...,z])
     ++ creates a set aggregate containing items x,y,...,z.
     ++ This form is considered obsolete. Use \axiomFun{set} instead.
   set         : () -> %
     ++ set()$D creates an empty set aggregate of type D.
   set         : List S -> %
     ++ set([x,y,...,z]) creates a set aggregate containing items x,y,...,z.
   intersect: (%, %) -> %
     ++ intersect(u,v) returns the set aggregate w consisting of
     ++ elements common to both set aggregates u and v.
     ++ Note: equivalent to the notation (not currently supported)
     ++ {x for x in u | member?(x,v)}.
   difference  : (%, %) -> %
```

```
      ++ difference(u,v) returns the set aggregate w consisting of
      ++ elements in set aggregate u but not in set aggregate v.
      ++ If u and v have no elements in common, \axiom{difference(u,v)}
      ++ returns a copy of u.
      ++ Note: equivalent to the notation (not currently supported)
      ++ \axiom{{x for x in u | not member?(x,v)}}.
    difference  : (%, S) -> %
      ++ difference(u,x) returns the set aggregate u with element x removed.
      ++ If u does not contain x, a copy of u is returned.
      ++ Note: \axiom{difference(s, x) = difference(s, {x})}.
    symmetricDifference : (%, %) -> %
      ++ symmetricDifference(u,v) returns the set aggregate of elements x
      ++ which are members of set aggregate u or set aggregate v but
      ++ not both. If u and v have no elements in common,
      ++ \axiom{symmetricDifference(u,v)} returns a copy of u.
      ++ Note: \axiom{symmetricDifference(u,v) =
      ++  union(difference(u,v),difference(v,u))}
    subset?     : (%, %) -> Boolean
      ++ subset?(u,v) tests if u is a subset of v.
      ++ Note: equivalent to
      ++ \axiom{reduce(and,{member?(x,v) for x in u},true,false)}.
    union       : (%, %) -> %
      ++ union(u,v) returns the set aggregate of elements which are members
      ++ of either set aggregate u or v.
    union       : (%, S) -> %
      ++ union(u,x) returns the set aggregate u with the element x added.
      ++ If u already contains x, \axiom{union(u,x)} returns a copy of u.
    union       : (S, %) -> %
      ++ union(x,u) returns the set aggregate u with the element x added.
      ++ If u already contains x, \axiom{union(x,u)} returns a copy of u.
  add
   symmetricDifference(x, y)    == union(difference(x, y), difference(y, x))
   union(s:%, x:S) == union(s, {x})
   union(x:S, s:%) == union(s, {x})
   difference(s:%, x:S) == difference(s, {x})
```

⟨*SETAGG.dotabb*⟩≡

```
"SETAGG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SETAGG"];
"SETAGG" -> "SETCAT"
"SETAGG" -> "CLAGG"
```

⟨*SETAGG.dotfull*⟩≡

```
"SetAggregate(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SETAGG"];
"SetAggregate(a:SetCategory)" -> "SetCategory()"
"SetAggregate(a:SetCategory)" -> "Collection(a:SetCategory)"
```

⟨*SETAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"SetAggregate(a:SetCategory)" [color=lightblue];
"SetAggregate(a:SetCategory)" -> "SetCategory()"
"SetAggregate(a:SetCategory)" -> "Collection(a:SetCategory)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Collection(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=CLAGG"];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

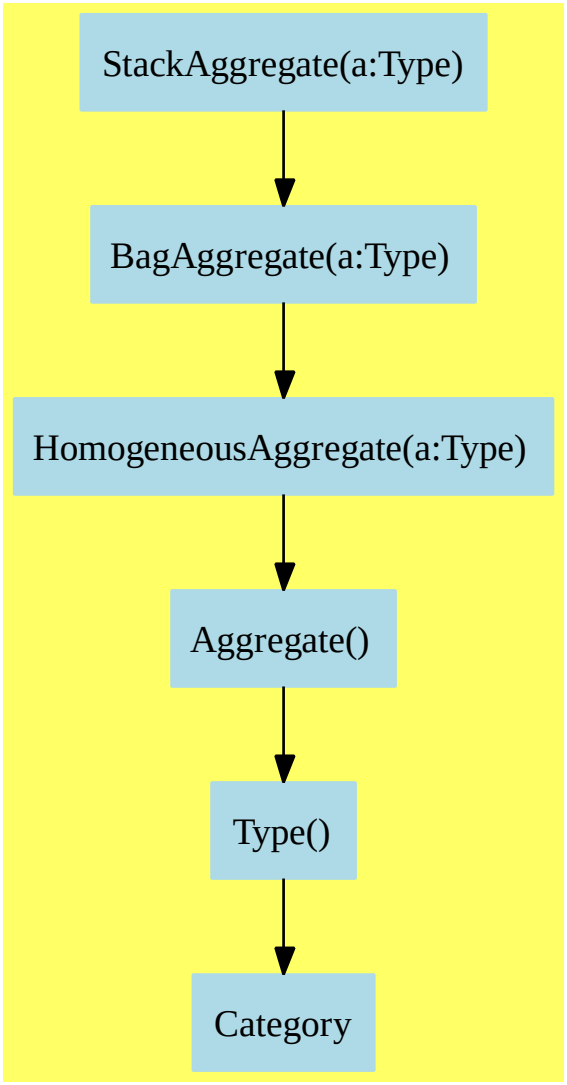
"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];

}
```

6.14 StackAggregate (SKAGG)



See:

⇒ “DequeueAggregate” (DQAGG) 7.4 on page 433
⇐ “BagAggregate” (BGAGG) 5.2 on page 211

Exports:

any?	bag	coerce	copy	count
depth	empty	empty?	eq?	eval
every?	extract!	hash	insert!	inspect
latex	less?	map	map!	member?
members	more?	parts	pop!	push!
sample	size?	top	#?	?=?
?~=?				

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
depth : % -> NonNegativeInteger
pop! : % -> S
push! : (S,%) -> S
top : % -> S
```

These exports come from (p211) BagAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
extract! : % -> S
hash : % -> SingleInteger if S has SETCAT
```

```
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category SKAGG StackAggregate*⟩≡

```
)abbrev category SKAGG StackAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A stack is a bag where the last item inserted is the first item extracted.
StackAggregate(S:Type): Category == BagAggregate S with
   finiteAggregate
   push_!: (S,%) -> S
     ++ push!(x,s) pushes x onto stack s, i.e. destructively changing s
     ++ so as to have a new first (top) element x.
     ++ Afterwards, pop!(s) produces x and pop!(s) produces the original s.
     ++
     ++X a:Stack INT:= stack [1,2,3,4,5]
     ++X push! a
     ++X a
   pop_!: % -> S
     ++ pop!(s) returns the top element x, destructively removing x from s.
     ++ Note: Use \axiom{top(s)} to obtain x without removing it from s.
     ++ Error: if s is empty.
     ++
     ++X a:Stack INT:= stack [1,2,3,4,5]
     ++X pop! a
     ++X a
```

```
    top: % -> S
      ++ top(s) returns the top element x from s; s remains unchanged.
      ++ Note: Use \axiom{pop!(s)} to obtain x and remove it from s.
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X top a
    depth: % -> NonNegativeInteger
      ++ depth(s) returns the number of elements of stack s.
      ++ Note: \axiom{depth(s) = #s}.
      ++
      ++X a:Stack INT:= stack [1,2,3,4,5]
      ++X depth a
```

⟨*SKAGG.dotabb*⟩≡

```
"SKAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=SKAGG"];
"SKAGG" -> "BGAGG"
```

⟨*SKAGG.dotfull*⟩≡

```
"StackAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SKAGG"];
"StackAggregate(a:Type)" -> "BagAggregate(a:Type)"

"StackAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=SKAGG"];
"StackAggregate(a:SetCategory)" -> "StackAggregate(a:Type)"
```

⟨*SKAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"StackAggregate(a:Type)" [color=lightblue];
"StackAggregate(a:Type)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

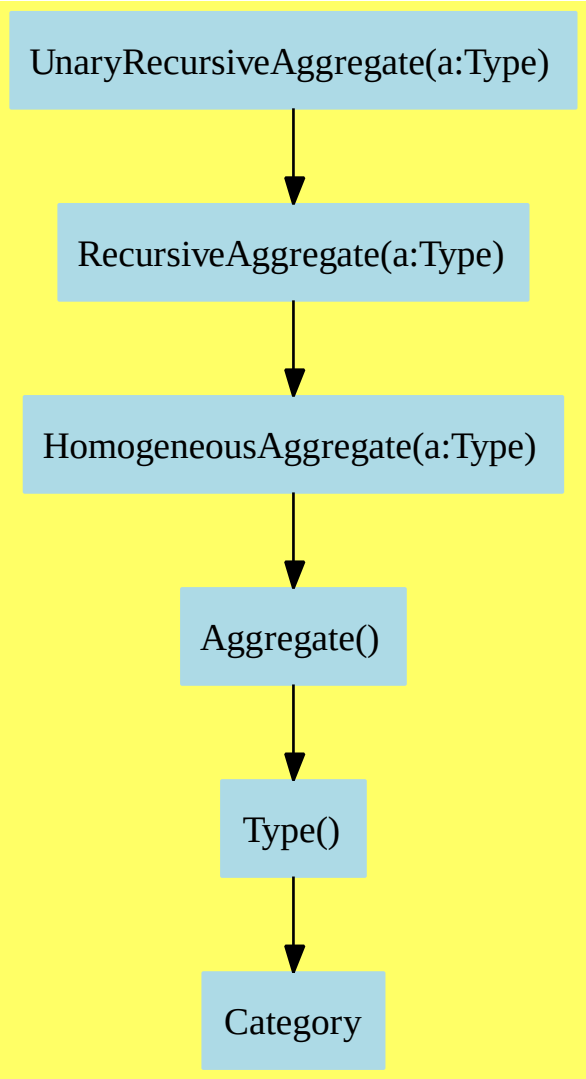
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

6.15 UnaryRecursiveAggregate (URAGG)



See:

⇒ “StreamAggregate” (STAGG) 7.11 on page 470
⇐ “RecursiveAggregate” (RCAGG) 5.12 on page 263

Exports:

any?	children	child?	coerce	concat
concat!	copy	count	cycleEntry	cycleLength
cycleSplit!	cycleTail	cyclic?	distance	empty
empty?	eq?	eval	every?	first
hash	last	latex	leaf?	leaves
less?	map	map!	member?	members
more?	nodes	node?	parts	rest
sample	second	setchildren!	setelt	setfirst!
setlast!	setrest!	setvalue!	size?	split!
tail	third	value	#?	?.last
?.rest	?.first	?.value	?=?	?~=?

Attributes exported:

- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
concat : (S,%) -> %
concat! : (%,S) -> % if $ has shallowlyMutable
concat! : (%,%) -> % if $ has shallowlyMutable
first : % -> S
first : (%,NonNegativeInteger) -> %
rest : % -> %
setfirst! : (%,S) -> S if $ has shallowlyMutable
```

These are implemented by this category:

```
children : % -> List %
concat : (%,%) -> %
cycleEntry : % -> %
cycleLength : % -> NonNegativeInteger
cycleSplit! : % -> % if $ has shallowlyMutable
cycleTail : % -> %
cyclic? : % -> Boolean
last : % -> S
last : (%,NonNegativeInteger) -> %
leaf? : % -> Boolean
less? : (%,NonNegativeInteger) -> Boolean
more? : (%,NonNegativeInteger) -> Boolean
```

```
nodes : % -> List %
node? : (%,%) -> Boolean if S has SETCAT
rest : (%,NonNegativeInteger) -> %
second : % -> S
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,first,S) -> S if $ has shallowlyMutable
setelt : (%,last,S) -> S if $ has shallowlyMutable
setelt : (%,rest,%) -> % if $ has shallowlyMutable
setlast! : (%,S) -> S if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
split! : (%,Integer) -> % if $ has shallowlyMutable
tail : % -> %
third : % -> S
value : % -> S
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?.first : (%,first) -> S
?.last : (%,last) -> S
?.rest : (%,rest) -> %
```

These exports come from (p263) RecursiveAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
child? : (%,%) -> Boolean if S has SETCAT
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
distance : (%,%) -> Integer
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
leaves : % -> List S
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
```

```
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
parts : % -> List S if $ has finiteAggregate
sample : () -> %
setelt : (%,value,S) -> S if $ has shallowlyMutable
setrest! : (%,%) -> % if $ has shallowlyMutable
?.value : (%,value) -> S
?~=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category URAGG UnaryRecursiveAggregate*⟩≡

```
)abbrev category URAGG UnaryRecursiveAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A unary-recursive aggregate is a one where nodes may have either
++ 0 or 1 children.
++ This aggregate models, though not precisely, a linked
++ list possibly with a single cycle.
++ A node with one children models a non-empty list, with the
++ \spadfun{value} of the list designating the head, or \spadfun{first},
++ of the list, and the child designating the tail, or \spadfun{rest},
++ of the list. A node with no child then designates the empty list.
++ Since these aggregates are recursive aggregates, they may be cyclic.
UnaryRecursiveAggregate(S:Type): Category == RecursiveAggregate S with
   concat: (%,%) -> %
      ++ concat(u,v) returns an aggregate w consisting of the elements of u
      ++ followed by the elements of v.
      ++ Note: \axiom{v = rest(w,#a)}.
   concat: (S,%) -> %
      ++ concat(x,u) returns aggregate consisting of x followed by
      ++ the elements of u.
      ++ Note: if \axiom{v = concat(x,u)} then \axiom{x = first v}
      ++ and \axiom{u = rest v}.
   first: % -> S
      ++ first(u) returns the first element of u
      ++ (equivalently, the value at the current node).
   elt: (%,"first") -> S
      ++ elt(u,"first") (also written: \axiom{u . first})
      ++ is equivalent to first u.
```

```
first: (%,NonNegativeInteger) -> %
   ++ first(u,n) returns a copy of the first n (\axiom{n >= 0})
   ++ elements of u.
rest: % -> %
   ++ rest(u) returns an aggregate consisting of all but the first
   ++ element of u
   ++ (equivalently, the next node of u).
elt: (%,"rest") -> %
   ++ elt(%,"rest") (also written: \axiom{u.rest}) is
   ++ equivalent to \axiom{rest u}.
rest: (%,NonNegativeInteger) -> %
   ++ rest(u,n) returns the \axiom{n}th (n >= 0) node of u.
   ++ Note: \axiom{rest(u,0) = u}.
last: % -> S
   ++ last(u) resturn the last element of u.
   ++ Note: for lists, \axiom{last(u) = u . (maxIndex u) = u . (# u - 1)}.
elt: (%,"last") -> S
   ++ elt(u,"last") (also written: \axiom{u . last}) is equivalent
   ++ to last u.
last: (%,NonNegativeInteger) -> %
   ++ last(u,n) returns a copy of the last n (\axiom{n >= 0}) nodes of u.
   ++ Note: \axiom{last(u,n)} is a list of n elements.
tail: % -> %
   ++ tail(u) returns the last node of u.
   ++ Note: if u is \axiom{shallowlyMutable},
   ++ \axiom{setrest(tail(u),v) = concat(u,v)}.
second: % -> S
   ++ second(u) returns the second element of u.
   ++ Note: \axiom{second(u) = first(rest(u))}.
third: % -> S
   ++ third(u) returns the third element of u.
   ++ Note: \axiom{third(u) = first(rest(rest(u)))}.
cycleEntry: % -> %
   ++ cycleEntry(u) returns the head of a top-level cycle contained in
   ++ aggregate u, or \axiom{empty()} if none exists.
cycleLength: % -> NonNegativeInteger
   ++ cycleLength(u) returns the length of a top-level cycle
   ++ contained  in aggregate u, or 0 is u has no such cycle.
cycleTail: % -> %
   ++ cycleTail(u) returns the last node in the cycle, or
   ++ empty if none exists.
if % has shallowlyMutable then
   concat_!: (%,%) -> %
     ++ concat!(u,v) destructively concatenates v to the end of u.
     ++ Note: \axiom{concat!(u,v) = setlast_!(u,v)}.
   concat_!: (%,S) -> %
```

```
        ++ concat!(u,x) destructively adds element x to the end of u.
        ++ Note: \axiom{concat!(a,x) = setlast!(a,[x])}.
      cycleSplit_!: % -> %
        ++ cycleSplit!(u) splits the aggregate by dropping off the cycle.
        ++ The value returned is the cycle entry, or nil if none exists.
        ++ For example, if \axiom{w = concat(u,v)} is the cyclic list where
        ++ v is the head of the cycle, \axiom{cycleSplit!(w)} will drop v
        ++ off w thus destructively changing w to u, and returning v.
      setfirst_!: (%,S) -> S
        ++ setfirst!(u,x) destructively changes the first element of a to x.
      setelt: (%,"first",S) -> S
        ++ setelt(u,"first",x) (also written: \axiom{u.first := x}) is
        ++ equivalent to \axiom{setfirst!(u,x)}.
      setrest_!: (%,%) -> %
        ++ setrest!(u,v) destructively changes the rest of u to v.
      setelt: (%,"rest",%) -> %
        ++ setelt(u,"rest",v) (also written: \axiom{u.rest := v}) is
        ++ equivalent to \axiom{setrest!(u,v)}.
      setlast_!: (%,S) -> S
        ++ setlast!(u,x) destructively changes the last element of u to x.
      setelt: (%,"last",S) -> S
        ++ setelt(u,"last",x) (also written: \axiom{u.last := b})
        ++ is equivalent to \axiom{setlast!(u,v)}.
      split_!: (%,Integer) -> %
         ++ split!(u,n) splits u into two aggregates: \axiom{v = rest(u,n)}
         ++ and \axiom{w = first(u,n)}, returning \axiom{v}.
         ++ Note: afterwards \axiom{rest(u,n)} returns \axiom{empty()}.
  add
   cycleMax ==> 1000

   findCycle: % -> %

   elt(x, "first") == first x
   elt(x,  "last") == last x
   elt(x,  "rest") == rest x
   second x        == first rest x
   third x         == first rest rest x
   cyclic? x       == not empty? x and not empty? findCycle x
   last x          == first tail x

   nodes x ==
     l := empty()$List(%)
     while not empty? x repeat
       l := concat(x, l)
       x := rest x
     reverse_! l
```

```
children x ==
  l := empty()$List(%)
  empty? x => l
  concat(rest x,l)

leaf? x == empty? x

value x ==
  empty? x => error "value of empty object"
  first x

less?(l, n) ==
  i := n::Integer
  while i > 0 and not empty? l repeat (l := rest l; i := i - 1)
  i > 0

more?(l, n) ==
  i := n::Integer
  while i > 0 and not empty? l repeat (l := rest l; i := i - 1)
  zero?(i) and not empty? l

size?(l, n) ==
  i := n::Integer
  while not empty? l and i > 0 repeat (l := rest l; i := i - 1)
  empty? l and zero? i

#x ==
  for k in 0.. while not empty? x repeat
    k = cycleMax and cyclic? x => error "cyclic list"
    x := rest x
  k

tail x ==
  empty? x => error "empty list"
  y := rest x
  for k in 0.. while not empty? y repeat
    k = cycleMax and cyclic? x => error "cyclic list"
    y := rest(x := y)
  x

findCycle x ==
  y := rest x
  while not empty? y repeat
    if eq?(x, y) then return x
    x := rest x
```

```
      y := rest y
      if empty? y then return y
      if eq?(x, y) then return y
      y := rest y
    y

  cycleTail x ==
    empty?(y := x := cycleEntry x) => x
    z := rest x
    while not eq?(x,z) repeat (y := z; z := rest z)
    y

  cycleEntry x ==
    empty? x => x
    empty?(y := findCycle x) => y
    z := rest y
    for l in 1.. while not eq?(y,z) repeat z := rest z
    y := x
    for k in 1..l repeat y := rest y
    while not eq?(x,y) repeat (x := rest x; y := rest y)
    x

  cycleLength x ==
    empty? x => 0
    empty?(x := findCycle x) => 0
    y := rest x
    for k in 1.. while not eq?(x,y) repeat y := rest y
    k

  rest(x, n) ==
    for i in 1..n repeat
      empty? x => error "Index out of range"
      x := rest x
    x

  if % has finiteAggregate then
    last(x, n) ==
      n > (m := #x) => error "index out of range"
      copy rest(x, (m - n)::NonNegativeInteger)

  if S has SetCategory then
    x = y ==
      eq?(x, y) => true
      for k in 0.. while not empty? x and not empty? y repeat
        k = cycleMax and cyclic? x => error "cyclic list"
        first x ^= first y => return false
```

```
      x := rest x
      y := rest y
    empty? x and empty? y

  node?(u, v) ==
    for k in 0.. while not empty? v repeat
      u = v => return true
      k = cycleMax and cyclic? v => error "cyclic list"
      v := rest v
    u=v

if % has shallowlyMutable then
  setelt(x, "first", a) == setfirst_!(x, a)
  setelt(x,  "last", a) == setlast_!(x, a)
  setelt(x,  "rest", a) == setrest_!(x, a)
  concat(x:%, y:%)      == concat_!(copy x, y)

  setlast_!(x, s) ==
    empty? x => error "setlast: empty list"
    setfirst_!(tail x, s)
    s

  setchildren_!(u,lv) ==
    #lv=1 => setrest_!(u, first lv)
    error "wrong number of children specified"

  setvalue_!(u,s) == setfirst_!(u,s)

  split_!(p, n) ==
    n < 1 => error "index out of range"
    p := rest(p, (n - 1)::NonNegativeInteger)
    q := rest p
    setrest_!(p, empty())
    q

  cycleSplit_! x ==
    empty?(y := cycleEntry x) or eq?(x, y) => y
    z := rest x
    while not eq?(z, y) repeat (x := z; z := rest z)
    setrest_!(x, empty())
    y
```

⟨*URAGG.dotabb*⟩≡
```
"URAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=URAGG"];
"URAGG" -> "RCAGG"
```

⟨*URAGG.dotfull*⟩≡
```
"UnaryRecursiveAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=URAGG"];
"UnaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"
```

⟨*URAGG.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UnaryRecursiveAggregate(a:Type)" [color=lightblue];
"UnaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

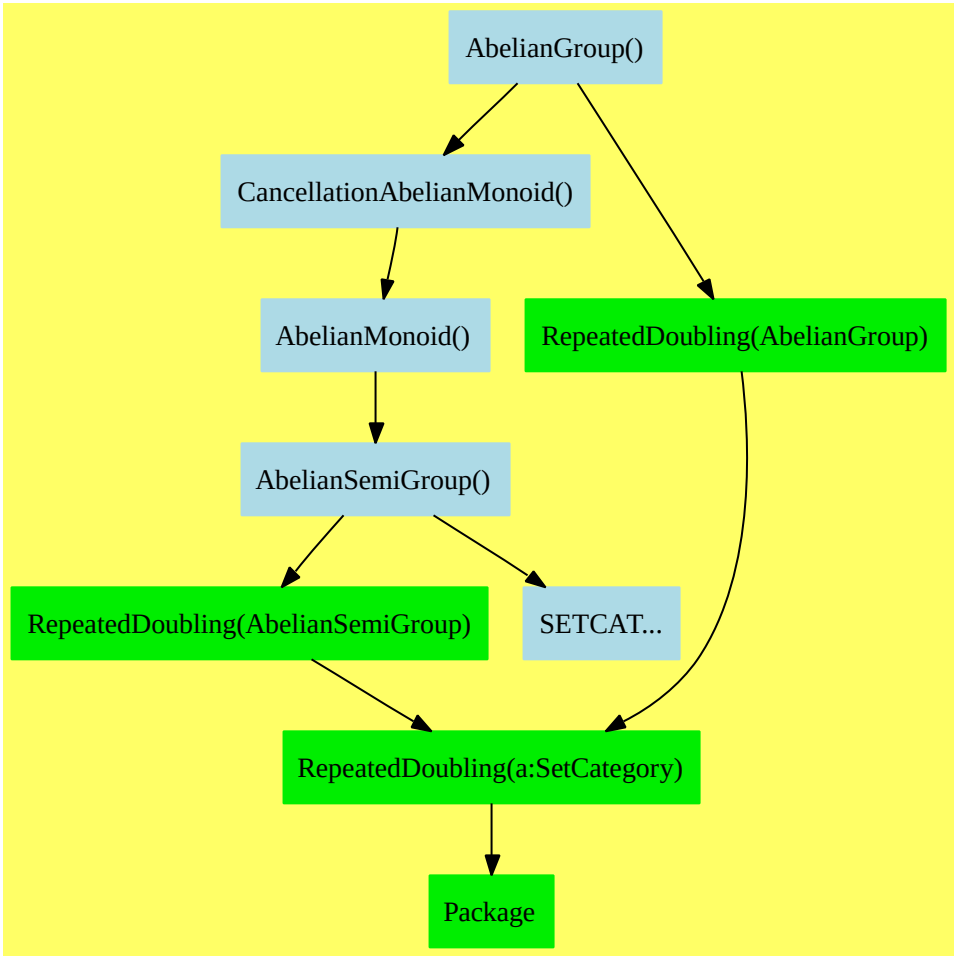
"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

Chapter 7

Category Layer 6

7.1 AbelianGroup (ABELGRP)



See:

⇒ "FiniteDivisorCategory" (FDIVCAT) 8.1 on page 498
⇒ "FunctionSpace" (FS) 17.5 on page 1095
⇒ "LeftModule" (LMODULE) 8.5 on page 533
⇒ "NonAssociativeRng" (NARNG) 8.8 on page 552
⇒ "OrderedAbelianGroup" (OAGROUP) 9.5 on page 617
⇒ "RightModule" (RMODULE) 8.12 on page 584
⇒ "Rng" (RNG) 8.13 on page 587
⇐ "CancellationAbelianMonoid" (CABMON) 6.2 on page 289

Exports:

0	coerce	hash	latex	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?-?	-?	?=?		

These are directly exported but not implemented:

```
-? : % -> %
```

These are implemented by this category:

```
subtractIfCan : (%,%) -> Union(%,"failed")
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
```

These exports come from (p289) CancellationAbelianMonoid():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
```

⟨*category ABELGRP AbelianGroup*⟩≡

```
)abbrev category ABELGRP AbelianGroup
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
```

```
++ Keywords:
++ References:
++ Description:
++ The class of abelian groups, i.e. additive monoids where
++ each element has an additive inverse.
++
++ Axioms:
++   \spad{-(-x) = x}
++   \spad{x+(-x) = 0}
-- following domain must be compiled with subsumption disabled
AbelianGroup(): Category == CancellationAbelianMonoid with
      "-": % -> %
        ++ -x is the additive inverse of x.
      "-": (%,%) -> %
        ++ x-y is the difference of x and y
        ++ i.e. \spad{x + (-y)}.
        -- subsumes the partial subtraction from previous
      "*": (Integer,%) -> %
        ++ n*x is the product of x by the integer n.
    add
      (x:% - y:%):% == x+(-y)
      subtractIfCan(x:%, y:%):Union(%, "failed") == (x-y)::Union(%,"failed")
      n:NonNegativeInteger * x:% == (n::Integer) * x
      import RepeatedDoubling(%)
      if not (% has Ring) then
        n:Integer * x:% ==
          zero? n => 0
          n>0 => double(n pretend PositiveInteger,x)
          double((-n) pretend PositiveInteger,-x)
```

⟨*ABELGRP.dotabb*⟩≡

```
"ABELGRP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ABELGRP"];
"ABELGRP" -> "CABMON"
```

⟨*ABELGRP.dotfull*⟩≡

```
"AbelianGroup()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ABELGRP"];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "RepeatedDoubling(AbelianGroup)"
```

⟨*ABELGRP.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "RepeatedDoubling(AbelianGroup)"

"RepeatedDoubling(AbelianGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

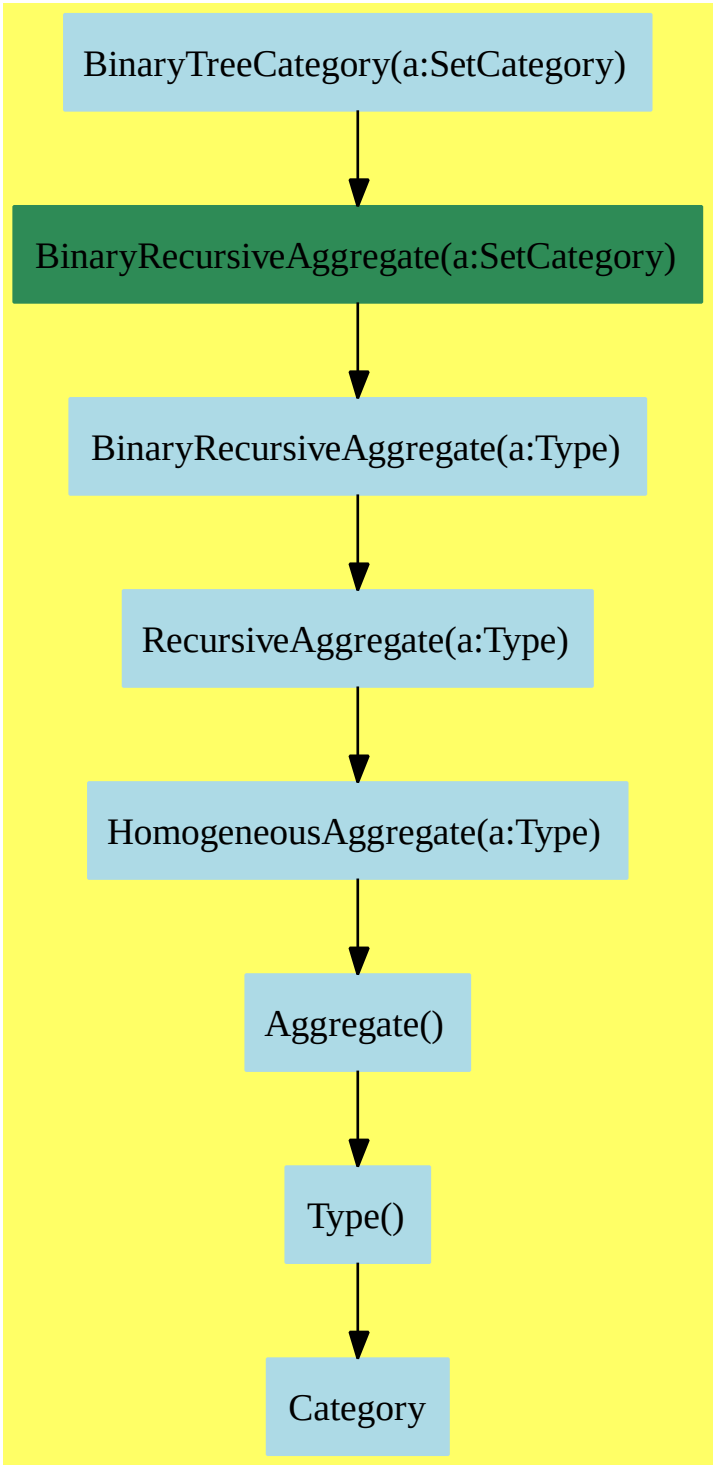
"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SETCAT..."
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SETCAT..." [color=lightblue];
}
```

7.2 BinaryTreeCategory (BTCAT)



See:

⇐ "BinaryRecursiveAggregate" (BRAGG) 6.1 on page 282

Exports:

any?	child?	children	coerce	copy
count	cyclic?	distance	empty	empty?
eq?	eval	every?	hash	latex
leaf?	leaves	less?	left	map
map!	member?	members	more?	node
node?	nodes	parts	right	sample
setchildren!	setelt	setleft!	setright!	setvalue!
size?	value	#?	?=?	?~=?
?.right	?.left	?.value		

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
node : (%,S,%) -> %
```

These are implemented by this category:

```
copy : % -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
#? : % -> NonNegativeInteger if $ has finiteAggregate
```

These exports come from (p282) BinaryRecursiveAggregate(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
child? : (%,%) -> Boolean if S has SETCAT
children : % -> List %
coerce : % -> OutputForm if S has SETCAT
count : (S,%) -> NonNegativeInteger if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
cyclic? : % -> Boolean
distance : (%,%) -> Integer
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
```

```
eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
leaf? : % -> Boolean
leaves : % -> List S
left : % -> %
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
member? : (S,%) -> Boolean if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
right : % -> %
sample : () -> %
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,value,S) -> S if $ has shallowlyMutable
setelt : (%,right,%) -> % if $ has shallowlyMutable
setelt : (%,left,%) -> % if $ has shallowlyMutable
setleft! : (%,%) -> % if $ has shallowlyMutable
setright! : (%,%) -> % if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
value : % -> S
?~=? : (%,%) -> Boolean if S has SETCAT
?.value : (%,value) -> S
?=? : (%,%) -> Boolean if S has SETCAT
?.right : (%,right) -> %
?.left : (%,left) -> %
```

⟨*category BTCAT BinaryTreeCategory*⟩≡

```
)abbrev category BTCAT BinaryTreeCategory
++ Author:W. H. Burge
++ Date Created:17 Feb 1992
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description: \spadtype{BinaryTreeCategory(S)} is the category of
++ binary trees: a tree which is either empty or else is a
++ \spadfun{node} consisting of a value and a \spadfun{left} and
++ \spadfun{right}, both binary trees.
```

```
BinaryTreeCategory(S: SetCategory): Category == _
      BinaryRecursiveAggregate(S) with
   shallowlyMutable
    ++ Binary trees have updateable components
   finiteAggregate
    ++ Binary trees have a finite number of components
   node: (%,S,%) -> %
    ++ node(left,v,right) creates a binary tree with value \spad{v}, a binary
    ++ tree \spad{left}, and a binary tree \spad{right}.
    ++
 add
     cycleTreeMax ==> 5

     copy t ==
       empty? t => empty()
       node(copy left t, value t, copy right t)

     if % has shallowlyMutable then
       map_!(f,t) ==
         empty? t => t
         t.value := f(t.value)
         map_!(f,left t)
         map_!(f,right t)
         t

     if % has finiteAggregate then
       treeCount : (%, NonNegativeInteger) -> NonNegativeInteger

       #t == treeCount(t,0)

       treeCount(t,k) ==
         empty? t => k
         k := k + 1
         k = cycleTreeMax and cyclic? t => error "cyclic binary tree"
         k := treeCount(left t,k)
         treeCount(right t,k)
```

⟨*BTCAT.dotabb*⟩≡

```
"BTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BTCAT"];
"BTCAT" -> "BRAGG"
```

⟨*BTCAT.dotfull*⟩≡
```
"BinaryTreeCategory(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BTCAT"];
"BinaryTreeCategory(a:SetCategory)" ->
   "BinaryRecursiveAggregate(a:SetCategory)"
```

⟨*BTCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"BinaryTreeCategory(a:SetCategory)" [color=lightblue];
"BinaryTreeCategory(a:SetCategory)" ->
   "BinaryRecursiveAggregate(a:SetCategory)"

"BinaryRecursiveAggregate(a:SetCategory)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=BRAGG"];
"BinaryRecursiveAggregate(a:SetCategory)" ->
   "BinaryRecursiveAggregate(a:Type)"

"BinaryRecursiveAggregate(a:Type)" [color=lightblue];
"BinaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

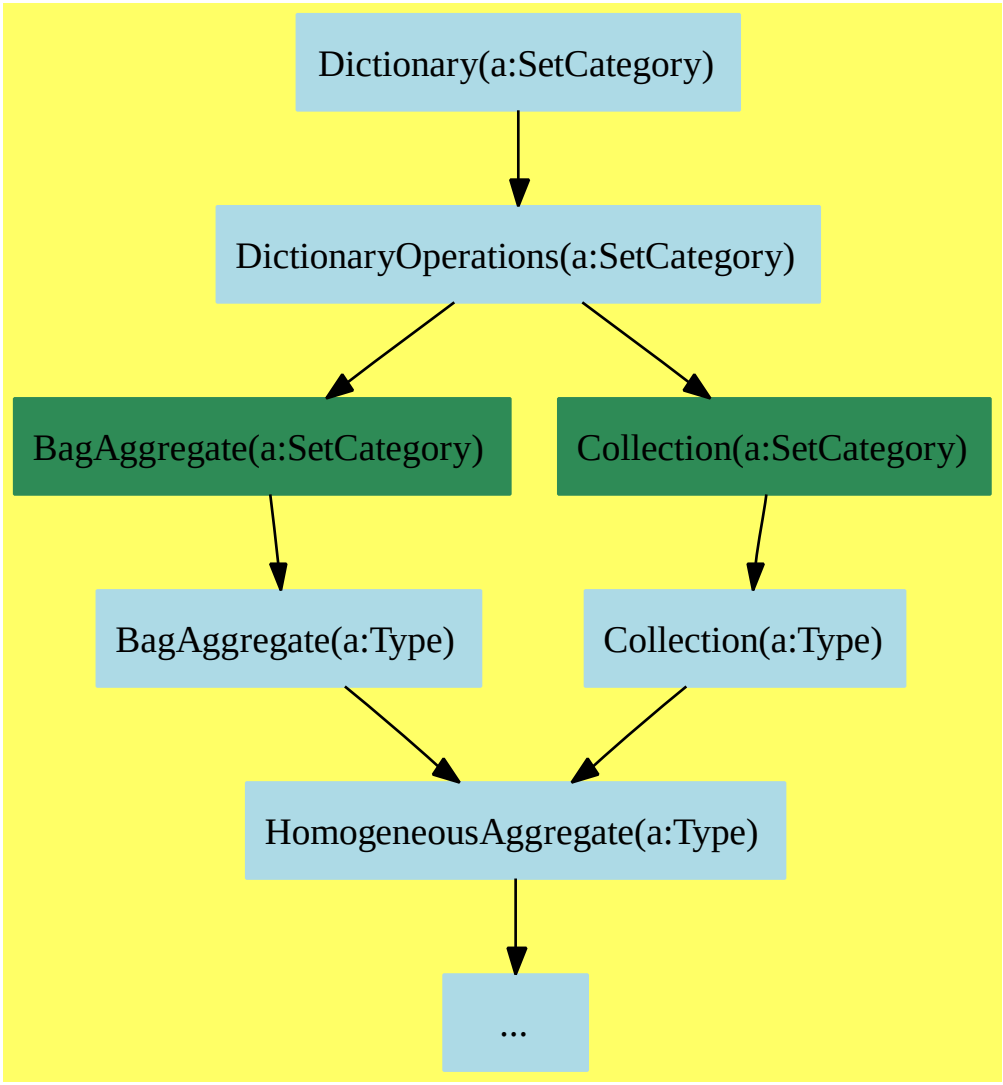
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

7.3 Dictionary (DIAGG)



See:

⇒ "FiniteSetAggregate" (FSAGG) 8.2 on page 503
⇒ "KeyedDictionary" (KDAGG) 8.3 on page 510
⇐ "DictionaryOperations" (DIOPS) 6.3 on page 293

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	empty	empty?
eq?	eval	every?	extract!	find
hash	insert!	inspect	latex	less?
map	map!	member?	members	more?
parts	reduce	remove	remove!	removeDuplicates
sample	select	select!	size?	#?
?~=?	?=?			

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are implemented by this category:

```
dictionary : List S -> %
select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

These exports come from (p293) DictionaryOperations(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
dictionary : () -> %
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
```

```
         if S has EVALAB S and S has SETCAT
 eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
 eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
 every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
 extract! : % -> S
 find : ((S -> Boolean),%) -> Union(S,"failed")
 hash : % -> SingleInteger if S has SETCAT
 insert! : (S,%) -> %
 inspect : % -> S
 latex : % -> String if S has SETCAT
 less? : (%,NonNegativeInteger) -> Boolean
 map : ((S -> S),%) -> %
 map! : ((S -> S),%) -> % if $ has shallowlyMutable
 member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
 members : % -> List S if $ has finiteAggregate
 more? : (%,NonNegativeInteger) -> Boolean
 parts : % -> List S if $ has finiteAggregate
 reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
 reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
 reduce : (((S,S) -> S),%,S,S) -> S
       if S has SETCAT and $ has finiteAggregate
 remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
 remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
 remove! : (S,%) -> % if $ has finiteAggregate
 removeDuplicates : % -> %
         if S has SETCAT and $ has finiteAggregate
 sample : () -> %
 select : ((S -> Boolean),%) -> % if $ has finiteAggregate
 size? : (%,NonNegativeInteger) -> Boolean
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?~=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category DIAGG Dictionary*⟩≡

```
 )abbrev category DIAGG Dictionary
 ++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
 ++ Date Created: August 87 through August 88
 ++ Date Last Updated: April 1991
 ++ Basic Operations:
 ++ Related Constructors:
 ++ Also See:
 ++ AMS Classifications:
 ++ Keywords:
 ++ References:
 ++ Description:
 ++ A dictionary is an aggregate in which entries can be inserted,
 ++ searched for and removed. Duplicates are thrown away on insertion.
```

```
++ This category models the usual notion of dictionary which involves
++ large amounts of data where copying is impractical.
++ Principal operations are thus destructive (non-copying) ones.
Dictionary(S:SetCategory): Category ==
 DictionaryOperations S add
   dictionary l ==
     d := dictionary()
     for x in l repeat insert_!(x, d)
     d

   if % has finiteAggregate then
    -- remove(f:S->Boolean,t:%)  == remove_!(f, copy t)
    -- select(f, t)        == select_!(f, copy t)
     select_!(f, t)      == remove_!((x:S):Boolean +-> not f(x), t)

     --extract_! d ==
     --  empty? d => error "empty dictionary"
     --  remove_!(x := first parts d, d, 1)
     --  x

     s = t ==
       eq?(s,t) => true
       #s ^= #t => false
       _and/[member?(x, t) for x in parts s]

     remove_!(f:S->Boolean, t:%) ==
       for m in parts t repeat if f m then remove_!(m, t)
       t
```

⟨*DIAGG.dotabb*⟩≡

```
"DIAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=DIAGG"];
"DIAGG" -> "DIOPS"
```

⟨*DIAGG.dotfull*⟩≡

```
"Dictionary(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIAGG"];
"Dictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"Dictionary(Record(a:SetCategory,b:SetCategory))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=DIAGG"];
"Dictionary(Record(a:SetCategory,b:SetCategory))" ->
    "Dictionary(a:SetCategory)"
```

⟨*DIAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Dictionary(a:SetCategory)" [color=lightblue];
"Dictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BagAggregate(a:SetCategory)"
"DictionaryOperations(a:SetCategory)" -> "Collection(a:SetCategory)"

"BagAggregate(a:SetCategory)" [color=seagreen];
"BagAggregate(a:SetCategory)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"Collection(a:SetCategory)" [color=seagreen];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

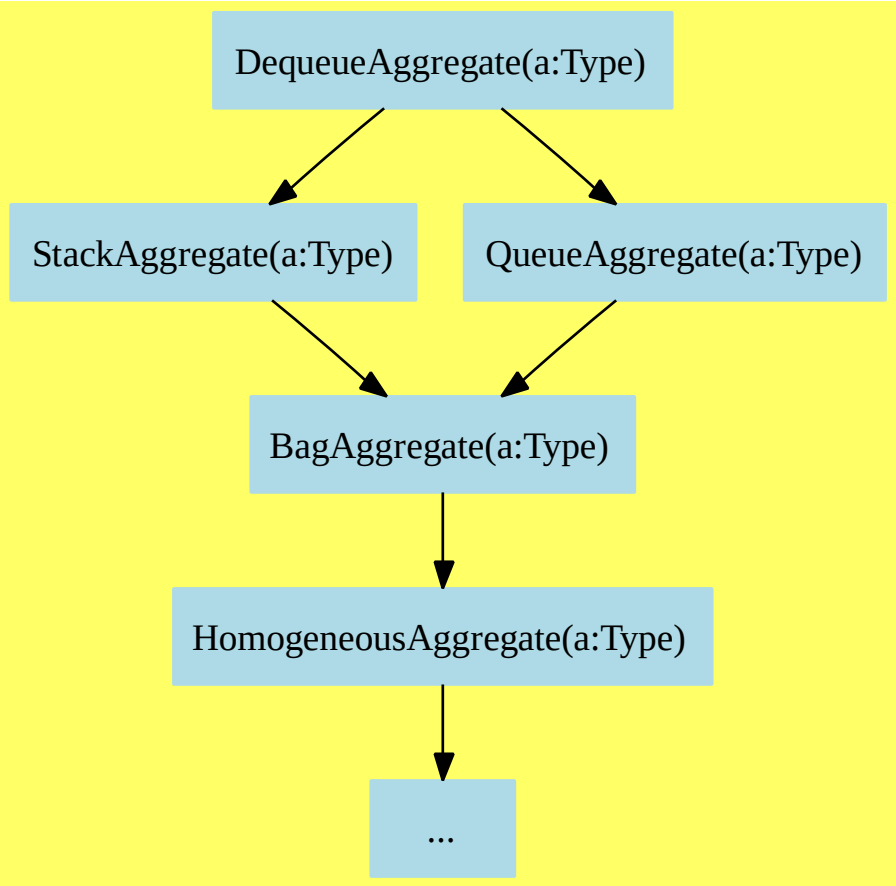
"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"Category" [color=lightblue];
}
```

7.4 DequeueAggregate (DQAGG)



See:

⇐ "QueueAggregate" (QUAGG) 6.12 on page 390
⇐ "StackAggregate" (SKAGG) 6.14 on page 402

Exports:

any?	back	bag	bottom!	coerce
copy	count	depth	dequeue	dequeue!
empty	empty?	enqueue!	eq?	eval
every?	extract!	extractBottom!	extractTop!	front
hash	height	insert!	insertBottom!	insertTop!
inspect	latex	length	less?	map
map!	members	member?	more?	parts
pop!	push!	reverse!	rotate!	sample
size?	top	top!	#?	?=?
?~=?				

Attributes exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
bottom! : % -> S
dequeue : () -> %
dequeue : List S -> %
extractBottom! : % -> S
extractTop! : % -> S
height : % -> NonNegativeInteger
insertBottom! : (S,%) -> S
insertTop! : (S,%) -> S
reverse! : % -> %
top! : % -> S
```

These exports come from (p402) StackAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
depth : % -> NonNegativeInteger
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
extract! : % -> S
```

```
hash : % -> SingleInteger if S has SETCAT
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
members : % -> List S if $ has finiteAggregate
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
pop! : % -> S
push! : (S,%) -> S
sample : () -> %
size? : (%,NonNegativeInteger) -> Boolean
top : % -> S
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p390) QueueAggregate(S:Type):

```
back : % -> S
dequeue! : % -> S
enqueue! : (S,%) -> S
front : % -> S
length : % -> NonNegativeInteger
rotate! : % -> %
```

⟨*category DQAGG DequeueAggregate*⟩≡

```
)abbrev category DQAGG DequeueAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A dequeue is a doubly ended stack, that is, a bag where first items
++ inserted are the first items extracted, at either the front or
++ the back end of the data structure.
DequeueAggregate(S:Type):
 Category == Join(StackAggregate S,QueueAggregate S) with
   dequeue: () -> %
     ++ dequeue()$D creates an empty dequeue of type D.
```

```
dequeue: List S -> %
  ++ dequeue([x,y,...,z]) creates a dequeue with first (top or front)
  ++ element x, second element y,...,and last (bottom or back) element z.
height: % -> NonNegativeInteger
  ++ height(d) returns the number of elements in dequeue d.
  ++ Note: \axiom{height(d) = # d}.
top_!: % -> S
  ++ top!(d) returns the element at the top (front) of the dequeue.
bottom_!: % -> S
  ++ bottom!(d) returns the element at the bottom (back) of the dequeue.
insertTop_!: (S,%) -> S
  ++ insertTop!(x,d) destructively inserts x into the dequeue d, that is,
  ++ at the top (front) of the dequeue.
  ++ The element previously at the top of the dequeue becomes the
  ++ second in the dequeue, and so on.
insertBottom_!: (S,%) -> S
  ++ insertBottom!(x,d) destructively inserts x into the dequeue d
  ++ at the bottom (back) of the dequeue.
extractTop_!: % -> S
  ++ extractTop!(d) destructively extracts the top (front) element
  ++ from the dequeue d.
  ++ Error: if d is empty.
extractBottom_!: % -> S
  ++ extractBottom!(d) destructively extracts the bottom (back) element
  ++ from the dequeue d.
  ++ Error: if d is empty.
reverse_!: % -> %
  ++ reverse!(d) destructively replaces d by its reverse dequeue, i.e.
  ++ the top (front) element is now the bottom (back) element, and so on.
```

⟨*DQAGG.dotabb*⟩≡

```
"DQAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=DQAGG"];
"DQAGG" -> "SKAGG"
"DQAGG" -> "QUAGG"
```

⟨*DQAGG.dotfull*⟩≡
```
  "DequeueAggregate(a:Type)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=DQAGG"];
  "DequeueAggregate(a:Type)" -> "StackAggregate(a:Type)"
  "DequeueAggregate(a:Type)" -> "QueueAggregate(a:Type)"

  "DequeueAggregate(a:SetCategory)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=DQAGG"];
  "DequeueAggregate(a:SetCategory)" -> "DequeueAggregate(a:Type)"
```

⟨*DQAGG.dotpic*⟩≡
```
  digraph pic {
   fontsize=10;
   bgcolor="#FFFF66";
   node [shape=box, color=white, style=filled];

  "DequeueAggregate(a:Type)" [color=lightblue];
  "DequeueAggregate(a:Type)" -> "StackAggregate(a:Type)"
  "DequeueAggregate(a:Type)" -> "QueueAggregate(a:Type)"

  "StackAggregate(a:Type)" [color=lightblue];
  "StackAggregate(a:Type)" -> "BagAggregate(a:Type)"

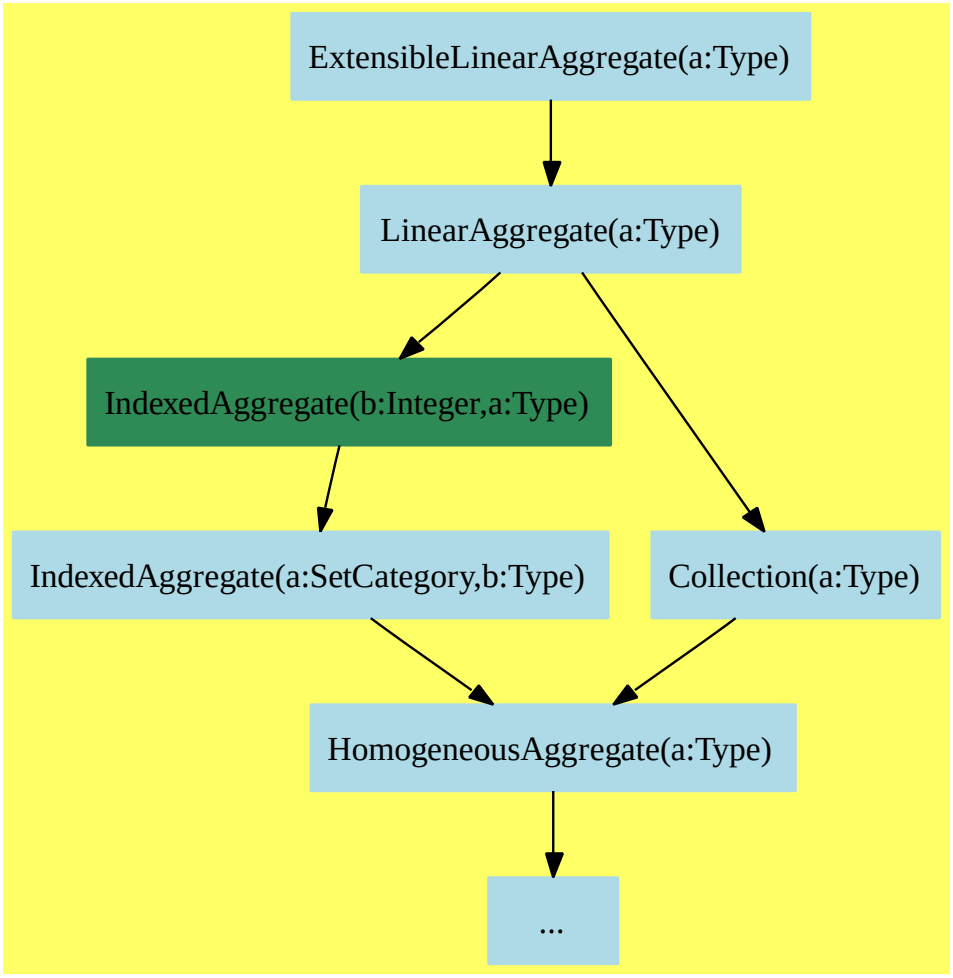
  "QueueAggregate(a:Type)" [color=lightblue];
  "QueueAggregate(a:Type)" -> "BagAggregate(a:Type)"

  "BagAggregate(a:Type)" [color=lightblue];
  "BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

  "HomogeneousAggregate(a:Type)" [color=lightblue];
  "HomogeneousAggregate(a:Type)" -> "..."

  "..." [color=lightblue];
  }
```

7.5 ExtensibleLinearAggregate (ELAGG)



See:

⇒ "ListAggregate" (LSAGG) 8.6 on page 536
⇐ "LinearAggregate" (LNAGG) 6.6 on page 308

Exports:

any?	coerce	concat	concat!	construct
copy	convert	count	delete	delete!
elt	empty	empty?	entries	entry?
eval	every?	eq?	fill!	find
first	hash	index?	indices	insert
insert!	latex	less?	map	map!
maxIndex	member?	members	merge!	minIndex
more?	new	parts	qelt	qsetelt!
reduce	remove	remove!	removeDuplicates	removeDuplicates!
sample	select	select!	setelt	size?
swap!	#?	?=?	?.?	?~=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
concat! : (%,S) -> %
delete! : (%,Integer) -> %
delete! : (%,UniversalSegment Integer) -> %
insert! : (%,%,Integer) -> %
insert! : (S,%,Integer) -> %
merge! : (((S,S) -> Boolean),%,%) -> %
remove! : ((S -> Boolean),%) -> %
removeDuplicates! : % -> % if S has SETCAT
select! : ((S -> Boolean),%) -> %
```

These are implemented by this category:

```
concat : (%,%) -> %
concat : (%,S) -> %
concat! : (%,%) -> %
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
insert : (%,%,Integer) -> %
insert : (S,%,Integer) -> %
merge! : (%,%) -> % if S has ORDSET
remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
remove! : (S,%) -> % if S has SETCAT
removeDuplicates : % -> %
         if S has SETCAT and $ has finiteAggregate
select : ((S -> Boolean),%) -> % if $ has finiteAggregate
```

These exports come from (p308) LinearAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
concat : List % -> %
concat : (S,%) -> %
construct : List S -> %
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
convert : % -> InputForm if S has KONVERT INFORM
elt : (%,Integer,S) -> S
empty : () -> %
empty? : % -> Boolean
entries : % -> List S
entry? : (S,%) -> Boolean
         if $ has finiteAggregate and S has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
index? : (Integer,%) -> Boolean
indices : % -> List Integer
fill! : (%,S) -> % if $ has shallowlyMutable
find : ((S -> Boolean),%) -> Union(S,"failed")
first : % -> S if Integer has ORDSET
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : (((S,S) -> S),%,%) -> %
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
maxIndex : % -> Integer if Integer has ORDSET
new : (NonNegativeInteger,S) -> %
members : % -> List S if $ has finiteAggregate
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S
```

```
          if $ has shallowlyMutable
 reduce : (((S,S) -> S),%) -> S
          if $ has finiteAggregate
 reduce : (((S,S) -> S),%,S) -> S
          if $ has finiteAggregate
 reduce : (((S,S) -> S),%,S,S) -> S
          if S has SETCAT and $ has finiteAggregate
 sample : () -> %
 setelt : (%,Integer,S) -> S if $ has shallowlyMutable
 setelt : (%,UniversalSegment Integer,S) -> S
          if $ has shallowlyMutable
 size? : (%,NonNegativeInteger) -> Boolean
 swap! : (%,Integer,Integer) -> Void
          if $ has shallowlyMutable
 #? : % -> NonNegativeInteger if $ has finiteAggregate
 ?=? : (%,%) -> Boolean if S has SETCAT
 ?~=? : (%,%) -> Boolean if S has SETCAT
 ?.? : (%,Integer) -> S
 ?.? : (%,UniversalSegment Integer) -> %
```

⟨*category ELAGG ExtensibleLinearAggregate*⟩≡

```
 )abbrev category ELAGG ExtensibleLinearAggregate
 ++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
 ++ Date Created: August 87 through August 88
 ++ Date Last Updated: April 1991
 ++ Basic Operations:
 ++ Related Constructors:
 ++ Also See:
 ++ AMS Classifications:
 ++ Keywords:
 ++ References:
 ++ Description:
 ++ An extensible aggregate is one which allows insertion and deletion of
 ++ entries. These aggregates are models of lists and streams which are
 ++ represented by linked structures so as to make insertion, deletion, and
 ++ concatenation efficient. However, access to elements of these
 ++ extensible aggregates is generally slow since access is made from the end.
 ++ See \spadtype{FlexibleArray} for an exception.
 ExtensibleLinearAggregate(S:Type):Category == LinearAggregate S with
    shallowlyMutable
    concat_!: (%,S) -> %
      ++ concat!(u,x) destructively adds element x to the end of u.
    concat_!: (%,%) -> %
      ++ concat!(u,v) destructively appends v to the end of u.
      ++ v is unchanged
    delete_!: (%,Integer) -> %
      ++ delete!(u,i) destructively deletes the \axiom{i}th element of u.
      ++
```

```
    ++E Data:=Record(age:Integer,gender:String)
    ++E a1:AssociationList(String,Data):=table()
    ++E a1."tim":=[55,"male"]$Data
    ++E delete!(a1,1)

  delete_!: (%,UniversalSegment(Integer)) -> %
    ++ delete!(u,i..j) destructively deletes elements u.i through u.j.
  remove_!: (S->Boolean,%) -> %
    ++ remove!(p,u) destructively removes all elements x of
    ++ u such that \axiom{p(x)} is true.
  insert_!: (S,%,Integer) -> %
    ++ insert!(x,u,i) destructively inserts x into u at position i.
  insert_!: (%,%,Integer) -> %
    ++ insert!(v,u,i) destructively inserts aggregate v into u
    ++ at position i.
  merge_!: ((S,S)->Boolean,%,%) -> %
    ++ merge!(p,u,v) destructively merges u and v using predicate p.
  select_!: (S->Boolean,%) -> %
    ++ select!(p,u) destructively changes u by keeping only values
    ++ x such that \axiom{p(x)}.
  if S has SetCategory then
    remove_!: (S,%) -> %
      ++ remove!(x,u) destructively removes all values x from u.
    removeDuplicates_!: % -> %
      ++ removeDuplicates!(u) destructively removes duplicates from u.
  if S has OrderedSet then merge_!: (%,%) -> %
      ++ merge!(u,v) destructively merges u and v in ascending order.
add
  delete(x:%, i:Integer)          == delete_!(copy x, i)
  delete(x:%, i:UniversalSegment(Integer))        == delete_!(copy x, i)
  remove(f:S -> Boolean, x:%)   == remove_!(f, copy x)
  insert(s:S, x:%, i:Integer)   == insert_!(s, copy x, i)
  insert(w:%, x:%, i:Integer)   == insert_!(copy w, copy x, i)
  select(f, x)            == select_!(f, copy x)
  concat(x:%, y:%)        == concat_!(copy x, y)
  concat(x:%, y:S)        == concat_!(copy x, new(1, y))
  concat_!(x:%, y:S)      == concat_!(x, new(1, y))
  if S has SetCategory then
    remove(s:S, x:%)        == remove_!(s, copy x)
    remove_!(s:S, x:%)      == remove_!(y +-> y = s, x)
    removeDuplicates(x:%)   == removeDuplicates_!(copy x)

  if S has OrderedSet then
    merge_!(x, y) == merge_!(_<$S, x, y)
```

⟨*ELAGG.dotabb*⟩≡
```
"ELAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=ELAGG"];
"ELAGG" -> "LNAGG"
```

⟨*ELAGG.dotfull*⟩≡
```
"ExtensibleLinearAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ELAGG"];
"ExtensibleLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"
```

⟨*ELAGG.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ExtensibleLinearAggregate(a:Type)" [color=lightblue];
"ExtensibleLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "Collection(a:Type)"

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

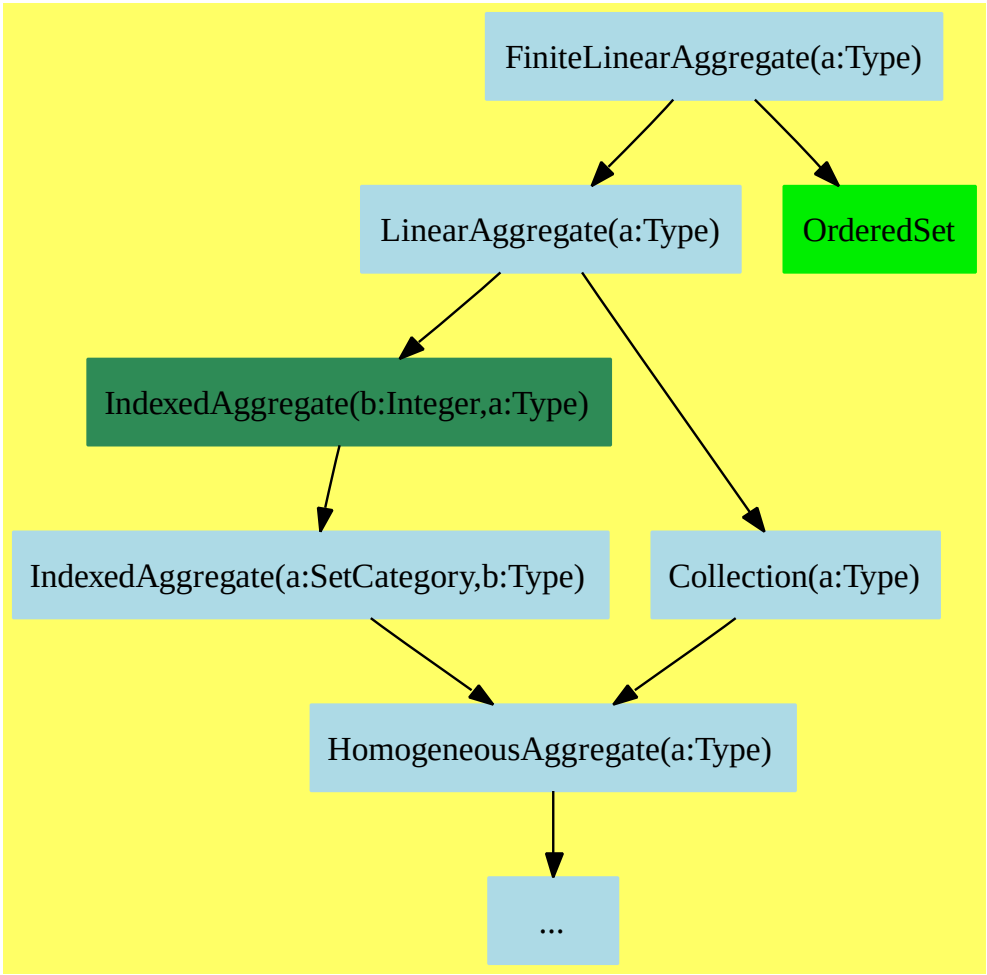
"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "HomogeneousAggregate(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "..."

"..." [color=lightblue];
}
```

7.6 FiniteLinearAggregate (FLAGG)



See:

⇒ “OneDimensionalArrayAggregate” (A1AGG) 8.9 on page 556
⇒ “ListAggregate” (LSAGG) 8.6 on page 536
⇐ “LinearAggregate” (LNAGG) 6.6 on page 308
⇐ “OrderedSet” (ORDSET) 4.19 on page 168

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	delete	elt
empty	empty?	entries	entry?	eq?
eval	every?	fill!	find	first
hash	index?	indices	insert	latex
less?	map	map!	max	maxIndex
member?	members	merge	min	minIndex
more?	new	parts	position	qelt
qsetelt!	reduce	remove	removeDuplicates	reverse
reverse!	sample	select	setelt	size?
sort	sort!	sorted?	swap!	# ?
?.?	?<?	?<=?	?=?	?>?
?>=?	?~=?			

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

Attributes Used:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are directly exported but not implemented:

```
copyInto! : (%,%,Integer) -> %
         if $ has shallowlyMutable
merge : (((S,S) -> Boolean),%,%) -> %
sorted? : (((S,S) -> Boolean),%) -> Boolean
position : (S,%,Integer) -> Integer if S has SETCAT
position : ((S -> Boolean),%) -> Integer
reverse! : % -> % if $ has shallowlyMutable
sort! : (((S,S) -> Boolean),%) -> %
         if $ has shallowlyMutable
```

These are implemented by this category:

```
merge : (%,%) -> % if S has ORDSET
position : (S,%) -> Integer if S has SETCAT
reverse : % -> %
sort : % -> % if S has ORDSET
sort : (((S,S) -> Boolean),%) -> %
sorted? : % -> Boolean if S has ORDSET
sort! : % -> %
         if S has ORDSET and $ has shallowlyMutable
```

These exports come from (p308) LinearAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
concat : List % -> %
concat : (%,%) -> %
concat : (S,%) -> %
concat : (%,S) -> %
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
delete : (%,UniversalSegment Integer) -> %
delete : (%,Integer) -> %
elt : (%,Integer,S) -> S
empty : () -> %
empty? : % -> Boolean
entries : % -> List S
entry? : (S,%) -> Boolean
         if $ has finiteAggregate and S has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
fill! : (%,S) -> % if $ has shallowlyMutable
find : ((S -> Boolean),%) -> Union(S,"failed")
first : % -> S if Integer has ORDSET
hash : % -> SingleInteger if S has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (S,%,Integer) -> %
insert : (%,%,Integer) -> %
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : (((S,S) -> S),%,%) -> %
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
maxIndex : % -> Integer if Integer has ORDSET
member? : (S,%) -> Boolean
```

```
          if S has SETCAT and $ has finiteAggregate
 members : % -> List S if $ has finiteAggregate
 minIndex : % -> Integer if Integer has ORDSET
 more? : (%,NonNegativeInteger) -> Boolean
 new : (NonNegativeInteger,S) -> %
 parts : % -> List S if $ has finiteAggregate
 qelt : (%,Integer) -> S
 qsetelt! : (%,Integer,S) -> S
          if $ has shallowlyMutable
 reduce : (((S,S) -> S),%) -> S
          if $ has finiteAggregate
 reduce : (((S,S) -> S),%,S) -> S
          if $ has finiteAggregate
 reduce : (((S,S) -> S),%,S,S) -> S
          if S has SETCAT and $ has finiteAggregate
 remove : ((S -> Boolean),%) -> %
          if $ has finiteAggregate
 remove : (S,%) -> %
          if S has SETCAT and $ has finiteAggregate
 removeDuplicates : % -> %
          if S has SETCAT and $ has finiteAggregate
 sample : () -> %
 select : ((S -> Boolean),%) -> %
          if $ has finiteAggregate
 setelt : (%,Integer,S) -> S
          if $ has shallowlyMutable
 setelt : (%,UniversalSegment Integer,S) -> S
          if $ has shallowlyMutable
 size? : (%,NonNegativeInteger) -> Boolean
 swap! : (%,Integer,Integer) -> Void
          if $ has shallowlyMutable
 #? : % -> NonNegativeInteger
          if $ has finiteAggregate
 ?.? : (%,Integer) -> S
 ?.? : (%,UniversalSegment Integer) -> %
 ?~=? : (%,%) -> Boolean if S has SETCAT
 ?=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p168) OrderedSet:

```
max : (%,%) -> % if S has ORDSET
min : (%,%) -> % if S has ORDSET
?<? : (%,%) -> Boolean if S has ORDSET
?<=? : (%,%) -> Boolean if S has ORDSET
?>? : (%,%) -> Boolean if S has ORDSET
?>=? : (%,%) -> Boolean if S has ORDSET
```

⟨*category FLAGG FiniteLinearAggregate*⟩≡

```
 )abbrev category FLAGG FiniteLinearAggregate
 ++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
```

```
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A finite linear aggregate is a linear aggregate of finite length.
++ The finite property of the aggregate adds several exports to the
++ list of exports from \spadtype{LinearAggregate} such as
++ \spadfun{reverse}, \spadfun{sort}, and so on.
FiniteLinearAggregate(S:Type): Category == LinearAggregate S with
   finiteAggregate
   merge: ((S,S)->Boolean,%,%) -> %
      ++ merge(p,a,b) returns an aggregate c which merges \axiom{a} and b.
      ++ The result is produced by examining each element x of \axiom{a}
      ++ and y of b successively. If \axiom{p(x,y)} is true, then x is
      ++ inserted into the result; otherwise y is inserted. If x is
      ++ chosen, the next element of \axiom{a} is examined, and so on.
      ++ When all the elements of one aggregate are examined, the
      ++ remaining elements of the other are appended.
      ++ For example, \axiom{merge(<,[1,3],[2,7,5])} returns
      ++ \axiom{[1,2,3,7,5]}.
   reverse: % -> %
      ++ reverse(a) returns a copy of \axiom{a} with elements
      ++ in reverse order.
   sort: ((S,S)->Boolean,%) -> %
      ++ sort(p,a) returns a copy of \axiom{a} sorted using total ordering
      ++ predicate p.
   sorted?: ((S,S)->Boolean,%) -> Boolean
      ++ sorted?(p,a) tests if \axiom{a} is sorted according to predicate p.
   position: (S->Boolean, %) -> Integer
      ++ position(p,a) returns the index i of the first x in \axiom{a}
      ++ such that \axiom{p(x)} is true, and \axiom{minIndex(a) - 1}
      ++ if there is no such x.
   if S has SetCategory then
      position: (S, %)  -> Integer
        ++ position(x,a) returns the index i of the first occurrence of
        ++ x in a, and \axiom{minIndex(a) - 1} if there is no such x.
      position: (S,%,Integer) -> Integer
        ++ position(x,a,n) returns the index i of the first occurrence of
        ++ x in \axiom{a} where \axiom{i >= n}, and \axiom{minIndex(a) - 1}
        ++ if no such x is found.
   if S has OrderedSet then
```

```
        OrderedSet
        merge: (%,%) -> %
          ++ merge(u,v) merges u and v in ascending order.
          ++ Note: \axiom{merge(u,v) = merge(<=,u,v)}.
        sort: % -> %
          ++ sort(u) returns an u with elements in ascending order.
          ++ Note: \axiom{sort(u) = sort(<=,u)}.
        sorted?: % -> Boolean
          ++ sorted?(u) tests if the elements of u are in ascending order.
      if % has shallowlyMutable then
        copyInto_!: (%,%,Integer) -> %
          ++ copyInto!(u,v,i) returns aggregate u containing a copy of
          ++ v inserted at element i.
        reverse_!: % -> %
          ++ reverse!(u) returns u with its elements in reverse order.
        sort_!: ((S,S)->Boolean,%) -> %
          ++ sort!(p,u) returns u with its elements ordered by p.
        if S has OrderedSet then sort_!: % -> %
          ++ sort!(u) returns u with its elements in ascending order.
    add
      if S has SetCategory then
        position(x:S, t:%) == position(x, t, minIndex t)

      if S has OrderedSet then
    --    sorted? l   == sorted?(_<$S, l)
        sorted? l   == sorted?((x,y) +-> x < y or x = y, l)
        merge(x, y) == merge(_<$S, x, y)
        sort l      == sort(_<$S, l)

      if % has shallowlyMutable then
        reverse x  == reverse_! copy x
        sort(f, l) == sort_!(f, copy l)

        if S has OrderedSet then
          sort_! l == sort_!(_<$S, l)
```

⟨*FLAGG.dotabb*⟩≡

```
"FLAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=FLAGG"];
"FLAGG" -> "LNAGG"
```

⟨*FLAGG.dotfull*⟩≡
```
"FiniteLinearAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FLAGG"];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"
```

⟨*FLAGG.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"
"FiniteLinearAggregate(a:Type)" -> "OrderedSet"

"OrderedSet" [color="#00EE00"];

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "Collection(a:Type)"

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

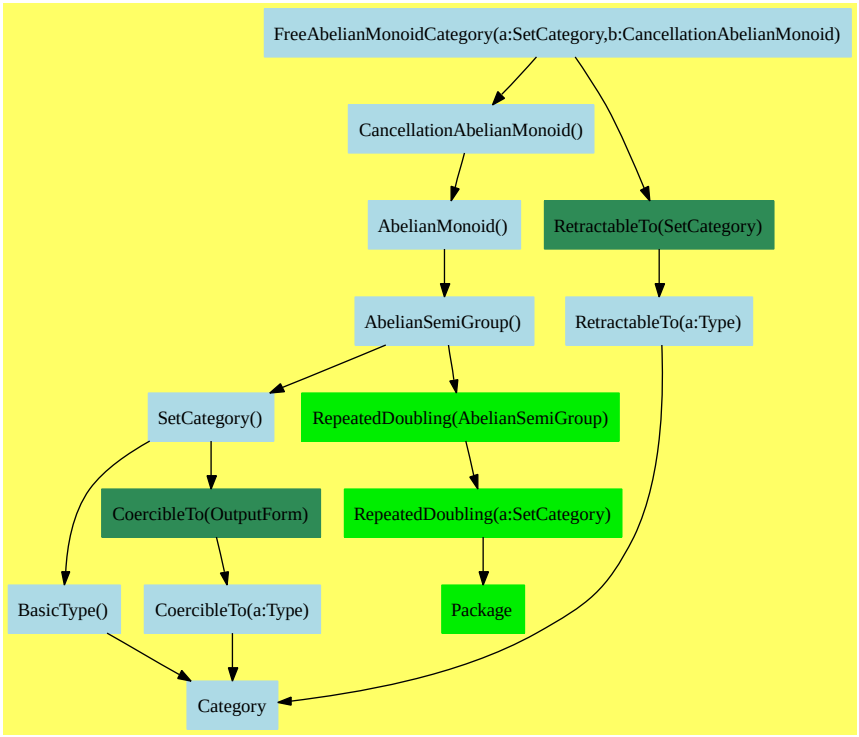
"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "HomogeneousAggregate(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "..."

"..." [color=lightblue];
}
```

7.7 FreeAbelianMonoidCategory (FAMONC)



See:

⇐ "CancellationAbelianMonoid" (CABMON) 6.2 on page 289
⇐ "RetractableTo" (RETRACT) 2.18 on page 44

Exports:

0	coefficient	coerce	hash	highCommonTerms
latex	mapCoef	mapGen	nthCoef	nthFactor
retract	retractIfCan	sample	size	subtractIfCan
terms	zero?	?*?	?+?	?=?
?~=?				

These are directly exported but not implemented:

```
coefficient : (S,%) -> E
highCommonTerms : (%,%) -> % if E has OAMON
mapCoef : ((E -> E),%) -> %
mapGen : ((S -> S),%) -> %
nthCoef : (%,Integer) -> E
nthFactor : (%,Integer) -> S
size : % -> NonNegativeInteger
terms : % -> List Record(gen: S,exp: E)
```

```
?+? : (S,%) -> %
?*? : (E,S) -> %
```

These exports come from (p289) CancellationAbelianMonoid():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
```

These exports come from (p44) RetractableTo(SetCategory):

```
coerce : S -> %
retract : % -> S
retractIfCan : % -> Union(S,"failed")
```

⟨*category FAMONC FreeAbelianMonoidCategory*⟩≡

```
)abbrev category FAMONC FreeAbelianMonoidCategory
++ Category for free abelian monoid on any set of generators
++ Author: Manuel Bronstein
++ Date Created: November 1989
++ Date Last Updated: 6 June 1991
++ Description:
++ A free abelian monoid on a set S is the monoid of finite sums of
++ the form \spad{reduce(+,[ni * si])} where the si's are in S, and the ni's
++ are in a given abelian monoid. The operation is commutative.
FreeAbelianMonoidCategory(S: SetCategory, E:CancellationAbelianMonoid): _
  Category == Join(CancellationAbelianMonoid, RetractableTo S) with
        "+"        : (S, $) -> $
          ++ s + x returns the sum of s and x.
        "*"        : (E, S) -> $
          ++ e * s returns e times s.
        size       : $ -> NonNegativeInteger
          ++ size(x) returns the number of terms in x.
          ++ mapGen(f, a1\^e1 ... an\^en) returns
          ++\spad{f(a1)\^e1 ... f(an)\^en}.
        terms      : $ -> List Record(gen: S, exp: E)
          ++ terms(e1 a1 + ... + en an) returns \spad{[[a1, e1],...,[an, en]]}.
        nthCoef    : ($, Integer) -> E
          ++ nthCoef(x, n) returns the coefficient of the n^th term of x.
```

```
        nthFactor  : ($, Integer) -> S
          ++ nthFactor(x, n) returns the factor of the n^th term of x.
        coefficient: (S, $) -> E
          ++ coefficient(s, e1 a1 + ... + en an) returns ei such that
          ++ ai = s, or 0 if s is not one of the ai's.
        mapCoef    : (E -> E, $) -> $
          ++ mapCoef(f, e1 a1 +...+ en an) returns
          ++ \spad{f(e1) a1 +...+ f(en) an}.
        mapGen     : (S -> S, $) -> $
          ++ mapGen(f, e1 a1 +...+ en an) returns
          ++ \spad{e1 f(a1) +...+ en f(an)}.
        if E has OrderedAbelianMonoid then
          highCommonTerms: ($, $) -> $
            ++ highCommonTerms(e1 a1 + ... + en an, f1 b1 + ... + fm bm)
            ++ returns \spad{reduce(+,[max(ei, fi) ci])}
            ++ where ci ranges in the intersection
            ++ of \spad{{a1,...,an}} and \spad{{b1,...,bm}}.
```

⟨*FAMONC.dotabb*⟩≡

```
"FAMONC"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=FAMONC"];
"FAMONC" -> "CABMON"
"FAMONC" -> "RETRACT"
```

⟨*FAMONC.dotfull*⟩≡

```
"FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelianMonoid)"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=FAMONC"];
"FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelianMonoid)" ->
   "CancellationAbelianMonoid()"
"FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelianMonoid)" ->
   "RetractableTo(SetCategory)"
```

⟨*FAMONC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelianMonoid)"
  [color=lightblue];
"FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelianMonoid)" ->
   "CancellationAbelianMonoid()"
"FreeAbelianMonoidCategory(a:SetCategory,b:CancellationAbelianMonoid)" ->
   "RetractableTo(SetCategory)"

"RetractableTo(SetCategory)" [color=seagreen];
"RetractableTo(SetCategory)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SetCategory()"
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" ->
    "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

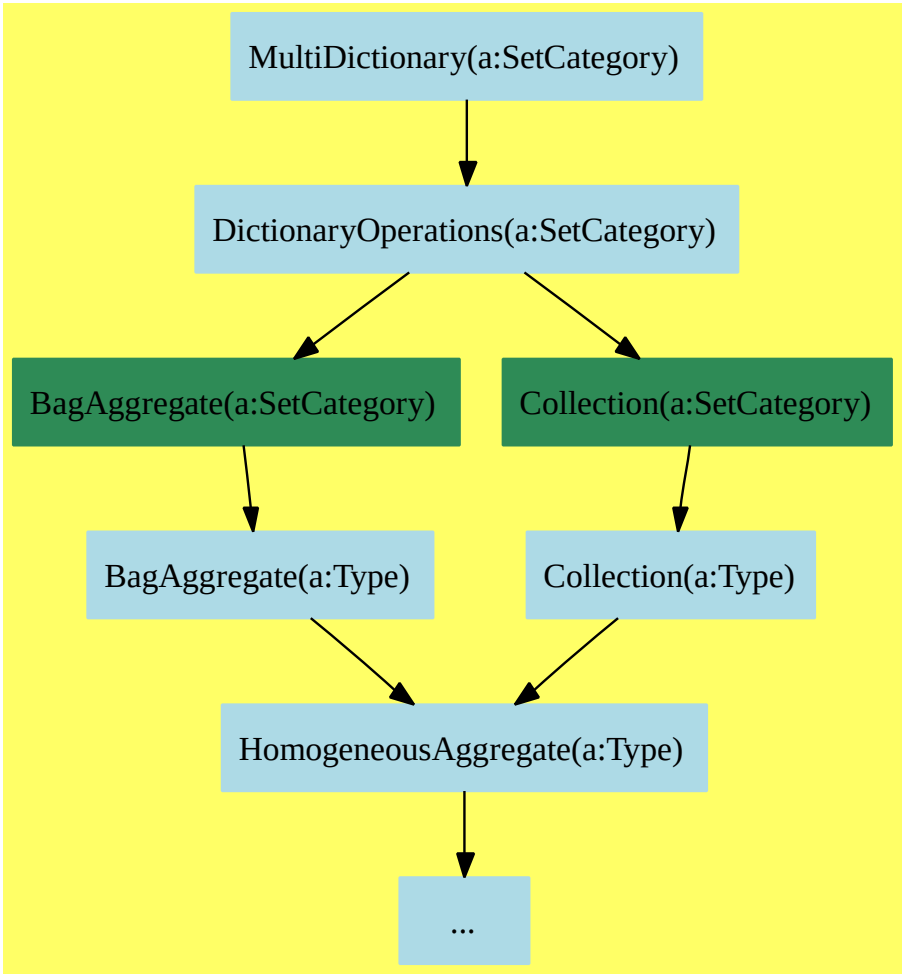
"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"
```

```
"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];
}
```

7.8 MultiDictionary (MDAGG)



See:

⇒ "MultisetAggregate" (MSETAGG) 8.7 on page 548
⇐ "DictionaryOperations" (DIOPS) 6.3 on page 293

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	duplicates	empty
empty?	eq?	eval	every?	extract!
find	hash	insert!	inspect	latex
less?	map	map!	member?	members
more?	parts	reduce	remove	remove!
removeDuplicates	removeDuplicates!	sample	select	select!
size?	#?	?=?	?~=?	

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
insert! : (S,%,NonNegativeInteger) -> %
removeDuplicates! : % -> %
duplicates : % -> List Record(entry:S,count:NonNegativeInteger)
```

These exports come from (p293) DictionaryOperations(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
dictionary : () -> %
dictionary : List S -> %
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S and S has SETCAT
```

```
every? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
extract! : % -> S
find : ((S -> Boolean),%) -> Union(S,"failed")
hash : % -> SingleInteger if S has SETCAT
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
         if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
remove : (S,%) -> %
         if S has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
         if S has SETCAT and $ has finiteAggregate
remove! : ((S -> Boolean),%) -> % if $ has finiteAggregate
remove! : (S,%) -> % if $ has finiteAggregate
select : ((S -> Boolean),%) -> % if $ has finiteAggregate
sample : () -> %
select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger
         if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category MDAGG MultiDictionary*⟩≡

```
)abbrev category MDAGG MultiDictionary
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A multi-dictionary is a dictionary which may contain duplicates.
```

```
  ++ As for any dictionary, its size is assumed large so that
  ++ copying (non-destructive) operations are generally to be avoided.
  MultiDictionary(S:SetCategory): Category == DictionaryOperations S with
  -- count: (S,%) -> NonNegativeInteger                  ++ multiplicity count
     insert_!: (S,%,NonNegativeInteger) -> %
       ++ insert!(x,d,n) destructively inserts n copies of x into dictionary d.
  -- remove_!: (S,%,NonNegativeInteger) -> %
  --   ++ remove!(x,d,n) destructively removes (up to) n copies of x from
  --   ++ dictionary d.
     removeDuplicates_!: % -> %
       ++ removeDuplicates!(d) destructively removes any duplicate values
       ++ in dictionary d.
     duplicates: % -> List Record(entry:S,count:NonNegativeInteger)
       ++ duplicates(d) returns a list of values which have duplicates in d
  --   ++ duplicates(d) returns a list of              ++ duplicates iterator
  -- to become duplicates: % -> Iterator(D,D)
```

⟨*MDAGG.dotabb*⟩≡

```
  "MDAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=MDAGG"];
  "MDAGG" -> "DIOPS"
```

⟨*MDAGG.dotfull*⟩≡

```
  "MultiDictionary(a:SetCategory)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=MDAGG"];
  "MultiDictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"
```

⟨*MDAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"MultiDictionary(a:SetCategory)" [color=lightblue];
"MultiDictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BagAggregate(a:SetCategory)"
"DictionaryOperations(a:SetCategory)" -> "Collection(a:SetCategory)"

"BagAggregate(a:SetCategory)" [color=seagreen];
"BagAggregate(a:SetCategory)" -> "BagAggregate(a:Type)"

"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

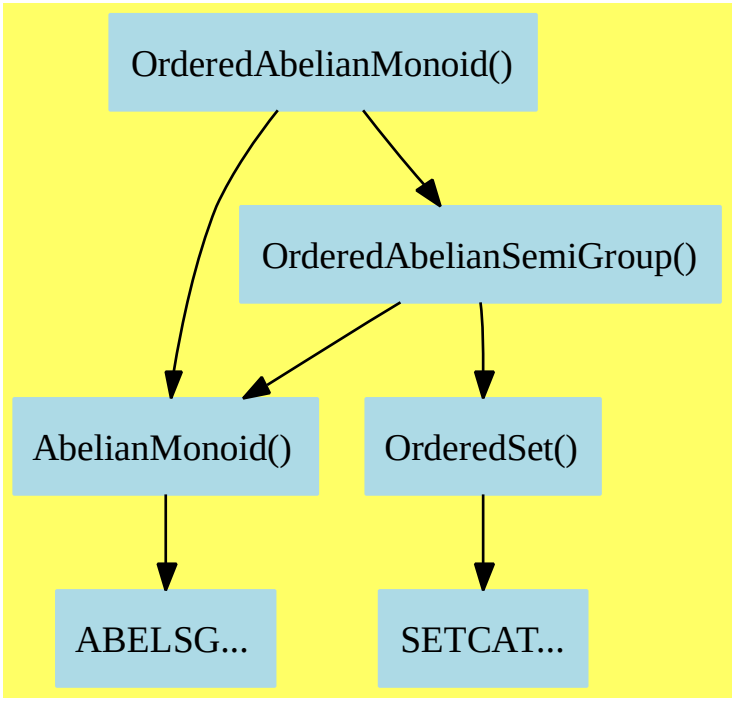
"Collection(a:SetCategory)" [color=seagreen];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "..."

"..." [color=lightblue];
}
```

7.9 OrderedAbelianMonoid (OAMON)



See:

⇒ "OrderedCancellationAbelianMonoid" (OCAMON) 8.10 on page 567
⇐ "AbelianMonoid" (ABELMON) 5.1 on page 207
⇐ "OrderedAbelianSemiGroup" (OASGP) 6.8 on page 364

Exports:

0	coerce	hash	latex	max
min	sample	zero?	?*?	?+?
?<?	?<=?	?=?	?>?	?>=?
?~=?				

These exports come from (p364) OrderedAbelianSemiGroup():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
```

```
?>=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

These exports come from (p207) AbelianMonoid():

```
0 : () -> %
sample : () -> %
zero? : % -> Boolean
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?*? : (NonNegativeInteger,%) -> %
```

⟨*category OAMON OrderedAbelianMonoid*⟩≡

```
)abbrev category OAMON OrderedAbelianMonoid
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Ordered sets which are also abelian monoids, such that the addition
++ preserves the ordering.
OrderedAbelianMonoid(): Category ==
        Join(OrderedAbelianSemiGroup, AbelianMonoid)
```

⟨*OAMON.dotabb*⟩≡

```
"OAMON" [color=lightblue,href="bookvol10.2.pdf#nameddest=OAMON"];
"OAMON" -> "OASGP"
"OAMON" -> "ABELMON"
```

⟨*OAMON.dotfull*⟩≡

```
"OrderedAbelianMonoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OAMON"];
"OrderedAbelianMonoid()" -> "OrderedAbelianSemiGroup()"
"OrderedAbelianMonoid()" -> "AbelianMonoid()"
```

⟨*OAMON.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedAbelianMonoid()" [color=lightblue];
"OrderedAbelianMonoid()" -> "OrderedAbelianSemiGroup()"
"OrderedAbelianMonoid()" -> "AbelianMonoid()"

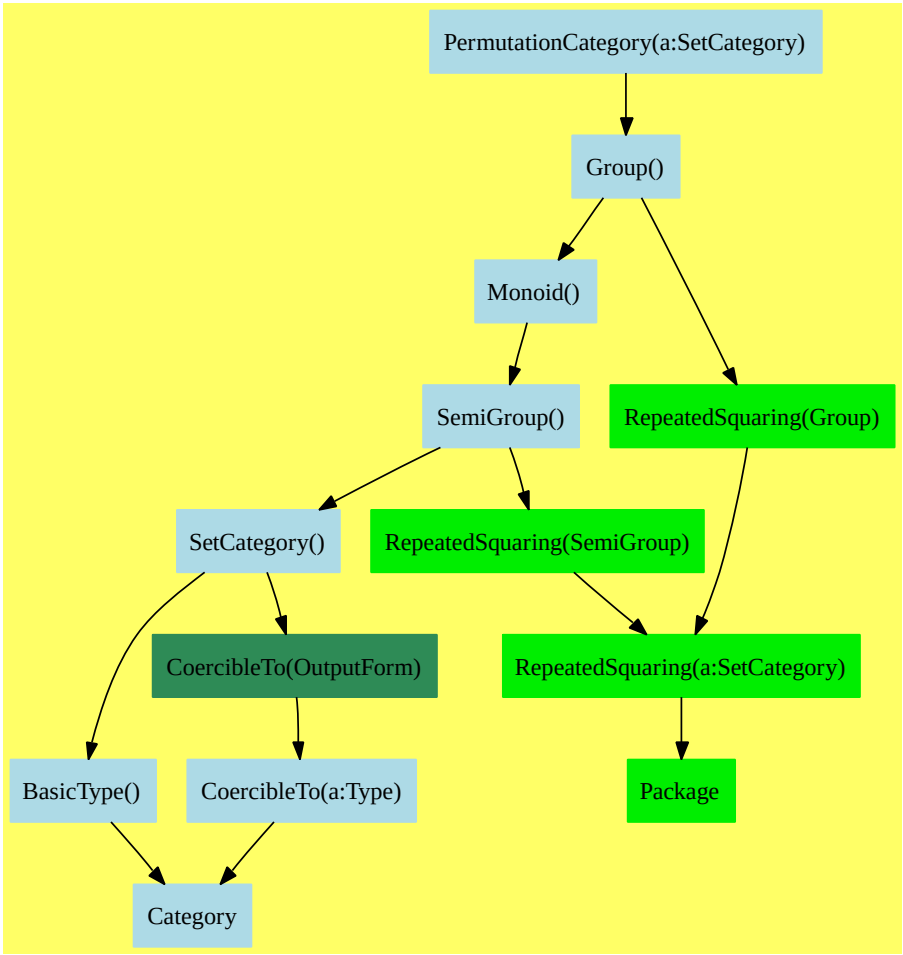
"OrderedAbelianSemiGroup()" [color=lightblue];
"OrderedAbelianSemiGroup()" -> "OrderedSet()"
"OrderedAbelianSemiGroup()" -> "AbelianMonoid()"

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SETCAT..."

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "ABELSG..."

"SETCAT..." [color=lightblue];
"ABELSG..." [color=lightblue];
}
```

7.10 PermutationCategory (PERMCAT)



See:

⇐ “Group” (GROUP) 6.5 on page 304

Exports:

1	coerce	commutator	conjugate	cycle
cycles	eval	hash	inv	latex
max	min	one?	orbit	recip
sample	?^?	?.?	?~=?	?**?
?<?	?<=?	?>?	?>=?	?*?
?/?	?=?			

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are directly exported but not implemented:

```
cycle : List S -> %
cycles : List List S -> %
eval : (%,S) -> S
orbit : (%,S) -> Set S
?<? : (%,%) -> Boolean
?.? : (%,S) -> S
```

These exports come from (p304) Group():

```
1 : () -> %
coerce : % -> OutputForm
commutator : (%,%) -> %
conjugate : (%,%) -> %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
?/? : (%,%) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,Integer) -> %
?*? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,Integer) -> %
```

These exports come from (p168) OrderedSet():

```
max : (%,%) -> % if S has ORDSET or S has FINITE
min : (%,%) -> % if S has ORDSET or S has FINITE
?>? : (%,%) -> Boolean if S has ORDSET or S has FINITE
?<=? : (%,%) -> Boolean if S has ORDSET or S has FINITE
?>=? : (%,%) -> Boolean if S has ORDSET or S has FINITE
```

⟨*category PERMCAT PermutationCategory*⟩≡

```
)abbrev category PERMCAT PermutationCategory
++ Authors:  Holger Gollan, Johannes Grabmeier, Gerhard Schneider
++ Date Created: 27 July 1989
++ Date Last Updated: 29 March 1990
```

```
++ Basic Operations: cycle, cycles, eval, orbit
++ Related Constructors: PermutationGroup, PermutationGroupExamples
++ Also See: RepresentationTheoryPackage1
++ AMS Classifications:
++ Keywords: permutation, symmetric group
++ References:
++ Description: PermutationCategory provides a categorial environment
++  for subgroups of bijections of a set (i.e. permutations)

PermutationCategory(S:SetCategory): Category  ==  Group with
    cycle   :  List S       ->  %
      ++ cycle(ls) coerces a cycle {\em ls}, i.e. a list with not
      ++ repetitions to a permutation, which maps {\em ls.i} to
      ++ {\em ls.i+1}, indices modulo the length of the list.
      ++ Error: if repetitions occur.
    cycles  :  List List S  ->  %
      ++ cycles(lls) coerces a list list of cycles {\em lls}
      ++ to a permutation, each cycle being a list with not
      ++ repetitions, is coerced to the permutation, which maps
      ++ {\em ls.i} to {\em ls.i+1}, indices modulo the length of the list,
      ++ then these permutations are mutiplied.
      ++ Error: if repetitions occur in one cycle.
    eval  :  (%,S)          ->  S
      ++ eval(p, el) returns the image of {\em el} under the
      ++ permutation p.
    elt  :  (%,S)          ->  S
      ++ elt(p, el) returns the image of {\em el} under the
      ++ permutation p.
    orbit :  (%,S)          ->  Set S
      ++ orbit(p, el) returns the orbit of {\em el} under the
      ++ permutation p, i.e. the set which is given by applications of
      ++ the powers of p to {\em el}.
    "<" : (%,%)             ->  Boolean
      ++ p < q is an order relation on permutations.
      ++ Note: this order is only total if and only if S is totally ordered
      ++ or S is finite.
    if S has OrderedSet then OrderedSet
    if S has Finite then OrderedSet
```

⟨*PERMCAT.dotabb*⟩≡

```
"PERMCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PERMCAT"];
"PERMCAT" -> "GROUP"
```

⟨*PERMCAT.dotfull*⟩≡

```
"PermutationCategory(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PERMCAT"];
"PermutationCategory(a:SetCategory)" -> "Group()"
```

⟨*PERMCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PermutationCategory(a:SetCategory)" [color=lightblue];
"PermutationCategory(a:SetCategory)" -> "Group()"

"Group()" [color=lightblue];
"Group()" -> "Monoid()"
"Group()" -> "RepeatedSquaring(Group)"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SetCategory()"
"SemiGroup()" -> "RepeatedSquaring(SemiGroup)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"RepeatedSquaring(Group)" [color="#00EE00"];
"RepeatedSquaring(Group)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(SemiGroup)" [color="#00EE00"];
"RepeatedSquaring(SemiGroup)" -> "RepeatedSquaring(a:SetCategory)"

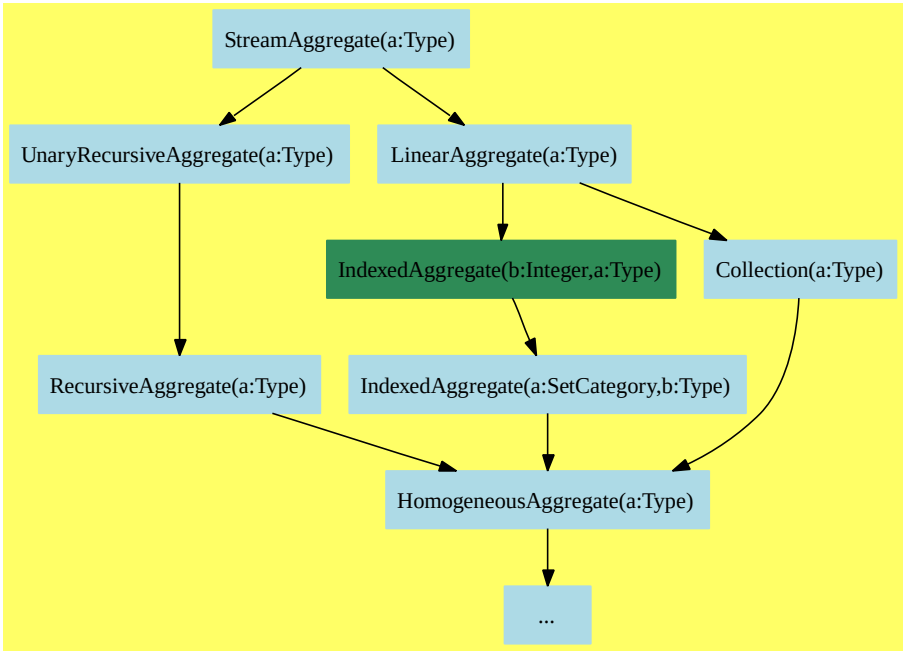
"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"Category" [color=lightblue];
```

```
}
```

7.11 StreamAggregate (STAGG)



See:

⇒ "LazyStreamAggregate" (LZSTAGG) 8.4 on page 516
⇐ "LinearAggregate" (LNAGG) 6.6 on page 308
⇐ "UnaryRecursiveAggregate" (URAGG) 6.15 on page 407

Exports:

any?	children	child?	coerce	concat
concat!	construct	convert	copy	count
cycleEntry	cycleLength	cycleSplit!	cycleTail	cyclic?
delete	distance	elt	empty	empty?
entries	entry?	eq?	eval	every?
explicitlyFinite?	fill!	find	first	hash
index?	indices	insert	last	latex
leaf?	leaves	less?	map	map!
maxIndex	member?	members	minIndex	more?
new	nodes	node?	parts	possiblyInfinite?
qelt	qsetelt!	reduce	remove	removeDuplicates
rest	sample	second	select	setchildren!
setelt	setfirst!	setlast!	setrest!	setvalue!
size?	split!	swap!	tail	third
value	#?	?=?	?.?	?.first
?.last	?.rest	?.value	?~=?	

Attributes exported:

- **nil**

Attributes Used:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.

These are implemented by this category:

```
concat : (%,%) -> %
concat : List % -> %
concat! : (%,%) -> % if $ has shallowlyMutable
fill! : (%,S) -> % if $ has shallowlyMutable
first : (%,NonNegativeInteger) -> %
explicitlyFinite? : % -> Boolean
map! : ((S -> S),%) -> % if $ has shallowlyMutable
possiblyInfinite? : % -> Boolean
setelt : (%,Integer,S) -> S if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,S) -> S
         if $ has shallowlyMutable
?.? : (%,Integer) -> S
?.? : (%,UniversalSegment Integer) -> %
```

These exports come from (p407) UnaryRecursiveAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
children : % -> List %
child? : (%,%) -> Boolean if S has SETCAT
coerce : % -> OutputForm if S has SETCAT
concat : (S,%) -> %
concat : (%,S) -> %
concat! : (%,S) -> % if $ has shallowlyMutable
copy : % -> %
count : (S,%) -> NonNegativeInteger
         if S has SETCAT
         and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
cycleEntry : % -> %
cycleLength : % -> NonNegativeInteger
cycleSplit! : % -> % if $ has shallowlyMutable
cycleTail : % -> %
cyclic? : % -> Boolean
distance : (%,%) -> Integer
empty : () -> %
```

```
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S
         and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S
         and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S
         and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S
         and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
first : % -> S
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
last : % -> S
last : (%,NonNegativeInteger) -> %
leaf? : % -> Boolean
leaves : % -> List S
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
member? : (S,%) -> Boolean
         if S has SETCAT
         and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
nodes : % -> List %
node? : (%,%) -> Boolean if S has SETCAT
parts : % -> List S if $ has finiteAggregate
rest : % -> %
rest : (%,NonNegativeInteger) -> %
sample : () -> %
second : % -> S
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,first,S) -> S if $ has shallowlyMutable
setelt : (%,last,S) -> S if $ has shallowlyMutable
setelt : (%,rest,%) -> % if $ has shallowlyMutable
setelt : (%,value,S) -> S if $ has shallowlyMutable
setfirst! : (%,S) -> S if $ has shallowlyMutable
setlast! : (%,S) -> S if $ has shallowlyMutable
setrest! : (%,%) -> % if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
split! : (%,Integer) -> % if $ has shallowlyMutable
tail : % -> %
third : % -> S
```

```
value : % -> S
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?.last : (%,last) -> S
?.rest : (%,rest) -> %
?.first : (%,first) -> S
?~=? : (%,%) -> Boolean if S has SETCAT
?.value : (%,value) -> S
```

These exports come from (p308) LinearAggregate(S:Type):

```
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
elt : (%,Integer,S) -> S
entry? : (S,%) -> Boolean
         if $ has finiteAggregate
         and S has SETCAT
entries : % -> List S
find : ((S -> Boolean),%) -> Union(S,"failed")
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (S,%,Integer) -> %
insert : (%,%,Integer) -> %
maxIndex : % -> Integer if Integer has ORDSET
map : (((S,S) -> S),%,%) -> %
minIndex : % -> Integer if Integer has ORDSET
new : (NonNegativeInteger,S) -> %
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S
         if $ has shallowlyMutable
reduce : (((S,S) -> S),%,S,S) -> S
         if S has SETCAT
         and $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
         if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
         if $ has finiteAggregate
remove : (S,%) -> %
         if S has SETCAT
         and $ has finiteAggregate
remove : ((S -> Boolean),%) -> %
         if $ has finiteAggregate
removeDuplicates : % -> %
         if S has SETCAT
         and $ has finiteAggregate
select : ((S -> Boolean),%) -> %
         if $ has finiteAggregate
swap! : (%,Integer,Integer) -> Void
```

```
        if $ has shallowlyMutable
```

⟨*category STAGG StreamAggregate*⟩≡

```
)abbrev category STAGG StreamAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A stream aggregate is a linear aggregate which possibly has an infinite
++ number of elements. A basic domain constructor which builds stream
++ aggregates is \spadtype{Stream}. From streams, a number of infinite
++ structures such power series can be built. A stream aggregate may
++ also be infinite since it may be cyclic.
++ For example, see \spadtype{DecimalExpansion}.
StreamAggregate(S:Type): Category ==
   Join(UnaryRecursiveAggregate S, LinearAggregate S) with
      explicitlyFinite?: % -> Boolean
        ++ explicitlyFinite?(s) tests if the stream has a finite
        ++ number of elements, and false otherwise.
        ++ Note: for many datatypes,
        ++ \axiom{explicitlyFinite?(s) = not possiblyInfinite?(s)}.
      possiblyInfinite?: % -> Boolean
        ++ possiblyInfinite?(s) tests if the stream s could possibly
        ++ have an infinite number of elements.
        ++ Note: for many datatypes,
        ++ \axiom{possiblyInfinite?(s) = not explictlyFinite?(s)}.
 add
   c2: (%, %) -> S

   explicitlyFinite? x == not cyclic? x
   possiblyInfinite? x == cyclic? x
   first(x, n)         == construct [c2(x, x := rest x) for i in 1..n]

   c2(x, r) ==
     empty? x => error "Index out of range"
     first x

   elt(x:%, i:Integer) ==
     i := i - minIndex x
     (i < 0) or empty?(x := rest(x, i::NonNegativeInteger)) => _
```

```
        error "index out of range"
    first x

  elt(x:%, i:UniversalSegment(Integer)) ==
    l := lo(i) - minIndex x
    l < 0 => error "index out of range"
    not hasHi i => copy(rest(x, l::NonNegativeInteger))
    (h := hi(i) - minIndex x) < l => empty()
    first(rest(x, l::NonNegativeInteger), (h - l + 1)::NonNegativeInteger)

  if % has shallowlyMutable then
    concat(x:%, y:%) == concat_!(copy x, y)

    concat l ==
      empty? l => empty()
      concat_!(copy first l, concat rest l)

    map_!(f, l) ==
      y := l
      while not empty? l repeat
        setfirst_!(l, f first l)
        l := rest l
      y

    fill_!(x, s) ==
      y := x
      while not empty? y repeat (setfirst_!(y, s); y := rest y)
      x

    setelt(x:%, i:Integer, s:S) ==
      i := i - minIndex x
      (i < 0) or empty?(x := rest(x,i::NonNegativeInteger)) => _
          error "index out of range"
      setfirst_!(x, s)

    setelt(x:%, i:UniversalSegment(Integer), s:S) ==
      (l := lo(i) - minIndex x) < 0 => error "index out of range"
      h := if hasHi i then hi(i) - minIndex x else maxIndex x
      h < l => s
      y := rest(x, l::NonNegativeInteger)
      z := rest(y, (h - l + 1)::NonNegativeInteger)
      while not eq?(y, z) repeat (setfirst_!(y, s); y := rest y)
      s

    concat_!(x:%, y:%) ==
      empty? x => y
```

```
        setrest_!(tail x, y)
        x
```

⟨*STAGG.dotabb*⟩≡

```
  "STAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=STAGG"];
  "STAGG" -> "RCAGG"
  "STAGG" -> "LNAGG"
```

⟨*STAGG.dotfull*⟩≡

```
  "StreamAggregate(a:Type)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=STAGG"];
  "StreamAggregate(a:Type)" -> "UnaryRecursiveAggregate(a:Type)"
  "StreamAggregate(a:Type)" -> "LinearAggregate(a:Type)"
```

⟨*STAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"StreamAggregate(a:Type)" [color=lightblue];
"StreamAggregate(a:Type)" -> "UnaryRecursiveAggregate(a:Type)"
"StreamAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"UnaryRecursiveAggregate(a:Type)" [color=lightblue];
"UnaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HomogeneousAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "Collection(a:Type)"

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" ->
    "HomogeneousAggregate(a:Type)"

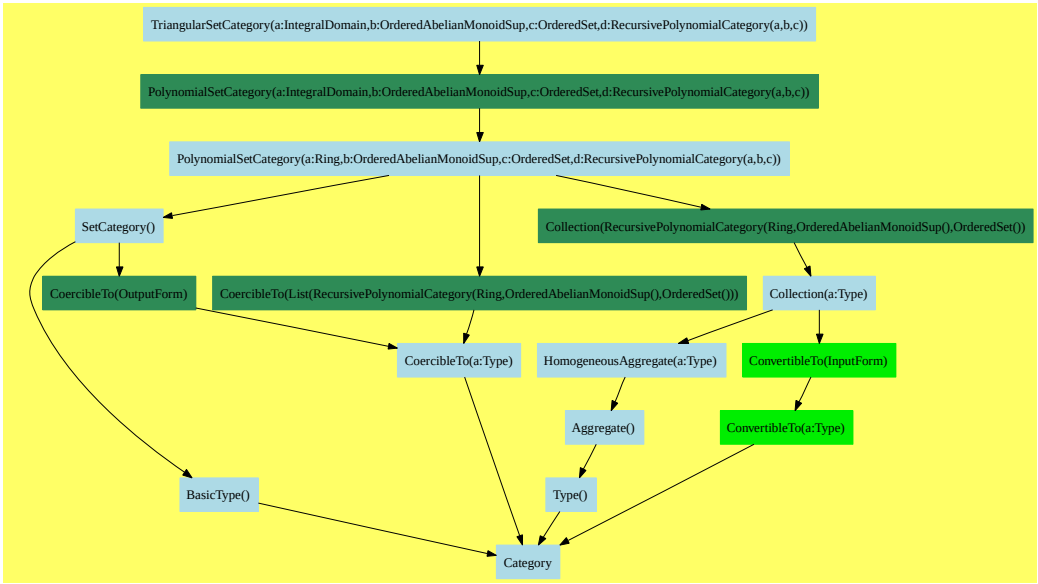
"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "..."

"..." [color=lightblue];

}
```

7.12 TriangularSetCategory (TSETCAT)



See:

⇒ “RegularTriangularSetCategory” (RSETCAT) 8.11 on page 570
⇐ “PolynomialSetCategory” (PSETCAT) 6.10 on page 373

Exports:

algebraic?	algebraicVariables
any?	autoReduced?
basicSet	coerce
coHeight	collect
collectQuasiMonic	collectUnder
collectUpper	construct
convert	copy
count	degree
empty	empty?
eq?	eval
every?	extend
extendIfCan	find
first	hash
headReduce	headReduced?
headRemainder	infRittWu?
initiallyReduce	initiallyReduced?
initials	last
latex	less?
mainVariable?	mainVariables
map	map!
member?	members
more?	mvar
normalized?	parts
quasiComponent	reduce
reduced?	reduceByQuasiMonic
remainder	remove
removeDuplicates	removeZero
rest	retract
retractIfCan	rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder	rewriteSetWithReduction
roughBase?	roughEqualIdeals?
roughSubIdeal?	roughUnitIdeal?
sample	select
size?	sort
stronglyReduce	stronglyReduced?
triangular?	trivialIdeal?
variables	zeroSetSplit
zeroSetSplitIntoTriangularSystems	#?
?=?	?~=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of

elements.

- **nil**

These are directly exported but not implemented:

```
extendIfCan : (%,P) -> Union(%,"failed")
zeroSetSplit : List P -> List %
zeroSetSplitIntoTriangularSystems :
  List P -> List Record(close: %,open: List P)
```

These are implemented by this category:

```
algebraic? : (V,%) -> Boolean
algebraicVariables : % -> List V
autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
basicSet :
  (List P,(P -> Boolean),((P,P) -> Boolean)) ->
    Union(Record(bas: %,top: List P),"failed")
basicSet : (List P,((P,P) -> Boolean)) ->
  Union(Record(bas: %,top: List P),"failed")
coerce : % -> List P
coHeight : % -> NonNegativeInteger if V has FINITE
collectQuasiMonic : % -> %
collectUnder : (%,V) -> %
collectUpper : (%,V) -> %
construct : List P -> %
convert : % -> InputForm if P has KONVERT INFORM
degree : % -> NonNegativeInteger
extend : (%,P) -> %
first : % -> Union(P,"failed")
headReduce : (P,%) -> P
headReduced? : % -> Boolean
headReduced? : (P,%) -> Boolean
infRittWu? : (%,%) -> Boolean
initiallyReduce : (P,%) -> P
initiallyReduced? : % -> Boolean
initiallyReduced? : (P,%) -> Boolean
initials : % -> List P
last : % -> Union(P,"failed")
mvar : % -> V
normalized? : % -> Boolean
normalized? : (P,%) -> Boolean
quasiComponent : % -> Record(close: List P,open: List P)
reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
reduceByQuasiMonic : (P,%) -> P
reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
removeZero : (P,%) -> P
rest : % -> Union(%,"failed")
retractIfCan : List P -> Union(%,"failed")
```

```
rewriteSetWithReduction :
   (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List P
select : (%,V) -> Union(P,"failed")
stronglyReduce : (P,%) -> P
stronglyReduced? : % -> Boolean
stronglyReduced? : (P,%) -> Boolean
?=? : (%,%) -> Boolean
```

These exports come from (p373) PolynomialSetCategory(R,E,V,P)
where R:IntegralDomain, E:OrderedAbelianMonoidSup,
V:OrderedSet, P:RecursivePolynomialCategory(R,E,V)):

```
any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
coerce : % -> OutputForm
collect : (%,V) -> %
copy : % -> %
count : ((P -> Boolean),%) -> NonNegativeInteger
  if $ has finiteAggregate
count : (P,%) -> NonNegativeInteger
  if P has SETCAT and $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
find : ((P -> Boolean),%) -> Union(P,"failed")
hash : % -> SingleInteger
headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
mainVariable? : (V,%) -> Boolean
mainVariables : % -> List V
map : ((P -> P),%) -> %
map! : ((P -> P),%) -> % if $ has shallowlyMutable
member? : (P,%) -> Boolean
  if P has SETCAT and $ has finiteAggregate
members : % -> List P if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List P if $ has finiteAggregate
reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P,P) -> P
  if P has SETCAT
  and $ has finiteAggregate
remainder : (P,%) -> Record(rnum: R,polnum: P,den: R)
  if R has INTDOM
remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
```

```
remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
  if P has SETCAT
  and $ has finiteAggregate
retract : List P -> %
rewriteIdealWithHeadRemainder : (List P,%) -> List P
  if R has INTDOM
rewriteIdealWithRemainder : (List P,%) -> List P
  if R has INTDOM
roughBase? : % -> Boolean if R has INTDOM
roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
roughUnitIdeal? : % -> Boolean if R has INTDOM
sample : () -> %
select : ((P -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
sort : (%,V) -> Record(under: %,floor: %,upper: %)
triangular? : % -> Boolean if R has INTDOM
trivialIdeal? : % -> Boolean
variables : % -> List V
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean
```

⟨*category TSETCAT TriangularSetCategory*⟩≡

```
)abbrev category TSETCAT TriangularSetCategory
++ Author: Marc Moreno Maza (marc@nag.co.uk)
++ Date Created: 04/26/1994
++ Date Last Updated: 12/15/1998
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: polynomial, multivariate, ordered variables set
++ Description:
++ The category of triangular sets of multivariate polynomials
++ with coefficients in an integral domain.
++ Let \axiom{R} be an integral domain and \axiom{V} a finite ordered set of
++ variables, say \axiom{X1 < X2 < ... < Xn}.
++ A set \axiom{S} of polynomials in \axiom{R[X1,X2,...,Xn]} is triangular
++ if no elements of \axiom{S} lies in \axiom{R}, and if two distinct
++ elements of \axiom{S} have distinct main variables.
++ Note that the empty set is a triangular set. A triangular set is not
++ necessarily a (lexicographical) Groebner basis and the notion of
++ reduction related to triangular sets is based on the recursive view
++ of polynomials. We recall this notion here and refer to [1] for more
++ details.
++ A polynomial \axiom{P} is reduced w.r.t a non-constant polynomial
++ \axiom{Q} if the degree of \axiom{P} in the main variable of \axiom{Q}
```

```
++ is less than the main degree of \axiom{Q}.
++ A polynomial \axiom{P} is reduced w.r.t a triangular set \axiom{T}
++ if it is reduced w.r.t. every polynomial of \axiom{T}. \newline
++ References :
++  [1] P. AUBRY, D. LAZARD and M. MORENO MAZA "On the Theories
++      of Triangular Sets" Journal of Symbol. Comp. (to appear)
++ Version: 4.

TriangularSetCategory(R:IntegralDomain,E:OrderedAbelianMonoidSup,_
 V:OrderedSet,P:RecursivePolynomialCategory(R,E,V)):
         Category ==
   PolynomialSetCategory(R,E,V,P) with
     finiteAggregate
     shallowlyMutable

     infRittWu? : ($,$) -> Boolean
       ++ \axiom{infRittWu?(ts1,ts2)} returns true iff \axiom{ts2} has
       ++ higher rank than \axiom{ts1} in Wu Wen Tsun sense.
     basicSet : (List P,((P,P)->Boolean)) -> _
      Union(Record(bas:$,top:List P),"failed")
       ++ \axiom{basicSet(ps,redOp?)} returns \axiom{[bs,ts]} where
       ++ \axiom{concat(bs,ts)} is \axiom{ps} and \axiom{bs}
       ++ is a basic set in Wu Wen Tsun sense of \axiom{ps} w.r.t
       ++ the reduction-test \axiom{redOp?}, if no non-zero constant
       ++ polynomial lie in \axiom{ps}, otherwise \axiom{"failed"} is returned.
     basicSet : (List P,(P->Boolean),((P,P)->Boolean)) -> _
      Union(Record(bas:$,top:List P),"failed")
       ++ \axiom{basicSet(ps,pred?,redOp?)} returns the same as
       ++ \axiom{basicSet(qs,redOp?)}
       ++ where \axiom{qs} consists of the polynomials of \axiom{ps}
       ++ satisfying property \axiom{pred?}.
     initials : $ -> List P
       ++ \axiom{initials(ts)} returns the list of the non-constant initials
       ++ of the members of \axiom{ts}.
     degree : $ -> NonNegativeInteger
       ++ \axiom{degree(ts)} returns the product of main degrees of the
       ++ members of \axiom{ts}.
     quasiComponent : $ -> Record(close:List P,open:List P)
       ++ \axiom{quasiComponent(ts)} returns \axiom{[lp,lq]} where \axiom{lp}
       ++ is the list of the members of \axiom{ts} and \axiom{lq}is
       ++ \axiom{initials(ts)}.
     normalized? : (P,$) -> Boolean
       ++ \axiom{normalized?(p,ts)} returns true iff \axiom{p} and all
       ++ its iterated initials have degree zero w.r.t. the main variables
       ++ of the polynomials of \axiom{ts}
     normalized? : $  -> Boolean
```

```
  ++ \axiom{normalized?(ts)} returns true iff for every axiom{p} in
  ++ \axiom{ts} we have \axiom{normalized?(p,us)} where \axiom{us}
  ++ is \axiom{collectUnder(ts,mvar(p))}.
reduced? : (P,$,((P,P) -> Boolean)) -> Boolean
  ++ \axiom{reduced?(p,ts,redOp?)} returns true iff \axiom{p} is reduced
  ++ w.r.t.in the sense of the operation \axiom{redOp?}, that is if for
  ++ every \axiom{t} in \axiom{ts} \axiom{redOp?(p,t)} holds.
stronglyReduced? : (P,$) -> Boolean
  ++ \axiom{stronglyReduced?(p,ts)} returns true iff \axiom{p}
  ++ is reduced w.r.t. \axiom{ts}.
headReduced? : (P,$) -> Boolean
  ++ \axiom{headReduced?(p,ts)} returns true iff the head of \axiom{p} is
  ++ reduced w.r.t. \axiom{ts}.
initiallyReduced? : (P,$) -> Boolean
  ++ \axiom{initiallyReduced?(p,ts)} returns true iff \axiom{p} and all
  ++ its iterated initials are reduced w.r.t. to the elements of
  ++ \axiom{ts} with the same main variable.
autoReduced? : ($,((P,List(P)) -> Boolean)) -> Boolean
  ++ \axiom{autoReduced?(ts,redOp?)} returns true iff every element of
  ++ \axiom{ts} is reduced w.r.t to every other in the sense of
  ++ \axiom{redOp?}
stronglyReduced? : $ -> Boolean
  ++ \axiom{stronglyReduced?(ts)} returns true iff every element of
  ++ \axiom{ts} is reduced w.r.t to any other element of \axiom{ts}.
headReduced? : $ -> Boolean
  ++ headReduced?(ts) returns true iff the head of every element of
  ++ \axiom{ts} is reduced w.r.t to any other element of \axiom{ts}.
initiallyReduced? : $ -> Boolean
  ++ initiallyReduced?(ts) returns true iff for every element \axiom{p}
  ++ of \axiom{ts}. \axiom{p} and all its iterated initials are reduced
  ++ w.r.t. to the other elements of \axiom{ts} with the same main
  ++ variable.
reduce : (P,$,((P,P) -> P),((P,P) -> Boolean) ) -> P
  ++ \axiom{reduce(p,ts,redOp,redOp?)} returns a polynomial \axiom{r}
  ++ such that \axiom{redOp?(r,p)} holds for every \axiom{p} of
  ++ \axiom{ts} and there exists some product \axiom{h} of the initials
  ++ of the members of \axiom{ts} such that \axiom{h*p - r} lies in the
  ++ ideal generated by \axiom{ts}. The operation \axiom{redOp} must
  ++ satisfy the following conditions. For every \axiom{p} and \axiom{q}
  ++ we have  \axiom{redOp?(redOp(p,q),q)} and there exists an integer
  ++ \axiom{e} and a polynomial \axiom{f} such that
  ++ \axiom{init(q)^e*p = f*q + redOp(p,q)}.
rewriteSetWithReduction : (List P,$,((P,P) -> P),((P,P) -> Boolean) ) ->_
 List P
  ++ \axiom{rewriteSetWithReduction(lp,ts,redOp,redOp?)} returns a list
  ++ \axiom{lq} of polynomials such that
```

```
    ++ \axiom{[reduce(p,ts,redOp,redOp?) for p in lp]} and \axiom{lp}
    ++ have the same zeros inside the regular zero set of \axiom{ts}.
    ++ Moreover, for every polynomial \axiom{q} in \axiom{lq} and every
    ++ polynomial \axiom{t} in \axiom{ts}
    ++ \axiom{redOp?(q,t)} holds and there exists a polynomial \axiom{p}
    ++ in the ideal generated by \axiom{lp} and a product \axiom{h} of
    ++ \axiom{initials(ts)} such that \axiom{h*p - r} lies in the ideal
    ++ generated by \axiom{ts}.
    ++ The operation \axiom{redOp} must satisfy the following conditions.
    ++ For every \axiom{p} and \axiom{q} we have
    ++ \axiom{redOp?(redOp(p,q),q)}
    ++ and there exists an integer \axiom{e} and a polynomial \axiom{f}
    ++ such that \axiom{init(q)^e*p = f*q + redOp(p,q)}.
  stronglyReduce : (P,$) -> P
    ++ \axiom{stronglyReduce(p,ts)} returns a polynomial \axiom{r} such that
    ++ \axiom{stronglyReduced?(r,ts)} holds and there exists some product
    ++ \axiom{h} of \axiom{initials(ts)}
    ++ such that \axiom{h*p - r} lies in the ideal generated by \axiom{ts}.
  headReduce : (P,$) -> P
    ++ \axiom{headReduce(p,ts)} returns a polynomial \axiom{r}  such that
    ++ \axiom{headReduce?(r,ts)} holds and there exists some product
    ++ \axiom{h} of \axiom{initials(ts)} such that \axiom{h*p - r} lies
    ++ in the ideal generated by \axiom{ts}.
  initiallyReduce : (P,$) -> P
    ++ \axiom{initiallyReduce(p,ts)} returns a polynomial \axiom{r}
    ++ such that  \axiom{initiallyReduced?(r,ts)}
    ++ holds and there exists some product \axiom{h} of \axiom{initials(ts)}
    ++ such that \axiom{h*p - r} lies in the ideal generated by \axiom{ts}.
  removeZero: (P, $) -> P
    ++ \axiom{removeZero(p,ts)} returns \axiom{0} if \axiom{p} reduces
    ++ to \axiom{0} by pseudo-division w.r.t \axiom{ts} otherwise
    ++ returns a polynomial \axiom{q} computed from \axiom{p}
    ++ by removing any coefficient in \axiom{p} reducing to \axiom{0}.
  collectQuasiMonic: $ -> $
    ++ \axiom{collectQuasiMonic(ts)} returns the subset of \axiom{ts}
    ++ consisting of the polynomials with initial in \axiom{R}.
  reduceByQuasiMonic: (P, $) -> P
    ++ \axiom{reduceByQuasiMonic(p,ts)} returns the same as
    ++ \axiom{remainder(p,collectQuasiMonic(ts)).polnum}.
  zeroSetSplit : List P -> List $
    ++ \axiom{zeroSetSplit(lp)} returns a list \axiom{lts} of triangular
    ++ sets such that the zero set of \axiom{lp} is the union of the
    ++ closures of the regular zero sets  of the members of \axiom{lts}.
  zeroSetSplitIntoTriangularSystems : List P -> _
   List Record(close:$,open:List P)
    ++ \axiom{zeroSetSplitIntoTriangularSystems(lp)} returns a list of
```

```
      ++ triangular systems \axiom{[[ts1,qs1],...,[tsn,qsn]]} such that the
      ++ zero set of \axiom{lp} is the union of the closures of the
      ++ \axiom{W_i} where \axiom{W_i} consists of the zeros of \axiom{ts}
      ++ which do not cancel any polynomial in \axiom{qsi}.
    first : $ -> Union(P,"failed")
      ++ \axiom{first(ts)} returns the polynomial of \axiom{ts} with
      ++ greatest main variable if \axiom{ts} is not empty, otherwise
      ++ returns \axiom{"failed"}.
    last : $ -> Union(P,"failed")
      ++ \axiom{last(ts)} returns the polynomial of \axiom{ts} with
      ++ smallest main variable if \axiom{ts} is not empty, otherwise
      ++ returns \axiom{"failed"}.
    rest : $ -> Union($,"failed")
      ++ \axiom{rest(ts)} returns the polynomials of \axiom{ts} with smaller
      ++ main variable than \axiom{mvar(ts)} if \axiom{ts} is not empty,
      ++ otherwise returns "failed"
    algebraicVariables : $ -> List(V)
      ++ \axiom{algebraicVariables(ts)} returns the decreasingly sorted
      ++ list of the main variables of the polynomials of \axiom{ts}.
    algebraic? : (V,$) -> Boolean
      ++ \axiom{algebraic?(v,ts)} returns true iff \axiom{v} is the
      ++ main variable of some polynomial in \axiom{ts}.
    select : ($,V) -> Union(P,"failed")
      ++ \axiom{select(ts,v)} returns the polynomial of \axiom{ts} with
      ++ \axiom{v} as main variable, if any.
    extendIfCan : ($,P) -> Union($,"failed")
      ++ \axiom{extendIfCan(ts,p)} returns a triangular set which encodes
      ++ the simple extension by \axiom{p} of the extension of the base
      ++ field defined by \axiom{ts}, according
      ++ to the properties of triangular sets of the current domain.
      ++ If the required properties do not hold then "failed" is returned.
      ++ This operation encodes in some sense the properties of the
      ++ triangular sets of the current category. Is is used to implement
      ++ the \axiom{construct} operation to guarantee that every triangular
      ++ set build from a list of polynomials has the required properties.
    extend : ($,P) -> $
      ++ \axiom{extend(ts,p)} returns a triangular set which encodes the
      ++ simple extension by \axiom{p} of the extension of the base field
      ++ defined by \axiom{ts}, according to the properties of triangular
      ++ sets of the current category. If the required properties do not
      ++ hold an error is returned.
    if V has Finite
    then
      coHeight : $ -> NonNegativeInteger
        ++ \axiom{coHeight(ts)} returns \axiom{size()\$V} minus \axiom{\#ts}.
  add
```

```
GPS ==> GeneralPolynomialSet(R,E,V,P)
B ==> Boolean
RBT ==> Record(bas:$,top:List P)

ts:$ = us:$ ==
  empty?(ts)$$ => empty?(us)$$
  empty?(us)$$ => false
  first(ts)::P =$P first(us)::P => rest(ts)::$ =$$ rest(us)::$
  false

infRittWu?(ts,us) ==
  empty?(us)$$ => not empty?(ts)$$
  empty?(ts)$$ => false
  p : P := (last(ts))::P
  q : P := (last(us))::P
  infRittWu?(p,q)$P => true
  supRittWu?(p,q)$P => false
  v : V := mvar(p)
  infRittWu?(collectUpper(ts,v),collectUpper(us,v))$$

reduced?(p,ts,redOp?) ==
  lp : List P := members(ts)
  while (not empty? lp) and (redOp?(p,first(lp))) repeat
    lp := rest lp
  empty? lp

basicSet(ps,redOp?) ==
  ps := remove(zero?,ps)
  any?(ground?,ps) => "failed"::Union(RBT,"failed")
  ps := sort(infRittWu?,ps)
  p,b : P
  bs := empty()$$
  ts : List P := []
  while not empty? ps repeat
    b := first(ps)
    bs := extend(bs,b)$$
    ps := rest ps
    while (not empty? ps) and _
          (not reduced?((p := first(ps)),bs,redOp?)) repeat
      ts := cons(p,ts)
      ps := rest ps
  ([bs,ts]$RBT)::Union(RBT,"failed")

basicSet(ps,pred?,redOp?) ==
  ps := remove(zero?,ps)
```

```
  any?(ground?,ps) => "failed"::Union(RBT,"failed")
  gps : List P := []
  bps : List P := []
  while not empty? ps repeat
    p := first ps
    ps := rest ps
    if pred?(p)
      then
        gps := cons(p,gps)
      else
        bps := cons(p,bps)
  gps := sort(infRittWu?,gps)
  p,b : P
  bs := empty()$$
  ts : List P := []
  while not empty? gps repeat
    b := first(gps)
    bs := extend(bs,b)$$
    gps := rest gps
    while (not empty? gps) and _
          (not reduced?((p := first(gps)),bs,redOp?)) repeat
      ts := cons(p,ts)
      gps := rest gps
  ts := sort(infRittWu?,concat(ts,bps))
  ([bs,ts]$RBT)::Union(RBT,"failed")

initials ts ==
  lip : List P := []
  empty? ts => lip
  lp := members(ts)
  while not empty? lp repeat
     p := first(lp)
     if not ground?((ip := init(p)))
       then
         lip := cons(primPartElseUnitCanonical(ip),lip)
     lp := rest lp
  removeDuplicates lip

degree ts ==
  empty? ts => 0$NonNegativeInteger
  lp := members ts
  d : NonNegativeInteger := mdeg(first lp)
  while not empty? (lp := rest lp) repeat
    d := d * mdeg(first lp)
  d
```

```
quasiComponent ts ==
  [members(ts),initials(ts)]

normalized?(p,ts) ==
  normalized?(p,members(ts))$P

stronglyReduced? (p,ts) ==
  reduced?(p,members(ts))$P

headReduced? (p,ts) ==
  stronglyReduced?(head(p),ts)

initiallyReduced? (p,ts) ==
  lp : List (P) := members(ts)
  red : Boolean := true
  while (not empty? lp) and (not ground?(p)$P) and red repeat
    while (not empty? lp) and (mvar(first(lp)) > mvar(p)) repeat
      lp := rest lp
    if (not empty? lp)
      then
        if  (mvar(first(lp)) = mvar(p))
          then
            if reduced?(p,first(lp))
              then
                lp := rest lp
                p := init(p)
              else
                red := false
          else
            p := init(p)
  red

reduce(p,ts,redOp,redOp?) ==
  (empty? ts) or (ground? p) => p
  ts0 := ts
  while (not empty? ts) and (not ground? p) repeat
     reductor := (first ts)::P
     ts := (rest ts)::$
     if not redOp?(p,reductor)
       then
         p := redOp(p,reductor)
         ts := ts0
  p

rewriteSetWithReduction(lp,ts,redOp,redOp?) ==
  trivialIdeal? ts => lp
```

```
  lp := remove(zero?,lp)
  empty? lp => lp
  any?(ground?,lp) => [1$P]
  rs : List P := []
  while not empty? lp repeat
    p := first lp
    lp := rest lp
    p := primPartElseUnitCanonical reduce(p,ts,redOp,redOp?)
    if not zero? p
      then
        if ground? p
          then
            lp := []
            rs := [1$P]
          else
            rs := cons(p,rs)
  removeDuplicates rs

stronglyReduce(p,ts) ==
  reduce (p,ts,lazyPrem,reduced?)

headReduce(p,ts) ==
  reduce (p,ts,headReduce,headReduced?)

initiallyReduce(p,ts) ==
  reduce (p,ts,initiallyReduce,initiallyReduced?)

removeZero(p,ts) ==
  (ground? p) or (empty? ts) => p
  v := mvar(p)
  ts_v_- := collectUnder(ts,v)
  if algebraic?(v,ts)
    then
      q := lazyPrem(p,select(ts,v)::P)
      zero? q => return q
      zero? removeZero(q,ts_v_-) => return 0
  empty? ts_v_- => p
  q: P := 0
  while positive? degree(p,v) repeat
     q := removeZero(init(p),ts_v_-) * mainMonomial(p) + q
     p := tail(p)
  q + removeZero(p,ts_v_-)

reduceByQuasiMonic(p, ts) ==
  (ground? p) or (empty? ts) => p
  remainder(p,collectQuasiMonic(ts)).polnum
```

```
autoReduced?(ts : $,redOp? : ((P,List(P)) -> Boolean)) ==
  empty? ts => true
  lp : List (P) := members(ts)
  p : P := first(lp)
  lp := rest lp
  while (not empty? lp) and redOp?(p,lp) repeat
     p := first lp
     lp := rest lp
  empty? lp

stronglyReduced? ts ==
  autoReduced? (ts, reduced?)

normalized? ts ==
  autoReduced? (ts,normalized?)

headReduced? ts ==
  autoReduced? (ts,headReduced?)

initiallyReduced?  ts ==
  autoReduced? (ts,initiallyReduced?)

mvar ts ==
  empty? ts => error"Error from TSETCAT in mvar : #1 is empty"
  mvar((first(ts))::P)$P

first ts ==
  empty? ts => "failed"::Union(P,"failed")
  lp : List(P) := sort(supRittWu?,members(ts))$(List P)
  first(lp)::Union(P,"failed")

last ts ==
  empty? ts => "failed"::Union(P,"failed")
  lp : List(P) := sort(infRittWu?,members(ts))$(List P)
  first(lp)::Union(P,"failed")

rest ts ==
  empty? ts => "failed"::Union($,"failed")
  lp : List(P) := sort(supRittWu?,members(ts))$(List P)
  construct(rest(lp))::Union($,"failed")

coerce (ts:$) : List(P) ==
  sort(supRittWu?,members(ts))$(List P)

algebraicVariables ts ==
```

```
    [mvar(p) for p in members(ts)]

  algebraic? (v,ts) ==
    member?(v,algebraicVariables(ts))

  select  (ts,v) ==
    lp : List (P) := sort(supRittWu?,members(ts))$(List P)
    while (not empty? lp) and (not (v = mvar(first lp))) repeat
      lp := rest lp
    empty? lp => "failed"::Union(P,"failed")
    (first lp)::Union(P,"failed")

  collectQuasiMonic ts ==
    lp: List(P) := members(ts)
    newlp: List(P) := []
    while (not empty? lp) repeat
      if ground? init(first(lp)) then newlp := cons(first(lp),newlp)
      lp := rest lp
    construct(newlp)

  collectUnder (ts,v) ==
    lp : List (P) := sort(supRittWu?,members(ts))$(List P)
    while (not empty? lp) and (not (v > mvar(first lp))) repeat
      lp := rest lp
    construct(lp)

  collectUpper  (ts,v) ==
    lp1 : List(P) := sort(supRittWu?,members(ts))$(List P)
    lp2 : List(P) := []
    while (not empty? lp1) and  (mvar(first lp1) > v) repeat
      lp2 := cons(first(lp1),lp2)
      lp1 := rest lp1
    construct(reverse lp2)

  construct(lp:List(P)) ==
    rif := retractIfCan(lp)@Union($,"failed")
    not (rif case $) => error"in construct : LP -> $ from TSETCAT : bad arg"
    rif::$

  retractIfCan(lp:List(P)) ==
    empty? lp => (empty()$$)::Union($,"failed")
    lp := sort(supRittWu?,lp)
    rif := retractIfCan(rest(lp))@Union($,"failed")
    not (rif case $) => _
     error "in retractIfCan : LP -> ... from TSETCAT : bad arg"
    extendIfCan(rif::$,first(lp))@Union($,"failed")
```

```
      extend(ts:$,p:P):$ ==
        eif := extendIfCan(ts,p)@Union($,"failed")
        not (eif case $) => error"in extend : ($,P) -> $ from TSETCAT : bad ars"
        eif::$

      if V has Finite
      then

        coHeight ts ==
          n := size()$V
          m := #(members ts)
          subtractIfCan(n,m)$NonNegativeInteger::NonNegativeInteger
```

⟨*TSETCAT.dotabb*⟩≡

```
"TSETCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=TSETCAT"];
"TSETCAT" -> "PSETCAT"
```

⟨*TSETCAT.dotfull*⟩≡

```
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
 [color=lightblue,href="bookvol10.2.pdf#nameddest=TSETCAT"];
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
  -> "PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recurs

"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom
 [color=seagreen,href="bookvol10.2.pdf#nameddest=TSETCAT"];
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom
 ->
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
```

⟨*TSETCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
 [color=lightblue];
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
  -> "PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedS

"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
 [color=seagreen];
"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
  -> "PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recur

"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 [color=lightblue];
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 -> "SetCategory()"
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 -> "Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),Ordere
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 -> "CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),

"CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),Orde
 [color=seagreen];
"CoercibleTo(List(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),Orde
  -> "CoercibleTo(a:Type)"

"Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet
"Collection(RecursivePolynomialCategory(Ring,OrderedAbelianMonoidSup(),OrderedSet
  -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HomogeneousAggregate(a:Type)"
"Collection(a:Type)" -> "ConvertibleTo(InputForm)"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
```

```
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"HomogeneousAggregate(a:Type)" [color=lightblue];
"HomogeneousAggregate(a:Type)" -> "Aggregate()"

"Aggregate()" [color=lightblue];
"Aggregate()" -> "Type()"

"Type()" [color=lightblue];
"Type()" -> "Category"

"ConvertibleTo(InputForm)" [color="#00EE00"];
"ConvertibleTo(InputForm)" -> "ConvertibleTo(a:Type)"

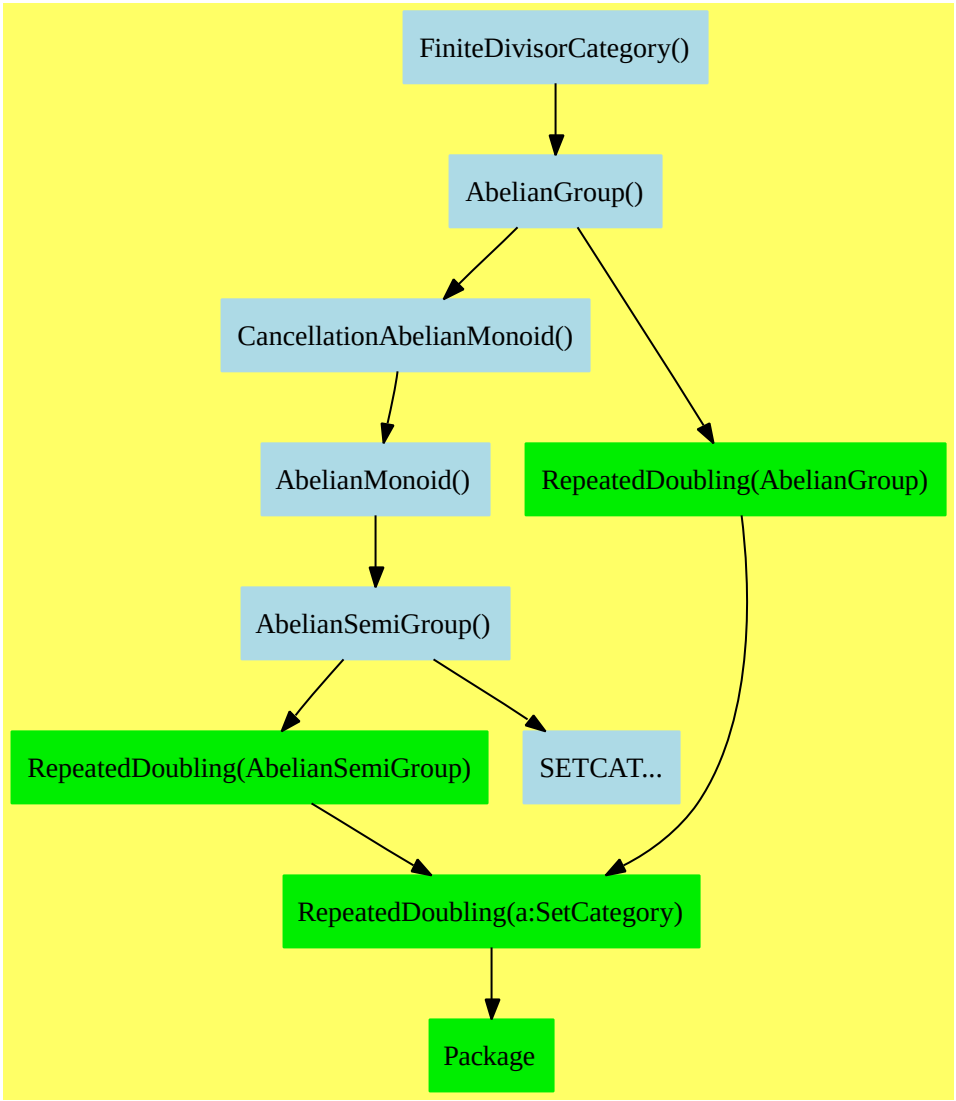
"ConvertibleTo(a:Type)" [color="#00EE00"];
"ConvertibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

Chapter 8

Category Layer 7

8.1 FiniteDivisorCategory (FDIVCAT)



See:

⇐ "AbelianGroup" (ABELGRP) 7.1 on page 418

Exports:

0	coerce	decompose	divisor	generator
hash	ideal	latex	principal?	reduce
sample	subtractIfCan	zero?	?~=?	?*?
?+?	?-?	-?	?=?	

These are directly exported but not implemented:

```
decompose : % ->
  Record(id: FractionalIdeal(UP,Fraction UP,UPUP,R),principalPart: R)
divisor : R -> %
divisor : FractionalIdeal(UP,Fraction UP,UPUP,R) -> %
divisor : (F,F) -> %
divisor : (F,F,Integer) -> %
divisor : (R,UP,UP,UP,F) -> %
generator : % -> Union(R,"failed")
ideal : % -> FractionalIdeal(UP,Fraction UP,UPUP,R)
reduce : % -> %
```

These are implemented by this category:

```
principal? : % -> Boolean
```

These exports come from (p418) AbelianGroup():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
-? : % -> %
?-? : (%,%) -> %
?~=? : (%,%) -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
```

⟨*category FDIVCAT FiniteDivisorCategory*⟩≡

```
)abbrev category FDIVCAT FiniteDivisorCategory
++ Category for finite rational divisors on a curve
++ Author: Manuel Bronstein
++ Date Created: 19 May 1993
```

```
++ Date Last Updated: 19 May 1993
++ Description:
++ This category describes finite rational divisors on a curve, that
++ is finite formal sums SUM(n * P) where the n's are integers and the
++ P's are finite rational points on the curve.
++ Keywords: divisor, algebraic, curve.
++ Examples: )r FDIV INPUT
FiniteDivisorCategory(F, UP, UPUP, R): Category == Result where
  F   : Field
  UP  : UnivariatePolynomialCategory F
  UPUP: UnivariatePolynomialCategory Fraction UP
  R   : FunctionFieldCategory(F, UP, UPUP)

  ID  ==> FractionalIdeal(UP, Fraction UP, UPUP, R)

  Result ==> AbelianGroup with
    ideal      : % -> ID
      ++ ideal(D) returns the ideal corresponding to a divisor D.
    divisor    : ID -> %
      ++ divisor(I) makes a divisor D from an ideal I.
    divisor    : R -> %
      ++ divisor(g) returns the divisor of the function g.
    divisor    : (F, F) -> %
      ++ divisor(a, b) makes the divisor P: \spad{(x = a, y = b)}.
      ++ Error: if P is singular.
    divisor    : (F, F, Integer) -> %
      ++ divisor(a, b, n) makes the divisor
      ++ \spad{nP} where P: \spad{(x = a, y = b)}.
      ++ P is allowed to be singular if n is a multiple of the rank.
    decompose  : % -> Record(id:ID, principalPart: R)
      ++ decompose(d) returns \spad{[id, f]} where \spad{d = (id) + div(f)}.
    reduce     : % -> %
      ++ reduce(D) converts D to some reduced form (the reduced forms can
      ++ be differents in different implementations).
    principal? : % -> Boolean
      ++ principal?(D) tests if the argument is the divisor of a function.
    generator  : % -> Union(R, "failed")
      ++ generator(d) returns f if \spad{(f) = d},
      ++ "failed" if d is not principal.
    divisor    : (R, UP, UP, UP, F) -> %
      ++ divisor(h, d, d', g, r) returns the sum of all the finite points
      ++ where \spad{h/d} has residue \spad{r}.
      ++ \spad{h} must be integral.
      ++ \spad{d} must be squarefree.
      ++ \spad{d'} is some derivative of \spad{d} (not necessarily dd/dx).
      ++ \spad{g = gcd(d,discriminant)} contains the ramified zeros of \spad{d}
```

```
    add
     principal? d == generator(d) case R
```

⟨*FDIVCAT.dotabb*⟩≡

```
  "FDIVCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FDIVCAT"];
  "FDIVCAT" -> "ABELGRP"
```

⟨*FDIVCAT.dotfull*⟩≡

```
  "FiniteDivisorCategory()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FDIVCAT"];
  "FiniteDivisorCategory()" -> "AbelianGroup()"
```

⟨*FDIVCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FiniteDivisorCategory()" [color=lightblue];
"FiniteDivisorCategory()" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "RepeatedDoubling(AbelianGroup)"

"RepeatedDoubling(AbelianGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(AbelianSemiGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianSemiGroup)" -> "RepeatedDoubling(a:SetCategory)"

"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

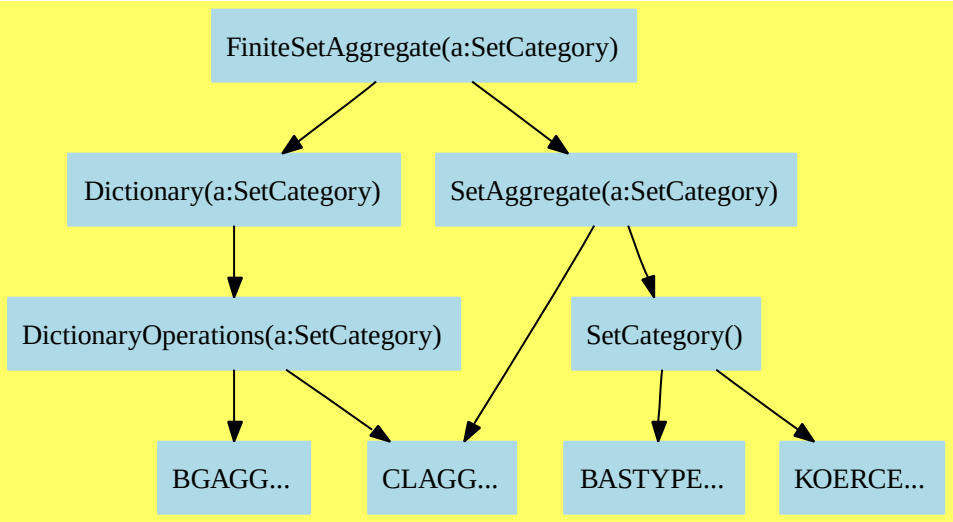
"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SETCAT..."
"AbelianSemiGroup()" -> "RepeatedDoubling(AbelianSemiGroup)"

"SETCAT..." [color=lightblue];
}
```

8.2 FiniteSetAggregate (FSAGG)



See:

⇐ "Dictionary" (DIAGG) 7.3 on page 428
⇐ "SetAggregate" (SETAGG) 6.13 on page 395

Exports:

any?	bag	brace	cardinality	coerce
complement	construct	convert	copy	count
dictionary	difference	empty	empty?	eq?
eval	every?	extract!	find	hash
index	insert!	inspect	intersect	latex
less?	lookup	map	map!	max
member?	members	min	more?	parts
random	reduce	remove	remove!	removeDuplicates
sample	select	select!	set	size
size?	subset?	symmetricDifference	union	universe
#?	?<?	?=?	?~=?	

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **partiallyOrderedSet** is true if a set with $<$ which is transitive, but not$(a < b$ or $a = b)$ does not necessarily imply $b < a$.

- **nil**

These are implemented by this category:

```
brace : List S -> %
cardinality : % -> NonNegativeInteger
coerce : % -> OutputForm
complement : % -> % if S has FINITE
construct : List S -> %
count : (S,%) -> NonNegativeInteger
    if S has SETCAT and $ has finiteAggregate
difference : (%,%) -> %
index : PositiveInteger -> % if S has FINITE
intersect : (%,%) -> %
lookup : % -> PositiveInteger if S has FINITE
max : % -> S if S has ORDSET
min : % -> S if S has ORDSET
random : () -> % if S has FINITE
set : List S -> %
size : () -> NonNegativeInteger if S has FINITE
subset? : (%,%) -> Boolean
symmetricDifference : (%,%) -> %
union : (%,%) -> %
universe : () -> % if S has FINITE
?<? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
```

These exports come from (p428) Dictionary(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
bag : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : ((S -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
dictionary : () -> %
dictionary : List S -> %
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,S,S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,Equation S) -> %
    if S has EVALAB S
    and S has SETCAT
```

```
eval : (%,List Equation S) -> %
    if S has EVALAB S
    and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
extract! : % -> S
find : ((S -> Boolean),%) -> Union(S,"failed")
hash : % -> SingleInteger
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
select! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> %
    if $ has shallowlyMutable
member? : (S,%) -> Boolean
    if S has SETCAT
    and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
    if S has SETCAT
    and $ has finiteAggregate
remove : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove : (S,%) -> %
    if S has SETCAT
    and $ has finiteAggregate
remove! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove! : (S,%) -> % if $ has finiteAggregate
removeDuplicates : % -> %
    if S has SETCAT
    and $ has finiteAggregate
sample : () -> %
select : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean
```

These exports come from (p395) SetAggregate(S:SetCategory):

```
brace : () -> %
difference : (%,S) -> %
set : () -> %
union : (%,S) -> %
union : (S,%) -> %
```

⟨*category FSAGG FiniteSetAggregate*⟩≡

```
)abbrev category FSAGG FiniteSetAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: 14 Oct, 1993 by RSS
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A finite-set aggregate models the notion of a finite set, that is,
++ a collection of elements characterized by membership, but not
++ by order or multiplicity.
++ See \spadtype{Set} for an example.
FiniteSetAggregate(S:SetCategory): Category ==
  Join(Dictionary S, SetAggregate S) with
    finiteAggregate
    cardinality: % -> NonNegativeInteger
      ++ cardinality(u) returns the number of elements of u.
      ++ Note: \axiom{cardinality(u) = #u}.
    if S has Finite then
      Finite
      complement: % -> %
        ++ complement(u) returns the complement of the set u,
        ++ i.e. the set of all values not in u.
      universe: () -> %
        ++ universe()$D returns the universal set for finite set aggregate D.
    if S has OrderedSet then
      max: % -> S
        ++ max(u) returns the largest element of aggregate u.
      min: % -> S
        ++ min(u) returns the smallest element of aggregate u.

  add
    s < t           == #s < #t and s = intersect(s,t)
    s = t           == #s = #t and empty? difference(s,t)
    brace l         == construct l
    set   l         == construct l
    cardinality s   == #s
```

```
construct l     == (s := set(); for x in l repeat insert_!(x,s); s)
count(x:S, s:%) == (member?(x, s) => 1; 0)
subset?(s, t)   == #s < #t and _and/[member?(x, t) for x in parts s]

coerce(s:%):OutputForm ==
  brace [x::OutputForm for x in parts s]$List(OutputForm)

intersect(s, t) ==
  i := {}
  for x in parts s | member?(x, t) repeat insert_!(x, i)
  i

difference(s:%, t:%) ==
  m := copy s
  for x in parts t repeat remove_!(x, m)
  m

symmetricDifference(s, t) ==
  d := copy s
  for x in parts t repeat
    if member?(x, s) then remove_!(x, d) else insert_!(x, d)
  d

union(s:%, t:%) ==
   u := copy s
   for x in parts t repeat insert_!(x, u)
   u

if S has Finite then
  universe()   == {index(i::PositiveInteger) for i in 1..size()$S}
  complement s == difference(universe(), s )
  size()  == 2 ** size()$S
  index i ==
    {index(j::PositiveInteger)$S for j in 1..size()$S | bit?(i-1,j-1)}
  random()  ==
    index((random()$Integer rem (size()$% + 1))::PositiveInteger)

  lookup s ==
    n:PositiveInteger := 1
    for x in parts s repeat _
      n := n + 2 ** ((lookup(x) - 1)::NonNegativeInteger)
    n

if S has OrderedSet then
  max s ==
    empty?(l := parts s) => error "Empty set"
```

```
        reduce("max", l)

      min s ==
        empty?(l := parts s) => error "Empty set"
        reduce("min", l)
```

⟨*FSAGG.dotabb*⟩≡

```
"FSAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=FSAGG"];
"FSAGG" -> "DIAGG"
"FSAGG" -> "SETAGG"
```

⟨*FSAGG.dotfull*⟩≡

```
"FiniteSetAggregate(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FSAGG"];
"FiniteSetAggregate(a:SetCategory)" -> "Dictionary(a:SetCategory)"
"FiniteSetAggregate(a:SetCategory)" -> "SetAggregate(a:SetCategory)"
```

⟨*FSAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FiniteSetAggregate(a:SetCategory)" [color=lightblue];
"FiniteSetAggregate(a:SetCategory)" -> "Dictionary(a:SetCategory)"
"FiniteSetAggregate(a:SetCategory)" -> "SetAggregate(a:SetCategory)"

"SetAggregate(a:SetCategory)" [color=lightblue];
"SetAggregate(a:SetCategory)" -> "SetCategory()"
"SetAggregate(a:SetCategory)" -> "CLAGG..."

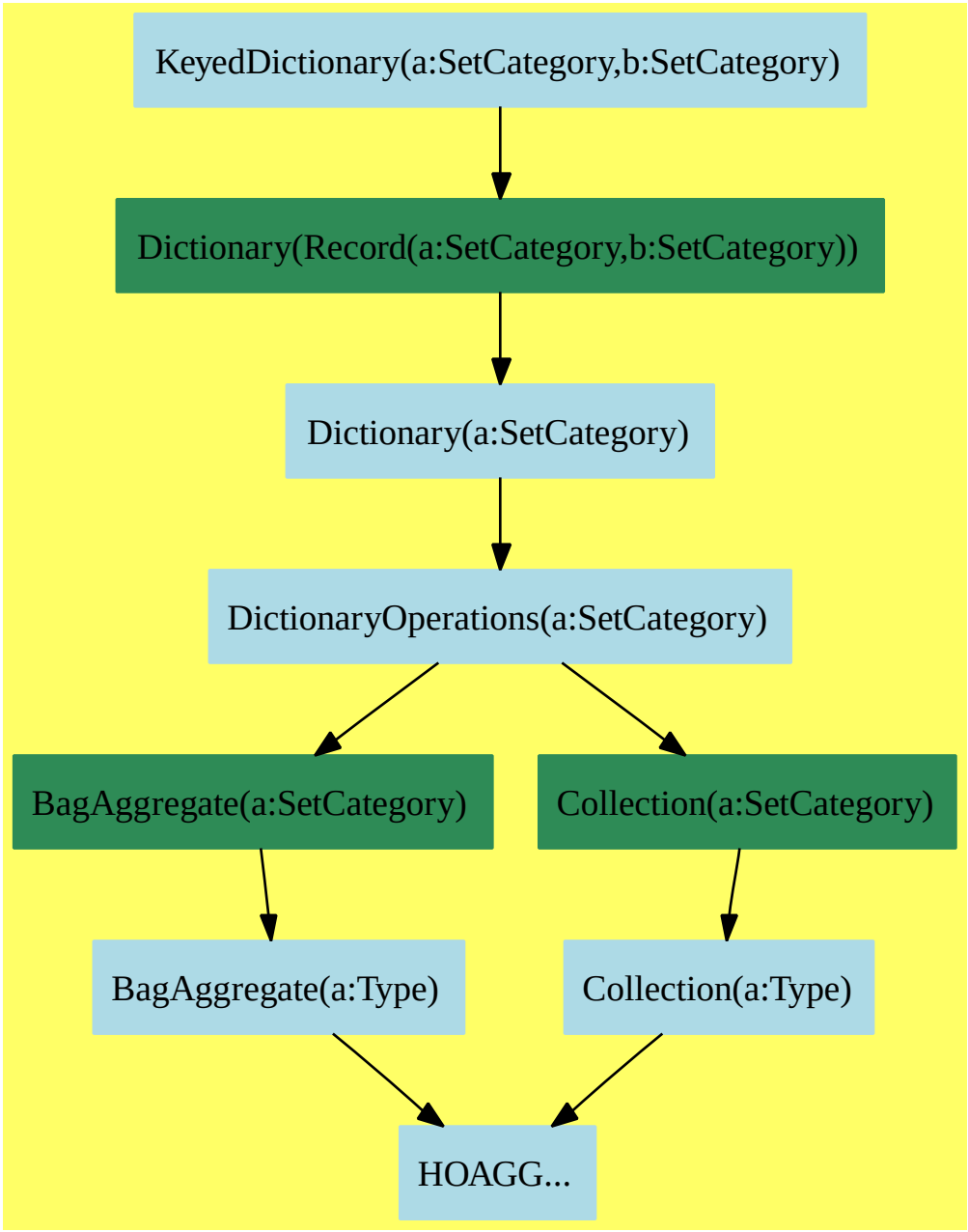
"SetCategory()" [color=lightblue];
"SetCategory()" -> "BASTYPE..."
"SetCategory()" -> "KOERCE..."

"Dictionary(a:SetCategory)" [color=lightblue];
"Dictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BGAGG..."
"DictionaryOperations(a:SetCategory)" -> "CLAGG..."

"BGAGG..." [color=lightblue];
"CLAGG..." [color=lightblue];
"BASTYPE..." [color=lightblue];
"KOERCE..." [color=lightblue];
}
```

8.3 KeyedDictionary (KDAGG)



See:

⇒ “TableAggregate” (TBAGG) 9.11 on page 649

⇐ "Dictionary" (DIAGG) 7.3 on page 428

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	empty	empty?
eq?	eval	every?	extract!	find
hash	insert!	inspect	key?	keys
latex	less?	map	map!	member?
members	more?	parts	reduce	remove
remove!	removeDuplicates	sample	search	select
select!	size?	#?	?=?	?~=?

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are directly exported but not implemented:

```
remove! : (Key,%) -> Union(Entry,"failed")
search : (Key,%) -> Union(Entry,"failed")
```

These are implemented by this category:

```
key? : (Key,%) -> Boolean
member? : (Record(key: Key,entry: Entry),%) -> Boolean
    if Record(key: Key,entry: Entry) has SETCAT
    and $ has finiteAggregate
keys : % -> List Key
```

These exports come from (p428) Dictionary(R)
where R=Record(a:SetCategory,b:SetCategory))
and S=Record(key: Key,entry: Entry)

```
any? : ((S) -> Boolean),%) -> Boolean
    if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm if S has SETCAT
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
```

```
count : (S,%) -> NonNegativeInteger
    if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
dictionary : () -> %
dictionary : List S -> %
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
    if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
    if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
    if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
    if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
extract! : % -> S
find : ((S -> Boolean),%) -> Union(S,"failed")
hash : % -> SingleInteger if S has SETCAT
insert! : (S,%) -> %
inspect : % -> S
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> %
    if $ has shallowlyMutable
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
    if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove : (S,%) -> %
    if S has SETCAT and $ has finiteAggregate
remove! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove! : (S,%) -> % if $ has finiteAggregate
removeDuplicates : % -> %
    if S has SETCAT and $ has finiteAggregate
sample : () -> %
select : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
```

```
select! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger
    if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category KDAGG KeyedDictionary*⟩≡

```
)abbrev category KDAGG KeyedDictionary
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A keyed dictionary is a dictionary of key-entry pairs for which there is
++ a unique entry for each key.
KeyedDictionary(Key:SetCategory, Entry:SetCategory): Category ==
  Dictionary Record(key:Key,entry:Entry) with
   key?: (Key, %) -> Boolean
     ++ key?(k,t) tests if k is a key in table t.
   keys: % -> List Key
     ++ keys(t) returns the list the keys in table t.
   -- to become keys: % -> Key* and keys: % -> Iterator(Entry,Entry)
   remove_!: (Key, %) -> Union(Entry,"failed")
     ++ remove!(k,t) searches the table t for the key k removing
     ++ (and return) the entry if there.
     ++ If t has no such key, \axiom{remove!(k,t)} returns "failed".
   search: (Key, %) -> Union(Entry,"failed")
     ++ search(k,t) searches the table t for the key k,
     ++ returning the entry stored in t for key k.
     ++ If t has no such key, \axiom{search(k,t)} returns "failed".
 add
   key?(k, t) == search(k, t) case Entry

   member?(p, t) ==
     r := search(p.key, t)
     r case Entry and r::Entry = p.entry

   if % has finiteAggregate then
     keys t == [x.key for x in parts t]
```

⟨*KDAGG.dotabb*⟩≡
```
"KDAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=KDAGG"];
"KDAGG" -> "DIAGG"
```

⟨*KDAGG.dotfull*⟩≡
```
"KeyedDictionary(a:SetCategory,b:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=KDAGG"];
"KeyedDictionary(a:SetCategory,b:SetCategory)" ->
    "Dictionary(Record(a:SetCategory,b:SetCategory))"
```

⟨*KDAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"KeyedDictionary(a:SetCategory,b:SetCategory)" [color=lightblue];
"KeyedDictionary(a:SetCategory,b:SetCategory)" ->
    "Dictionary(Record(a:SetCategory,b:SetCategory))"

"Dictionary(Record(a:SetCategory,b:SetCategory))" [color=seagreen];
"Dictionary(Record(a:SetCategory,b:SetCategory))" ->
    "Dictionary(a:SetCategory)"

"Dictionary(a:SetCategory)" [color=lightblue];
"Dictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BagAggregate(a:SetCategory)"
"DictionaryOperations(a:SetCategory)" -> "Collection(a:SetCategory)"

"BagAggregate(a:SetCategory)" [color=seagreen];
"BagAggregate(a:SetCategory)" -> "BagAggregate(a:Type)"

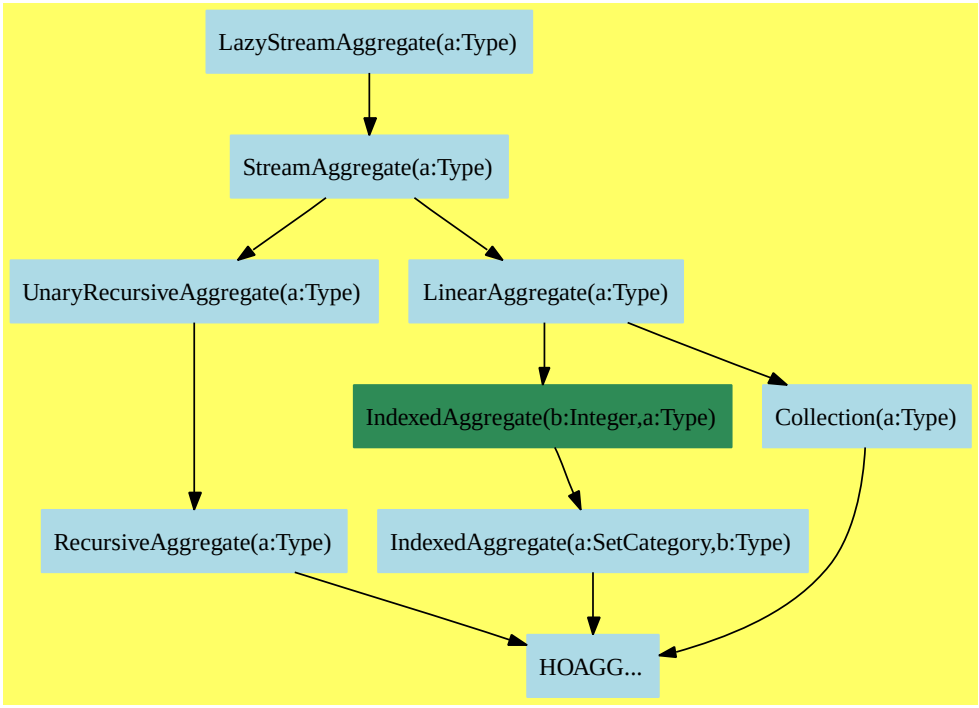
"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HOAGG..."

"Collection(a:SetCategory)" [color=seagreen];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HOAGG..."

"HOAGG..." [color=lightblue];
}
```

8.4 LazyStreamAggregate (LZSTAGG)



See:

⇐ "StreamAggregate" (STAGG) 7.11 on page 470

Exports:

any?	child?	children	coerce
complete	concat	concat!	construct
copy	convert	count	cycleEntry
cycleLength	cycleSplit!	cycleTail	cyclic?
delete	distance	elt	empty
empty?	entry?	entries	eq?
explicitEntries?	explicitlyEmpty?	explicitlyFinite?	extend
eval	every?	fill!	find
first	frst	hash	index?
indices	insert	last	latex
lazy?	lazyEvaluate	leaf?	leaves
less?	map	map!	maxIndex
member?	members	minIndex	more?
new	node?	nodes	numberOfComputedEntries
parts	possiblyInfinite?	qelt	qsetelt!
reduce	remove	removeDuplicates	rest
rst	sample	second	select
setchildren!	setelt	setfirst!	setlast!
setrest!	setvalue!	size?	split!
swap!	tail	third	value
#?	?=?	?.?	?.last
?.rest	?.first	?.value	?~=?

Attributes exported:

- **nil**

These are directly exported but not implemented:

```
explicitEntries? : % -> Boolean
explicitlyEmpty? : % -> Boolean
frst : % -> S
lazy? : % -> Boolean
lazyEvaluate : % -> %
numberOfComputedEntries : % -> NonNegativeInteger
remove : ((S -> Boolean),%) -> %
rst : % -> %
select : ((S -> Boolean),%) -> %
```

These are implemented by this category:

```
any? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
child? : (%,%) -> Boolean if S has SETCAT
children : % -> List %
complete : % -> %
construct : List S -> %
cycleEntry : % -> %
```

```
cycleLength : % -> NonNegativeInteger
cycleTail : % -> %
cyclic? : % -> Boolean
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
distance : (%,%) -> Integer
elt : (%,Integer,S) -> S
entries : % -> List S
every? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
explicitlyFinite? : % -> Boolean
extend : (%,Integer) -> %
first : (%,NonNegativeInteger) -> %
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (S,%,Integer) -> %
insert : (%,%,Integer) -> %
last : % -> S
last : (%,NonNegativeInteger) -> %
leaf? : % -> Boolean
less? : (%,NonNegativeInteger) -> Boolean
maxIndex : % -> Integer if Integer has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
node? : (%,%) -> Boolean if S has SETCAT
nodes : % -> List %
possiblyInfinite? : % -> Boolean
rest : % -> %
rest : (%,NonNegativeInteger) -> %
value : % -> S
size? : (%,NonNegativeInteger) -> Boolean
tail : % -> %
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean if S has SETCAT
?.? : (%,UniversalSegment Integer) -> %
?.? : (%,Integer) -> S
?.first : (%,first) -> S
?.last : (%,last) -> S
?.rest : (%,rest) -> %
```

These exports come from (p470) StreamAggregate(S:Type):

```
coerce : % -> OutputForm if S has SETCAT
concat : (%,%) -> %
concat : (%,S) -> %
concat : (S,%) -> %
concat : List % -> %
concat! : (%,%) -> % if $ has shallowlyMutable
concat! : (%,S) -> % if $ has shallowlyMutable
convert : % -> InputForm if S has KONVERT INFORM
```

```
copy : % -> %
count : (S,%) -> NonNegativeInteger
    if S has SETCAT
    and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
cycleSplit! : % -> % if $ has shallowlyMutable
empty : () -> %
empty? : % -> Boolean
entry? : (S,%) -> Boolean
    if $ has finiteAggregate
    and S has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,S,S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,Equation S) -> %
    if S has EVALAB S
    and S has SETCAT
eval : (%,List Equation S) -> %
    if S has EVALAB S
    and S has SETCAT
fill! : (%,S) -> % if $ has shallowlyMutable
find : ((S -> Boolean),%) -> Union(S,"failed")
first : % -> S
hash : % -> SingleInteger if S has SETCAT
latex : % -> String if S has SETCAT
leaves : % -> List S
map : (((S,S) -> S),%,%) -> %
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
    if S has SETCAT
    and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
new : (NonNegativeInteger,S) -> %
parts : % -> List S if $ has finiteAggregate
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
reduce : (((S,S) -> S),%,S,S) -> S
    if S has SETCAT
    and $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S if $ has finiteAggregate
remove : (S,%) -> % if S has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
    if S has SETCAT
```

```
    and $ has finiteAggregate
sample : () -> %
second : % -> S
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,Integer,S) -> S if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,S) -> S
    if $ has shallowlyMutable
setelt : (%,last,S) -> S if $ has shallowlyMutable
setelt : (%,rest,%) -> % if $ has shallowlyMutable
setelt : (%,first,S) -> S if $ has shallowlyMutable
setelt : (%,value,S) -> S if $ has shallowlyMutable
setfirst! : (%,S) -> S if $ has shallowlyMutable
setlast! : (%,S) -> S if $ has shallowlyMutable
setrest! : (%,%) -> % if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
split! : (%,Integer) -> % if $ has shallowlyMutable
swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
third : % -> S
?.value : (%,value) -> S
?~=? : (%,%) -> Boolean if S has SETCAT
```

⟨*category LZSTAGG LazyStreamAggregate*⟩≡

```
)abbrev category LZSTAGG LazyStreamAggregate
++ Category of streams with lazy evaluation
++ Author: Clifton J. Williamson
++ Date Created: 22 November 1989
++ Date Last Updated: 20 July 1990
++ Keywords: stream, infinite list, infinite sequence
++ Description:
++ LazyStreamAggregate is the category of streams with lazy
++ evaluation.  It is understood that the function 'empty?' will
++ cause lazy evaluation if necessary to determine if there are
++ entries.  Functions which call 'empty?', e.g. 'first' and 'rest',
++ will also cause lazy evaluation if necessary.

LazyStreamAggregate(S:Type): Category == StreamAggregate(S) with
  remove: (S -> Boolean,%) -> %
    ++ remove(f,st) returns a stream consisting of those elements of stream
    ++ st which do not satisfy the predicate f.
    ++ Note: \spad{remove(f,st) = [x for x in st | not f(x)]}.
    ++
    ++X m:=[i for i in 1..]
    ++X f(i:PositiveInteger):Boolean == even? i
    ++X remove(f,m)

  select: (S -> Boolean,%) -> %
    ++ select(f,st) returns a stream consisting of those elements of stream
    ++ st satisfying the predicate f.
```

```
    ++ Note: \spad{select(f,st) = [x for x in st | f(x)]}.
    ++
    ++X m:=[i for i in 0..]
    ++X select(x+->prime? x,m)

  explicitEntries?: % -> Boolean
    ++ explicitEntries?(s) returns true if the stream s has
    ++ explicitly computed entries, and false otherwise.
    ++
    ++X m:=[i for i in 0..]
    ++X explicitEntries? m

  explicitlyEmpty?: % -> Boolean
    ++ explicitlyEmpty?(s) returns true if the stream is an
    ++ (explicitly) empty stream.
    ++ Note: this is a null test which will not cause lazy evaluation.
    ++
    ++X m:=[i for i in 0..]
    ++X explicitlyEmpty? m

  lazy?: % -> Boolean
    ++ lazy?(s) returns true if the first node of the stream s
    ++ is a lazy evaluation mechanism which could produce an
    ++ additional entry to s.
    ++
    ++X m:=[i for i in 0..]
    ++X lazy? m

  lazyEvaluate: % -> %
    ++ lazyEvaluate(s) causes one lazy evaluation of stream s.
    ++ Caution: the first node must be a lazy evaluation mechanism
    ++ (satisfies \spad{lazy?(s) = true}) as there is no error check.
    ++ Note: a call to this function may
    ++ or may not produce an explicit first entry
  frst: % -> S
    ++ frst(s) returns the first element of stream s.
    ++ Caution: this function should only be called after a \spad{empty?}
    ++ test has been made since there no error check.
    ++
    ++X m:=[i for i in 0..]
    ++X frst m

  rst: % -> %
    ++ rst(s) returns a pointer to the next node of stream s.
    ++ Caution: this function should only be called after a \spad{empty?}
    ++ test has been made since there no error check.
```

```
    ++
    ++X m:=[i for i in 0..]
    ++X rst m

  numberOfComputedEntries: % -> NonNegativeInteger
    ++ numberOfComputedEntries(st) returns the number of explicitly
    ++ computed entries of stream st which exist immediately prior to the
    ++ time this function is called.
    ++
    ++X m:=[i for i in 0..]
    ++X numberOfComputedEntries m

  extend: (%,Integer) -> %
    ++ extend(st,n) causes entries to be computed, if necessary,
    ++ so that 'st' will have at least 'n' explicit entries or so
    ++ that all entries of 'st' will be computed if 'st' is finite
    ++ with length <= n.
    ++
    ++X m:=[i for i in 0..]
    ++X numberOfComputedEntries m
    ++X extend(m,20)
    ++X numberOfComputedEntries m

  complete: % -> %
    ++ complete(st) causes all entries of 'st' to be computed.
    ++ this function should only be called on streams which are
    ++ known to be finite.
    ++
    ++X m:=[i for i in 1..]
    ++X n:=filterUntil(i+->i>100,m)
    ++X numberOfComputedEntries n
    ++X complete n
    ++X numberOfComputedEntries n

  add

  MIN ==> 1  -- minimal stream index

  I   ==> Integer
  NNI ==> NonNegativeInteger
  L   ==> List
  U   ==> UniversalSegment Integer

  indexx?            : (Integer,%) -> Boolean
  cycleElt           : % -> Union(%,"failed")
  computeCycleLength : % -> NNI
```

```
    computeCycleEntry  : (%,%) -> %

  --% SETCAT functions

    if S has SetCategory then

      x = y ==
        eq?(x,y) => true
        explicitlyFinite? x and explicitlyFinite? y =>
          entries x = entries y
        explicitEntries? x and explicitEntries? y =>
          frst x = frst y and EQ(rst x, rst y)$Lisp
        -- treat cyclic streams
        false

  --% HOAGG functions

    --null x == empty? x

    less?(x,n) ==
      n = 0    => false
      empty? x => true
      less?(rst x,(n-1) :: NNI)

    more?(x,n) ==
      empty? x => false
      n = 0    => true
      more?(rst x,(n-1) :: NNI)

    size?(x,n) ==
      empty? x => n = 0
      size?(rst x,(n-1) :: NNI)

    # x ==
      -- error if stream is not finite
      y := x
      for i in 0.. repeat
        explicitlyEmpty? y  => return i
        lazy? y => error "#: infinite stream"
        y := rst y
        if odd? i then x := rst x
        eq?(x,y) => error "#: infinite stream"

  --% CLAGG functions

    any?(f,x) ==
```

```
      -- error message only when x is a stream with lazy
      -- evaluation and f(s) = false for all stream elements
      -- 's' which have been computed when the function is
      -- called
      y := x
      for i in 0.. repeat
        explicitlyEmpty? y  => return false
        lazy? y => error "any?: infinite stream"
        f frst y => return true
        y := rst y
        if odd? i then x := rst x
        eq?(x,y) => return false

    every?(f,x) ==
      -- error message only when x is a stream with lazy
      -- evaluation and f(s) = true for all stream elements
      -- 's' which have been computed when the function is
      -- called
      y := x
      for i in 0.. repeat
        explicitlyEmpty? y => return true
        lazy? y => error "every?: infinite stream"
        not f frst y => return false
        y := rst y
        if odd? i then x := rst x
        eq?(x,y) => return true

  -- following ops count and member? are only exported if $ has finiteAggregate

  --  count(f:S -> Boolean,x:%) ==
  --    -- error if stream is not finite
  --    count : NNI := 0
  --    y := x
  --    for i in 0.. repeat
  --      explicitlyEmpty? y  => return count
  --      lazy? y => error "count: infinite stream"
  --      if f frst y then count := count + 1
  --      y := rst y
  --      if odd? i then x := rst x
  --      eq?(x,y) => error "count: infinite stream"


  --  if S has SetCategory then

  --      count(s:S,x:%) == count(#1 = s,x)
  --        -- error if stream is not finite
```

```
--    member?(s,x) ==
--      -- error message only when x is a stream with lazy
--      -- evaluation and 's' is not among the stream elements
--      -- which have been computed when the function is called
--      y := x
--      for i in 0.. repeat
--        explicitlyEmpty? y  => return false
--        lazy? y => error "member?: infinite stream"
--        frst y = s => return true
--        y := rst y
--        if odd? i then x := rst x
--        eq?(x,y) => return false

  entries x ==
    -- returns a list of elements which have been computed
    -- error if infinite
    y := x
    l : L S := empty()
    for i in 0.. repeat
      explicitlyEmpty? y  => return reverse_! l
      lazy? y => error "infinite stream"
      l := concat(frst y,l)
      y := rst y
      if odd? i then x := rst x
      eq?(x,y) => error "infinite stream"

--% CNAGG functions

  construct l ==
    empty? l => empty()
    concat(first l, construct rest l)

  --entries x ==
    -- returns a list of the stream elements
    -- error if the stream is not finite
    --members x

--% ELTAGG functions

  elt(x:%,n:I) ==
    n < MIN or empty? x => error "elt: no such element"
    n = MIN => frst x
    elt(rst x,n - 1)

  elt(x:%,n:I,s:S) ==
```

```
      n < MIN or empty? x => s
      n = MIN => frst x
      elt(rst x,n - 1)

  --% IXAGG functions

  -- following assumes % has finiteAggregate and S has SetCategory
  --  entry?(s,x) ==
  --    -- error message only when x is a stream with lazy
  --    -- evaluation and 's' is not among the stream elements
  --    -- which have been computed when the function is called
  --    member?(s,x)

    --entries x ==
      -- error if the stream is not finite
      --members x

    indexx?(n,x) ==
      empty? x => false
      n = MIN => true
      indexx?(n-1,rst x)

    index?(n,x) ==
      -- returns 'true' iff 'n' is the index of an entry which
      -- may or may not have been computed when the function is
      -- called
      -- additional entries are computed if necessary
      n < MIN => false
      indexx?(n,x)

    indices x ==
      -- error if stream is not finite
      y := x
      l : L I := empty()
      for i in MIN.. repeat
        explicitlyEmpty? y  => return reverse_! l
        lazy? y => error "indices: infinite stream"
        l := concat(i,l)
        y := rst y
        if odd? i then x := rst x
        eq?(x,y) => error "indices: infinite stream"

    maxIndex x ==
      -- error if stream is not finite
      empty? x =>
        error "maxIndex: no maximal index for empty stream"
```

```
      y := rst x
      for i in MIN.. repeat
        explicitlyEmpty? y  => return i
        lazy? y => error "maxIndex: infinite stream"
        y := rst y
        if odd? i then x := rst x
        eq?(x,y) => error "maxIndex: infinite stream"

    minIndex x ==
      empty? x => error "minIndex: no minimal index for empty stream"
      MIN

  --% LNAGG functions

    delete(x:%,n:I) ==
    -- non-destructive
      not index?(n,x) => error "delete: index out of range"
      concat(first(x,(n - MIN) :: NNI), rest(x,(n - MIN + 1) :: NNI))

    delete(x:%,seg:U) ==
      low := lo seg
      hasHi seg =>
        high := hi seg
        high < low => copy x
        (not index?(low,x)) or (not index?(high,x)) =>
          error "delete: index out of range"
        concat(first(x,(low - MIN) :: NNI),rest(x,(high - MIN + 1) :: NNI))
      not index?(low,x) => error "delete: index out of range"
      first(x,(low - MIN) :: NNI)

    elt(x:%,seg:U) ==
      low := lo seg
      hasHi seg =>
        high := hi seg
        high < low => empty()
        (not index?(low,x)) or (not index?(high,x)) =>
          error "elt: index out of range"
        first(rest(x,(low - MIN) :: NNI),(high - low + 1) :: NNI)
      not index?(low,x) => error "elt: index out of range"
      rest(x,(low - MIN) :: NNI)

    insert(s:S,x:%,n:I) ==
      not index?(n,x) => error "insert: index out of range"
      nn := (n - MIN) :: NNI
      concat([first(x,nn), concat(s, empty()), rest(x,nn)])
```

```
    insert(y:%,x:%,n:I) ==
      not index?(n,x) => error "insert: index out of range"
      nn := (n - MIN) :: NNI
      concat([first(x,nn), y, rest(x,nn)])

  --% RCAGG functions

    cycleElt x == cycleElt(x)$CyclicStreamTools(S,%)

    cyclic? x ==
      cycleElt(x) case "failed" => false
      true

    if S has SetCategory then
      child?(x,y) ==
        empty? y => error "child: no children"
        x = rst y

    children x ==
      empty? x => error "children: no children"
      [rst x]

    distance(x,z) ==
      y := x
      for i in 0.. repeat
        eq?(y,z) => return i
        (explicitlyEmpty? y) or (lazy? y) =>
          error "distance: 2nd arg not a descendent of the 1st"
        y := rst y
        if odd? i then x := rst x
        eq?(x,y) =>
          error "distance: 2nd arg not a descendent of the 1st"

    if S has SetCategory then
      node?(z,x) ==
        -- error message only when x is a stream with lazy
        -- evaluation and 'y' is not a node of 'x'
        -- which has been computed when the function is called
        y := x
        for i in 0.. repeat
          z = y => return true
          explicitlyEmpty? y => return false
          lazy? y => error "node?: infinite stream"
          y := rst y
          if odd? i then x := rst x
          eq?(x,y) => return false
```

```
  nodes x ==
    y := x
    l : L % := []
    for i in 0.. repeat
      explicitlyEmpty? y => return reverse_! l
      lazy? y => error "nodes: infinite stream"
      l := concat(y,l)
      y := rst y
      if odd? i then x := rst x
      eq?(x,y) => error "nodes: infinite stream"
    l -- @#$%^& compiler

  leaf? x == empty? rest x

  value x == first x

--% URAGG functions

  computeCycleLength cycElt ==
    computeCycleLength(cycElt)$CyclicStreamTools(S,%)

  computeCycleEntry(x,cycElt) ==
    computeCycleEntry(x,cycElt)$CyclicStreamTools(S,%)

  cycleEntry x ==
    cycElt := cycleElt x
    cycElt case "failed" =>
      error "cycleEntry: non-cyclic stream"
    computeCycleEntry(x,cycElt::%)

  cycleLength x ==
    cycElt := cycleElt x
    cycElt case "failed" =>
      error "cycleLength: non-cyclic stream"
    computeCycleLength(cycElt::%)

  cycleTail x ==
    cycElt := cycleElt x
    cycElt case "failed" =>
      error "cycleTail: non-cyclic stream"
    y := x := computeCycleEntry(x,cycElt::%)
    z := rst x
    repeat
      eq?(x,z) => return y
      y := z ; z := rst z
```

```
elt(x,"first") == first x

first(x,n) ==
-- former name: take
  n = 0 or empty? x => empty()
  concat(frst x, first(rst x,(n-1) :: NNI))

rest x ==
  empty? x => error "Can't take the rest of an empty stream."
  rst x

elt(x,"rest") == rest x

rest(x,n) ==
-- former name: drop
  n = 0 or empty? x => x
  rest(rst x,(n-1) :: NNI)

last x ==
  -- error if stream is not finite
  empty? x => error "last: empty stream"
  y1 := x
  y2 := rst x
  for i in 0.. repeat
    explicitlyEmpty? y2 => return frst y1
    lazy? y2 => error "last: infinite stream"
    y1 := y2
    y2 := rst y2
    if odd? i then x := rst x
    eq?(x,y2) => error "last: infinite stream"

if % has finiteAggregate then -- # is only defined for finiteAggregates
  last(x,n) ==
    possiblyInfinite? x => error "last: infinite stream"
    m := # x
    m < n => error "last: index out of range"
    copy rest(x,(m-n)::NNI)

elt(x,"last") == last x

tail x ==
  -- error if stream is not finite
  empty? x => error "tail: empty stream"
  y1 := x
  y2 := rst x
```

```
      for i in 0.. repeat
        explicitlyEmpty? y2 => return y1
        lazy? y2 => error "tail: infinite stream"
        y1 := y2
        y2 := rst y2
        if odd? i then x := rst x
        eq?(x,y2) => error "tail: infinite stream"

  --% STAGG functions

    possiblyInfinite? x ==
      y := x
      for i in 0.. repeat
        explicitlyEmpty? y  => return false
        lazy? y => return true
        if odd? i then x := rst x
        y := rst y
        eq?(x,y) => return true

    explicitlyFinite? x == not possiblyInfinite? x

  --% LZSTAGG functions

    extend(x,n) ==
      y := x
      for i in 1..n while not empty? y repeat y := rst y
      x

    complete x ==
      y := x
      while not empty? y repeat y := rst y
      x
```

⟨*LZSTAGG.dotabb*⟩≡

```
  "LZSTAGG"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=LZSTAGG"];
  "LZSTAGG" -> "STAGG"
```

⟨*LZSTAGG.dotfull*⟩≡

```
  "LazyStreamAggregate(a:Type)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=LZSTAGG"];
  "LazyStreamAggregate(a:Type)" -> "StreamAggregate(a:Type)"
```

⟨*LZSTAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"LazyStreamAggregate(a:Type)" [color=lightblue];
"LazyStreamAggregate(a:Type)" -> "StreamAggregate(a:Type)"

"StreamAggregate(a:Type)" [color=lightblue];
"StreamAggregate(a:Type)" -> "UnaryRecursiveAggregate(a:Type)"
"StreamAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"UnaryRecursiveAggregate(a:Type)" [color=lightblue];
"UnaryRecursiveAggregate(a:Type)" -> "RecursiveAggregate(a:Type)"

"RecursiveAggregate(a:Type)" [color=lightblue];
"RecursiveAggregate(a:Type)" -> "HOAGG..."

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "Collection(a:Type)"

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

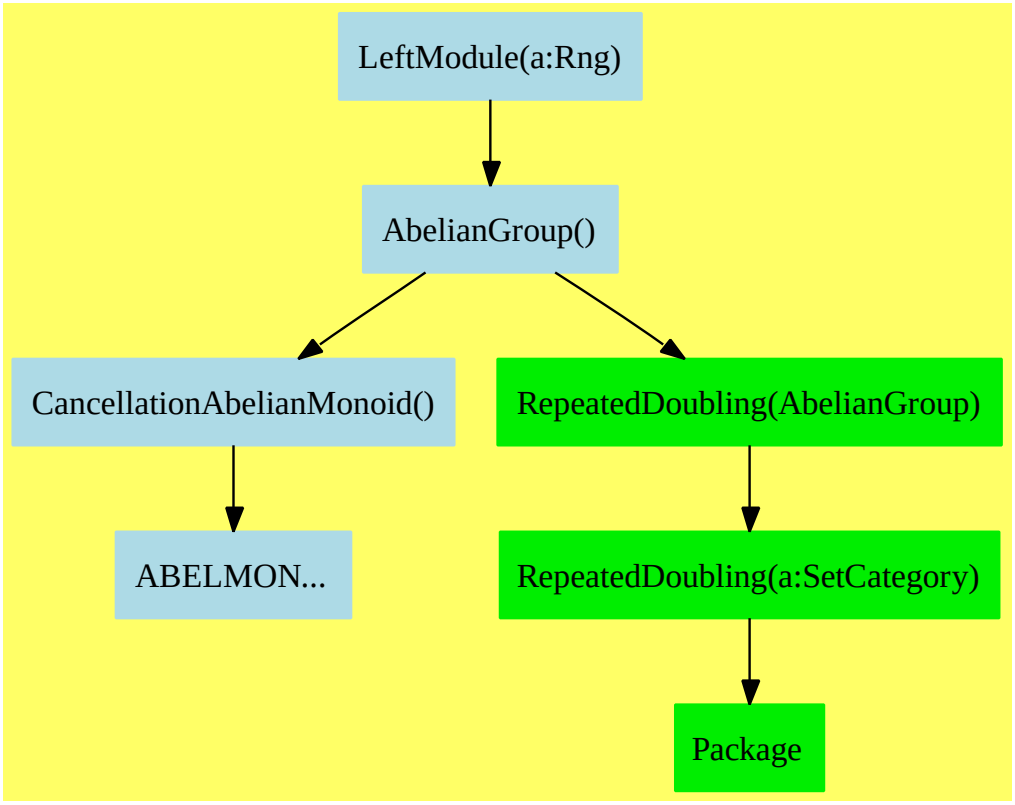
"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" -> "HOAGG..."

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HOAGG..."

"HOAGG..." [color=lightblue];

}
```

8.5 LeftModule (LMODULE)



See:

⇒ "BiModule" (BMODULE) 9.1 on page 592
⇒ "LeftAlgebra" (LALG) 10.8 on page 704
⇒ "Ring" (RING) 9.8 on page 628
⇐ "AbelianGroup" (ABELGRP) 7.1 on page 418

Exports:

0	coerce	hash	latex	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?-?	-?	?=?		

These are directly exported but not implemented:

```
?*? : (R,%) -> %
```

These exports come from (p418) AbelianGroup():

```
0 : () -> %
coerce : % -> OutputForm
```

```
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

⟨*category LMODULE LeftModule*⟩≡

```
)abbrev category LMODULE LeftModule
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of left modules over an rng (ring not necessarily with unit).
++ This is an abelian group which supports left multiplation by elements of
++ the rng.
++
++ Axioms:
++   \spad{ (a*b)*x = a*(b*x) }
++   \spad{ (a+b)*x = (a*x)+(b*x) }
++   \spad{ a*(x+y) = (a*x)+(a*y) }
LeftModule(R:Rng):Category == AbelianGroup with
   "*": (R,%) -> %
     ++ r*x returns the left multiplication of the module element x
     ++ by the ring element r.
```

⟨*LMODULE.dotabb*⟩≡

```
"LMODULE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LMODULE"];
"LMODULE" -> "ABELGRP"
```

⟨*LMODULE.dotfull*⟩≡
```
"LeftModule(a:Rng)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LMODULE"];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"LeftModule(a:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=LMODULE"];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"
```

⟨*LMODULE.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "RepeatedDoubling(AbelianGroup)"

"RepeatedDoubling(AbelianGroup)" [color="#00EE00"];
"RepeatedDoubling(AbelianGroup)" -> "RepeatedDoubling(a:SetCategory)"

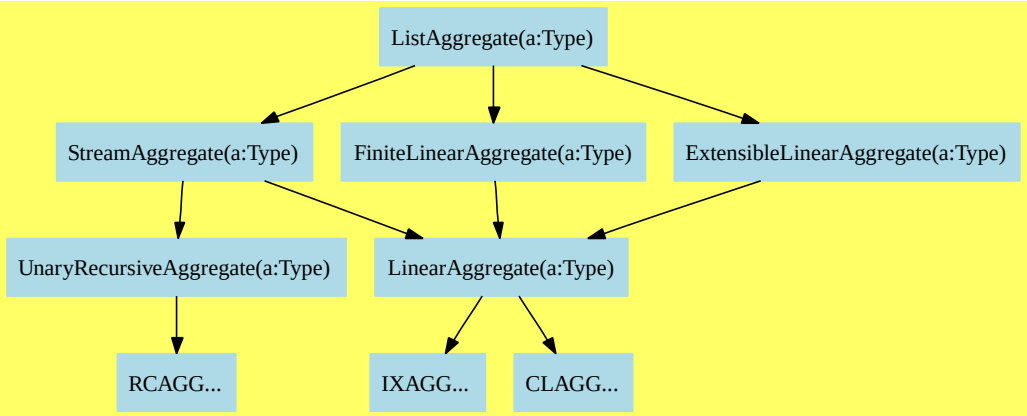
"RepeatedDoubling(a:SetCategory)" [color="#00EE00"];
"RepeatedDoubling(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "ABELMON..."

"ABELMON..." [color=lightblue];
}
```

8.6 ListAggregate (LSAGG)



See:

⇒ “AssociationListAggregate” (ALAGG) 10.1 on page 667
⇐ “ExtensibleLinearAggregate” (ELAGG) 7.5 on page 438
⇐ “FiniteLinearAggregate” (FLAGG) 7.6 on page 444

Exports:

any?	children	child?	coerce
concat	concat!	construct	convert
copy	copyInto!	count	cycleEntry
cycleLength	cycleSplit!	cycleTail	cyclic?
delete	delete!	distance	elt
empty	empty?	entries	entry?
eq?	eval	every?	explicitlyFinite?
fill!	find	first	hash
index?	indices	insert	insert!
last	latex	leaf?	leaves
less?	list	map	map!
max	maxIndex	member?	members
merge	merge!	min	minIndex
more?	new	nodes	node?
parts	position	possiblyInfinite?	qelt
qsetelt!	reduce	remove	remove!
removeDuplicates	removeDuplicates!	rest	reverse
reverse!	sample	second	select
select!	setchildren!	setelt	setfirst!
setlast!	setrest!	setvalue!	size?
sort	sort!	sorted?	split!
swap!	tail	third	value
#?	?.?	?.last	?.rest
?.first	?.value	?<?	?<=?
?=?	?>?	?>=?	?~=?

Attributes exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are implemented by this category:

```
copy : % -> %
copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
delete! : (%,Integer) -> %
delete! : (%,UniversalSegment Integer) -> %
find : ((S -> Boolean),%) -> Union(S,"failed")
insert! : (S,%,Integer) -> %
insert! : (%,%,Integer) -> %
list : S -> %
map : (((S,S) -> S),%,%) -> %
```

```
merge : (((S,S) -> Boolean),%,%) -> %
merge! : (((S,S) -> Boolean),%,%) -> %
new : (NonNegativeInteger,S) -> %
position : ((S -> Boolean),%) -> Integer
position : (S,%,Integer) -> Integer if S has SETCAT
reduce : (((S,S) -> S),%,S) -> S
         if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
         if S has SETCAT
         and $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
         if $ has finiteAggregate
remove! : ((S -> Boolean),%) -> %
removeDuplicates! : % -> % if S has SETCAT
reverse! : % -> % if $ has shallowlyMutable
select : ((S -> Boolean),%) -> %
         if $ has finiteAggregate
sort! : (((S,S) -> Boolean),%) -> %
         if $ has shallowlyMutable
sorted? : (((S,S) -> Boolean),%) -> Boolean
?<? : (%,%) -> Boolean if S has ORDSET
```

These exports come from (p470) StreamAggregate(S:Type):

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
children : % -> List %
child? : (%,%) -> Boolean if S has SETCAT
coerce : % -> OutputForm if S has SETCAT
concat : (%,S) -> %
concat : List % -> %
concat : (S,%) -> %
concat : (%,%) -> %
concat! : (%,S) -> %
concat! : (%,%) -> %
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
count : (S,%) -> NonNegativeInteger
         if S has SETCAT
         and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
cycleEntry : % -> %
cycleLength : % -> NonNegativeInteger
cycleSplit! : % -> % if $ has shallowlyMutable
cycleTail : % -> %
cyclic? : % -> Boolean
delete : (%,UniversalSegment Integer) -> %
delete : (%,Integer) -> %
distance : (%,%) -> Integer
```

```
elt : (%,Integer,S) -> S
empty : () -> %
empty? : % -> Boolean
entry? : (S,%) -> Boolean
         if $ has finiteAggregate
         and S has SETCAT
entries : % -> List S
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
         if S has EVALAB S
         and S has SETCAT
eval : (%,S,S) -> %
         if S has EVALAB S
         and S has SETCAT
eval : (%,Equation S) -> %
         if S has EVALAB S
         and S has SETCAT
eval : (%,List Equation S) -> %
         if S has EVALAB S
         and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
explicitlyFinite? : % -> Boolean
fill! : (%,S) -> % if $ has shallowlyMutable
first : % -> S
first : (%,NonNegativeInteger) -> %
hash : % -> SingleInteger if S has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (S,%,Integer) -> %
insert : (%,%,Integer) -> %
last : % -> S
last : (%,NonNegativeInteger) -> %
latex : % -> String if S has SETCAT
leaf? : % -> Boolean
leaves : % -> List S
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
maxIndex : % -> Integer if Integer has ORDSET
member? : (S,%) -> Boolean
         if S has SETCAT
         and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
nodes : % -> List %
node? : (%,%) -> Boolean if S has SETCAT
parts : % -> List S if $ has finiteAggregate
possiblyInfinite? : % -> Boolean
```

```
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S if $ has shallowlyMutable
remove : (S,%) -> %
         if S has SETCAT
         and $ has finiteAggregate
remove : ((S -> Boolean),%) -> % if $ has finiteAggregate
removeDuplicates : % -> %
         if S has SETCAT
         and $ has finiteAggregate
rest : % -> %
rest : (%,NonNegativeInteger) -> %
sample : () -> %
second : % -> S
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,Integer,S) -> S if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,S) -> S
         if $ has shallowlyMutable
setelt : (%,last,S) -> S if $ has shallowlyMutable
setelt : (%,rest,%) -> % if $ has shallowlyMutable
setelt : (%,first,S) -> S if $ has shallowlyMutable
setelt : (%,value,S) -> S if $ has shallowlyMutable
setfirst! : (%,S) -> S if $ has shallowlyMutable
setlast! : (%,S) -> S if $ has shallowlyMutable
setrest! : (%,%) -> % if $ has shallowlyMutable
setvalue! : (%,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
split! : (%,Integer) -> % if $ has shallowlyMutable
swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
tail : % -> %
third : % -> S
value : % -> S
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.last : (%,last) -> S
?.rest : (%,rest) -> %
?.first : (%,first) -> S
?.value : (%,value) -> S
?.? : (%,Integer) -> S
?.? : (%,UniversalSegment Integer) -> %
?=? : (%,%) -> Boolean if S has SETCAT
?~=? : (%,%) -> Boolean if S has SETCAT
```

These exports come from (p444) FiniteLinearAggregate(S:Type)

```
max : (%,%) -> % if S has ORDSET
merge : (%,%) -> % if S has ORDSET
min : (%,%) -> % if S has ORDSET
position : (S,%) -> Integer if S has SETCAT
reverse : % -> %
sort : (((S,S) -> Boolean),%) -> %
sort : % -> % if S has ORDSET
```

```
sort! : % -> % if S has ORDSET and $ has shallowlyMutable
sorted? : % -> Boolean if S has ORDSET
?<=? : (%,%) -> Boolean if S has ORDSET
?>? : (%,%) -> Boolean if S has ORDSET
?>=? : (%,%) -> Boolean if S has ORDSET
```

These exports come from (p438) ExtensibleLinearAggregate(S:Type):

```
merge! : (%,%) -> % if S has ORDSET
remove! : (S,%) -> % if S has SETCAT
select! : ((S -> Boolean),%) -> %
```

⟨*category LSAGG ListAggregate*⟩≡

```
)abbrev category LSAGG ListAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A list aggregate is a model for a linked list data structure.
++ A linked list is a versatile
++ data structure. Insertion and deletion are efficient and
++ searching is a linear operation.
ListAggregate(S:Type): Category == Join(StreamAggregate S,
   FiniteLinearAggregate S, ExtensibleLinearAggregate S) with
      list: S -> %
        ++ list(x) returns the list of one element x.
 add
   cycleMax ==> 1000

   mergeSort: ((S, S) -> Boolean, %, Integer) -> %

   sort_!(f, l)       == mergeSort(f, l, #l)
   list x                  == concat(x, empty())
   reduce(f, x)            ==
     empty? x => _
       error "reducing over an empty list needs the 3 argument form"
     reduce(f, rest x, first x)
   merge(f, p, q)          == merge_!(f, copy p, copy q)

   select_!(f, x) ==
     while not empty? x and not f first x repeat x := rest x
```

```
    empty? x => x
    y := x
    z := rest y
    while not empty? z repeat
      if f first z then (y := z; z := rest z)
                   else (z := rest z; setrest_!(y, z))
    x

  merge_!(f, p, q) ==
    empty? p => q
    empty? q => p
    eq?(p, q) => error "cannot merge a list into itself"
    if f(first p, first q)
      then (r := t := p; p := rest p)
      else (r := t := q; q := rest q)
    while not empty? p and not empty? q repeat
      if f(first p, first q)
        then (setrest_!(t, p); t := p; p := rest p)
        else (setrest_!(t, q); t := q; q := rest q)
    setrest_!(t, if empty? p then q else p)
    r

  insert_!(s:S, x:%, i:Integer) ==
    i < (m := minIndex x) => error "index out of range"
    i = m => concat(s, x)
    y := rest(x, (i - 1 - m)::NonNegativeInteger)
    z := rest y
    setrest_!(y, concat(s, z))
    x

  insert_!(w:%, x:%, i:Integer) ==
    i < (m := minIndex x) => error "index out of range"
    i = m => concat_!(w, x)
    y := rest(x, (i - 1 - m)::NonNegativeInteger)
    z := rest y
    setrest_!(y, w)
    concat_!(y, z)
    x

  remove_!(f:S -> Boolean, x:%) ==
    while not empty? x and f first x repeat x := rest x
    empty? x => x
    p := x
    q := rest x
    while not empty? q repeat
      if f first q then q := setrest_!(p, rest q)
```

```
                else (p := q; q := rest q)
  x

delete_!(x:%, i:Integer) ==
  i < (m := minIndex x) => error "index out of range"
  i = m => rest x
  y := rest(x, (i - 1 - m)::NonNegativeInteger)
  setrest_!(y, rest(y, 2))
  x

delete_!(x:%, i:UniversalSegment(Integer)) ==
  (l := lo i) < (m := minIndex x) => error "index out of range"
  h := if hasHi i then hi i else maxIndex x
  h < l => x
  l = m => rest(x, (h + 1 - m)::NonNegativeInteger)
  t := rest(x, (l - 1 - m)::NonNegativeInteger)
  setrest_!(t, rest(t, (h - l + 2)::NonNegativeInteger))
  x

find(f, x) ==
  while not empty? x and not f first x repeat x := rest x
  empty? x => "failed"
  first x

position(f:S -> Boolean, x:%) ==
  for k in minIndex(x).. while not empty? x and not f first x repeat
    x := rest x
  empty? x => minIndex(x) - 1
  k

mergeSort(f, p, n) ==
  if n = 2 and f(first rest p, first p) then p := reverse_! p
  n < 3 => p
  l := (n quo 2)::NonNegativeInteger
  q := split_!(p, l)
  p := mergeSort(f, p, l)
  q := mergeSort(f, q, n - l)
  merge_!(f, p, q)

sorted?(f, l) ==
  empty? l => true
  p := rest l
  while not empty? p repeat
    not f(first l, first p) => return false
    p := rest(l := p)
  true
```

```
    reduce(f, x, i) ==
      r := i
      while not empty? x repeat (r := f(r, first x); x := rest x)
      r

    if S has SetCategory then
       reduce(f, x, i,a) ==
         r := i
         while not empty? x and r ^= a repeat
           r := f(r, first x)
           x := rest x
         r

    new(n, s) ==
      l := empty()
      for k in 1..n repeat l := concat(s, l)
      l

    map(f, x, y) ==
      z := empty()
      while not empty? x and not empty? y repeat
        z := concat(f(first x, first y), z)
        x := rest x
        y := rest y
      reverse_! z

-- map(f, x, y, d) ==
--   z := empty()
--   while not empty? x and not empty? y repeat
--     z := concat(f(first x, first y), z)
--     x := rest x
--     y := rest y
--   z := reverseInPlace z
--   if not empty? x then
--      z := concat_!(z, map(f(#1, d), x))
--   if not empty? y then
--      z := concat_!(z, map(f(d, #1), y))
--   z

    reverse_! x ==
      empty? x => x
      empty?(y := rest x) => x
      setrest_!(x, empty())
      while not empty? y repeat
        z := rest y
```

```
    setrest_!(y, x)
    x := y
    y := z
  x

copy x ==
  y := empty()
  for k in 0.. while not empty? x repeat
    k = cycleMax and cyclic? x => error "cyclic list"
    y := concat(first x, y)
    x := rest x
  reverse_! y

copyInto_!(y, x, s) ==
  s < (m := minIndex y) => error "index out of range"
  z := rest(y, (s - m)::NonNegativeInteger)
  while not empty? z and not empty? x repeat
    setfirst_!(z, first x)
    x := rest x
    z := rest z
  y

if S has SetCategory then
  position(w, x, s) ==
    s < (m := minIndex x) => error "index out of range"
    x := rest(x, (s - m)::NonNegativeInteger)
    for k in s.. while not empty? x and w ^= first x repeat
      x := rest x
    empty? x => minIndex x - 1
    k

  removeDuplicates_! l ==
    p := l
    while not empty? p repeat
      p := setrest_!(p, remove_!((x:S):Boolean +-> x = first p, rest p))
    l

if S has OrderedSet then
  x < y ==
     while not empty? x and not empty? y repeat
       first x ^= first y => return(first x < first y)
       x := rest x
       y := rest y
     empty? x => not empty? y
     false
```

⟨*LSAGG.dotabb*⟩≡
```
"LSAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=LSAGG"];
"LSAGG" -> "FLAGG"
"LSAGG" -> "ELAGG"
```

⟨*LSAGG.dotfull*⟩≡
```
"ListAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LSAGG"];
"ListAggregate(a:Type)" -> "StreamAggregate(a:Type)"
"ListAggregate(a:Type)" -> "FiniteLinearAggregate(a:Type)"
"ListAggregate(a:Type)" -> "ExtensibleLinearAggregate(a:Type)"

"ListAggregate(Record(a:SetCategory,b:SetCategory))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=LSAGG"];
"ListAggregate(Record(a:SetCategory,b:SetCategory))" ->
    "ListAggregate(a:Type)"
```

⟨*LSAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ListAggregate(a:Type)" [color=lightblue];
"ListAggregate(a:Type)" -> "StreamAggregate(a:Type)"
"ListAggregate(a:Type)" -> "FiniteLinearAggregate(a:Type)"
"ListAggregate(a:Type)" -> "ExtensibleLinearAggregate(a:Type)"

"StreamAggregate(a:Type)" [color=lightblue];
"StreamAggregate(a:Type)" -> "UnaryRecursiveAggregate(a:Type)"
"StreamAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

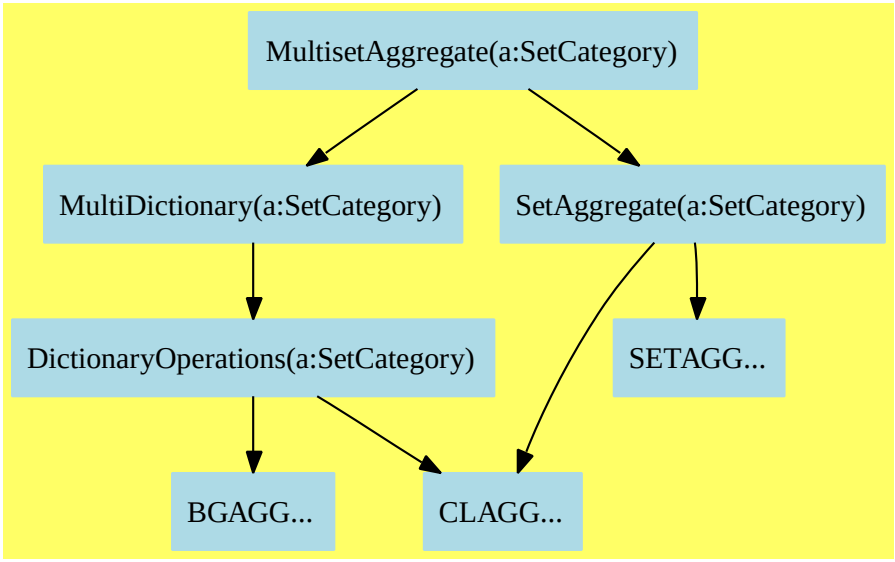
"ExtensibleLinearAggregate(a:Type)" [color=lightblue];
"ExtensibleLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"UnaryRecursiveAggregate(a:Type)" [color=lightblue];
"UnaryRecursiveAggregate(a:Type)" -> "RCAGG..."

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IXAGG..."
"LinearAggregate(a:Type)" -> "CLAGG..."

"CLAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
"RCAGG..." [color=lightblue];
}
```

8.7 MultisetAggregate (MSETAGG)



See:

⇒ "OrderedMultisetAggregate" (OMSAGG) 9.7 on page 623
⇐ "MultiDictionary" (MDAGG) 7.8 on page 456
⇐ "SetAggregate" (SETAGG) 6.13 on page 395

Exports:

any?	bag	brace	coerce
construct	convert	copy	count
dictionary	difference	duplicates	empty
empty?	eq?	eval	every?
extract!	find	hash	insert!
inspect	intersect	latex	less?
map	map!	member?	members
more?	parts	reduce	remove
remove!	removeDuplicates	removeDuplicates!	sample
select	select!	set	size?
subset?	symmetricDifference	union	#?
?~=?	?<?	?=?	

Attributes exported:

- **partiallyOrderedSet** is true if a set with $<$ which is transitive, but not$(a < b$ or $a = b)$ does not necessarily imply $b < a$.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain

are irrevelant to the shallowlyMutable proper.

- **nil**

These exports come from (p456) MultiDictionary(S:SetCategory):

```
any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
bag : List S -> %
coerce : % -> OutputForm
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : (S,%) -> NonNegativeInteger
    if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
dictionary : List S -> %
dictionary : () -> %
duplicates : % ->
    List Record(entry: S,count: NonNegativeInteger)
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> % if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> % if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> % if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> % if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
extract! : % -> S
find : ((S -> Boolean),%) -> Union(S,"failed")
hash : % -> SingleInteger
insert! : (S,%) -> %
insert! : (S,%,NonNegativeInteger) -> %
inspect : % -> S
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
member? : (S,%) -> Boolean
    if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
    if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> %
```

```
    if $ has finiteAggregate
remove : (S,%) -> %
    if S has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
    if S has SETCAT and $ has finiteAggregate
removeDuplicates! : % -> %
remove! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove! : (S,%) -> % if $ has finiteAggregate
select : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
select! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
```

These exports come from (p395) SetAggregate(S:SetCategory):

```
brace : () -> %
brace : List S -> %
difference : (%,S) -> %
intersect : (%,%) -> %
sample : () -> %
set : () -> %
set : List S -> %
subset? : (%,%) -> Boolean
symmetricDifference : (%,%) -> %
union : (%,S) -> %
union : (S,%) -> %
union : (%,%) -> %
?<? : (%,%) -> Boolean
```

⟨*category MSETAGG MultisetAggregate*⟩≡

```
)abbrev category MSETAGG MultisetAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A multi-set aggregate is a set which keeps track of the multiplicity
++ of its elements.
MultisetAggregate(S:SetCategory):
```

```
Category == Join(MultiDictionary S, SetAggregate S)
```

⟨*MSETAGG.dotabb*⟩≡

```
"MSETAGG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MSETAGG"];
"MSETAGG" -> "MDAGG"
"MSETAGG" -> "SETAGG"
```

⟨*MSETAGG.dotfull*⟩≡

```
"MultisetAggregate(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MSETAGG"];
"MultisetAggregate(a:SetCategory)" -> "MultiDictionary(a:SetCategory)"
"MultisetAggregate(a:SetCategory)" -> "SetAggregate(a:SetCategory)"
```

⟨*MSETAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"MultisetAggregate(a:SetCategory)" [color=lightblue];
"MultisetAggregate(a:SetCategory)" -> "MultiDictionary(a:SetCategory)"
"MultisetAggregate(a:SetCategory)" -> "SetAggregate(a:SetCategory)"

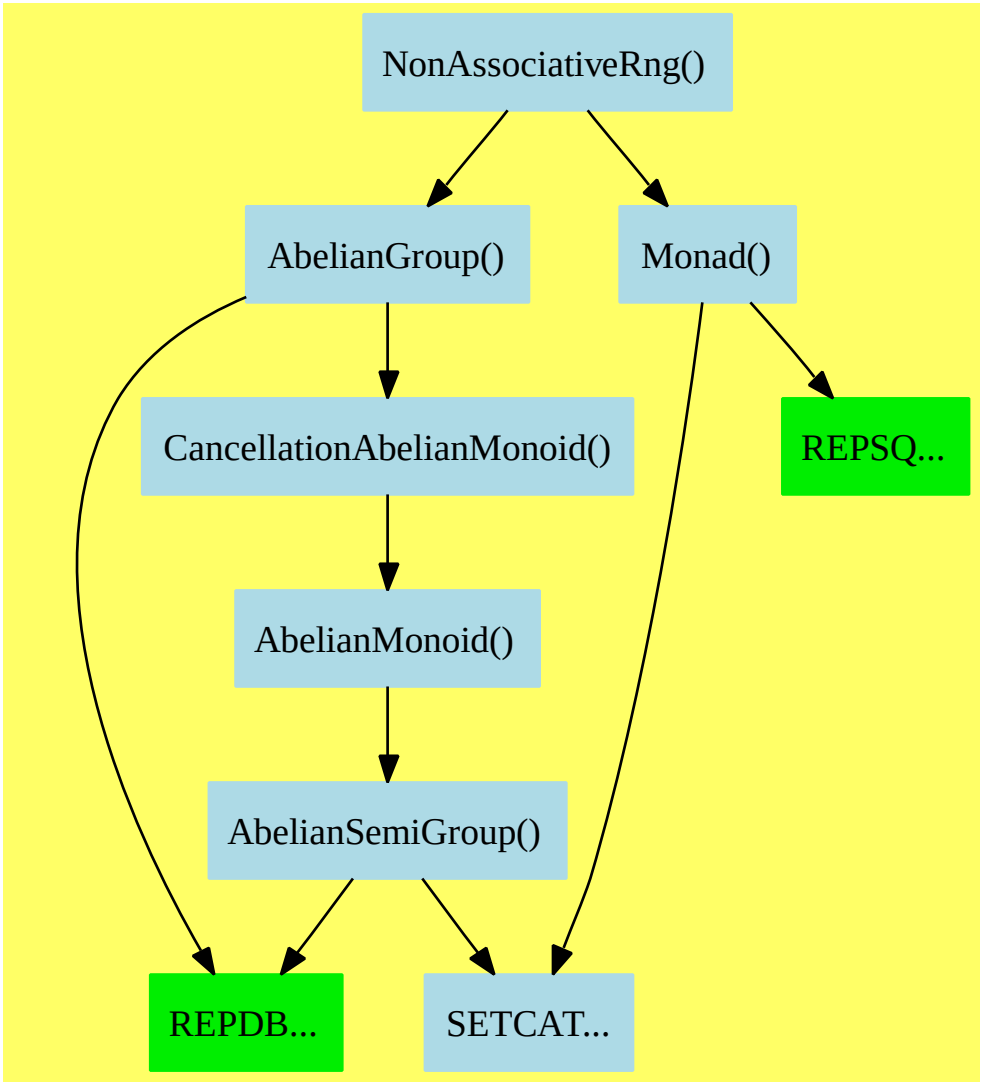
"MultiDictionary(a:SetCategory)" [color=lightblue];
"MultiDictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"SetAggregate(a:SetCategory)" [color=lightblue];
"SetAggregate(a:SetCategory)" -> "SETAGG..."
"SetAggregate(a:SetCategory)" -> "CLAGG..."

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BGAGG..."
"DictionaryOperations(a:SetCategory)" -> "CLAGG..."

"BGAGG..." [color=lightblue];
"CLAGG..." [color=lightblue];
"SETAGG..." [color=lightblue];
}
```

8.8 NonAssociativeRng (NARNG)



See:

⇒ "NonAssociativeAlgebra" (NAALG) 11.6 on page 798
⇒ "NonAssociativeRing" (NASRING) 9.3 on page 604
⇐ "AbelianGroup" (ABELGRP) 7.1 on page 418
⇐ "Monad" (MONAD) 4.15 on page 152

Exports:

0	antiCommutator	associator	coerce	commutator
hash	latex	leftPower	rightPower	sample
subtractIfCan	zero?	?*?	?**?	?+?
?-?	-?	?=?	?~=?	

These are implemented by this category:

```
antiCommutator : (%,%) -> %
associator : (%,%,%) -> %
commutator : (%,%) -> %
```

These exports come from (p418) AbelianGroup():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

These exports come from (p152) Monad():

```
leftPower : (%,PositiveInteger) -> %
rightPower : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
```

⟨*category NARNG NonAssociativeRng*⟩≡

```
)abbrev category NARNG NonAssociativeRng
++ Author: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 03 July 1991
++ Basic Operations: +, *, -, **
++ Related Constructors: Rng, Ring, NonAssociativeRing
++ Also See:
++ AMS Classifications:
++ Keywords: not associative ring
++ Reference:
++  R.D. Schafer: An Introduction to Nonassociative Algebras
++  Academic Press, New York, 1966
```

```
++ Description:
++  NonAssociativeRng is a basic ring-type structure, not necessarily
++  commutative or associative, and not necessarily with unit.
++  Axioms
++    x*(y+z) = x*y + x*z
++    (x+y)*z = x*z + y*z
++  Common Additional Axioms
++    noZeroDivisors  ab = 0 => a=0 or b=0
NonAssociativeRng(): Category == Join(AbelianGroup,Monad)  with
    associator: (%,%,%) -> %
      ++ associator(a,b,c) returns \spad{(a*b)*c-a*(b*c)}.
    commutator: (%,%) -> %
      ++ commutator(a,b) returns \spad{a*b-b*a}.
    antiCommutator: (%,%) -> %
      ++ antiCommutator(a,b) returns \spad{a*b+b*a}.
  add
    associator(x,y,z) == (x*y)*z - x*(y*z)
    commutator(x,y) == x*y - y*x
    antiCommutator(x,y) == x*y + y*x
```

⟨*NARNG.dotabb*⟩≡

```
"NARNG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NARNG"];
"NARNG" -> "ABELGRP"
"NARNG" -> "MONAD"
```

⟨*NARNG.dotfull*⟩≡

```
"NonAssociativeRng()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NARNG"];
"NonAssociativeRng()" -> "AbelianGroup()"
"NonAssociativeRng()" -> "Monad()"
```

⟨*NARNG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"NonAssociativeRng()" [color=lightblue];
"NonAssociativeRng()" -> "AbelianGroup()"
"NonAssociativeRng()" -> "Monad()"

"Monad()" [color=lightblue];
"Monad()" -> "SETCAT..."
"Monad()" -> "REPSQ..."

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "REPDB..."

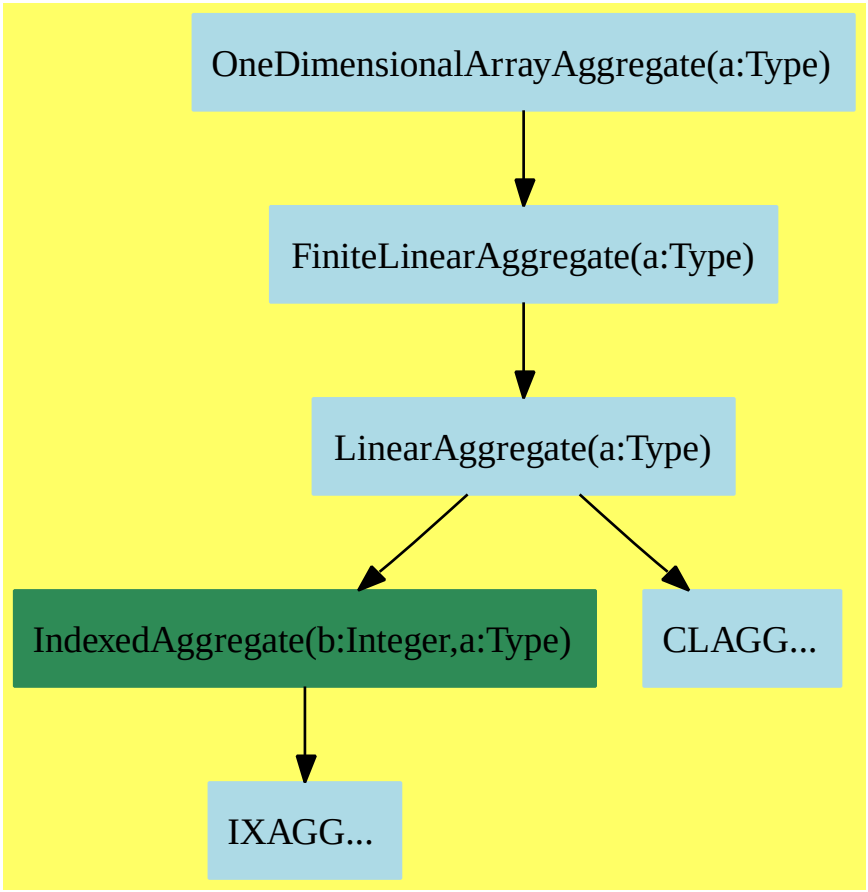
"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SETCAT..."
"AbelianSemiGroup()" -> "REPDB..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
}
```

8.9 OneDimensionalArrayAggregate (A1AGG)



See:

⇒ "BitAggregate" (BTAGG) 9.2 on page 596
⇒ "StringAggregate" (SRAGG) 9.10 on page 640
⇒ "VectorCategory" (VECTCAT) 9.12 on page 659
⇐ "FiniteLinearAggregate" (FLAGG) 7.6 on page 444

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	delete	elt
empty	empty?	entries	entry?	eq?
eval	every?	fill!	find	first
hash	index?	indices	insert	latex
less?	map	map!	max	maxIndex
member?	members	merge	min	minIndex
more?	new	parts	position	qelt
qsetelt!	reduce	remove	removeDuplicates	reverse
reverse!	sample	select	setelt	size?
sort	sort!	sorted?	swap!	#?
?.?	?<?	?<=?	?~=?	?=?
?>?	?>=?			

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are implemented by this category:

```
any? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if S has SETCAT
concat : (%,%) -> %
concat : List % -> %
construct : List S -> %
copy : % -> %
copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
count : ((S -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
delete : (%,UniversalSegment Integer) -> %
delete : (%,Integer) -> %
every? : ((S -> Boolean),%) -> Boolean
         if $ has finiteAggregate
find : ((S -> Boolean),%) -> Union(S,"failed")
insert : (%,%,Integer) -> %
map : (((S,S) -> S),%,%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
merge : (((S,S) -> Boolean),%,%) -> %
parts : % -> List S if $ has finiteAggregate
position : ((S -> Boolean),%) -> Integer
position : (S,%,Integer) -> Integer
```

```
          if S has SETCAT
reduce : (((S,S) -> S),%) -> S
          if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
          if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
          if S has SETCAT and $ has finiteAggregate
reverse! : % -> % if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,S) -> S
          if $ has shallowlyMutable
sort! : (((S,S) -> Boolean),%) -> %
          if $ has shallowlyMutable
sorted? : (((S,S) -> Boolean),%) -> Boolean
?.? : (%,UniversalSegment Integer) -> %
?=? : (%,%) -> Boolean if S has SETCAT
?<? : (%,%) -> Boolean if S has ORDSET
```

These exports come from (p444) FiniteLinearAggregate(S:Type):

```
concat : (S,%) -> %
concat : (%,S) -> %
convert : % -> InputForm if S has KONVERT INFORM
count : (S,%) -> NonNegativeInteger
          if S has SETCAT and $ has finiteAggregate
elt : (%,Integer,S) -> S
empty : () -> %
empty? : % -> Boolean
entries : % -> List S
entry? : (S,%) -> Boolean
          if $ has finiteAggregate and S has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
          if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
          if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
          if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
          if S has EVALAB S and S has SETCAT
fill! : (%,S) -> % if $ has shallowlyMutable
first : % -> S if Integer has ORDSET
hash : % -> SingleInteger if S has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (S,%,Integer) -> %
latex : % -> String if S has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
max : (%,%) -> % if S has ORDSET
maxIndex : % -> Integer if Integer has ORDSET
```

```
member? : (S,%) -> Boolean
         if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
merge : (%,%) -> % if S has ORDSET
min : (%,%) -> % if S has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,S) -> %
position : (S,%) -> Integer if S has SETCAT
qelt : (%,Integer) -> S
qsetelt! : (%,Integer,S) -> S
         if $ has shallowlyMutable
remove : ((S -> Boolean),%) -> %
         if $ has finiteAggregate
remove : (S,%) -> %
         if S has SETCAT and $ has finiteAggregate
removeDuplicates : % -> %
         if S has SETCAT and $ has finiteAggregate
reverse : % -> %
sample : () -> %
select : ((S -> Boolean),%) -> %
         if $ has finiteAggregate
setelt : (%,Integer,S) -> S if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
sort : % -> % if S has ORDSET
sort : (((S,S) -> Boolean),%) -> %
sort! : % -> %
         if S has ORDSET and $ has shallowlyMutable
sorted? : % -> Boolean if S has ORDSET
swap! : (%,Integer,Integer) -> Void
         if $ has shallowlyMutable
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.? : (%,Integer) -> S
?~=? : (%,%) -> Boolean if S has SETCAT
?>? : (%,%) -> Boolean if S has ORDSET
?>=? : (%,%) -> Boolean if S has ORDSET
?<=? : (%,%) -> Boolean if S has ORDSET
```

⟨*category A1AGG OneDimensionalArrayAggregate*⟩≡

```
)abbrev category A1AGG OneDimensionalArrayAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
```

```
++ Description:
++ One-dimensional-array aggregates serves as models for one-dimensional
++ arrays. Categorically, these aggregates are finite linear aggregates
++ with the \spadatt{shallowlyMutable} property, that is, any component of
++ the array may be changed without affecting the
++ identity of the overall array.
++ Array data structures are typically represented by a fixed area in
++ storage and cannot efficiently grow or shrink on demand as can list
++ structures (see however \spadtype{FlexibleArray} for a data structure
++ which is a cross between a list and an array).
++ Iteration over, and access to, elements of arrays is extremely fast
++ (and often can be optimized to open-code).
++ Insertion and deletion however is generally slow since an entirely new
++ data structure must be created for the result.
OneDimensionalArrayAggregate(S:Type): Category ==
    FiniteLinearAggregate S with shallowlyMutable
  add
    parts x         == [qelt(x, i) for i in minIndex x .. maxIndex x]
    sort_!(f, a) == quickSort(f, a)$FiniteLinearAggregateSort(S, %)

    any?(f, a) ==
      for i in minIndex a .. maxIndex a repeat
        f qelt(a, i) => return true
      false

    every?(f, a) ==
      for i in minIndex a .. maxIndex a repeat
        not(f qelt(a, i)) => return false
      true

    position(f:S -> Boolean, a:%) ==
      for i in minIndex a .. maxIndex a repeat
        f qelt(a, i) => return i
      minIndex(a) - 1

    find(f, a) ==
      for i in minIndex a .. maxIndex a repeat
        f qelt(a, i) => return qelt(a, i)
      "failed"

    count(f:S->Boolean, a:%) ==
      n:NonNegativeInteger := 0
      for i in minIndex a .. maxIndex a repeat
        if f(qelt(a, i)) then n := n+1
      n
```

```
    map_!(f, a) ==
      for i in minIndex a .. maxIndex a repeat
        qsetelt_!(a, i, f qelt(a, i))
      a

    setelt(a:%, s:UniversalSegment(Integer), x:S) ==
      l := lo s; h := if hasHi s then hi s else maxIndex a
      l < minIndex a or h > maxIndex a => error "index out of range"
      for k in l..h repeat qsetelt_!(a, k, x)
      x

    reduce(f, a) ==
      empty? a => error "cannot reduce an empty aggregate"
      r := qelt(a, m := minIndex a)
      for k in m+1 .. maxIndex a repeat r := f(r, qelt(a, k))
      r

    reduce(f, a, identity) ==
      for k in minIndex a .. maxIndex a repeat
        identity := f(identity, qelt(a, k))
      identity

    if S has SetCategory then
       reduce(f, a, identity,absorber) ==
         for k in minIndex a .. maxIndex a while identity ^= absorber
                repeat identity := f(identity, qelt(a, k))
         identity

-- this is necessary since new has disappeared.
    stupidnew: (NonNegativeInteger, %, %) -> %
    stupidget: List % -> S
-- a and b are not both empty if n > 0
    stupidnew(n, a, b) ==
      zero? n => empty()
      new(n, (empty? a => qelt(b, minIndex b); qelt(a, minIndex a)))
-- at least one element of l must be non-empty
    stupidget l ==
      for a in l repeat
        not empty? a => return first a
      error "Should not happen"

    map(f, a, b) ==
      m := max(minIndex a, minIndex b)
      n := min(maxIndex a, maxIndex b)
      l := max(0, n - m + 1)::NonNegativeInteger
      c := stupidnew(l, a, b)
```

```
      for i in minIndex(c).. for j in m..n repeat
        qsetelt_!(c, i, f(qelt(a, j), qelt(b, j)))
      c

--  map(f, a, b, x) ==
--    m := min(minIndex a, minIndex b)
--    n := max(maxIndex a, maxIndex b)
--    l := (n - m + 1)::NonNegativeInteger
--    c := new l
--    for i in minIndex(c).. for j in m..n repeat
--      qsetelt_!(c, i, f(a(j, x), b(j, x)))
--    c

    merge(f, a, b) ==
      r := stupidnew(#a + #b, a, b)
      i := minIndex a
      m := maxIndex a
      j := minIndex b
      n := maxIndex b
      for k in minIndex(r).. while i <= m and j <= n repeat
        if f(qelt(a, i), qelt(b, j)) then
          qsetelt_!(r, k, qelt(a, i))
          i := i+1
        else
          qsetelt_!(r, k, qelt(b, j))
          j := j+1
      for k in k.. for i in i..m repeat qsetelt_!(r, k, elt(a, i))
      for k in k.. for j in j..n repeat qsetelt_!(r, k, elt(b, j))
      r

    elt(a:%, s:UniversalSegment(Integer)) ==
      l := lo s
      h := if hasHi s then hi s else maxIndex a
      l < minIndex a or h > maxIndex a => error "index out of range"
      r := stupidnew(max(0, h - l + 1)::NonNegativeInteger, a, a)
      for k in minIndex r.. for i in l..h repeat
        qsetelt_!(r, k, qelt(a, i))
      r

    insert(a:%, b:%, i:Integer) ==
      m := minIndex b
      n := maxIndex b
      i < m or i > n => error "index out of range"
      y := stupidnew(#a + #b, a, b)
      for k in minIndex y.. for j in m..i-1 repeat
        qsetelt_!(y, k, qelt(b, j))
```

```
      for k in k.. for j in minIndex a .. maxIndex a repeat
        qsetelt_!(y, k, qelt(a, j))
      for k in k.. for j in i..n repeat qsetelt_!(y, k, qelt(b, j))
      y

    copy x ==
      y := stupidnew(#x, x, x)
      for i in minIndex x .. maxIndex x for j in minIndex y .. repeat
        qsetelt_!(y, j, qelt(x, i))
      y

    copyInto_!(y, x, s) ==
      s < minIndex y or s + #x > maxIndex y + 1 =>
                                              error "index out of range"
      for i in minIndex x .. maxIndex x for j in s.. repeat
        qsetelt_!(y, j, qelt(x, i))
      y

    construct l ==
--    a := new(#l)
      empty? l => empty()
      a := new(#l, first l)
      for i in minIndex(a).. for x in l repeat qsetelt_!(a, i, x)
      a

    delete(a:%, s:UniversalSegment(Integer)) ==
      l := lo s; h := if hasHi s then hi s else maxIndex a
      l < minIndex a or h > maxIndex a => error "index out of range"
      h < l => copy a
      r := stupidnew((#a - h + l - 1)::NonNegativeInteger, a, a)
      for k in minIndex(r).. for i in minIndex a..l-1 repeat
        qsetelt_!(r, k, qelt(a, i))
      for k in k.. for i in h+1 .. maxIndex a repeat
        qsetelt_!(r, k, qelt(a, i))
      r

    delete(x:%, i:Integer) ==
      i < minIndex x or i > maxIndex x => error "index out of range"
      y := stupidnew((#x - 1)::NonNegativeInteger, x, x)
      for i in minIndex(y).. for j in minIndex x..i-1 repeat
        qsetelt_!(y, i, qelt(x, j))
      for i in i .. for j in i+1 .. maxIndex x repeat
        qsetelt_!(y, i, qelt(x, j))
      y

    reverse_! x ==
```

```
    m := minIndex x
    n := maxIndex x
    for i in 0..((n-m) quo 2) repeat swap_!(x, m+i, n-i)
    x

  concat l ==
    empty? l => empty()
    n := _+/[#a for a in l]
    i := minIndex(r := new(n, stupidget l))
    for a in l repeat
      copyInto_!(r, a, i)
      i := i + #a
    r

  sorted?(f, a) ==
    for i in minIndex(a)..maxIndex(a)-1 repeat
      not f(qelt(a, i), qelt(a, i + 1)) => return false
    true

  concat(x:%, y:%) ==
    z := stupidnew(#x + #y, x, y)
    copyInto_!(z, x, i := minIndex z)
    copyInto_!(z, y, i + #x)
    z

  if S has SetCategory then
    x = y ==
      #x ^= #y => false
      for i in minIndex x .. maxIndex x repeat
        not(qelt(x, i) = qelt(y, i)) => return false
      true

    coerce(r:%):OutputForm ==
      bracket commaSeparate
            [qelt(r, k)::OutputForm for k in minIndex r .. maxIndex r]

    position(x:S, t:%, s:Integer) ==
      n := maxIndex t
      s < minIndex t or s > n => error "index out of range"
      for k in s..n repeat
        qelt(t, k) = x => return k
      minIndex(t) - 1

  if S has OrderedSet then
    a < b ==
      for i in minIndex a .. maxIndex a
```

```
            for j in minIndex b .. maxIndex b repeat
              qelt(a, i) ^= qelt(b, j) => return a.i < b.j
          #a < #b
```

⟨*A1AGG.dotabb*⟩≡

```
"A1AGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=A1AGG"];
"A1AGG" -> "FLAGG"
```

⟨*A1AGG.dotfull*⟩≡

```
"OneDimensionalArrayAggregate(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=A1AGG"];
"OneDimensionalArrayAggregate(a:Type)" ->
    "FiniteLinearAggregate(a:Type)"

"OneDimensionalArrayAggregate(Character)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=A1AGG"];
"OneDimensionalArrayAggregate(Character)" ->
    "OneDimensionalArrayAggregate(a:Type)"

"OneDimensionalArrayAggregate(Boolean)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=A1AGG"];
"OneDimensionalArrayAggregate(Boolean)" ->
    "OneDimensionalArrayAggregate(a:Type)"
```

⟨*A1AGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OneDimensionalArrayAggregate(a:Type)" [color=lightblue];
"OneDimensionalArrayAggregate(a:Type)" ->
    "FiniteLinearAggregate(a:Type)"

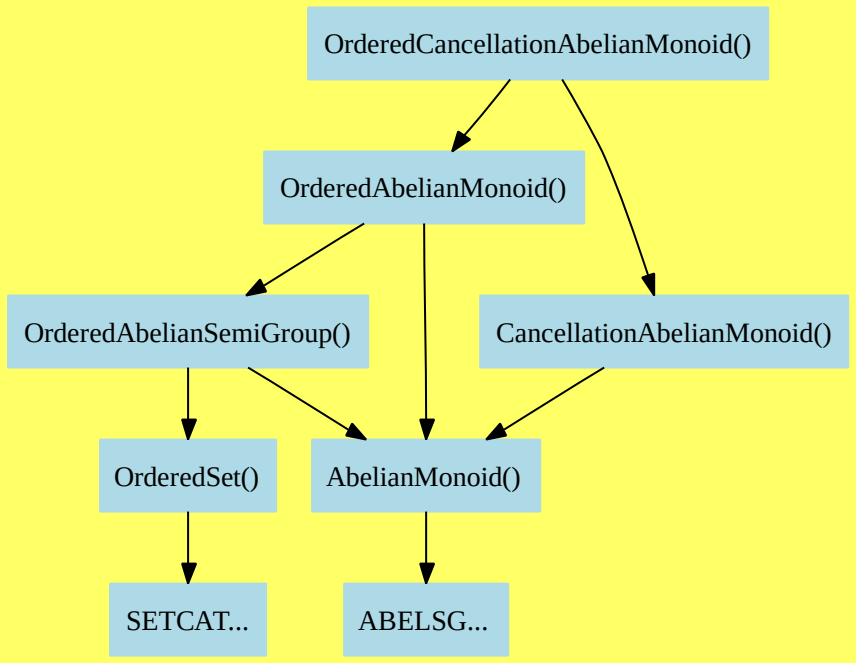
"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "CLAGG..."

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" -> "IXAGG..."

"CLAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
}
```

8.10 OrderedCancellationAbelianMonoid (OCAMON)



See:

⇒ "OrderedAbelianGroup" (OAGROUP) 9.5 on page 617
⇒ "OrderedAbelianMonoidSup" (OAMONS) 9.6 on page 620
⇐ "CancellationAbelianMonoid" (CABMON) 6.2 on page 289
⇐ "OrderedAbelianMonoid" (OAMON) 7.9 on page 461

Exports:

0	coerce	hash	latex	max
min	sample	subtractIfCan	zero?	?~=?
?*?	?+?	?<?	?<=?	?=?
?>?	?>=?			

These exports come from (p461) OrderedAbelianMonoid():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
sample : () -> %
```

```
zero? : % -> Boolean
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
```

These exports come from (p289) CancellationAbelianMonoid():

```
subtractIfCan : (%,%) -> Union(%,"failed")
```

⟨*category OCAMON OrderedCancellationAbelianMonoid*⟩≡

```
)abbrev category OCAMON OrderedCancellationAbelianMonoid
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Ordered sets which are also abelian cancellation monoids,
++ such that the addition preserves the ordering.
OrderedCancellationAbelianMonoid(): Category ==
        Join(OrderedAbelianMonoid, CancellationAbelianMonoid)
```

⟨*OCAMON.dotabb*⟩≡

```
"OCAMON"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OCAMON"];
"OCAMON" -> "OAMON"
"OCAMON" -> "CABMON"
```

⟨*OCAMON.dotfull*⟩≡

```
"OrderedCancellationAbelianMonoid()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OCAMON"];
"OrderedCancellationAbelianMonoid()" -> "OrderedAbelianMonoid()"
"OrderedCancellationAbelianMonoid()" -> "CancellationAbelianMonoid()"
```

⟨*OCAMON.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedCancellationAbelianMonoid()" [color=lightblue];
"OrderedCancellationAbelianMonoid()" -> "OrderedAbelianMonoid()"
"OrderedCancellationAbelianMonoid()" -> "CancellationAbelianMonoid()"

"OrderedAbelianMonoid()" [color=lightblue];
"OrderedAbelianMonoid()" -> "OrderedAbelianSemiGroup()"
"OrderedAbelianMonoid()" -> "AbelianMonoid()"

"OrderedAbelianSemiGroup()" [color=lightblue];
"OrderedAbelianSemiGroup()" -> "OrderedSet()"
"OrderedAbelianSemiGroup()" -> "AbelianMonoid()"

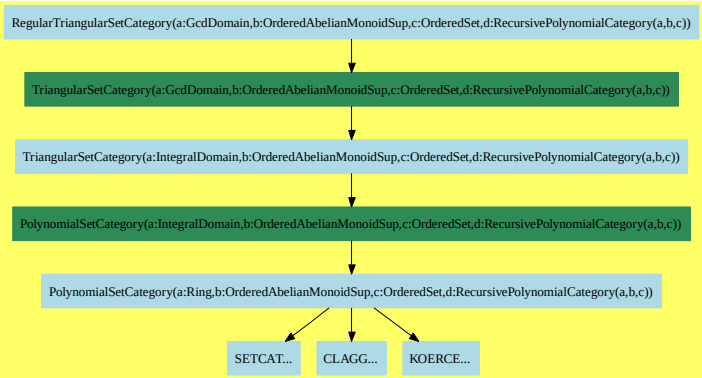
"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SETCAT..."

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "ABELSG..."

"SETCAT..." [color=lightblue];
"ABELSG..." [color=lightblue];
}
```

8.11 RegularTriangularSetCategory (RSETCAT)



See:

⇒ “NormalizedTriangularSetCategory” (NTSCAT) 9.4 on page 608
⇒ “SquareFreeRegularTriangularSetCategory” (SFRTCAT) 9.9 on page 632
⇐ “TriangularSetCategory” (TSETCAT) 7.12 on page 478

Exports:

algebraicCoefficients?	algebraicVariables	any?
augment	autoReduced?	basicSet
coerce	coHeight	collect
collectQuasiMonic	collectUnder	collectUpper
construct	convert	copy
count	degree	empty
empty?	eq?	eval
every?	extend	extendIfCan
find	first	hash
headReduce	headReduced?	headRemainder
infRittWu?	initiallyReduce	initiallyReduced?
initials	internalAugment	intersect
invertible?	invertibleElseSplit?	invertibleSet
last	lastSubResultant	lastSubResultantElseSplit
less?	latex	mainVariable?
mainVariables	map	map!
member?	members	more?
mvar	normalized?	parts
purelyAlgebraic?	purelyAlgebraicLeadingMonomial?	purelyTranscendental?
quasiComponent	reduce	reduceByQuasiMonic
reduced?	remainder	remove
removeDuplicates	removeZero	rest
retract	retractIfCan	rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder	rewriteSetWithReduction	roughBase?
roughEqualIdeals?	roughSubIdeal?	roughUnitIdeal?
sample	select	size?
sort	squareFreePart	stronglyReduce
stronglyReduced?	triangular?	trivialIdeal?
variables	zeroSetSplit	zeroSetSplitIntoTriangularSystems
#?	?=?	?~=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
augment : (P,%) -> List %
extend : (P,%) -> List %
internalAugment : (P,%) -> %
internalAugment : (List P,%) -> %
```

```
intersect : (P,%) -> List %
invertibleElseSplit? : (P,%) -> Union(Boolean,List %)
invertible? : (P,%) -> List Record(val: Boolean,tower: %)
invertible? : (P,%) -> Boolean
invertibleSet : (P,%) -> List %
lastSubResultant : (P,P,%) -> List Record(val: P,tower: %)
lastSubResultantElseSplit : (P,P,%) -> Union(P,List %)
squareFreePart : (P,%) -> List Record(val: P,tower: %)
zeroSetSplit : (List P,Boolean) -> List %
```

These are implemented by this category:

```
algebraicCoefficients? : (P,%) -> Boolean
augment : (P,List %) -> List %
augment : (List P,%) -> List %
augment : (List P,List %) -> List %
extend : (P,List %) -> List %
extend : (List P,%) -> List %
extend : (List P,List %) -> List %
intersect : (List P,List %) -> List %
intersect : (List P,%) -> List %
intersect : (P,List %) -> List %
purelyAlgebraic? : % -> Boolean
purelyAlgebraic? : (P,%) -> Boolean
purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
purelyTranscendental? : (P,%) -> Boolean
```

These exports come from (p478) TriangularSetCategory(R,E,V,P) where R:GcdDomain, E:OrderedAbelianMonoidSup, V:OrderedSet, P:RecursivePolynomialCategory(R,E,V)):

```
algebraic? : (V,%) -> Boolean
algebraicVariables : % -> List V
any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
basicSet :
  (List P,((P,P) -> Boolean)) ->
    Union(Record(bas: %,top: List P),"failed")
basicSet :
  (List P,(P -> Boolean),((P,P) -> Boolean)) ->
     Union(Record(bas: %,top: List P),"failed")
coerce : % -> List P
coerce : % -> OutputForm
coHeight : % -> NonNegativeInteger if V has FINITE
collect : (%,V) -> %
collectQuasiMonic : % -> %
collectUnder : (%,V) -> %
collectUpper : (%,V) -> %
construct : List P -> %
```

```
convert : % -> InputForm if P has KONVERT INFORM
copy : % -> %
count : ((P -> Boolean),%) -> NonNegativeInteger
  if $ has finiteAggregate
count : (P,%) -> NonNegativeInteger
  if P has SETCAT
  and $ has finiteAggregate
degree : % -> NonNegativeInteger
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
extend : (%,P) -> %
extendIfCan : (%,P) -> Union(%,"failed")
find : ((P -> Boolean),%) -> Union(P,"failed")
first : % -> Union(P,"failed")
hash : % -> SingleInteger
headReduce : (P,%) -> P
headReduced? : % -> Boolean
headReduced? : (P,%) -> Boolean
headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
infRittWu? : (%,%) -> Boolean
initiallyReduce : (P,%) -> P
initiallyReduced? : % -> Boolean
initiallyReduced? : (P,%) -> Boolean
initials : % -> List P
last : % -> Union(P,"failed")
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
mainVariable? : (V,%) -> Boolean
mainVariables : % -> List V
map : ((P -> P),%) -> %
map! : ((P -> P),%) -> % if $ has shallowlyMutable
member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
members : % -> List P if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
mvar : % -> V
normalized? : % -> Boolean
normalized? : (P,%) -> Boolean
parts : % -> List P if $ has finiteAggregate
quasiComponent : % -> Record(close: List P,open: List P)
reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P,P) -> P
  if P has SETCAT
  and $ has finiteAggregate
```

```
reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
reduceByQuasiMonic : (P,%) -> P
reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
removeZero : (P,%) -> P
rest : % -> Union(%,"failed")
retract : List P -> %
rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
retractIfCan : List P -> Union(%,"failed")
rewriteSetWithReduction :
   (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List P
roughBase? : % -> Boolean if R has INTDOM
roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
roughUnitIdeal? : % -> Boolean if R has INTDOM
sample : () -> %
select : ((P -> Boolean),%) -> % if $ has finiteAggregate
select : (%,V) -> Union(P,"failed")
size? : (%,NonNegativeInteger) -> Boolean
sort : (%,V) -> Record(under: %,floor: %,upper: %)
stronglyReduce : (P,%) -> P
stronglyReduced? : (P,%) -> Boolean
stronglyReduced? : % -> Boolean
triangular? : % -> Boolean if R has INTDOM
trivialIdeal? : % -> Boolean
variables : % -> List V
zeroSetSplit : List P -> List %
zeroSetSplitIntoTriangularSystems :
   List P -> List Record(close: %,open: List P)
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
```

⟨*category RSETCAT RegularTriangularSetCategory*⟩≡

```
)abbrev category RSETCAT RegularTriangularSetCategory
++ Author: Marc Moreno Maza
++ Date Created: 09/03/1998
++ Date Last Updated: 12/15/1998
++ Basic Functions:
++ Related Constructors:
++ Also See: essai Graphisme
++ AMS Classifications:
++ Keywords: polynomial, multivariate, ordered variables set
++ Description:
++ The category of regular triangular sets, introduced under
```

```
++ the name regular chains in [1] (and other papers).
++ In [3] it is proved that regular triangular sets and towers of simple
++ extensions of a field are equivalent notions.
++ In the following definitions, all polynomials and ideals
++ are taken from the polynomial ring \spad{k[x1,...,xn]} where \spad{k}
++ is the fraction field of \spad{R}.
++ The triangular set \spad{[t1,...,tm]} is regular
++ iff for every \spad{i} the initial of \spad{ti+1} is invertible
++ in the tower of simple extensions associated with \spad{[t1,...,ti]}.
++ A family \spad{[T1,...,Ts]} of regular triangular sets
++ is a split of Kalkbrener of a given ideal \spad{I}
++ iff the radical of \spad{I} is equal to the intersection
++ of the radical ideals generated by the saturated ideals
++ of the \spad{[T1,...,Ti]}.
++ A family \spad{[T1,...,Ts]} of regular triangular sets
++ is a split of Kalkbrener of a given triangular set \spad{T}
++ iff it is a split of Kalkbrener of the saturated ideal of \spad{T}.
++ Let \spad{K} be an algebraic closure of \spad{k}.
++ Assume that \spad{V} is finite with cardinality
++ \spad{n} and let \spad{A} be the affine space \spad{K^n}.
++ For a regular triangular set \spad{T} let denote by \spad{W(T)} the
++ set of regular zeros of \spad{T}.
++ A family \spad{[T1,...,Ts]} of regular triangular sets
++ is a split of Lazard of a given subset \spad{S} of \spad{A}
++ iff the union of the \spad{W(Ti)} contains \spad{S} and
++ is contained in the closure of \spad{S} (w.r.t. Zariski topology).
++ A family \spad{[T1,...,Ts]} of regular triangular sets
++ is a split of Lazard of a given triangular set \spad{T}
++ if it is a split of Lazard of \spad{W(T)}.
++ Note that if \spad{[T1,...,Ts]} is a split of Lazard of
++ \spad{T} then it is also a split of Kalkbrener of \spad{T}.
++ The converse is false.
++ This category provides operations related to both kinds of
++ splits, the former being related to ideals decomposition whereas
++ the latter deals with varieties decomposition.
++ See the example illustrating the \spadtype{RegularTriangularSet} constructor
++ for more explanations about decompositions by means of regular triangular sets. \newline
++ References :
++  [1] M. KALKBRENER "Three contributions to elimination theory"
++      Phd Thesis, University of Linz, Austria, 1991.
++  [2] M. KALKBRENER "Algorithmic properties of polynomial rings"
++      Journal of Symbol. Comp. 1998
++  [3] P. AUBRY, D. LAZARD and M. MORENO MAZA "On the Theories
++      of Triangular Sets" Journal of Symbol. Comp. (to appear)
++  [4] M. MORENO MAZA "A new algorithm for computing triangular
++      decomposition of algebraic varieties" NAG Tech. Rep. 4/98.
```

```
++ Version: 2

RegularTriangularSetCategory(R:GcdDomain, E:OrderedAbelianMonoidSup,_
  V:OrderedSet,P:RecursivePolynomialCategory(R,E,V)):
        Category ==
   TriangularSetCategory(R,E,V,P) with
    purelyAlgebraic?: (P,$) -> Boolean
      ++ \spad{purelyAlgebraic?(p,ts)} returns \spad{true} iff every
      ++ variable of \spad{p} is algebraic w.r.t. \spad{ts}.
    purelyTranscendental? : (P,$) -> Boolean
      ++ \spad{purelyTranscendental?(p,ts)} returns \spad{true} iff every
      ++ variable of \spad{p} is not algebraic w.r.t. \spad{ts}
    algebraicCoefficients?  : (P,$) -> Boolean
      ++ \spad{algebraicCoefficients?(p,ts)} returns \spad{true} iff every
      ++ variable of \spad{p} which is not the main one of \spad{p}
      ++ is algebraic w.r.t. \spad{ts}.
    purelyAlgebraic?: $ -> Boolean
      ++ \spad{purelyAlgebraic?(ts)} returns true iff for every algebraic
      ++ variable \spad{v} of \spad{ts} we have
      ++ \spad{algebraicCoefficients?(t_v,ts_v_-)} where \spad{ts_v}
      ++ is \axiomOpFrom{select}{TriangularSetCategory}(ts,v) and
      ++ \spad{ts_v_-} is
      ++ \axiomOpFrom{collectUnder}{TriangularSetCategory}(ts,v).
    purelyAlgebraicLeadingMonomial?: (P, $) -> Boolean
      ++ \spad{purelyAlgebraicLeadingMonomial?(p,ts)} returns true iff
      ++ the main variable of any non-constant iterarted initial
      ++ of \spad{p} is algebraic w.r.t. \spad{ts}.
    invertibleElseSplit? : (P,$) -> Union(Boolean,List $)
      ++ \spad{invertibleElseSplit?(p,ts)} returns \spad{true} (resp.
      ++ \spad{false}) if \spad{p} is invertible in the tower
      ++ associated with \spad{ts} or returns a split of Kalkbrener
      ++ of \spad{ts}.
    invertible? : (P,$) -> List Record(val : Boolean, tower : $)
      ++ \spad{invertible?(p,ts)} returns \spad{lbwt} where \spad{lbwt.i}
      ++ is the result of \spad{invertibleElseSplit?(p,lbwt.i.tower)} and
      ++ the list of the \spad{(lqrwt.i).tower} is a split of Kalkbrener of
      ++ \spad{ts}.
    invertible?: (P,$) -> Boolean
      ++ \spad{invertible?(p,ts)} returns true iff \spad{p} is invertible
      ++ in the tower associated with \spad{ts}.
    invertibleSet: (P,$) -> List $
      ++ \spad{invertibleSet(p,ts)} returns a split of Kalkbrener of the
      ++ quotient ideal of the ideal \axiom{I}  by \spad{p} where \spad{I} is
      ++ the radical of saturated of \spad{ts}.
    lastSubResultantElseSplit: (P, P, $) -> Union(P,List $)
      ++ \spad{lastSubResultantElseSplit(p1,p2,ts)} returns either
```

```
    ++ \spad{g} a quasi-monic gcd of \spad{p1} and \spad{p2} w.r.t.
    ++ the \spad{ts} or a split of Kalkbrener of \spad{ts}.
    ++ This assumes that \spad{p1} and \spad{p2} have the same maim
    ++ variable and that this variable is greater that any variable
    ++ occurring in \spad{ts}.
  lastSubResultant: (P, P, $) -> List Record(val : P, tower : $)
    ++ \spad{lastSubResultant(p1,p2,ts)} returns \spad{lpwt} such that
    ++ \spad{lpwt.i.val} is a quasi-monic gcd of \spad{p1} and \spad{p2}
    ++ w.r.t. \spad{lpwt.i.tower}, for every \spad{i}, and such
    ++ that the list of the \spad{lpwt.i.tower} is a split of Kalkbrener of
    ++ \spad{ts}. Moreover, if \spad{p1} and \spad{p2} do not
    ++ have a non-trivial gcd w.r.t. \spad{lpwt.i.tower} then
    ++ \spad{lpwt.i.val} is the resultant of these polynomials w.r.t.
    ++ \spad{lpwt.i.tower}. This assumes that \spad{p1} and \spad{p2} have
    ++ the same main variable and that this variable is greater that any
    ++ variable occurring in \spad{ts}.
  squareFreePart: (P,$) -> List Record(val : P, tower : $)
    ++ \spad{squareFreePart(p,ts)} returns \spad{lpwt} such that
    ++ \spad{lpwt.i.val} is a square-free polynomial
    ++ w.r.t. \spad{lpwt.i.tower}, this polynomial being associated with
    ++ \spad{p} modulo \spad{lpwt.i.tower}, for every \spad{i}. Moreover,
    ++ the list of the \spad{lpwt.i.tower} is a split
    ++ of Kalkbrener of \spad{ts}.
    ++ WARNING: This assumes that \spad{p} is a non-constant polynomial
    ++ such that if \spad{p} is added to \spad{ts}, then the resulting set
    ++ is a regular triangular set.
  intersect: (P,$) -> List $
    ++ \spad{intersect(p,ts)} returns the same as
    ++ \spad{intersect([p],ts)}
  intersect: (List P, $) -> List $
    ++ \spad{intersect(lp,ts)} returns \spad{lts} a split of Lazard
    ++ of the intersection of the affine variety associated
    ++ with \spad{lp} and the regular zero set of \spad{ts}.
  intersect: (List P, List $) -> List $
    ++ \spad{intersect(lp,lts)} returns the same as
    ++ \spad{concat([intersect(lp,ts) for ts in lts])|}
  intersect: (P, List $) -> List $
    ++ \spad{intersect(p,lts)} returns the same as
    ++ \spad{intersect([p],lts)}
  augment: (P,$) -> List $
    ++ \spad{augment(p,ts)} assumes that \spad{p} is a non-constant
    ++ polynomial whose main variable is greater than any variable
    ++ of \spad{ts}. This operation assumes also that if \spad{p} is
    ++ added to \spad{ts} the resulting set, say \spad{ts+p}, is a
    ++ regular triangular set. Then it returns a split of Kalkbrener
    ++ of \spad{ts+p}. This may not be \spad{ts+p} itself, if for
```

```
      ++ instance \spad{ts+p} is required to be square-free.
    augment: (P,List $) -> List $
      ++ \spad{augment(p,lts)} returns the same as
      ++ \spad{concat([augment(p,ts) for ts in lts])}
    augment: (List P,$) -> List $
      ++ \spad{augment(lp,ts)} returns \spad{ts} if \spad{empty? lp},
      ++ \spad{augment(p,ts)} if \spad{lp = [p]}, otherwise
      ++ \spad{augment(first lp, augment(rest lp, ts))}
    augment: (List P,List $) -> List $
      ++ \spad{augment(lp,lts)} returns the same as
      ++ \spad{concat([augment(lp,ts) for ts in lts])}
    internalAugment: (P, $) -> $
      ++ \spad{internalAugment(p,ts)} assumes that \spad{augment(p,ts)}
      ++ returns a singleton and returns it.
    internalAugment: (List P, $) -> $
      ++ \spad{internalAugment(lp,ts)} returns \spad{ts} if \spad{lp}
      ++ is empty otherwise returns
      ++ \spad{internalAugment(rest lp, internalAugment(first lp, ts))}
    extend: (P,$) -> List $
      ++ \spad{extend(p,ts)} assumes that \spad{p} is a non-constant
      ++ polynomial whose main variable is greater than any variable
      ++ of \spad{ts}. Then it returns a split of Kalkbrener
      ++ of \spad{ts+p}. This may not be \spad{ts+p} itself, if for
      ++ instance \spad{ts+p} is not a regular triangular set.
    extend: (P, List $)  -> List $
      ++ \spad{extend(p,lts)} returns the same as
      ++ \spad{concat([extend(p,ts) for ts in lts])|}
    extend: (List P,$) -> List $
      ++ \spad{extend(lp,ts)} returns \spad{ts} if \spad{empty? lp}
      ++ \spad{extend(p,ts)} if \spad{lp = [p]} else
      ++ \spad{extend(first lp, extend(rest lp, ts))}
    extend: (List P,List $) -> List $
      ++ \spad{extend(lp,lts)} returns the same as
      ++ \spad{concat([extend(lp,ts) for ts in lts])|}
    zeroSetSplit: (List P, Boolean) -> List $
      ++ \spad{zeroSetSplit(lp,clos?)} returns \spad{lts} a split of
      ++ Kalkbrener of the radical ideal associated with \spad{lp}.
      ++ If \spad{clos?} is false, it is also a decomposition of the
      ++ variety associated with \spad{lp} into the regular zero set of the
      ++ \spad{ts} in \spad{lts} (or, in other words, a split of Lazard of
      ++ this variety). See the example illustrating the
      ++ \spadtype{RegularTriangularSet} constructor for more explanations
      ++ about decompositions by means of regular triangular sets.

  add
```

```
NNI ==> NonNegativeInteger
INT ==> Integer
LP ==> List P
PWT ==> Record(val : P, tower : $)
LpWT ==> Record(val : (List P), tower : $)
Split ==> List $
pack ==> PolynomialSetUtilitiesPackage(R,E,V,P)

purelyAlgebraic?(p: P, ts: $): Boolean ==
  ground? p => true
  not algebraic?(mvar(p),ts) => false
  algebraicCoefficients?(p,ts)

purelyTranscendental?(p:P,ts:$): Boolean  ==
  empty? ts => true
  lv : List V := variables(p)$P
  while (not empty? lv) and (not algebraic?(first(lv),ts)) repeat _
    lv := rest lv
  empty? lv

purelyAlgebraicLeadingMonomial?(p: P, ts: $): Boolean  ==
  ground? p => true
  algebraic?(mvar(p),ts) and purelyAlgebraicLeadingMonomial?(init(p), ts)

algebraicCoefficients?(p:P,ts:$): Boolean  ==
  ground? p => true
  (not ground? init(p)) and not (algebraic?(mvar(init(p)),ts)) => false
  algebraicCoefficients?(init(p),ts) =>
    ground? tail(p) => true
    mvar(tail(p)) = mvar(p) =>
      algebraicCoefficients?(tail(p),ts)
    algebraic?(mvar(tail(p)),ts) =>
      algebraicCoefficients?(tail(p),ts)
    false
  false

if V has Finite
then
 purelyAlgebraic?(ts: $): Boolean ==
    empty? ts => true
    size()$V = #ts => true
    lp: LP := sort(infRittWu?,members(ts))
    i: NonNegativeInteger := size()$V
    for p in lp repeat
      v: V := mvar(p)
      (i = (lookup(v)$V)::NNI) =>
```

```
        i := subtractIfCan(i,1)::NNI
      univariate?(p)$pack =>
        i := subtractIfCan(i,1)::NNI
      not algebraicCoefficients?(p,collectUnder(ts,v)) =>
        return false
      i := subtractIfCan(i,1)::NNI
    true

else

  purelyAlgebraic?(ts: $): Boolean ==
    empty? ts => true
    v: V := mvar(ts)
    p: P := select(ts,v)::P
    ts := collectUnder(ts,v)
    empty? ts => univariate?(p)$pack
    not purelyAlgebraic?(ts) => false
    algebraicCoefficients?(p,ts)

augment(p:P,lts:List $) ==
  toSave: Split := []
  while not empty? lts repeat
    ts := first lts
    lts := rest lts
    toSave := concat(augment(p,ts),toSave)
  toSave

augment(lp:LP,ts:$) ==
  toSave: Split := [ts]
  empty? lp => toSave
  lp := sort(infRittWu?,lp)
  while not empty? lp repeat
    p := first lp
    lp := rest lp
    toSave := augment(p,toSave)
  toSave

augment(lp:LP,lts:List $) ==
  empty? lp => lts
  toSave: Split := []
  while not empty? lts repeat
    ts := first lts
    lts := rest lts
    toSave := concat(augment(lp,ts),toSave)
  toSave
```

```
extend(p:P,lts:List $) ==
  toSave : Split := []
  while not empty? lts repeat
    ts := first lts
    lts := rest lts
    toSave := concat(extend(p,ts),toSave)
  toSave

extend(lp:LP,ts:$) ==
  toSave: Split := [ts]
  empty? lp => toSave
  lp := sort(infRittWu?,lp)
  while not empty? lp repeat
    p := first lp
    lp := rest lp
    toSave := extend(p,toSave)
  toSave

extend(lp:LP,lts:List $) ==
  empty? lp => lts
  toSave: Split := []
  while not empty? lts repeat
    ts := first lts
    lts := rest lts
    toSave := concat(extend(lp,ts),toSave)
  toSave

intersect(lp:LP,lts:List $): List $  ==
  -- A VERY GENERAL default algorithm
  (empty? lp) or (empty? lts) => lts
  lp := [primitivePart(p) for p in lp]
  lp := removeDuplicates lp
  lp := remove(zero?,lp)
  any?(ground?,lp) => []
  toSee: List LpWT := [[lp,ts]$LpWT for ts in lts]
  toSave: List $ := []
  lp: LP
  p: P
  ts: $
  lus: List $
  while (not empty? toSee) repeat
    lpwt := first toSee; toSee := rest toSee
    lp := lpwt.val; ts := lpwt.tower
    empty? lp => toSave := cons(ts, toSave)
    p := first lp;  lp := rest lp
    lus := intersect(p,ts)
```

```
          toSee := concat([[lp,us]$LpWT for us in lus], toSee)
        toSave

      intersect(lp: LP,ts: $): List $  ==
        intersect(lp,[ts])

      intersect(p: P,lts: List $): List $  ==
        intersect([p],lts)
```

⟨*RSETCAT.dotabb*⟩≡

```
"RSETCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RSETCAT"];
"RSETCAT" -> "TSETCAT"
```

⟨*RSETCAT.dotfull*⟩≡

```
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RSETCAT"];
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,
  ->
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recur
```

⟨*RSETCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recursive
 [color=lightblue];
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recursive
  ->
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom

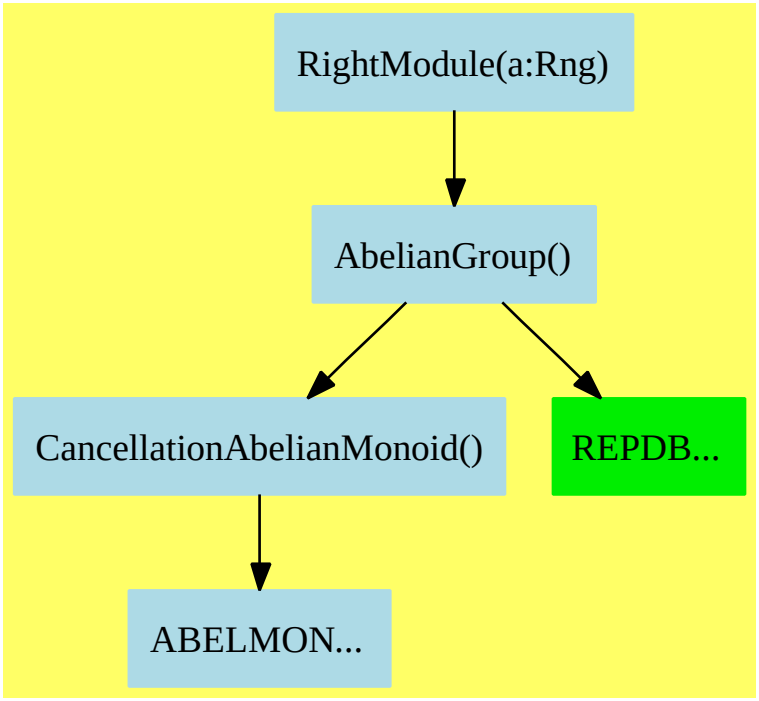
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom
 [color=seagreen];
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom
 ->
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
 [color=lightblue];
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
  -> "PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recurs

"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
 [color=seagreen];
"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
  -> "PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom

"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 [color=lightblue];
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 -> "SETCAT..."
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 -> "CLAGG..."
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 -> "KOERCE..."

"SETCAT..." [color=lightblue];
"KOERCE..." [color=lightblue];
"CLAGG..." [color=lightblue];
}
```

8.12 RightModule (RMODULE)



See:

⇒ "BiModule" (BMODULE) 9.1 on page 592
⇐ "AbelianGroup" (ABELGRP) 7.1 on page 418

Exports:

0	coerce	hash	latex	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?-?	-?	?=?		

These are directly exported but not implemented:

```
?*? : (%,R) -> %
```

These exports come from (p418) AbelianGroup():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
```

```
?~=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

⟨*category RMODULE RightModule*⟩≡

```
)abbrev category RMODULE RightModule
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of right modules over an rng (ring not necessarily
++ with unit). This is an abelian group which supports right
++ multiplication by elements of the rng.
++
++ Axioms:
++   \spad{ x*(a*b) = (x*a)*b }
++   \spad{ x*(a+b) = (x*a)+(x*b) }
++   \spad{ (x+y)*x = (x*a)+(y*a) }
RightModule(R:Rng):Category == AbelianGroup with
   "*": (%,R) -> %
     ++ x*r returns the right multiplication of the module element x
     ++ by the ring element r.
```

⟨*RMODULE.dotabb*⟩≡

```
"RMODULE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RMODULE"];
"RMODULE" -> "ABELGRP"
```

⟨*RMODULE.dotfull*⟩≡
```
"RightModule(a:Rng)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RMODULE"];
"RightModule(a:Rng)" -> "AbelianGroup()"

"RightModule(a:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RMODULE"];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"
```

⟨*RMODULE.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

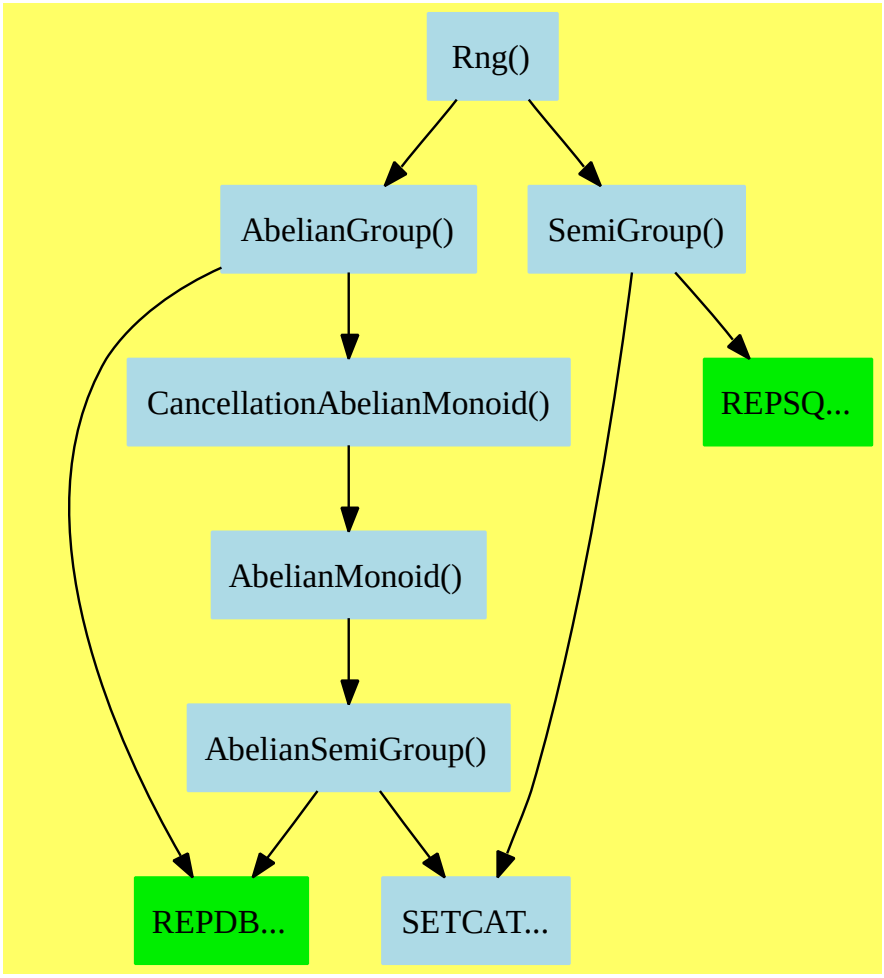
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "REPDB..."

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "ABELMON..."

"ABELMON..." [color=lightblue];
"REPDB..." [color="#00EE00"];
}
```

8.13 Rng (RNG)



Rng is a Ring that does not necessarily have a unit.

See:

⇒ "Ring" (RING) 9.8 on page 628
⇐ "AbelianGroup" (ABELGRP) 7.1 on page 418
⇐ "SemiGroup" (SGROUP) 4.24 on page 186

Exports:

0	coerce	hash	latex	sample
zero?	subtractIfCan	?*?	?**?	?+?
?-?	-?	?=?	?~=?	?^?

These exports come from (p418) AbelianGroup():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

These exports come from (p186) SemiGroup():

```
?*? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

⟨*category RNG Rng*⟩≡

```
)abbrev category RNG Rng
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of associative rings, not necessarily commutative, and not
++ necessarily with a 1. This is a combination of an abelian group
++ and a semigroup, with multiplication distributing over addition.
++
++ Axioms:
++   \spad{ x*(y+z) = x*y + x*z}
++   \spad{ (x+y)*z = x*z + y*z }
++
++ Conditional attributes:
++   \spadnoZeroDivisors\tab{25}\spad{  ab = 0 => a=0 or b=0}
Rng(): Category == Join(AbelianGroup,SemiGroup)
```

⟨*RNG.dotabb*⟩≡
```
"RNG" [color=lightblue,href="bookvol10.2.pdf#nameddest=RNG"];
"RNG" -> "ABELGRP"
"RNG" -> "SGROUP"
```

⟨*RNG.dotfull*⟩≡
```
"Rng()" [color=lightblue,href="bookvol10.2.pdf#nameddest=RNG"];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"
```

⟨*RNG.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "REPDB..."

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SETCAT..."
"AbelianSemiGroup()" -> "REPDB..."

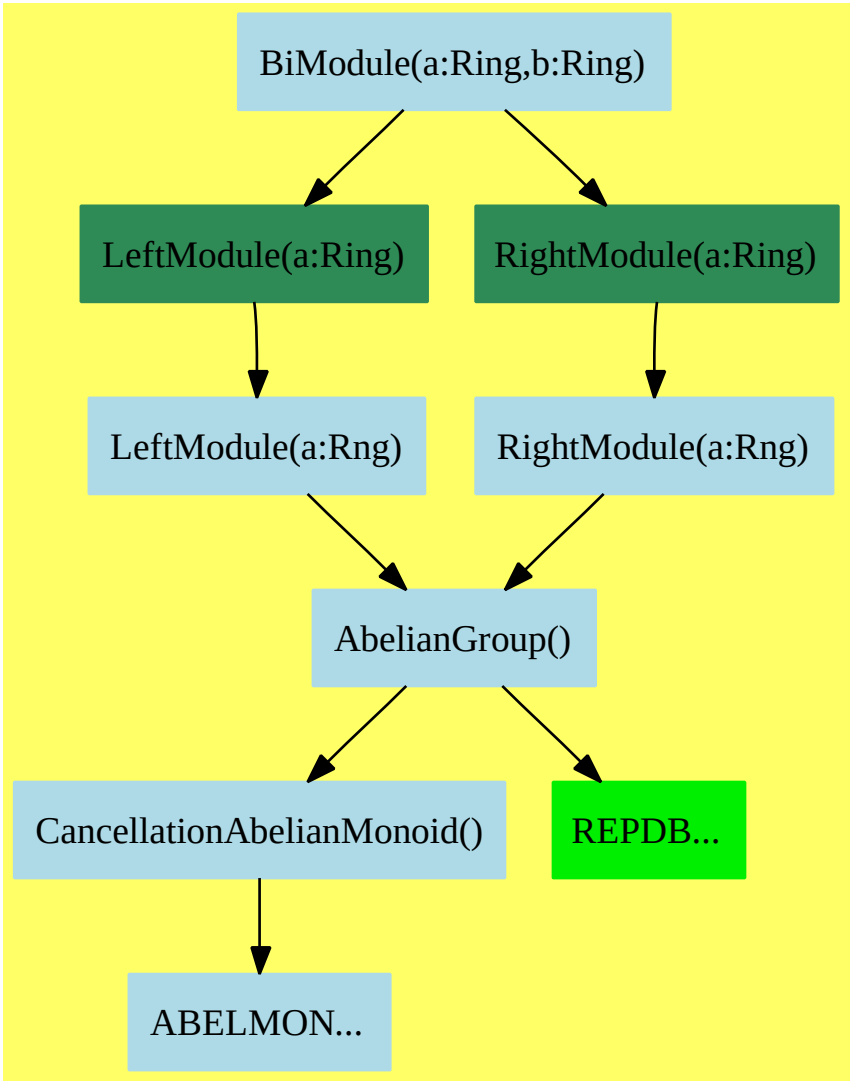
"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
}
```

Chapter 9

Category Layer 8

9.1 BiModule (BMODULE)



See:

⇒ "AbelianMonoidRing" (AMR) 13.1 on page 905
⇒ "CommutativeRing" (COMRING) 10.4 on page 685
⇒ "DirectProductCategory" (DIRPCAT) 12.1 on page 815
⇒ "EntireRing" (ENTIRER) 10.6 on page 694
⇒ "FreeModuleCat" (FMCAT) 10.7 on page 699
⇒ "Module" (MODULE) 10.10 on page 711
⇒ "MonogenicLinearOperator" (MLO) 12.6 on page 862
⇒ "RectangularMatrixCategory" (RMATCAT) 10.14 on page 732
⇒ "SquareMatrixCategory" (SMATCAT) 12.9 on page 887
⇒ "UnivariateSkewPolynomialCategory" (OREPCAT) 10.17 on page 754
⇒ "XAlgebra" (XALG) 10.18 on page 765
⇐ "LeftModule" (LMODULE) 8.5 on page 533
⇐ "RightModule" (RMODULE) 8.12 on page 584

Exports:

0	coerce	hash	latex	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?-?	-?	?=?		

Attributes Exported:

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p533) LeftModule(R:Ring):

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (R,%) -> %
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

These exports come from (p584) RightModule(S:Ring):

```
?*? : (%,S) -> %
```

⟨*category BMODULE BiModule*⟩≡
```
)abbrev category BMODULE BiModule
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A \spadtype{BiModule} is both a left and right module with respect
++ to potentially different rings.
++
++ Axiom:
++   \spad{ r*(x*s) = (r*x)*s }
BiModule(R:Ring,S:Ring):Category ==
  Join(LeftModule(R),RightModule(S)) with
     leftUnitary
       ++ \spad{1 * x = x}
     rightUnitary
       ++ \spad{x * 1 = x}
```

⟨*BMODULE.dotabb*⟩≡
```
"BMODULE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BMODULE"];
"BMODULE" -> "LMODULE"
"BMODULE" -> "RMODULE"
```

⟨*BMODULE.dotfull*⟩≡
```
"BiModule(a:Ring,b:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BMODULE"];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"BiModule(a:CommutativeRing,b:CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=BMODULE"];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:OrderedAbelianMonoid)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=BMODULE"];
"BiModule(a:Ring,b:OrderedAbelianMonoid)" -> "BiModule(a:Ring,b:Ring)"
```

⟨*BMODULE.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "AbelianGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

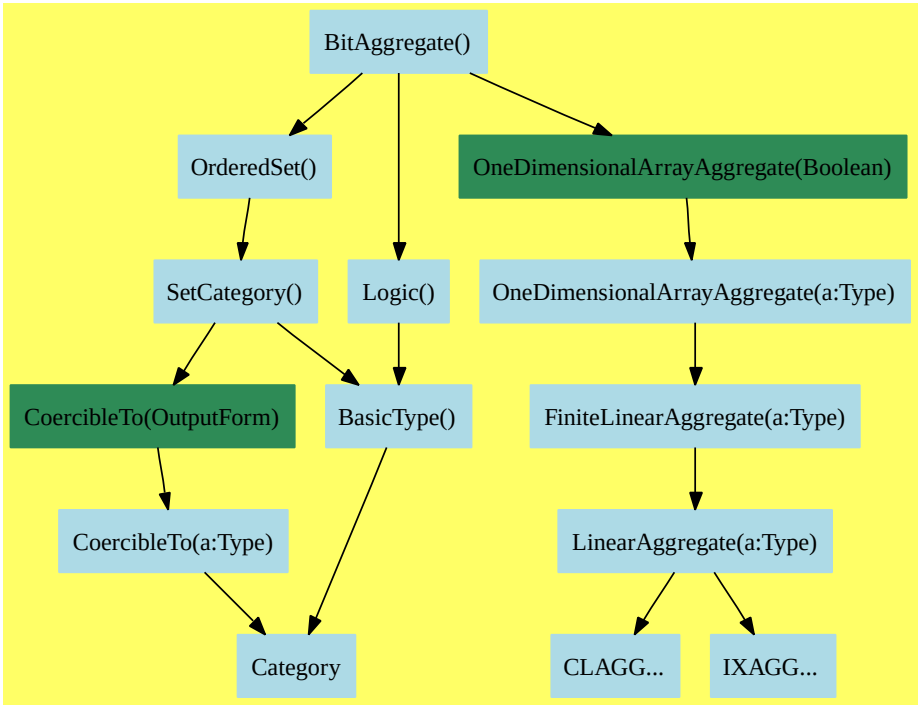
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "REPDB..."

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "ABELMON..."

"ABELMON..." [color=lightblue];
"REPDB..." [color="#00EE00"];
}
```

9.2 BitAggregate (BTAGG)



See:

⇐ “Logic” (LOGIC) 3.8 on page 77
⇐ “OneDimensionalArrayAggregate” (A1AGG) 8.9 on page 556
⇐ “OrderedSet” (ORDSET) 4.19 on page 168

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	delete	elt
empty	empty?	entry?	entries	eq?
eval	every?	fill!	find	first
hash	index?	indices	insert	latex
less?	map	map!	max	maxIndex
member?	members	merge	min	minIndex
more?	nand	new	nor	not?
parts	position	qelt	qsetelt!	reduce
remove	removeDuplicates	reverse	reverse!	sample
select	setelt	size?	sort	sort!
sorted?	swap!	xor	#?	?/\?
?<?	?<=?	?=?	?>?	?>=?
?\/?	^?	?and?	?.?	?or?
~?	?~=?			

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
?and? : (%,%) -> %
?or? : (%,%) -> %
xor : (%,%) -> %
```

These are implemented by this category:

```
not? : % -> %
^? : % -> %
~? : % -> %
?/\? : (%,%) -> %
?\/? : (%,%) -> %
nand : (%,%) -> %
nor : (%,%) -> %
```

These exports come from (p168) OrderedSet():

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
```

```
min : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

TPDHERE: Note that none of the exports of Logic are needed. Perhaps this can be eliminated.

These exports come from (p556) OneDimensionalArrayAggregate(Boolean):

```
any? : ((Boolean -> Boolean),%) -> Boolean
    if $ has finiteAggregate
concat : (%,%) -> %
concat : List % -> %
concat : (%,Boolean) -> %
concat : (Boolean,%) -> %
construct : List Boolean -> %
convert : % -> InputForm
    if Boolean has KONVERT INFORM
copy : % -> %
copyInto! : (%,%,Integer) -> %
    if $ has shallowlyMutable
count : (Boolean,%) -> NonNegativeInteger
    if Boolean has SETCAT and $ has finiteAggregate
count : ((Boolean -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
delete : (%,UniversalSegment Integer) -> %
delete : (%,Integer) -> %
elt : (%,Integer,Boolean) -> Boolean
empty : () -> %
empty? : % -> Boolean
entry? : (Boolean,%) -> Boolean
    if $ has finiteAggregate and Boolean has SETCAT
entries : % -> List Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation Boolean) -> %
    if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
eval : (%,Equation Boolean) -> %
    if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
eval : (%,Boolean,Boolean) -> %
    if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
eval : (%,List Boolean,List Boolean) -> %
    if Boolean has EVALAB BOOLEAN and Boolean has SETCAT
every? : ((Boolean -> Boolean),%) -> Boolean
    if $ has finiteAggregate
fill! : (%,Boolean) -> %
    if $ has shallowlyMutable
```

```
find : ((Boolean -> Boolean),%) -> Union(Boolean,"failed")
first : % -> Boolean if Integer has ORDSET
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (Boolean,%,Integer) -> %
insert : (%,%,Integer) -> %
less? : (%,NonNegativeInteger) -> Boolean
map : ((Boolean -> Boolean),%) -> %
map : (((Boolean,Boolean) -> Boolean),%,%) -> %
map! : ((Boolean -> Boolean),%) -> %
    if $ has shallowlyMutable
maxIndex : % -> Integer if Integer has ORDSET
member? : (Boolean,%) -> Boolean
    if Boolean has SETCAT and $ has finiteAggregate
members : % -> List Boolean if $ has finiteAggregate
merge : (%,%) -> % if Boolean has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,Boolean) -> %
merge : (((Boolean,Boolean) -> Boolean),%,%) -> %
parts : % -> List Boolean if $ has finiteAggregate
position : ((Boolean -> Boolean),%) -> Integer
position : (Boolean,%,Integer) -> Integer
    if Boolean has SETCAT
position : (Boolean,%) -> Integer
    if Boolean has SETCAT
qelt : (%,Integer) -> Boolean
qsetelt! : (%,Integer,Boolean) -> Boolean
    if $ has shallowlyMutable
reverse : % -> %
reduce : (((Boolean,Boolean) -> Boolean),%) -> Boolean
    if $ has finiteAggregate
reduce : (((Boolean,Boolean) -> Boolean),%,Boolean) -> Boolean
    if $ has finiteAggregate
reduce :
 (((Boolean,Boolean) -> Boolean),%,Boolean,Boolean) -> Boolean
    if Boolean has SETCAT and $ has finiteAggregate
remove : (Boolean,%) -> %
    if Boolean has SETCAT and $ has finiteAggregate
remove : ((Boolean -> Boolean),%) -> %
    if $ has finiteAggregate
removeDuplicates : % -> %
    if Boolean has SETCAT and $ has finiteAggregate
reverse! : % -> % if $ has shallowlyMutable
sample : () -> %
setelt : (%,UniversalSegment Integer,Boolean) -> Boolean
    if $ has shallowlyMutable
select : ((Boolean -> Boolean),%) -> %
    if $ has finiteAggregate
setelt : (%,Integer,Boolean) -> Boolean
```

```
    if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
sort : (((Boolean,Boolean) -> Boolean),%) -> %
sort : % -> % if Boolean has ORDSET
sort! : % -> %
    if Boolean has ORDSET and $ has shallowlyMutable
sort! : (((Boolean,Boolean) -> Boolean),%) -> %
    if $ has shallowlyMutable
sorted? : % -> Boolean if Boolean has ORDSET
sorted? : (((Boolean,Boolean) -> Boolean),%) -> Boolean
swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
?.? : (%,UniversalSegment Integer) -> %
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.? : (%,Integer) -> Boolean
```

⟨*category BTAGG BitAggregate*⟩≡

```
)abbrev category BTAGG BitAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The bit aggregate category models aggregates representing large
++ quantities of Boolean data.
BitAggregate(): Category ==
  Join(OrderedSet, Logic, OneDimensionalArrayAggregate Boolean) with
    "not": % -> %
      ++ not(b) returns the logical {\em not} of bit aggregate
      ++ \axiom{b}.
    "^"  : % -> %
      ++ ^ b returns the logical {\em not} of bit aggregate
      ++ \axiom{b}.
    nand : (%, %) -> %
      ++ nand(a,b) returns the logical {\em nand} of bit aggregates
      ++ \axiom{a} and \axiom{b}.
    nor  : (%, %) -> %
      ++ nor(a,b) returns the logical {\em nor} of bit aggregates
      ++ \axiom{a} and \axiom{b}.
    _and : (%, %) -> %
      ++ a and b returns the logical {\em and} of bit aggregates
      ++ \axiom{a} and \axiom{b}.
    _or  : (%, %) -> %
```

```
        ++ a or b returns the logical {\em or} of bit aggregates
        ++ \axiom{a} and \axiom{b}.
      xor  : (%, %) -> %
        ++ xor(a,b) returns the logical {\em exclusive-or} of bit aggregates
        ++ \axiom{a} and \axiom{b}.

    add
      not v      == map(_not, v)
      _^ v       == map(_not, v)
      _~(v)      == map(_~, v)
      _/_\(v, u) == map(_/_\, v, u)
      _\_/(v, u) == map(_\_/, v, u)
      nand(v, u) == map(nand, v, u)
      nor(v, u)  == map(nor, v, u)
```

⟨*BTAGG.dotabb*⟩≡

```
"BTAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=BTAGG"];
"BTAGG" -> "ORDSET"
"BTAGG" -> "LOGIC"
"BTAGG" -> "A1AGG"
```

⟨*BTAGG.dotfull*⟩≡

```
"BitAggregate()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=BTAGG"];
"BitAggregate()" -> "OrderedSet()"
"BitAggregate()" -> "Logic()"
"BitAggregate()" -> "OneDimensionalArrayAggregate(Boolean)"
```

⟨*BTAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"BitAggregate()" [color=lightblue];
"BitAggregate()" -> "OrderedSet()"
"BitAggregate()" -> "Logic()"
"BitAggregate()" -> "OneDimensionalArrayAggregate(Boolean)"

"OneDimensionalArrayAggregate(Boolean)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=A1AGG"];
"OneDimensionalArrayAggregate(Boolean)" ->
    "OneDimensionalArrayAggregate(a:Type)"

"OneDimensionalArrayAggregate(a:Type)" [color=lightblue];
"OneDimensionalArrayAggregate(a:Type)" ->
    "FiniteLinearAggregate(a:Type)"

"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IXAGG..."
"LinearAggregate(a:Type)" -> "CLAGG..."

"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SetCategory()"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

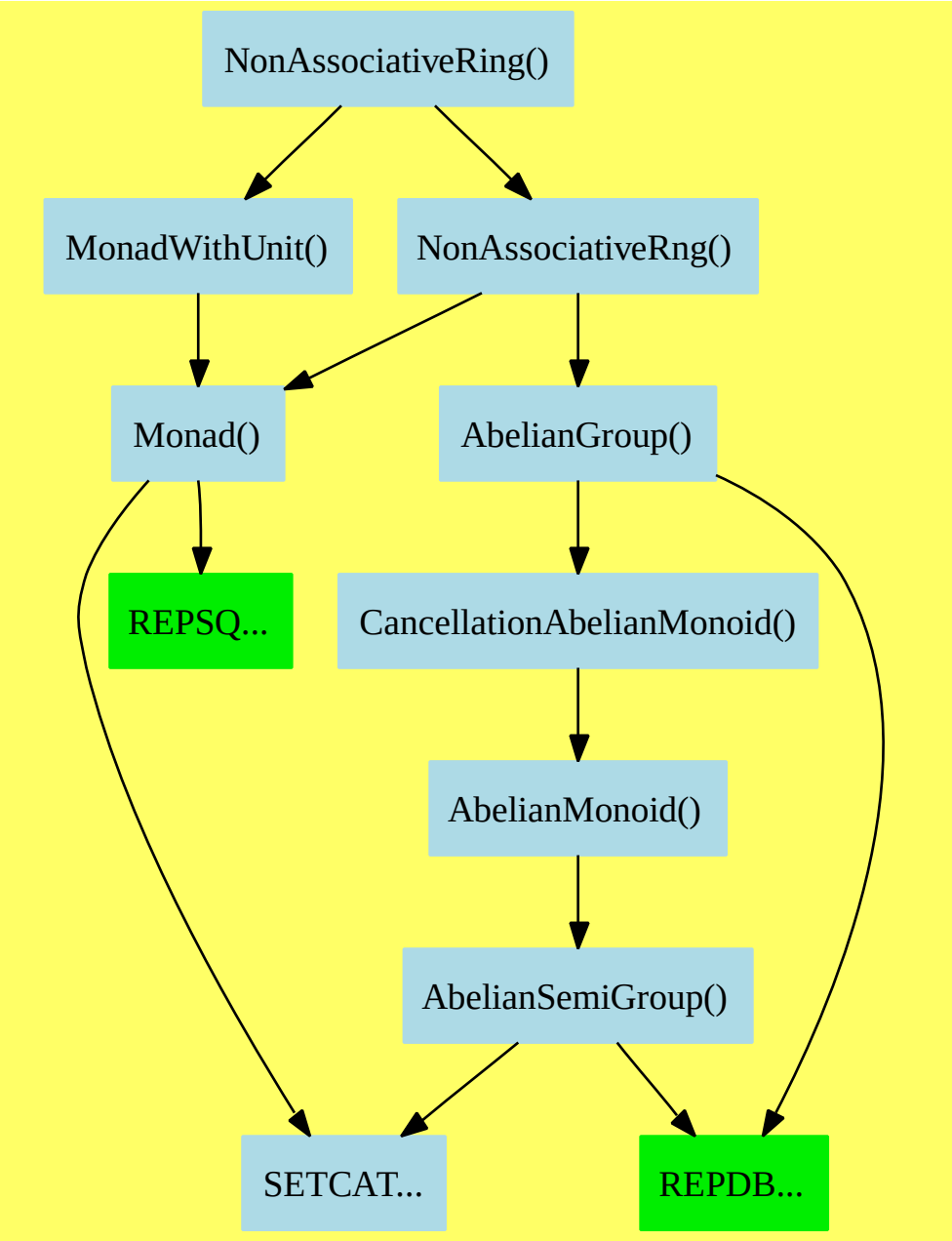
"Logic()" [color=lightblue];
"Logic()" -> "BasicType()"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"
```

```
"Category" [color=lightblue];

"CLAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
}
```

9.3 NonAssociativeRing (NASRING)



See:

⇐ "MonadWithUnit" (MONADWU) 5.9 on page 252

⇐ "NonAssociativeRng" (NARNG) 8.8 on page 552

Exports:

0	1	antiCommutator	associator	characteristic
coerce	commutator	hash	latex	leftPower
leftRecip	one?	recip	rightPower	rightRecip
sample	subtractIfCan	zero?	?*?	?~=?
?**?	?+?	?-?	-?	?=?

These are directly exported but not implemented:

```
characteristic : () -> NonNegativeInteger
```

These are implemented by this category:

```
coerce : Integer -> %
```

These exports come from (p552) NonAssociativeRng():

```
0 : () -> %
antiCommutator : (%,%) -> %
associator : (%,%,%) -> %
coerce : % -> OutputForm
commutator : (%,%) -> %
hash : % -> SingleInteger
latex : % -> String
leftPower : (%,PositiveInteger) -> %
rightPower : (%,PositiveInteger) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p252) MonadWithUnit():

```
1 : () -> %
leftPower : (%,NonNegativeInteger) -> %
leftRecip : % -> Union(%,"failed")
one? : % -> Boolean
recip : % -> Union(%,"failed")
rightPower : (%,NonNegativeInteger) -> %
```

```
 rightRecip : % -> Union(%,"failed")
 ?**? : (%,NonNegativeInteger) -> %
```

⟨*category NASRING NonAssociativeRing*⟩≡

```
 )abbrev category NASRING NonAssociativeRing
 ++ Author: J. Grabmeier, R. Wisbauer
 ++ Date Created: 01 March 1991
 ++ Date Last Updated: 11 June 1991
 ++ Basic Operations: +, *, -, **
 ++ Related Constructors: NonAssociativeRng, Rng, Ring
 ++ Also See:
 ++ AMS Classifications:
 ++ Keywords: non-associative ring with unit
 ++ Reference:
 ++  R.D. Schafer: An Introduction to Nonassociative Algebras
 ++  Academic Press, New York, 1966
 ++ Description:
 ++  A NonAssociativeRing is a non associative rng which has a unit,
 ++  the multiplication is not necessarily commutative or associative.
 NonAssociativeRing(): Category == Join(NonAssociativeRng,MonadWithUnit) with
       characteristic: -> NonNegativeInteger
         ++ characteristic() returns the characteristic of the ring.
         --we can not make this a constant, since some domains are mutable
       coerce: Integer -> %
         ++ coerce(n) coerces the integer n to an element of the ring.
    add
       n:Integer
       coerce(n) == n * 1$%
```

⟨*NASRING.dotabb*⟩≡

```
 "NASRING"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=NASRING"];
 "NASRING" -> "MONADWU"
 "NASRING" -> "NARNG"
```

⟨*NASRING.dotfull*⟩≡

```
 "NonAssociativeRing()"
  [color=lightblue,href="bookvol10.2.pdf#nameddest=NASRING"];
 "NonAssociativeRing()" -> "NonAssociativeRng()"
 "NonAssociativeRing()" -> "MonadWithUnit()"
```

⟨*NASRING.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"NonAssociativeRing()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NASRING"];
"NonAssociativeRing()" -> "NonAssociativeRng()"
"NonAssociativeRing()" -> "MonadWithUnit()"

"MonadWithUnit()" [color=lightblue];
"MonadWithUnit()" -> "Monad()"

"NonAssociativeRng()" [color=lightblue];
"NonAssociativeRng()" -> "AbelianGroup()"
"NonAssociativeRng()" -> "Monad()"

"Monad()" [color=lightblue];
"Monad()" -> "SETCAT..."
"Monad()" -> "REPSQ..."

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CancellationAbelianMonoid()"
"AbelianGroup()" -> "REPDB..."

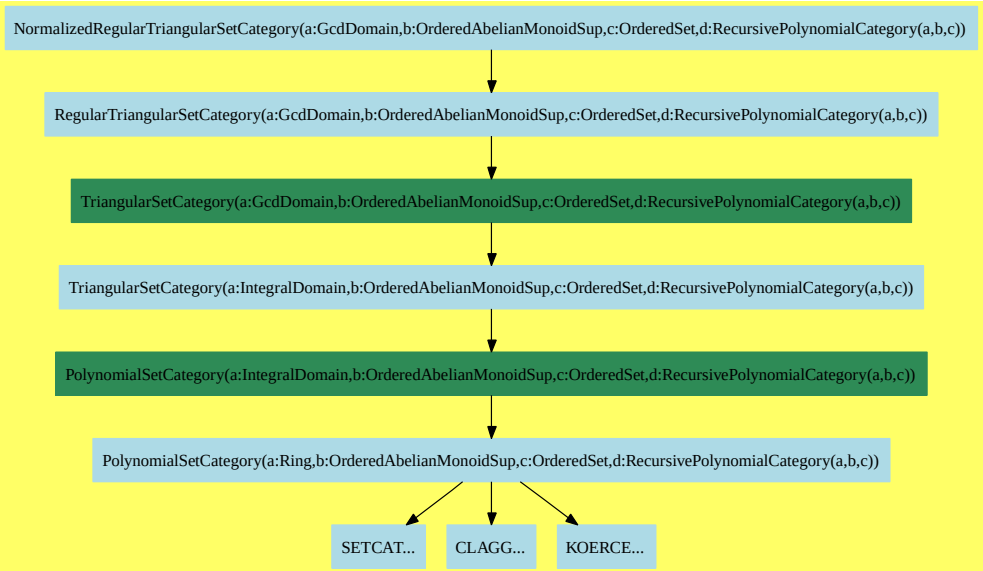
"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "AbelianSemiGroup()"

"AbelianSemiGroup()" [color=lightblue];
"AbelianSemiGroup()" -> "SETCAT..."
"AbelianSemiGroup()" -> "REPDB..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
}
```

9.4 NormalizedTriangularSetCategory (NTSCAT)



See:

⇒ “SquareFreeNormalizedTriangularSetCategory” (SNTSCAT) 10.15 on page 740

⇐ “RegularTriangularSetCategory” (RSETCAT) 8.11 on page 570

Exports:

algebraic?
algebraicCoefficients?
algebraicVariables
any?
augment
autoReduced?
basicSet
coerce
coHeight
collect
collectQuasiMonic
collectUnder
collectUpper
construct
copy
convert
count
degree
empty
empty?
eq?
eval
every?
extend
extendIfCan
find
first
hash
headReduce
headReduced?
headRemainder
infRittWu?
initiallyReduce
initiallyReduced?
initials
internalAugment
intersect
invertible?
invertibleElseSplit?
invertibleSet
last
lastSubResultant
lastSubResultantElseSplit
latex
less?
mainVariable?
mainVariables
map
map!
member?
members
more?
mvar
normalized?
parts
purelyAlgebraic?
purelyAlgebraicLeadingMonomial?
purelyTranscendental?
quasiComponent
reduce
reduceByQuasiMonic
reduced?
remainder
remove
removeDuplicates
removeZero
rest
retract
retractIfCan
rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder
rewriteSetWithReduction
roughBase?
roughEqualIdeals?
roughSubIdeal?
roughUnitIdeal?
sample
select
size?
sort
squareFreePart
stronglyReduce
stronglyReduced?
triangular?
trivialIdeal?
variables
zeroSetSplit
zeroSetSplitIntoTriangularSystems
#?
?=?
?~=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These exports come from (p570) RegularTriangularSetCategory(R,E,V,P) where R:GcdDomain, E:OrderedAbelianMonoidSup, V:OrderedSet, P:RecursivePolynomialCategory(R,E,V)):

```
algebraic? : (V,%) -> Boolean
algebraicCoefficients? : (P,%) -> Boolean
algebraicVariables : % -> List V
any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
augment : (List P,List %) -> List %
augment : (List P,%) -> List %
augment : (P,List %) -> List %
augment : (P,%) -> List %
autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
basicSet :
 (List P,(P -> Boolean),((P,P) -> Boolean)) ->
   Union(Record(bas: %,top: List P),"failed")
basicSet :
 (List P,((P,P) -> Boolean)) ->
   Union(Record(bas: %,top: List P),"failed")
coerce : % -> List P
coerce : % -> OutputForm
coHeight : % -> NonNegativeInteger if V has FINITE
collect : (%,V) -> %
collectQuasiMonic : % -> %
collectUnder : (%,V) -> %
collectUpper : (%,V) -> %
construct : List P -> %
copy : % -> %
convert : % -> InputForm if P has KONVERT INFORM
count : ((P -> Boolean),%) -> NonNegativeInteger
  if $ has finiteAggregate
count : (P,%) -> NonNegativeInteger
  if P has SETCAT
  and $ has finiteAggregate
degree : % -> NonNegativeInteger
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
```

```
eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
extend : (List P,List %) -> List %
extend : (List P,%) -> List %
extend : (P,List %) -> List %
extend : (P,%) -> List %
extend : (%,P) -> %
extendIfCan : (%,P) -> Union(%,"failed")
find : ((P -> Boolean),%) -> Union(P,"failed")
first : % -> Union(P,"failed")
hash : % -> SingleInteger
headReduce : (P,%) -> P
headReduced? : % -> Boolean
headReduced? : (P,%) -> Boolean
headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
infRittWu? : (%,%) -> Boolean
initiallyReduce : (P,%) -> P
initiallyReduced? : % -> Boolean
initiallyReduced? : (P,%) -> Boolean
initials : % -> List P
internalAugment : (P,%) -> %
internalAugment : (List P,%) -> %
intersect : (P,List %) -> List %
intersect : (List P,%) -> List %
intersect : (P,%) -> List %
intersect : (List P,List %) -> List %
invertible? : (P,%) -> Boolean
invertible? : (P,%) -> List Record(val: Boolean,tower: %)
invertibleElseSplit? : (P,%) -> Union(Boolean,List %)
invertibleSet : (P,%) -> List %
last : % -> Union(P,"failed")
lastSubResultant : (P,P,%) -> List Record(val: P,tower: %)
lastSubResultantElseSplit : (P,P,%) -> Union(P,List %)
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
mainVariable? : (V,%) -> Boolean
mainVariables : % -> List V
map : ((P -> P),%) -> %
map! : ((P -> P),%) -> % if $ has shallowlyMutable
member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
members : % -> List P if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
mvar : % -> V
normalized? : % -> Boolean
normalized? : (P,%) -> Boolean
parts : % -> List P if $ has finiteAggregate
purelyAlgebraic? : (P,%) -> Boolean
purelyAlgebraic? : % -> Boolean
```

```
purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
purelyTranscendental? : (P,%) -> Boolean
quasiComponent : % -> Record(close: List P,open: List P)
reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P,P) -> P
  if P has SETCAT
  and $ has finiteAggregate
reduceByQuasiMonic : (P,%) -> P
reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
remainder : (P,%) -> Record(rnum: R,polnum: P,den: R)
  if R has INTDOM
remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
removeZero : (P,%) -> P
rest : % -> Union(%,"failed")
retract : List P -> %
retractIfCan : List P -> Union(%,"failed")
rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
rewriteSetWithReduction :
   (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List P
roughBase? : % -> Boolean if R has INTDOM
roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
roughUnitIdeal? : % -> Boolean if R has INTDOM
sample : () -> %
select : (%,V) -> Union(P,"failed")
select : ((P -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
sort : (%,V) -> Record(under: %,floor: %,upper: %)
squareFreePart : (P,%) -> List Record(val: P,tower: %)
stronglyReduce : (P,%) -> P
stronglyReduced? : (P,%) -> Boolean
stronglyReduced? : % -> Boolean
triangular? : % -> Boolean if R has INTDOM
trivialIdeal? : % -> Boolean
variables : % -> List V
zeroSetSplit : List P -> List %
zeroSetSplit : (List P,Boolean) -> List %
zeroSetSplitIntoTriangularSystems :
  List P -> List Record(close: %,open: List P)
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category NTSCAT NormalizedTriangularSetCategory*⟩≡

```
)abbrev category NTSCAT NormalizedTriangularSetCategory
```

```
++ Author: Marc Moreno Maza
++ Date Created: 10/07/1998
++ Date Last Updated: 12/12/1998
++ Basic Functions:
++ Related Constructors:
++ Also See: essai Graphisme
++ AMS Classifications:
++ Keywords: polynomial, multivariate, ordered variables set
++ Description:
++ The category of normalized triangular sets. A triangular
++ set \spad{ts} is said normalized if for every algebraic
++ variable \spad{v} of \spad{ts} the polynomial \spad{select(ts,v)}
++ is normalized w.r.t. every polynomial in \spad{collectUnder(ts,v)}.
++ A polynomial \spad{p} is said normalized w.r.t. a non-constant
++ polynomial \spad{q} if \spad{p} is constant or \spad{degree(p,mdeg(q)) = 0}
++ and \spad{init(p)} is normalized w.r.t. \spad{q}. One of the important
++ features of normalized triangular sets is that they are regular sets.\newline
++ References :
++  [1] D. LAZARD "A new method for solving algebraic systems of
++      positive dimension" Discr. App. Math. 33:147-160,1991
++  [2] P. AUBRY, D. LAZARD and M. MORENO MAZA "On the Theories
++      of Triangular Sets" Journal of Symbol. Comp. (to appear)
++  [3] M. MORENO MAZA and R. RIOBOO "Computations of gcd over
++      algebraic towers of simple extensions" In proceedings of AAECC11
++      Paris, 1995.
++  [4] M. MORENO MAZA "Calculs de pgcd au-dessus des tours
++      d'extensions simples et resolution des systemes d'equations
++      algebriques" These, Universite P.etM. Curie, Paris, 1997.


NormalizedTriangularSetCategory(R:GcdDomain,E:OrderedAbelianMonoidSup,_
 V:OrderedSet,P:RecursivePolynomialCategory(R,E,V)):
         Category ==  RegularTriangularSetCategory(R,E,V,P)
```

⟨*NTSCAT.dotabb*⟩≡

```
"NTSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NTSCAT"];
"NTSCAT" -> "RSETCAT"
```

⟨*NTSCAT.dotfull*⟩≡

```
"NormalizedRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:O
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NTSCAT"];
"NormalizedRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:O
 ->
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,
```

⟨*NTSCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"NormalizedRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,
 [color=lightblue];
"NormalizedRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,
 ->
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recursive

"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recursive
 [color=lightblue];
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recursive
  ->
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom

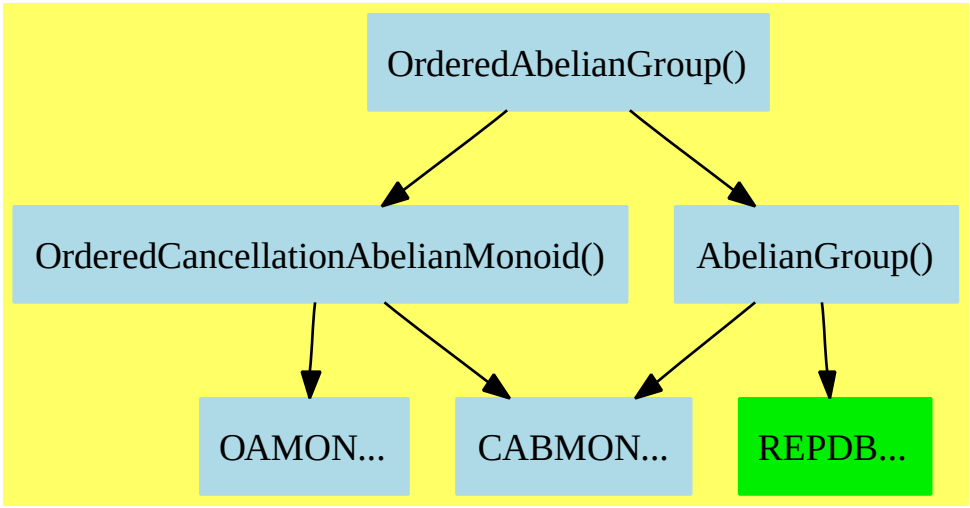
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom
 [color=seagreen];
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom
 ->
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
 [color=lightblue];
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
  -> "PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recurs

"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
 [color=seagreen];
"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePo
  -> "PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynom

"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 [color=lightblue];
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 -> "SETCAT..."
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 -> "CLAGG..."
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursivePolynomialCa
 -> "KOERCE..."

"SETCAT..." [color=lightblue];
"KOERCE..." [color=lightblue];
"CLAGG..." [color=lightblue];
}
```

9.5 OrderedAbelianGroup (OAGROUP)



See:

⇒ "OrderedRing" (ORDRING) 10.11 on page 715
⇐ "AbelianGroup" (ABELGRP) 7.1 on page 418
⇐ "OrderedCancellationAbelianMonoid" (OCAMON) 8.10 on page 567

Exports:

0	coerce	hash	latex	max
min	sample	subtractIfCan	zero?	?~=?
?*?	?+?	-?	?-?	?<?
?<=?	?=?	?>?	?>=?	

These exports come from (p567) OrderedCancellationAbelianMonoid():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
```

```
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
```

These exports come from (p418) AbelianGroup():

```
-? : % -> %
?*? : (Integer,%) -> %
?-? : (%,%) -> %
```

⟨*category OAGROUP OrderedAbelianGroup*⟩≡

```
)abbrev category OAGROUP OrderedAbelianGroup
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Ordered sets which are also abelian groups, such that the
++ addition preserves the ordering.
OrderedAbelianGroup(): Category ==
        Join(OrderedCancellationAbelianMonoid, AbelianGroup)
```

⟨*OAGROUP.dotabb*⟩≡

```
"OAGROUP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OAGROUP"];
"OAGROUP" -> "OCAMON"
"OAGROUP" -> "ABELGRP"
```

⟨*OAGROUP.dotfull*⟩≡

```
"OrderedAbelianGroup()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OAGROUP"];
"OrderedAbelianGroup()" -> "OrderedCancellationAbelianMonoid()"
"OrderedAbelianGroup()" -> "AbelianGroup()"
```

⟨*OAGROUP.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

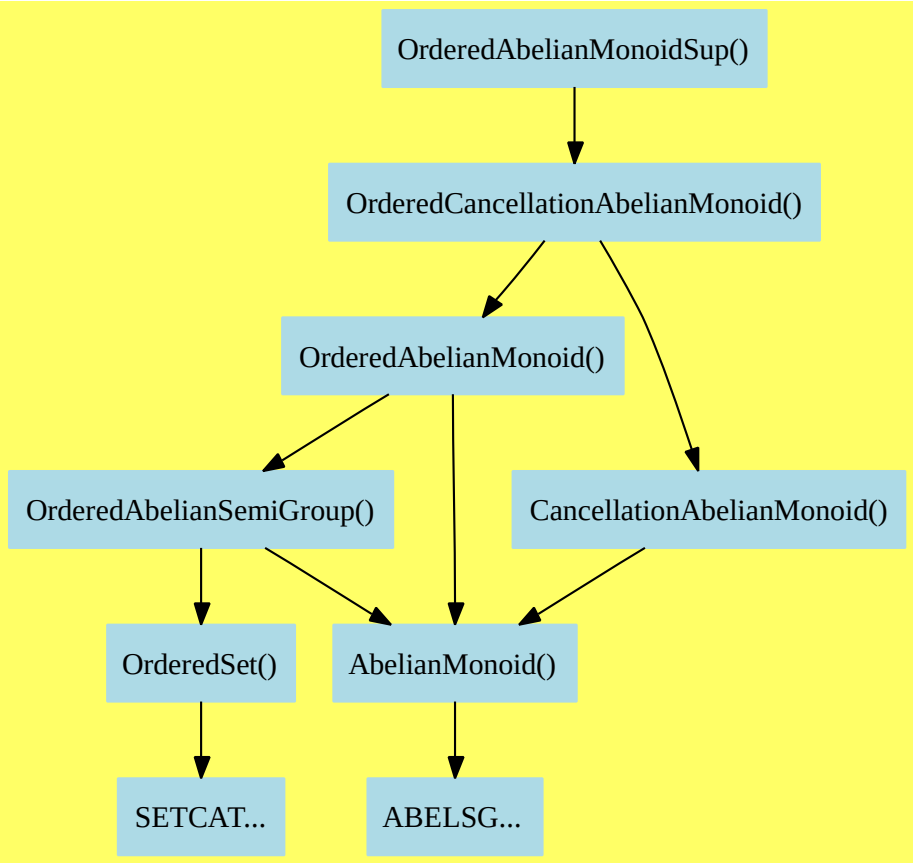
"OrderedAbelianGroup()" [color=lightblue];
"OrderedAbelianGroup()" -> "OrderedCancellationAbelianMonoid()"
"OrderedAbelianGroup()" -> "AbelianGroup()"

"OrderedCancellationAbelianMonoid()" [color=lightblue];
"OrderedCancellationAbelianMonoid()" -> "OAMON..."
"OrderedCancellationAbelianMonoid()" -> "CABMON..."

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"REPDB..." [color="#00EE00"];
"OAMON..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

9.6 OrderedAbelianMonoidSup (OAMONS)



See:

⇐ "DirectProductCategory" (DIRPCAT) 12.1 on page 815
⇐ "OrderedCancellationAbelianMonoid" (OCAMON) 8.10 on page 567

Exports:

0	coerce	hash	latex	max
min	sample	subtractIfCan	sup	zero?
?~=?	?*?	?<=?	?+?	?<?
?=?	?>?	?>=?		

These are directly exported but not implemented:

```
sup : (%,%) -> %
```

These exports come from (p567) OrderedCancellationAbelianMonoid():

```
0 : () -> %
```

```
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

⟨*category OAMONS OrderedAbelianMonoidSup*⟩≡

```
)abbrev category OAMONS OrderedAbelianMonoidSup
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This domain is an OrderedAbelianMonoid with a \spadfun{sup}
++ operation added. The purpose of the \spadfun{sup} operator
++ in this domain is to act as a supremum with respect to the
++ partial order imposed by \spadop{-}, rather than with respect to
++ the total \spad{>} order (since that is "max").
++
++ Axioms:
++   \spad{sup(a,b)-a \~~= "failed"}
++   \spad{sup(a,b)-b \~~= "failed"}
++   \spad{x-a \~~= "failed" and x-b \~~= "failed" => x >= sup(a,b)}

OrderedAbelianMonoidSup(): Category == OrderedCancellationAbelianMonoid with
  sup: (%,%) -> %
    ++ sup(x,y) returns the least element from which both
    ++ x and y can be subtracted.
```

⟨*OAMONS.dotabb*⟩≡
```
"OAMONS"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OAMONS"];
"OAMONS" -> "OCAMON"
```

⟨*OAMONS.dotfull*⟩≡
```
"OrderedAbelianMonoidSup()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OAMONS"];
"OrderedAbelianMonoidSup()" -> "OrderedCancellationAbelianMonoid()"
```

⟨*OAMONS.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedAbelianMonoidSup()" [color=lightblue];
"OrderedAbelianMonoidSup()" -> "OrderedCancellationAbelianMonoid()"

"OrderedCancellationAbelianMonoid()" [color=lightblue];
"OrderedCancellationAbelianMonoid()" -> "OrderedAbelianMonoid()"
"OrderedCancellationAbelianMonoid()" -> "CancellationAbelianMonoid()"

"OrderedAbelianMonoid()" [color=lightblue];
"OrderedAbelianMonoid()" -> "OrderedAbelianSemiGroup()"
"OrderedAbelianMonoid()" -> "AbelianMonoid()"

"OrderedAbelianSemiGroup()" [color=lightblue];
"OrderedAbelianSemiGroup()" -> "OrderedSet()"
"OrderedAbelianSemiGroup()" -> "AbelianMonoid()"

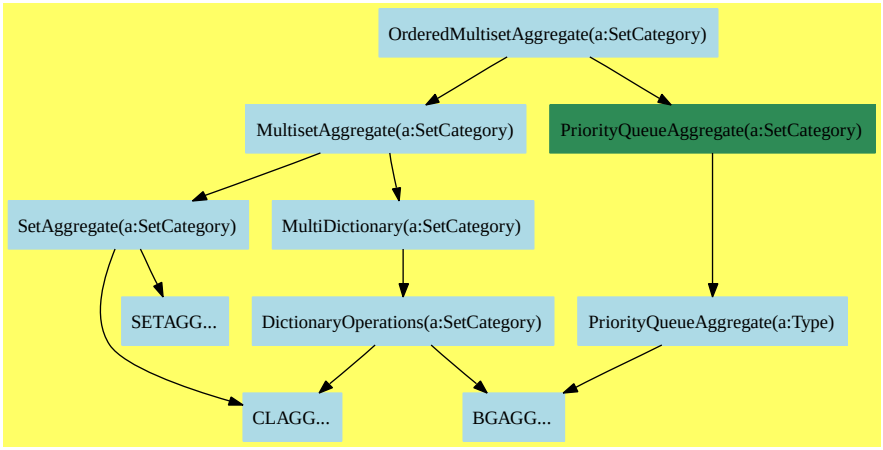
"OrderedSet()" [color=lightblue];
"OrderedSet()" -> "SETCAT..."

"CancellationAbelianMonoid()" [color=lightblue];
"CancellationAbelianMonoid()" -> "AbelianMonoid()"

"AbelianMonoid()" [color=lightblue];
"AbelianMonoid()" -> "ABELSG..."

"SETCAT..." [color=lightblue];
"ABELSG..." [color=lightblue];
}
```

9.7 OrderedMultisetAggregate (OMSAGG)



See:

⇐ "MultisetAggregate" (MSETAGG) 8.7 on page 548
⇐ "PriorityQueueAggregate" (PRQAGG) 6.11 on page 386

Exports:

any?	bag	brace	coerce
construct	convert	copy	count
dictionary	difference	empty	empty?
eq?	duplicates	eval	every?
extract!	find	hash	insert!
inspect	intersect	latex	less?
map	map!	max	member?
members	merge	merge!	min
more?	parts	reduce	remove
remove!	removeDuplicates	removeDuplicates!	sample
select	select!	set	size?
subset?	symmetricDifference	union	# ?
?<?	?=?	?~?	

Attributes exported:

- **partiallyOrderedSet** is true if a set with $<$ which is transitive, but not$(a < b$ or $a = b)$ does not necessarily imply $b < a$.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

- **nil**

These are directly exported but not implemented:

```
min : % -> S
```

These exports come from (p548) MultisetAggregate(S:OrderedSet):

```
any? : ((S -> Boolean),%) -> Boolean if $ has finiteAggregate
bag : List S -> %
brace : () -> %
brace : List S -> %
coerce : % -> OutputForm
construct : List S -> %
convert : % -> InputForm if S has KONVERT INFORM
copy : % -> %
count : (S,%) -> NonNegativeInteger
    if S has SETCAT and $ has finiteAggregate
count : ((S -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
dictionary : List S -> %
dictionary : () -> %
difference : (%,S) -> %
difference : (%,%) -> %
duplicates : % ->
    List Record(entry: S,count: NonNegativeInteger)
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List S,List S) -> %
    if S has EVALAB S and S has SETCAT
eval : (%,S,S) -> %
    if S has EVALAB S and S has SETCAT
eval : (%,Equation S) -> %
    if S has EVALAB S and S has SETCAT
eval : (%,List Equation S) -> %
    if S has EVALAB S and S has SETCAT
every? : ((S -> Boolean),%) -> Boolean
    if $ has finiteAggregate
extract! : % -> S
find : ((S -> Boolean),%) -> Union(S,"failed")
hash : % -> SingleInteger
insert! : (S,%) -> %
insert! : (S,%,NonNegativeInteger) -> %
inspect : % -> S
intersect : (%,%) -> %
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
map : ((S -> S),%) -> %
map! : ((S -> S),%) -> % if $ has shallowlyMutable
```

```
member? : (S,%) -> Boolean
    if S has SETCAT and $ has finiteAggregate
members : % -> List S if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List S if $ has finiteAggregate
reduce : (((S,S) -> S),%) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S) -> S
    if $ has finiteAggregate
reduce : (((S,S) -> S),%,S,S) -> S
    if S has SETCAT and $ has finiteAggregate
remove : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove : (S,%) -> %
    if S has SETCAT and $ has finiteAggregate
remove! : ((S -> Boolean),%) -> %
    if $ has finiteAggregate
remove! : (S,%) -> %
    if $ has finiteAggregate
removeDuplicates : % -> %
    if S has SETCAT and $ has finiteAggregate
removeDuplicates! : % -> %
sample : () -> %
select : ((S -> Boolean),%) -> % if $ has finiteAggregate
select! : ((S -> Boolean),%) -> % if $ has finiteAggregate
set : () -> %
set : List S -> %
size? : (%,NonNegativeInteger) -> Boolean
subset? : (%,%) -> Boolean
symmetricDifference : (%,%) -> %
union : (%,%) -> %
union : (%,S) -> %
union : (S,%) -> %
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
```

These exports come from (p386) PriorityQueueAggregate(S:OrderedSet):

```
max : % -> S
merge : (%,%) -> %
merge! : (%,%) -> %
```

⟨*category OMSAGG OrderedMultisetAggregate*⟩≡

```
)abbrev category OMSAGG OrderedMultisetAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
```

```
  ++ Related Constructors:
  ++ Also See:
  ++ AMS Classifications:
  ++ Keywords:
  ++ References:
  ++ Description:
  ++ An ordered-multiset aggregate is a multiset built over an ordered set S
  ++ so that the relative sizes of its entries can be assessed.
  ++ These aggregates serve as models for priority queues.
  OrderedMultisetAggregate(S:OrderedSet): Category ==
     Join(MultisetAggregate S,PriorityQueueAggregate S) with
     -- max: % -> S                      ++ smallest entry in the set
     -- duplicates: % -> List Record(entry:S,count:NonNegativeInteger)
          ++ to become an in order iterator
     -- parts: % -> List S               ++ in order iterator
        min: % -> S
          ++ min(u) returns the smallest entry in the multiset aggregate u.
```

⟨*OMSAGG.dotabb*⟩≡

```
  "OMSAGG"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=OMSAGG"];
  "OMSAGG" -> "MSETAGG"
  "OMSAGG" -> "PRQAGG"
```

⟨*OMSAGG.dotfull*⟩≡

```
  "OrderedMultisetAggregate(a:SetCategory)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=OMSAGG"];
  "OrderedMultisetAggregate(a:SetCategory)" ->
      "MultisetAggregate(a:SetCategory)"
  "OrderedMultisetAggregate(a:SetCategory)" ->
     "PriorityQueueAggregate(a:SetCategory)"
```

⟨*OMSAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedMultisetAggregate(a:SetCategory)" [color=lightblue];
"OrderedMultisetAggregate(a:SetCategory)" ->
    "MultisetAggregate(a:SetCategory)"
"OrderedMultisetAggregate(a:SetCategory)" ->
   "PriorityQueueAggregate(a:SetCategory)"

"MultisetAggregate(a:SetCategory)" [color=lightblue];
"MultisetAggregate(a:SetCategory)" -> "MultiDictionary(a:SetCategory)"
"MultisetAggregate(a:SetCategory)" -> "SetAggregate(a:SetCategory)"

"MultiDictionary(a:SetCategory)" [color=lightblue];
"MultiDictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"SetAggregate(a:SetCategory)" [color=lightblue];
"SetAggregate(a:SetCategory)" -> "SETAGG..."
"SetAggregate(a:SetCategory)" -> "CLAGG..."

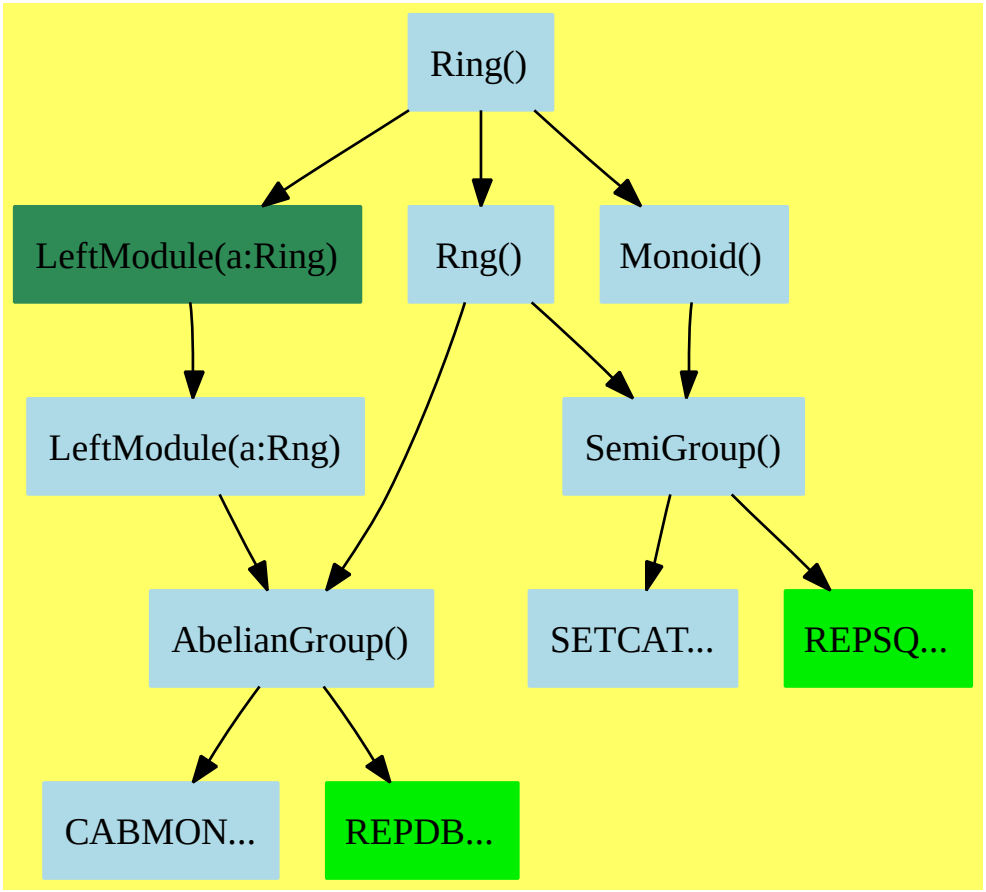
"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BGAGG..."
"DictionaryOperations(a:SetCategory)" -> "CLAGG..."

"PriorityQueueAggregate(a:SetCategory)" [color=seagreen];
"PriorityQueueAggregate(a:SetCategory)" -> "PriorityQueueAggregate(a:Type)"

"PriorityQueueAggregate(a:Type)" [color=lightblue];
"PriorityQueueAggregate(a:Type)" -> "BGAGG..."

"BGAGG..." [color=lightblue];
"CLAGG..." [color=lightblue];
"SETAGG..." [color=lightblue];
}
```

9.8 Ring (RING)



See:

⇒ “Algebra” (ALGEBRA) 11.1 on page 771
⇒ “AbelianMonoidRing” (AMR) 13.1 on page 905
⇒ “CharacteristicNonZero” (CHARNZ) 10.2 on page 677
⇒ “CharacteristicZero” (CHARZ) 10.3 on page 681
⇒ “CommutativeRing” (COMRING) 10.4 on page 685
⇒ “DifferentialExtension” (DIFEXT) 11.2 on page 777
⇒ “DifferentialRing” (DIFRING) 10.5 on page 690
⇒ “EntireRing” (ENTIRER) 10.6 on page 694
⇒ “FunctionSpace” (FS) 17.5 on page 1095
⇒ “LeftAlgebra” (LALG) 10.8 on page 704
⇒ “LinearlyExplicitRingOver” (LINEXP) 10.9 on page 707
⇒ “MonogenicLinearOperator” (MLO) 12.6 on page 862
⇒ “OrderedRing” (ORDRING) 10.11 on page 715

⇒ "PartialDifferentialRing" (PDRING) 10.12 on page 720
⇒ "UnivariateSkewPolynomialCategory" (OREPCAT) 10.17 on page 754
⇒ "XAlgebra" (XALG) 10.18 on page 765
⇒ "XFreeAlgebra" (XFALG) 11.8 on page 807
⇐ "LeftModule" (LMODULE) 8.5 on page 533
⇐ "Monoid" (MONOID) 5.10 on page 256
⇐ "Rng" (RNG) 8.13 on page 587

Exports:

1	0	characteristic	coerce	hash
latex	one?	recip	sample	subtractIfCan
zero?	?~=?	?*?	?**?	?^?
?+?	?-?	-?	?=?	

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are directly exported but not implemented:

```
characteristic : () -> NonNegativeInteger
```

These are implemented by this category:

```
coerce : Integer -> %
```

These exports come from (p587) Rng():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p256) Monoid():

```
1 : () -> %
one? : % -> Boolean
recip : % -> Union(%,"failed")
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

⟨*category RING Ring*⟩≡

```
)abbrev category RING Ring
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of rings with unity, always associative, but
++ not necessarily commutative.
Ring(): Category == Join(Rng,Monoid,LeftModule(%)) with
      characteristic: () -> NonNegativeInteger
        ++ characteristic() returns the characteristic of the ring
        ++ this is the smallest positive integer n such that
        ++ \spad{n*x=0} for all x in the ring, or zero if no such n
        ++ exists.
        --We can not make this a constant, since some domains are mutable
      coerce: Integer -> %
        ++ coerce(i) converts the integer i to a member of the given domain.
--    recip: % -> Union(%,"failed") -- inherited from Monoid
      unitsKnown
        ++ recip truly yields
        ++ reciprocal or "failed" if not a unit.
        ++ Note: \spad{recip(0) = "failed"}.
   add
      n:Integer
      coerce(n) == n * 1$%
```

⟨*RING.dotabb*⟩≡

```
"RING"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RING"];
"RING" -> "RNG"
"RING" -> "MONOID"
"RING" -> "LMODULE"
```

⟨*RING.dotfull*⟩≡
```
"Ring()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RING"];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"
```

⟨*RING.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

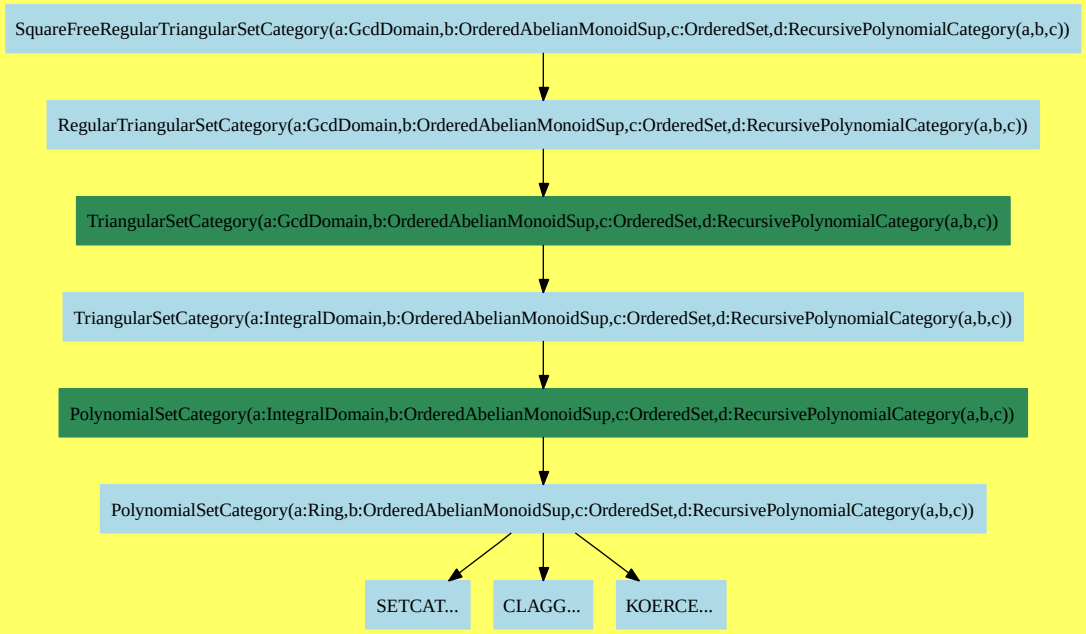
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

9.9 SquareFreeRegularTriangularSetCategory (SFRTCAT)



See:

⇒ “SquareFreeNormalizedTriangularSetCategory” (SNTSCAT) 10.15 on page 740

⇐ “RegularTriangularSetCategory” (RSETCAT) 8.11 on page 570

Exports:

algebraic?
algebraicCoefficients?
algebraicVariables
any?
augment
autoReduced?
basicSet
coerce
coHeight
collect
collectQuasiMonic
collectUnder
collectUpper
construct
copy
convert
count
degree
empty
empty?
eq?
eval
every?
extend
extendIfCan
find
first
hash
headReduce
headReduced?
headRemainder
infRittWu?
initiallyReduce
initiallyReduced?
initials
internalAugment
intersect
invertible?
invertibleElseSplit?
invertibleSet
last
lastSubResultant
lastSubResultantElseSplit
latex
less?
mainVariable?
mainVariables
map
map!
member?
members
more?
mvar
normalized?
parts
purelyAlgebraic?
purelyAlgebraicLeadingMonomial?
purelyTranscendental?
quasiComponent
reduce
reduceByQuasiMonic
reduced?
remainder
remove
removeDuplicates
removeZero
rest
retract
retractIfCan
rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder
rewriteSetWithReduction
roughBase?
roughEqualIdeals?
roughSubIdeal?
roughUnitIdeal?
sample
select
size?
sort
squareFreePart
stronglyReduce
stronglyReduced?
triangular?
trivialIdeal?
variables
zeroSetSplit
zeroSetSplitIntoTriangularSystems
#?
?=?
?~=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These exports come from (p570) RegularTriangularSetCategory(R,E,V,P) where R:GcdDomain, E:OrderedAbelianMonoidSup, V:OrderedSet, P:RecursivePolynomialCategory(R,E,V)):

```
algebraic? : (V,%) -> Boolean
algebraicCoefficients? : (P,%) -> Boolean
algebraicVariables : % -> List V
any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
augment : (List P,List %) -> List %
augment : (List P,%) -> List %
augment : (P,List %) -> List %
augment : (P,%) -> List %
autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
basicSet :
 (List P,(P -> Boolean),((P,P) -> Boolean)) ->
   Union(Record(bas: %,top: List P),"failed")
basicSet :
 (List P,((P,P) -> Boolean)) ->
   Union(Record(bas: %,top: List P),"failed")
coerce : % -> List P
coerce : % -> OutputForm
coHeight : % -> NonNegativeInteger if V has FINITE
collect : (%,V) -> %
collectQuasiMonic : % -> %
collectUnder : (%,V) -> %
collectUpper : (%,V) -> %
construct : List P -> %
convert : % -> InputForm if P has KONVERT INFORM
copy : % -> %
count : ((P -> Boolean),%) -> NonNegativeInteger
  if $ has finiteAggregate
count : (P,%) -> NonNegativeInteger
  if P has SETCAT
  and $ has finiteAggregate
degree : % -> NonNegativeInteger
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
```

```
eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
extend : (List P,List %) -> List %
extend : (List P,%) -> List %
extend : (P,List %) -> List %
extend : (P,%) -> List %
extend : (%,P) -> %
extendIfCan : (%,P) -> Union(%,"failed")
find : ((P -> Boolean),%) -> Union(P,"failed")
first : % -> Union(P,"failed")
hash : % -> SingleInteger
headReduce : (P,%) -> P
headReduced? : % -> Boolean
headReduced? : (P,%) -> Boolean
headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
infRittWu? : (%,%) -> Boolean
initiallyReduce : (P,%) -> P
initiallyReduced? : % -> Boolean
initiallyReduced? : (P,%) -> Boolean
initials : % -> List P
internalAugment : (P,%) -> %
internalAugment : (List P,%) -> %
intersect : (P,List %) -> List %
intersect : (List P,%) -> List %
intersect : (P,%) -> List %
intersect : (List P,List %) -> List %
invertible? : (P,%) -> Boolean
invertible? : (P,%) -> List Record(val: Boolean,tower: %)
invertibleElseSplit? : (P,%) -> Union(Boolean,List %)
invertibleSet : (P,%) -> List %
last : % -> Union(P,"failed")
lastSubResultant : (P,P,%) -> List Record(val: P,tower: %)
lastSubResultantElseSplit : (P,P,%) -> Union(P,List %)
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
mainVariable? : (V,%) -> Boolean
mainVariables : % -> List V
map : ((P -> P),%) -> %
map! : ((P -> P),%) -> % if $ has shallowlyMutable
member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
members : % -> List P if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
mvar : % -> V
normalized? : % -> Boolean
normalized? : (P,%) -> Boolean
parts : % -> List P if $ has finiteAggregate
purelyAlgebraic? : (P,%) -> Boolean
purelyAlgebraic? : % -> Boolean
```

```
purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
purelyTranscendental? : (P,%) -> Boolean
quasiComponent : % -> Record(close: List P,open: List P)
reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P,P) -> P
  if P has SETCAT
  and $ has finiteAggregate
reduceByQuasiMonic : (P,%) -> P
reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
remainder : (P,%) -> Record(rnum: R,polnum: P,den: R)
  if R has INTDOM
remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
removeZero : (P,%) -> P
rest : % -> Union(%,"failed")
retract : List P -> %
retractIfCan : List P -> Union(%,"failed")
rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
rewriteSetWithReduction :
   (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List P
roughBase? : % -> Boolean if R has INTDOM
roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
roughUnitIdeal? : % -> Boolean if R has INTDOM
sample : () -> %
select : (%,V) -> Union(P,"failed")
select : ((P -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
sort : (%,V) -> Record(under: %,floor: %,upper: %)
squareFreePart : (P,%) -> List Record(val: P,tower: %)
stronglyReduce : (P,%) -> P
stronglyReduced? : (P,%) -> Boolean
stronglyReduced? : % -> Boolean
triangular? : % -> Boolean if R has INTDOM
trivialIdeal? : % -> Boolean
variables : % -> List V
zeroSetSplit : List P -> List %
zeroSetSplit : (List P,Boolean) -> List %
zeroSetSplitIntoTriangularSystems :
  List P -> List Record(close: %,open: List P)
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category SFRTCAT SquareFreeRegularTriangularSetCategory*⟩≡

```
)abbrev category SFRTCAT SquareFreeRegularTriangularSetCategory
```

```
++ Author: Marc Moreno Maza
++ Date Created: 09/03/1996
++ Date Last Updated: 09/10/1998
++ Basic Functions:
++ Related Constructors:
++ Also See: essai Graphisme
++ AMS Classifications:
++ Keywords: polynomial, multivariate, ordered variables set
++ Description:
++ The category of square-free regular triangular sets.
++ A regular triangular set \spad{ts} is square-free if
++ the gcd of any polynomial \spad{p} in \spad{ts} and
++ \spad{differentiate(p,mvar(p))} w.r.t.
++ \axiomOpFrom{collectUnder}{TriangularSetCategory}
++                  (ts,\axiomOpFrom{mvar}{RecursivePolynomialCategory}(p))
++ has degree zero w.r.t. \spad{mvar(p)}. Thus any square-free regular
++ set defines a tower of square-free simple extensions.\newline
++ References :
++  [1] D. LAZARD "A new method for solving algebraic systems of
++      positive dimension" Discr. App. Math. 33:147-160,1991
++  [2] M. KALKBRENER "Algorithmic properties of polynomial rings"
++      Habilitation Thesis, ETZH, Zurich, 1995.
++  [3] M. MORENO MAZA "A new algorithm for computing triangular
++      decomposition of algebraic varieties" NAG Tech. Rep. 4/98.
SquareFreeRegularTriangularSetCategory(R:GcdDomain,E:OrderedAbelianMonoidSup,_
  V:OrderedSet,P:RecursivePolynomialCategory(R,E,V)):
        Category ==
   RegularTriangularSetCategory(R,E,V,P)
```

⟨*SFRTCAT.dotabb*⟩≡

```
"SFRTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SFRTCAT"];
"SFRTCAT" -> "RSETCAT"
```

⟨*SFRTCAT.dotfull*⟩≡

```
"SquareFreeRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SFRTCAT"];
"SquareFreeRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,
 ->
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recursive
```

⟨*SFRTCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"SquareFreeRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:O
 [color=lightblue];
"SquareFreeRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:O
 ->
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,

"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,
 [color=lightblue];
"RegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,
  ->
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recur

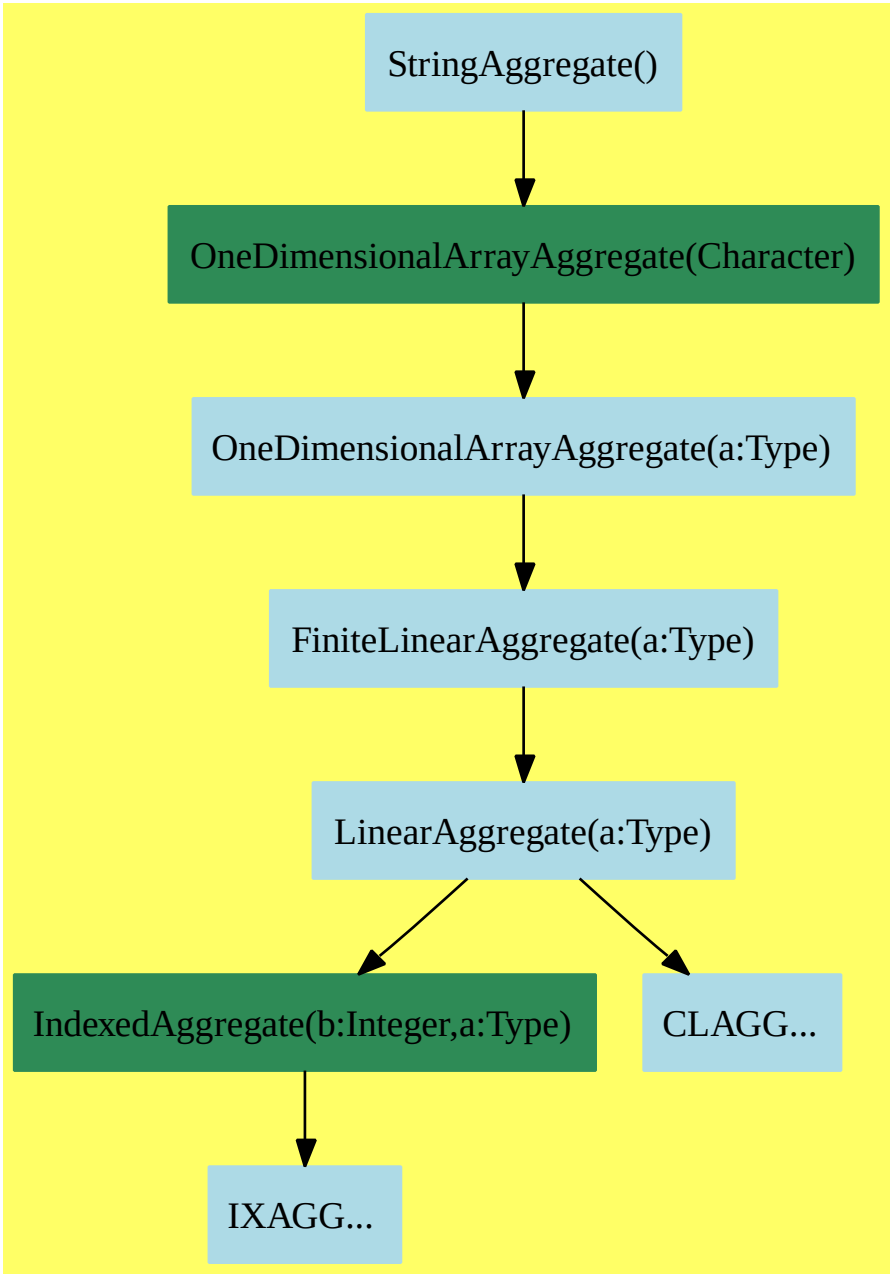
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recur
 [color=seagreen];
"TriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recur
 ->
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
 [color=lightblue];
"TriangularSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
  -> "PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedS

"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
 [color=seagreen];
"PolynomialSetCategory(a:IntegralDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,d:
  -> "PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:Recur

"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 [color=lightblue];
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 -> "SETCAT..."
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 -> "CLAGG..."
"PolynomialSetCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet,d:RecursiveP
 -> "KOERCE..."

"SETCAT..." [color=lightblue];
"KOERCE..." [color=lightblue];
"CLAGG..." [color=lightblue];
}
```

9.10 StringAggregate (SRAGG)



See:

⇒ "StringCategory" (STRICAT) 10.16 on page 747

⇐ "OneDimensionalArrayAggregate" (A1AGG) 8.9 on page 556

Exports:

any?	coerce	concat	construct	copy
convert	copyInto!	count	delete	elt
empty	empty?	entries	entry?	eval
every?	eq?	fill!	find	first
hash	index?	indices	insert	latex
leftTrim	less?	lowerCase	lowerCase!	map
map!	match	match?	max	maxIndex
member?	members	merge	min	minIndex
more?	new	parts	position	prefix?
qelt	qsetelt!	reduce	remove	removeDuplicates
replace	reverse	reverse!	rightTrim	sample
setelt	size?	sort	sort!	sorted?
split	substring?	suffix?	swap!	trim
upperCase	upperCase!	#	?~=?	?.?
?<?	?<=?	?=?	?>?	?>=?

Attributes exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

These are directly exported but not implemented:

```
leftTrim : (%,Character) -> %
leftTrim : (%,CharacterClass) -> %
lowerCase! : % -> %
match : (%,%,Character) -> NonNegativeInteger
match? : (%,%,Character) -> Boolean
position : (CharacterClass,%,Integer) -> Integer
position : (%,%,Integer) -> Integer
replace : (%,UniversalSegment Integer,%) -> %
rightTrim : (%,Character) -> %
rightTrim : (%,CharacterClass) -> %
split : (%,Character) -> List %
split : (%,CharacterClass) -> List %
substring? : (%,%,Integer) -> Boolean
suffix? : (%,%) -> Boolean
upperCase! : % -> %
```

These are implemented by this category:

```
coerce : Character -> %
lowerCase : % -> %
prefix? : (%,%) -> Boolean
trim : (%,CharacterClass) -> %
trim : (%,Character) -> %
upperCase : % -> %
?.? : (%,%) -> %
```

These exports come from (p556) OneDimensionalArrayAggregate(Character):

```
any? : ((Character -> Boolean),%) -> Boolean
         if $ has finiteAggregate
coerce : % -> OutputForm if Character has SETCAT
concat : List % -> %
concat : (%,%) -> %
concat : (Character,%) -> %
concat : (%,Character) -> %
construct : List Character -> %
convert : % -> InputForm
         if Character has KONVERT INFORM
copy : % -> %
copyInto! : (%,%,Integer) -> %
         if $ has shallowlyMutable
count : (Character,%) -> NonNegativeInteger
         if Character has SETCAT
         and $ has finiteAggregate
count : ((Character -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
elt : (%,Integer,Character) -> Character
empty : () -> %
empty? : % -> Boolean
entries : % -> List Character
entry? : (Character,%) -> Boolean
         if $ has finiteAggregate
         and Character has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List Character,List Character) -> %
         if Character has EVALAB CHAR
         and Character has SETCAT
eval : (%,Character,Character) -> %
         if Character has EVALAB CHAR
         and Character has SETCAT
eval : (%,Equation Character) -> %
         if Character has EVALAB CHAR
         and Character has SETCAT
eval : (%,List Equation Character) -> %
         if Character has EVALAB CHAR
         and Character has SETCAT
```

```
every? : ((Character -> Boolean),%) -> Boolean
         if $ has finiteAggregate
fill! : (%,Character) -> %
         if $ has shallowlyMutable
find : ((Character -> Boolean),%) -> Union(Character,"failed")
first : % -> Character
         if Integer has ORDSET
hash : % -> SingleInteger
         if Character has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (Character,%,Integer) -> %
insert : (%,%,Integer) -> %
latex : % -> String if Character has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((Character -> Character),%) -> %
map : (((Character,Character) -> Character),%,%) -> %
map! : ((Character -> Character),%) -> %
         if $ has shallowlyMutable
max : (%,%) -> % if Character has ORDSET
maxIndex : % -> Integer if Integer has ORDSET
member? : (Character,%) -> Boolean
         if Character has SETCAT
         and $ has finiteAggregate
members : % -> List Character
         if $ has finiteAggregate
merge : (%,%) -> % if Character has ORDSET
merge : (((Character,Character) -> Boolean),%,%) -> %
min : (%,%) -> % if Character has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,Character) -> %
parts : % -> List Character if $ has finiteAggregate
position : (Character,%) -> Integer
         if Character has SETCAT
position : ((Character -> Boolean),%) -> Integer
position : (Character,%,Integer) -> Integer
         if Character has SETCAT
qelt : (%,Integer) -> Character
qsetelt! : (%,Integer,Character) -> Character
         if $ has shallowlyMutable
reduce : (((Character,Character) -> Character),%) -> Character
         if $ has finiteAggregate
reduce :
  (((Character,Character) -> Character),%,Character) -> Character
         if $ has finiteAggregate
reduce :
  (((Character,Character) -> Character),%,Character,Character)
      -> Character
         if Character has SETCAT and $ has finiteAggregate
```

```
remove : ((Character -> Boolean),%) -> %
         if $ has finiteAggregate
remove : (Character,%) -> %
         if Character has SETCAT
         and $ has finiteAggregate
removeDuplicates : % -> %
         if Character has SETCAT
         and $ has finiteAggregate
reverse : % -> %
reverse! : % -> % if $ has shallowlyMutable
sample : () -> %
select : ((Character -> Boolean),%) -> %
         if $ has finiteAggregate
setelt : (%,UniversalSegment Integer,Character) -> Character
         if $ has shallowlyMutable
sort! : (((Character,Character) -> Boolean),%) -> %
         if $ has shallowlyMutable
sorted? : (((Character,Character) -> Boolean),%) -> Boolean
setelt : (%,Integer,Character) -> Character
         if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
sort : % -> % if Character has ORDSET
sort : (((Character,Character) -> Boolean),%) -> %
sort! : % -> %
         if Character has ORDSET
         and $ has shallowlyMutable
sorted? : % -> Boolean if Character has ORDSET
swap! : (%,Integer,Integer) -> Void
         if $ has shallowlyMutable
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean if Character has SETCAT
?.? : (%,UniversalSegment Integer) -> %
?.? : (%,Integer) -> Character
?<? : (%,%) -> Boolean if Character has ORDSET
?<=? : (%,%) -> Boolean if Character has ORDSET
?=? : (%,%) -> Boolean if Character has SETCAT
?>? : (%,%) -> Boolean if Character has ORDSET
?>=? : (%,%) -> Boolean if Character has ORDSET
```

⟨*category SRAGG StringAggregate*⟩≡

```
)abbrev category SRAGG StringAggregate
++ Author: Stephen Watt and Michael Monagan.
++ revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
```

```
++ Keywords:
++ References:
++ Description:
++ A string aggregate is a category for strings, that is,
++ one dimensional arrays of characters.
StringAggregate: Category == OneDimensionalArrayAggregate Character with
    lowerCase       : % -> %
      ++ lowerCase(s) returns the string with all characters in lower case.
    lowerCase_!: % -> %
      ++ lowerCase!(s) destructively replaces the alphabetic characters
      ++ in s by lower case.
    upperCase       : % -> %
      ++ upperCase(s) returns the string with all characters in upper case.
    upperCase_!: % -> %
      ++ upperCase!(s) destructively replaces the alphabetic characters
      ++ in s by upper case characters.
    prefix?         : (%, %) -> Boolean
      ++ prefix?(s,t) tests if the string s is the initial substring of t.
      ++ Note: \axiom{prefix?(s,t) ==
      ++   reduce(and,[s.i = t.i for i in 0..maxIndex s])}.
    suffix?         : (%, %) -> Boolean
      ++ suffix?(s,t) tests if the string s is the final substring of t.
      ++ Note: \axiom{suffix?(s,t) ==
      ++  reduce(and,[s.i = t.(n - m + i) for i in 0..maxIndex s])}
      ++ where m and n denote the maxIndex of s and t respectively.
    substring?: (%, %, Integer) -> Boolean
      ++ substring?(s,t,i) tests if s is a substring of t beginning at
      ++ index i.
      ++ Note: \axiom{substring?(s,t,0) = prefix?(s,t)}.
    match: (%, %, Character) -> NonNegativeInteger
      ++ match(p,s,wc) tests if pattern \axiom{p} matches subject \axiom{s}
      ++ where \axiom{wc} is a wild card character. If no match occurs,
      ++ the index \axiom{0} is returned; otheriwse, the value returned
      ++ is the first index of the first character in the subject matching
      ++ the subject (excluding that matched by an initial wild-card).
      ++ For example, \axiom{match("*to*","yorktown","*")} returns \axiom{5}
      ++ indicating a successful match starting at index \axiom{5} of
      ++ \axiom{"yorktown"}.
    match?: (%, %, Character) -> Boolean
      ++ match?(s,t,c) tests if s matches t except perhaps for
      ++ multiple and consecutive occurrences of character c.
      ++ Typically c is the blank character.
    replace         : (%, UniversalSegment(Integer), %) -> %
      ++ replace(s,i..j,t) replaces the substring \axiom{s(i..j)}
      ++ of s by string t.
    position        : (%, %, Integer) -> Integer
```

```
      ++ position(s,t,i) returns the position j of the substring s in
      ++ string t, where \axiom{j >= i} is required.
    position        : (CharacterClass, %, Integer) -> Integer
      ++ position(cc,t,i) returns the position \axiom{j >= i} in t of
      ++ the first character belonging to cc.
    coerce          : Character -> %
      ++ coerce(c) returns c as a string s with the character c.
    split: (%, Character) -> List %
      ++ split(s,c) returns a list of substrings delimited by character c.
    split: (%, CharacterClass) -> List %
      ++ split(s,cc) returns a list of substrings delimited by
      ++ characters in cc.
    trim: (%, Character) -> %
      ++ trim(s,c) returns s with all characters c deleted from right
      ++ and left ends.
      ++ For example, \axiom{trim(" abc ", char " ")} returns \axiom{"abc"}.
    trim: (%, CharacterClass) -> %
      ++ trim(s,cc) returns s with all characters in cc deleted from right
      ++ and left ends.
      ++ For example, \axiom{trim("(abc)", charClass "()")}
      ++ returns \axiom{"abc"}.
    leftTrim: (%, Character) -> %
      ++ leftTrim(s,c) returns s with all leading characters c deleted.
      ++ For example, \axiom{leftTrim("  abc  ", char " ")}
      ++ returns \axiom{"abc  "}.
    leftTrim: (%, CharacterClass) -> %
      ++ leftTrim(s,cc) returns s with all leading characters in cc deleted.
      ++ For example, \axiom{leftTrim("(abc)", charClass "()")}
      ++ returns \axiom{"abc)"}.
    rightTrim: (%, Character) -> %
      ++ rightTrim(s,c) returns s with all trailing occurrences of c deleted.
      ++ For example, \axiom{rightTrim("  abc  ", char " ")}
      ++ returns \axiom{"  abc"}.
    rightTrim: (%, CharacterClass) -> %
      ++ rightTrim(s,cc) returns s with all trailing occurences of
      ++ characters in cc deleted.
      ++ For example, \axiom{rightTrim("(abc)", charClass "()")}
      ++ returns \axiom{"(abc"}.
    elt: (%, %) -> %
      ++ elt(s,t) returns the concatenation of s and t. It is provided to
      ++ allow juxtaposition of strings to work as concatenation.
      ++ For example, \axiom{"smoo" "shed"} returns \axiom{"smooshed"}.
  add
    trim(s: %, c:  Character)      == leftTrim(rightTrim(s, c),  c)
    trim(s: %, cc: CharacterClass) == leftTrim(rightTrim(s, cc), cc)
    lowerCase s           == lowerCase_! copy s
```

```
    upperCase s           == upperCase_! copy s
    prefix?(s, t)         == substring?(s, t, minIndex t)
    coerce(c:Character):% == new(1, c)
    elt(s:%, t:%): %      == concat(s,t)$%
```

⟨*SRAGG.dotabb*⟩≡

```
"SRAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=SRAGG"];
"SRAGG" -> "A1AGG"
```

⟨*SRAGG.dotfull*⟩≡

```
"StringAggregate()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SRAGG"];
"StringAggregate()" -> "OneDimensionalArrayAggregate(Character)"
```

⟨*SRAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"StringAggregate()" [color=lightblue];
"StringAggregate()" -> "OneDimensionalArrayAggregate(Character)"

"OneDimensionalArrayAggregate(Character)" [color=seagreen];
"OneDimensionalArrayAggregate(Character)" ->
    "OneDimensionalArrayAggregate(a:Type)"

"OneDimensionalArrayAggregate(a:Type)" [color=lightblue];
"OneDimensionalArrayAggregate(a:Type)" ->
    "FiniteLinearAggregate(a:Type)"

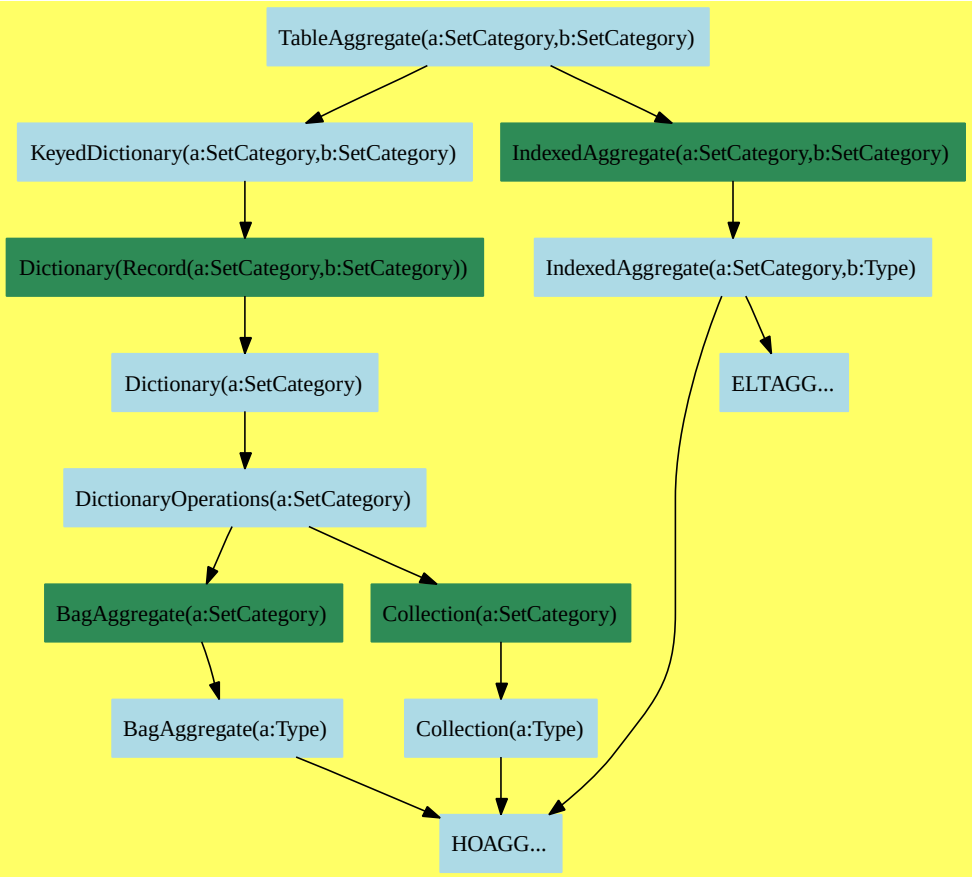
"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "CLAGG..."

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" -> "IXAGG..."

"CLAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
}
```

9.11 TableAggregate (TBAGG)



See:

⇒ “AssociationListAggregate” (ALAGG) 10.1 on page 667
⇐ “IndexedAggregate” (IXAGG) 5.8 on page 246
⇐ “KeyedDictionary” (KDAGG) 8.3 on page 510

Exports:

any?	bag	coerce	construct	convert
copy	count	dictionary	elt	empty
empty?	entries	entry?	eq?	eval
every?	extract!	fill!	find	first
hash	index?	indices	insert!	inspect
key?	keys	latex	less?	map
map!	maxIndex	member?	members	minIndex
more?	parts	qelt	qsetelt!	reduce
remove	remove!	removeDuplicates	sample	search
select	select!	setelt	size?	swap!
table	#?	?=?	?.?	?~=?

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **nil**

Attributes Used:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are directly exported but not implemented:

```
setelt : (%,Key,Entry) -> Entry
```

These are implemented by this category:

```
any? : ((Entry -> Boolean),%) -> Boolean
    if $ has finiteAggregate
coerce : % -> OutputForm
         if Entry has SETCAT
         or Record(key: Key,entry: Entry) has SETCAT
count : ((Entry -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
elt : (%,Key,Entry) -> Entry
entries : % -> List Entry
every? : ((Entry -> Boolean),%) -> Boolean
        if $ has finiteAggregate
extract! : % -> Record(key: Key,entry: Entry)
find :
  ((Record(key: Key,entry: Entry) -> Boolean),%)
     -> Union(Record(key: Key,entry: Entry),"failed")
index? : (Key,%) -> Boolean
indices : % -> List Key
insert! : (Record(key: Key,entry: Entry),%) -> %
```

```
inspect : % -> Record(key: Key,entry: Entry)
map : (((Entry,Entry) -> Entry),%,%) -> %
map : ((Record(key: Key,entry: Entry)
      -> Record(key: Key,entry: Entry)),%) -> %
map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
map! :
  ((Record(key: Key,entry: Entry)
      -> Record(key: Key,entry: Entry)),%)
       -> % if $ has shallowlyMutable
parts : % -> List Entry if $ has finiteAggregate
parts : % -> List Record(key: Key,entry: Entry)
         if $ has finiteAggregate
remove! : (Key,%) -> Union(Entry,"failed")
table : () -> %
table : List Record(key: Key,entry: Entry) -> %
?.? : (%,Key) -> Entry
?=? : (%,%) -> Boolean
         if Entry has SETCAT
         or Record(key: Key,entry: Entry) has SETCAT
```

These exports come from (p510) KeyedDictionary(Key,Entry)
where Key:SetCategory and Entry:SetCategory
and RecKE=Record(key: Key,entry: Entry):

```
any? : ((RecKE -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List RecKE -> %
construct : List RecKE -> %
convert : % -> InputForm if RecKE has KONVERT INFORM
copy : % -> %
count : (Entry,%) -> NonNegativeInteger
         if Entry has SETCAT
         and $ has finiteAggregate
count : ((RecKE -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
dictionary : () -> %
dictionary : List RecKE -> %
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List RecKE,List RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,RecKE,RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,Equation RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,List Equation RecKE) -> %
```

```
          if RecKE has EVALAB RecKE
          and RecKE has SETCAT
every? : ((RecKE -> Boolean),%) -> Boolean
          if $ has finiteAggregate
key? : (Key,%) -> Boolean
keys : % -> List Key
hash : % -> SingleInteger
          if Entry has SETCAT
          or RecKE has SETCAT
latex : % -> String
          if Entry has SETCAT
          or RecKE has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
member? : (RecKE,%) -> Boolean
    if RecKE has SETCAT
     and $ has finiteAggregate
members : % -> List RecKE if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
reduce :
 (((RecKE,RecKE) -> RecKE),%) -> RecKE
          if $ has finiteAggregate
reduce :
 (((RecKE,RecKE)->RecKE),%,RecKE) -> RecKE
          if $ has finiteAggregate
reduce :
 (((RecKE,RecKE)->RecKE),%,RecKE,RecKE)
   -> RecKE
          if RecKE has SETCAT
          and $ has finiteAggregate
remove : ((RecKE -> Boolean),%) -> %
          if $ has finiteAggregate
remove : (RecKE,%) -> %
          if RecKE has SETCAT
          and $ has finiteAggregate
remove! : ((RecKE -> Boolean),%) -> %
          if $ has finiteAggregate
remove! : (RecKE,%) -> % if $ has finiteAggregate
removeDuplicates : % -> %
          if RecKE has SETCAT
          and $ has finiteAggregate
sample : () -> %
search : (Key,%) -> Union(Entry,"failed")
select : ((RecKE -> Boolean),%) -> %
          if $ has finiteAggregate
select! : ((RecKE -> Boolean),%) -> %
          if $ has finiteAggregate
#? : % -> NonNegativeInteger if $ has finiteAggregate
?~=? : (%,%) -> Boolean
          if Entry has SETCAT
          or RecKE has SETCAT
```

These exports come from (p246) IndexedAggregate(Key,Entry))
where Key:SetCategory and Entry:SetCategory
and RecKE=Record(key: Key,entry: Entry):

```
count : (RecKE,%) -> NonNegativeInteger
         if RecKE has SETCAT
         and $ has finiteAggregate
entry? : (Entry,%) -> Boolean
         if $ has finiteAggregate
         and Entry has SETCAT
eval : (%,List Equation Entry) -> %
         if Entry has EVALAB Entry
         and Entry has SETCAT
eval : (%,Equation Entry) -> %
         if Entry has EVALAB Entry
         and Entry has SETCAT
eval : (%,Entry,Entry) -> %
         if Entry has EVALAB Entry
         and Entry has SETCAT
eval : (%,List Entry,List Entry) -> %
         if Entry has EVALAB Entry
         and Entry has SETCAT
fill! : (%,Entry) -> % if $ has shallowlyMutable
first : % -> Entry if Key has ORDSET
map : ((Entry -> Entry),%) -> %
maxIndex : % -> Key if Key has ORDSET
member? : (Entry,%) -> Boolean
         if Entry has SETCAT
         and $ has finiteAggregate
members : % -> List Entry if $ has finiteAggregate
minIndex : % -> Key if Key has ORDSET
qelt : (%,Key) -> Entry
qsetelt! : (%,Key,Entry) -> Entry if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
swap! : (%,Key,Key) -> Void if $ has shallowlyMutable
```

⟨*category TBAGG TableAggregate*⟩≡

```
)abbrev category TBAGG TableAggregate
++ Author: Michael Monagan, Stephen Watt;
++ revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
```

```
++ A table aggregate is a model of a table, i.e. a discrete many-to-one
++ mapping from keys to entries.
TableAggregate(Key:SetCategory, Entry:SetCategory): Category ==
  Join(KeyedDictionary(Key,Entry),IndexedAggregate(Key,Entry)) with
   setelt: (%,Key,Entry) -> Entry          -- setelt_! later
     ++ setelt(t,k,e) (also written \axiom{t.k := e}) is equivalent
     ++ to \axiom{(insert([k,e],t); e)}.
   table: () -> %
     ++ table()$T creates an empty table of type T.
     ++
     ++E Data:=Record(age:Integer,gender:String)
     ++E a1:AssociationList(String,Data):=table()
     ++E a1."tim":=[55,"male"]$Data
   table: List Record(key:Key,entry:Entry) -> %
     ++ table([x,y,...,z]) creates a table consisting of entries
     ++ \axiom{x,y,...,z}.
   -- to become table: Record(key:Key,entry:Entry)* -> %
   map: ((Entry, Entry) -> Entry, %, %) -> %
     ++ map(fn,t1,t2) creates a new table t from given tables t1 and t2 with
     ++ elements fn(x,y) where x and y are corresponding elements from t1
     ++ and t2 respectively.
 add
   table()             == empty()
   table l             == dictionary l
-- empty()             == dictionary()

   insert_!(p, t)      == (t(p.key) := p.entry; t)
   indices t           == keys t

   coerce(t:%):OutputForm ==
     prefix("table"::OutputForm,
                   [k::OutputForm = (t.k)::OutputForm for k in keys t])

   elt(t, k) ==
      (r := search(k, t)) case Entry => r::Entry
      error "key not in table"

   elt(t, k, e) ==
      (r := search(k, t)) case Entry => r::Entry
      e

   map_!(f, t) ==
      for k in keys t repeat t.k := f t.k
      t

   map(f:(Entry, Entry) -> Entry, s:%, t:%) ==
```

```
        z := table()
        for k in keys s | key?(k, t) repeat z.k := f(s.k, t.k)
        z

  -- map(f, s, t, x) ==
  --    z := table()
  --    for k in keys s repeat z.k := f(s.k, t(k, x))
  --    for k in keys t | not key?(k, s) repeat z.k := f(t.k, x)
  --    z

     if % has finiteAggregate then
       parts(t:%):List Record(key:Key,entry:Entry) ==
           [[k, t.k] for k in keys t]
       parts(t:%):List Entry   == [t.k for k in keys t]
       entries(t:%):List Entry == parts(t)

       s:% = t:% ==
         eq?(s,t) => true
         #s ^= #t => false
         for k in keys s repeat
           (e := search(k, t)) _
             case "failed" or (e::Entry) ^= s.k => return false
         true

       map(f: Record(key:Key,entry:Entry)->Record(key:Key,entry:Entry),t:%):%==
         z := table()
         for k in keys t repeat
           ke: Record(key:Key,entry:Entry) := f [k, t.k]
           z ke.key := ke.entry
         z
       map_!(f:Record(key:Key,entry:Entry)->Record(key:Key,entry:Entry),t:%):%_
        ==
         lke: List Record(key:Key,entry:Entry) := nil()
         for k in keys t repeat
           lke := cons(f [k, remove_!(k,t)::Entry], lke)
         for ke in lke repeat
           t ke.key := ke.entry
         t

       inspect(t: %): Record(key:Key,entry:Entry) ==
         ks := keys t
         empty? ks => error "Cannot extract from an empty aggregate"
         [first ks, t first ks]

       find(f: Record(key:Key,entry:Entry)->Boolean, t:%):_
             Union(Record(key:Key,entry:Entry), "failed") ==
```

```
      for ke in parts(t)@List(Record(key:Key,entry:Entry)) _
         repeat if f ke then return ke
      "failed"

    index?(k: Key, t: %): Boolean ==
      search(k,t) case Entry

    remove_!(x:Record(key:Key,entry:Entry), t:%) ==
      if member?(x, t) then remove_!(x.key, t)
      t
    extract_!(t: %): Record(key:Key,entry:Entry) ==
      k: Record(key:Key,entry:Entry) := inspect t
      remove_!(k.key, t)
      k

    any?(f: Entry->Boolean, t: %): Boolean ==
      for k in keys t | f t k repeat return true
      false
    every?(f: Entry->Boolean, t: %): Boolean ==
      for k in keys t | not f t k repeat return false
      true
    count(f: Entry->Boolean, t: %): NonNegativeInteger ==
      tally: NonNegativeInteger := 0
      for k in keys t | f t k repeat tally := tally + 1
      tally
```

⟨*TBAGG.dotabb*⟩≡

```
"TBAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=TBAGG"];
"TBAGG" -> "KDAGG"
"TBAGG" -> "IXAGG"
```

⟨*TBAGG.dotfull*⟩≡

```
"TableAggregate(a:SetCategory,b:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=TBAGG"];
"TableAggregate(a:SetCategory,b:SetCategory)" ->
    "KeyedDictionary(a:SetCategory,b:SetCategory)"
"TableAggregate(a:SetCategory,b:SetCategory)" ->
    "IndexedAggregate(a:SetCategory,b:SetCategory)"
```

⟨*TBAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"TableAggregate(a:SetCategory,b:SetCategory)" [color=lightblue];
"TableAggregate(a:SetCategory,b:SetCategory)" ->
    "KeyedDictionary(a:SetCategory,b:SetCategory)"
"TableAggregate(a:SetCategory,b:SetCategory)" ->
    "IndexedAggregate(a:SetCategory,b:SetCategory)"

"IndexedAggregate(a:SetCategory,b:SetCategory)" [color=seagreen];
"IndexedAggregate(a:SetCategory,b:SetCategory)" ->
    "IndexedAggregate(a:SetCategory,b:Type)"

"IndexedAggregate(a:SetCategory,b:Type)" [color=lightblue];
"IndexedAggregate(a:SetCategory,b:Type)" -> "HOAGG..."
"IndexedAggregate(a:SetCategory,b:Type)" -> "ELTAGG..."

"KeyedDictionary(a:SetCategory,b:SetCategory)" [color=lightblue];
"KeyedDictionary(a:SetCategory,b:SetCategory)" ->
    "Dictionary(Record(a:SetCategory,b:SetCategory))"

"Dictionary(Record(a:SetCategory,b:SetCategory))" [color=seagreen];
"Dictionary(Record(a:SetCategory,b:SetCategory))" ->
    "Dictionary(a:SetCategory)"

"Dictionary(a:SetCategory)" [color=lightblue];
"Dictionary(a:SetCategory)" -> "DictionaryOperations(a:SetCategory)"

"DictionaryOperations(a:SetCategory)" [color=lightblue];
"DictionaryOperations(a:SetCategory)" -> "BagAggregate(a:SetCategory)"
"DictionaryOperations(a:SetCategory)" -> "Collection(a:SetCategory)"

"BagAggregate(a:SetCategory)" [color=seagreen];
"BagAggregate(a:SetCategory)" -> "BagAggregate(a:Type)"

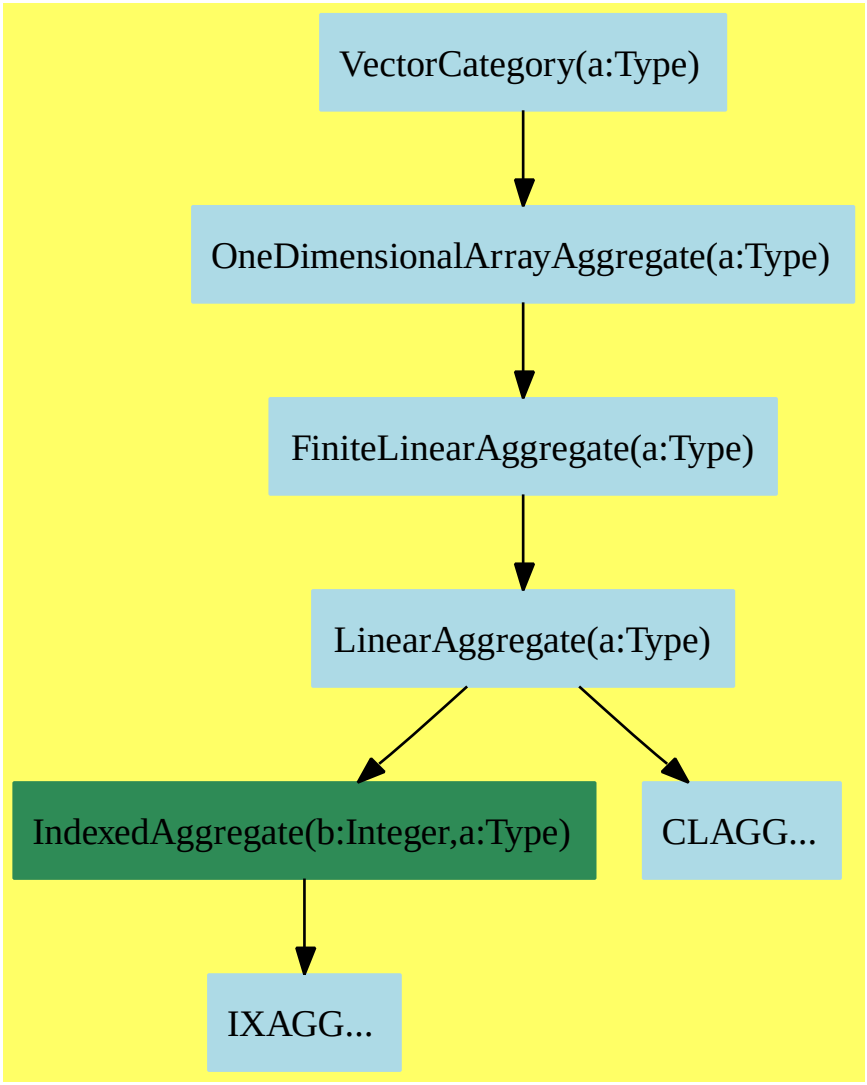
"BagAggregate(a:Type)" [color=lightblue];
"BagAggregate(a:Type)" -> "HOAGG..."

"Collection(a:SetCategory)" [color=seagreen];
"Collection(a:SetCategory)" -> "Collection(a:Type)"

"Collection(a:Type)" [color=lightblue];
"Collection(a:Type)" -> "HOAGG..."
```

```
"ELTAGG..." [color=lightblue];
"HOAGG..." [color=lightblue];
}
```

9.12 VectorCategory (VECTCAT)



See:

⇒ "PointCategory" (PTCAT) 10.13 on page 726
⇐ "OneDimensionalArrayAggregate" (A1AGG) 8.9 on page 556

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	cross	delete
dot	entry?	elt	empty	empty?
entries	eq?	eval	every?	fill!
find	first	hash	index?	indices
insert	latex	length	less?	magnitude
map	map!	max	maxIndex	member?
members	merge	min	minIndex	more?
new	outerProduct	parts	position	qelt
qsetelt!	reduce	remove	removeDuplicates	reverse
reverse!	sample	select	setelt	size?
sort	sort!	sorted?	swap!	zero
#?	?*?	?+?	?-?	?<?
?<=?	?=?	?>?	?>=?	-?
?.?	?~=?			

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.

These are implemented by this category:

```
cross : (%,%) -> % if R has RING
dot : (%,%) -> R if R has RING
length : % -> R if R has RING and R has RADCAT
magnitude : % -> R if R has RING and R has RADCAT
outerProduct : (%,%) -> Matrix R if R has RING
zero : NonNegativeInteger -> % if R has ABELMON
?*? : (Integer,%) -> % if R has ABELGRP
?*? : (%,R) -> % if R has MONOID
?*? : (R,%) -> % if R has MONOID
?-? : (%,%) -> % if R has ABELGRP
-? : % -> % if R has ABELGRP
?+? : (%,%) -> % if R has ABELSG
```

These exports come from (p556) OneDimensionalArrayAggregate(R:Type):

```
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
coerce : % -> OutputForm if R has SETCAT
concat : (R,%) -> %
concat : (%,R) -> %
concat : List % -> %
concat : (%,%) -> %
construct : List R -> %
```

```
convert : % -> InputForm if R has KONVERT INFORM
copy : % -> %
copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
elt : (%,Integer,R) -> R
empty : () -> %
empty? : % -> Boolean
entries : % -> List R
entry? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
fill! : (%,R) -> % if $ has shallowlyMutable
find : ((R -> Boolean),%) -> Union(R,"failed")
first : % -> R if Integer has ORDSET
hash : % -> SingleInteger if R has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (%,%,Integer) -> %
insert : (R,%,Integer) -> %
latex : % -> String if R has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((R -> R),%) -> %
map : (((R,R) -> R),%,%) -> %
map! : ((R -> R),%) -> % if $ has shallowlyMutable
max : (%,%) -> % if R has ORDSET
maxIndex : % -> Integer if Integer has ORDSET
member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
members : % -> List R if $ has finiteAggregate
merge : (%,%) -> % if R has ORDSET
merge : (((R,R) -> Boolean),%,%) -> %
min : (%,%) -> % if R has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,R) -> %
parts : % -> List R if $ has finiteAggregate
position : (R,%) -> Integer if R has SETCAT
position : ((R -> Boolean),%) -> Integer
position : (R,%,Integer) -> Integer if R has SETCAT
qelt : (%,Integer) -> R
qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
reduce : (((R,R) -> R),%) -> R if $ has finiteAggregate
reduce : (((R,R) -> R),%,R) -> R if $ has finiteAggregate
reduce : (((R,R) -> R),%,R,R) -> R if R has SETCAT and $ has finiteAggregate
```

```
remove : ((R -> Boolean),%) -> % if $ has finiteAggregate
remove : (R,%) -> % if R has SETCAT and $ has finiteAggregate
removeDuplicates : % -> % if R has SETCAT and $ has finiteAggregate
reverse : % -> %
reverse! : % -> % if $ has shallowlyMutable
sample : () -> %
select : ((R -> Boolean),%) -> % if $ has finiteAggregate
setelt : (%,Integer,R) -> R if $ has shallowlyMutable
setelt : (%,UniversalSegment Integer,R) -> R if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
sort : % -> % if R has ORDSET
sort : (((R,R) -> Boolean),%) -> %
sort! : % -> % if R has ORDSET and $ has shallowlyMutable
sort! : (((R,R) -> Boolean),%) -> % if $ has shallowlyMutable
sorted? : % -> Boolean if R has ORDSET
sorted? : (((R,R) -> Boolean),%) -> Boolean
swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.? : (%,Integer) -> R
?.? : (%,UniversalSegment Integer) -> %
?=? : (%,%) -> Boolean if R has SETCAT
?<? : (%,%) -> Boolean if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
?~=? : (%,%) -> Boolean if R has SETCAT
```

⟨*category VECTCAT VectorCategory*⟩≡

```
)abbrev category VECTCAT VectorCategory
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors: DirectProductCategory, Vector, IndexedVector
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ \spadtype{VectorCategory} represents the type of vector like objects,
++ i.e. finite sequences indexed by some finite segment of the
++ integers. The operations available on vectors depend on the structure
++ of the underlying components. Many operations from the component domain
++ are defined for vectors componentwise. It can by assumed that extraction or
++ updating components can be done in constant time.

VectorCategory(R:Type): Category == OneDimensionalArrayAggregate R with
    if R has AbelianSemiGroup then
```

```
    _+ : (%, %) -> %
      ++ x + y returns the component-wise sum of the vectors x and y.
      ++ Error: if x and y are not of the same length.
  if R has AbelianMonoid then
    zero: NonNegativeInteger -> %
      ++ zero(n) creates a zero vector of length n.
  if R has AbelianGroup then
    _- : % -> %
      ++ -x negates all components of the vector x.
    _- : (%, %) -> %
      ++ x - y returns the component-wise difference of the vectors x and y.
      ++ Error: if x and y are not of the same length.
    _* : (Integer, %) -> %
      ++ n * y multiplies each component of the vector y by the integer n.
  if R has Monoid then
    _* : (R, %) -> %
      ++ r * y multiplies the element r times each component of the vector y.
    _* : (%, R) -> %
      ++ y * r multiplies each component of the vector y by the element r.
  if R has Ring then
    dot: (%, %) -> R
      ++ dot(x,y) computes the inner product of the two vectors x and y.
      ++ Error: if x and y are not of the same length.
    outerProduct: (%, %) -> Matrix R
      ++ outerProduct(u,v) constructs the matrix whose (i,j)'th element is
      ++ u(i)*v(j).
    cross: (%, %) -> %
      ++ vectorProduct(u,v) constructs the cross product of u and v.
      ++ Error: if u and v are not of length 3.
  if R has RadicalCategory and R has Ring then
    length: % -> R
      ++ length(v) computes the sqrt(dot(v,v)), i.e. the magnitude
    magnitude: % -> R
      ++ magnitude(v) computes the sqrt(dot(v,v)), i.e. the length
add
  if R has AbelianSemiGroup then
    u + v ==
      (n := #u) ^= #v => error "Vectors must be of the same length"
      map(_+ , u, v)

  if R has AbelianMonoid then
    zero n == new(n, 0)

  if R has AbelianGroup then
    - u             == map(x +-> -x, u)
```

```
    n:Integer * u:% == map(x +-> n * x, u)

    u - v           == u + (-v)

  if R has Monoid then
    u:% * r:R       == map(x +-> x * r, u)

    r:R * u:%       == map(x +-> r * x, u)

  if R has Ring then
    dot(u, v) ==
      #u ^= #v => error "Vectors must be of the same length"
      _+/[qelt(u, i) * qelt(v, i) for i in minIndex u .. maxIndex u]

    outerProduct(u, v) ==
      matrix [[qelt(u, i) * qelt(v,j) for i in minIndex u .. maxIndex u] _
              for j in minIndex v .. maxIndex v]

    cross(u, v) ==
      #u ^= 3 or #v ^= 3 => error "Vectors must be of length 3"
      construct [qelt(u, 2)*qelt(v, 3) - qelt(u, 3)*qelt(v, 2) , _
                 qelt(u, 3)*qelt(v, 1) - qelt(u, 1)*qelt(v, 3) , _
                 qelt(u, 1)*qelt(v, 2) - qelt(u, 2)*qelt(v, 1) ]

  if R has RadicalCategory and R has Ring then
    length p ==
       sqrt(dot(p,p))

    magnitude p ==
       sqrt(dot(p,p))
```

⟨*VECTCAT.dotabb*⟩≡

```
"VECTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=VECTCAT"];
"VECTCAT" -> "A1AGG"
```

⟨*VECTCAT.dotfull*⟩≡
```
"VectorCategory(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=VECTCAT"];
"VectorCategory(a:Type)" -> "OneDimensionalArrayAggregate(a:Type)"

"VectorCategory(a:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=VECTCAT"];
"VectorCategory(a:Ring)" -> "VectorCategory(a:Type)"
```

⟨*VECTCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"VectorCategory(a:Type)" [color=lightblue];
"VectorCategory(a:Type)" -> "OneDimensionalArrayAggregate(a:Type)"

"OneDimensionalArrayAggregate(a:Type)" [color=lightblue];
"OneDimensionalArrayAggregate(a:Type)" ->
    "FiniteLinearAggregate(a:Type)"

"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "CLAGG..."

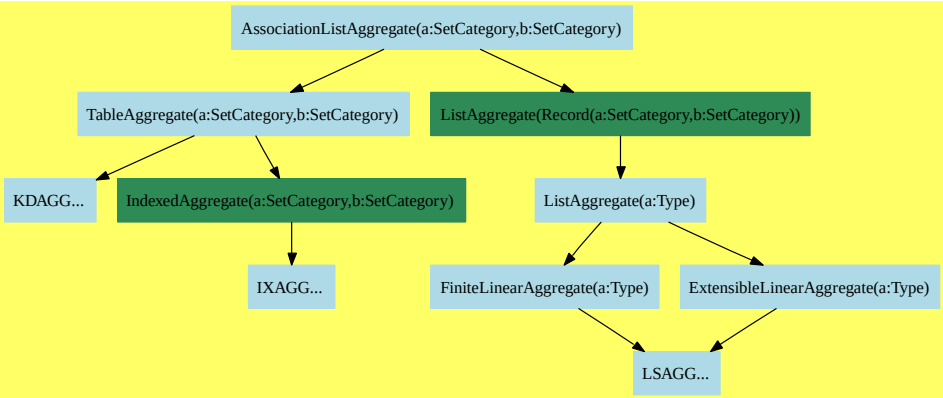
"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" -> "IXAGG..."

"CLAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
}
```

Chapter 10

Category Layer 9

10.1 AssociationListAggregate (ALAGG)



See:

⇐ "ListAggregate" (LSAGG) 8.6 on page 536
⇐ "TableAggregate" (TBAGG) 9.11 on page 649

Exports:

any?	assoc	bag	children
child?	coerce	concat	concat!
construct	convert	copy	copyInto!
count	cycleEntry	cycleLength	cycleSplit!
cycleTail	cyclic?	delete	delete!
dictionary	distance	elt	empty
empty?	entries	entry?	eq?
eval	every?	explicitlyFinite?	extract!
fill!	find	first	hash
index?	indices	insert	insert!
inspect	key?	keys	last
latex	leaf?	leaves	less?
list	map	map!	max
maxIndex	member?	members	merge
merge!	min	minIndex	more?
new	nodes	node?	parts
position	possiblyInfinite?	qelt	qsetelt!
reduce	remove	remove!	removeDuplicates
removeDuplicates!	rest	reverse	reverse!
sample	search	second	select
select!	setchildren!	setelt	setfirst!
setlast!	setrest!	setvalue!	size?
sort	sort!	sorted?	split!
swap!	table	tail	third
value	#?	?<?	?<=?
?=?	?>?	?>=?	?~=?
?.rest	?.value	?.first	?.last
?.?			

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
assoc : (Key,%) -> Union(Record(key: Key,entry: Entry),"failed")
```

These exports come from (p649) TableAggregate(Key, Entry)
where Key:SetCategory and Entry:SetCategory
and RecKE = Record(key: Key,entry: Entry)

```
any? : ((RecKE -> Boolean),%) -> Boolean
         if $ has finiteAggregate
any? : ((Entry -> Boolean),%) -> Boolean
         if $ has finiteAggregate
any? : ((RecKE -> Boolean),%) -> Boolean
         if $ has finiteAggregate
bag : List RecKE -> %
construct : List RecKE -> %
convert : % -> InputForm
         if RecKE has KONVERT INFORM
         or RecKE has KONVERT INFORM
copy : % -> %
count :
 ((RecKE -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
count : (RecKE,%) -> NonNegativeInteger
         if RecKE has SETCAT
         and $ has finiteAggregate
count : ((Entry -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
count : (Entry,%) -> NonNegativeInteger
         if Entry has SETCAT
         and $ has finiteAggregate
count : (RecKE,%) -> NonNegativeInteger
         if RecKE has SETCAT
         and $ has finiteAggregate
count :
 ((RecKE -> Boolean),%) -> NonNegativeInteger
         if $ has finiteAggregate
dictionary : () -> %
dictionary : List RecKE -> %
elt : (%,Key,Entry) -> Entry
elt : (%,Integer,RecKE) -> RecKE
empty : () -> %
empty? : % -> Boolean
entries : % -> List Entry
entry? : (Entry,%) -> Boolean
         if $ has finiteAggregate
         and Entry has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List Equation RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,Equation RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,RecKE,RecKE) -> %
         if RecKE has EVALAB RecKE
         and RecKE has SETCAT
eval : (%,List RecKE,List RecKE) -> %
```

```
          if RecKE has EVALAB RecKE
          and RecKE has SETCAT
eval : (%,List Equation Entry) -> %
          if Entry has EVALAB Entry
          and Entry has SETCAT
eval : (%,Equation Entry) -> %
          if Entry has EVALAB Entry
          and Entry has SETCAT
eval : (%,Entry,Entry) -> %
          if Entry has EVALAB Entry
          and Entry has SETCAT
eval : (%,List Entry,List Entry) -> %
          if Entry has EVALAB Entry
          and Entry has SETCAT
eval : (%,List RecKE,List RecKE) -> %
          if RecKE has EVALAB RecKE
          and RecKE has SETCAT
eval : (%,RecKE,RecKE) -> %
          if RecKE has EVALAB RecKE
          and RecKE has SETCAT
eval : (%,Equation RecKE) -> %
          if RecKE has EVALAB RecKE
          and RecKE has SETCAT
eval : (%,List Equation RecKE) -> %
          if RecKE has EVALAB RecKE
          and RecKE has SETCAT
every? : ((RecKE -> Boolean),%) -> Boolean
          if $ has finiteAggregate
every? : ((Entry -> Boolean),%) -> Boolean
          if $ has finiteAggregate
extract! : % -> RecKE
fill! : (%,Entry) -> % if $ has shallowlyMutable
find : ((RecKE -> Boolean),%) -> Union(RecKE,"failed")
first : % -> Entry if Key has ORDSET
hash : % -> SingleInteger
          if RecKE has SETCAT
          or Entry has SETCAT
          or RecKE has SETCAT
index? : (Key,%) -> Boolean
indices : % -> List Key
insert! : (RecKE,%) -> %
inspect : % -> RecKE
key? : (Key,%) -> Boolean
keys : % -> List Key
latex : % -> String
          if RecKE has SETCAT
          or Entry has SETCAT
          or RecKE has SETCAT
less? : (%,NonNegativeInteger) -> Boolean
map : ((Entry -> Entry),%) -> %
```

```
map : ((RecKE -> RecKE),%) -> %
map : (((Entry,Entry) -> Entry),%,%) -> %
map! : ((RecKE -> RecKE),%) -> %
         if $ has shallowlyMutable
map! : ((Entry -> Entry),%) -> % if $ has shallowlyMutable
map! : ((RecKE -> RecKE),%) -> %
         if $ has shallowlyMutable
maxIndex : % -> Key if Key has ORDSET
member? : (RecKE,%) -> Boolean
         if RecKE has SETCAT
         and $ has finiteAggregate
member? : (Entry,%) -> Boolean
         if Entry has SETCAT
         and $ has finiteAggregate
members : % -> List RecKE if $ has finiteAggregate
members : % -> List Entry if $ has finiteAggregate
members : % -> List RecKE if $ has finiteAggregate
minIndex : % -> Key if Key has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List Entry if $ has finiteAggregate
parts : % -> List RecKE if $ has finiteAggregate
qelt : (%,Key) -> Entry
qsetelt! : (%,Key,Entry) -> Entry if $ has shallowlyMutable
reduce :
 (((RecKE,RecKE) -> RecKE),%)
   -> RecKE
         if $ has finiteAggregate
reduce :
 (((RecKE,RecKE) -> RecKE),%,RecKE)
   -> RecKE
         if $ has finiteAggregate
reduce :
 (((RecKE,RecKE) -> RecKE),%,RecKE,RecKE)
   -> RecKE
         if RecKE has SETCAT
         and $ has finiteAggregate
remove : ((RecKE -> Boolean),%) -> % if $ has finiteAggregate
remove : (RecKE,%) -> %
         if RecKE has SETCAT
         and $ has finiteAggregate
remove! : (Key,%) -> Union(Entry,"failed")
remove! : (RecKE,%) -> % if RecKE has SETCAT
remove! : (RecKE,%) -> % if $ has finiteAggregate
removeDuplicates : % -> %
         if RecKE has SETCAT
         and $ has finiteAggregate
         or RecKE has SETCAT
         and $ has finiteAggregate
sample : () -> %
search : (Key,%) -> Union(Entry,"failed")
```

```
select : ((RecKE -> Boolean),%) -> %
         if $ has finiteAggregate
select! : ((RecKE -> Boolean),%) -> %
         if $ has finiteAggregate
setelt : (%,Key,Entry) -> Entry
size? : (%,NonNegativeInteger) -> Boolean
swap! : (%,Key,Key) -> Void if $ has shallowlyMutable
table : () -> %
table : List RecKE -> %
?~=? : (%,%) -> Boolean
         if RecKE has SETCAT
         or Entry has SETCAT
         or RecKE has SETCAT
?.? : (%,Key) -> Entry
```

These exports come from (p536) ListAggregate(a)
where a is Record(key:Key,entry:Entry)
and RecKE=Record(key: Key,entry: Entry)

```
children : % -> List %
child? : (%,%) -> Boolean if RecKE has SETCAT
coerce : % -> OutputForm
         if RecKE has SETCAT
         or Entry has SETCAT
         or RecKE has SETCAT
concat : (%,%) -> %
concat : List % -> %
concat : (RecKE,%) -> %
concat : (%,RecKE) -> %
concat! : (%,%) -> %
concat! : (%,RecKE) -> %
copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
cycleEntry : % -> %
cycleLength : % -> NonNegativeInteger
cycleSplit! : % -> % if $ has shallowlyMutable
cycleTail : % -> %
cyclic? : % -> Boolean
delete : (%,Integer) -> %
delete! : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
delete! : (%,UniversalSegment Integer) -> %
distance : (%,%) -> Integer
entries : % -> List RecKE
entry? : (RecKE,%) -> Boolean
         if $ has finiteAggregate
         and RecKE has SETCAT
explicitlyFinite? : % -> Boolean
fill! : (%,RecKE) -> % if $ has shallowlyMutable
first : % -> RecKE
first : (%,NonNegativeInteger) -> %
```

```
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (%,%,Integer) -> %
insert : (RecKE,%,Integer) -> %
insert! : (%,%,Integer) -> %
insert! : (RecKE,%,Integer) -> %
last : % -> RecKE
last : (%,NonNegativeInteger) -> %
leaf? : % -> Boolean
leaves : % -> List RecKE
list : RecKE -> %
map : (((RecKE,RecKE) -> RecKE),%,%) -> %
max : (%,%) -> % if RecKE has ORDSET
maxIndex : % -> Integer if Integer has ORDSET
merge : (%,%) -> % if RecKE has ORDSET
merge : (((RecKE,RecKE) -> Boolean),%,%) -> %
merge! : (%,%) -> % if RecKE has ORDSET
merge! : (((RecKE,RecKE) -> Boolean),%,%) -> %
min : (%,%) -> % if RecKE has ORDSET
minIndex : % -> Integer if Integer has ORDSET
new : (NonNegativeInteger,RecKE) -> %
nodes : % -> List %
node? : (%,%) -> Boolean if RecKE has SETCAT
position : (RecKE,%,Integer) -> Integer
         if RecKE has SETCAT
position : (RecKE,%) -> Integer
         if RecKE has SETCAT
position : ((RecKE -> Boolean),%) -> Integer
possiblyInfinite? : % -> Boolean
qelt : (%,Integer) -> RecKE
qsetelt! : (%,Integer,RecKE) -> RecKE
         if $ has shallowlyMutable
remove! : ((RecKE -> Boolean),%) -> %
remove! : ((RecKE -> Boolean),%) -> %
         if $ has finiteAggregate
removeDuplicates! : % -> % if RecKE has SETCAT
rest : % -> %
rest : (%,NonNegativeInteger) -> %
reverse : % -> %
reverse! : % -> % if $ has shallowlyMutable
second : % -> RecKE
select! : ((RecKE -> Boolean),%) -> %
setchildren! : (%,List %) -> % if $ has shallowlyMutable
setelt : (%,value,RecKE) -> RecKE
         if $ has shallowlyMutable
setelt : (%,first,RecKE) -> RecKE
         if $ has shallowlyMutable
setelt : (%,rest,%) -> % if $ has shallowlyMutable
setelt : (%,last,RecKE) -> RecKE
         if $ has shallowlyMutable
```

```
setelt : (%,UniversalSegment Integer,RecKE) -> RecKE
         if $ has shallowlyMutable
setelt : (%,Integer,RecKE) -> RecKE
         if $ has shallowlyMutable
setfirst! : (%,RecKE) -> RecKE
         if $ has shallowlyMutable
setlast! : (%,RecKE) -> RecKE
         if $ has shallowlyMutable
setrest! : (%,%) -> % if $ has shallowlyMutable
setvalue! : (%,RecKE) -> RecKE
         if $ has shallowlyMutable
sort : % -> % if RecKE has ORDSET
sort : (((RecKE,RecKE) -> Boolean),%) -> %
sort! : % -> %
         if RecKE has ORDSET
         and $ has shallowlyMutable
sort! : (((RecKE,RecKE) -> Boolean),%) -> %
         if $ has shallowlyMutable
sorted? : % -> Boolean if RecKE has ORDSET
sorted? : (((RecKE,RecKE) -> Boolean),%) -> Boolean
split! : (%,Integer) -> % if $ has shallowlyMutable
swap! : (%,Integer,Integer) -> Void
         if $ has shallowlyMutable
tail : % -> %
third : % -> RecKE
value : % -> RecKE
#? : % -> NonNegativeInteger if $ has finiteAggregate
?<? : (%,%) -> Boolean if RecKE has ORDSET
?<=? : (%,%) -> Boolean if RecKE has ORDSET
?=? : (%,%) -> Boolean
         if RecKE has SETCAT
         or Entry has SETCAT
         or RecKE has SETCAT
?>? : (%,%) -> Boolean if RecKE has ORDSET
?>=? : (%,%) -> Boolean if RecKE has ORDSET
?.value : (%,value) -> RecKE
?.first : (%,first) -> RecKE
?.last : (%,last) -> RecKE
?.rest : (%,rest) -> %
?.? : (%,UniversalSegment Integer) -> %
?.? : (%,Integer) -> RecKE
```

⟨*category ALAGG AssociationListAggregate*⟩≡

```
)abbrev category ALAGG AssociationListAggregate
++ Author: Michael Monagan; revised by Manuel Bronstein and Richard Jenks
++ Date Created: August 87 through August 88
++ Date Last Updated: April 1991
++ Basic Operations:
++ Related Constructors:
++ Also See:
```

```
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ An association list is a list of key entry pairs which may be viewed
++ as a table.  It is a poor mans version of a table:
++ searching for a key is a linear operation.
AssociationListAggregate(Key:SetCategory,Entry:SetCategory): Category ==
   Join(TableAggregate(Key, Entry), _
         ListAggregate Record(key:Key,entry:Entry)) with
      assoc: (Key, %) -> Union(Record(key:Key,entry:Entry), "failed")
        ++ assoc(k,u) returns the element x in association list u stored
        ++ with key k, or "failed" if u has no key k.
```

⟨*ALAGG.dotabb*⟩≡

```
"ALAGG" [color=lightblue,href="bookvol10.2.pdf#nameddest=ALAGG"];
"ALAGG" -> "TBAGG"
"ALAGG" -> "LSAGG"
```

⟨*ALAGG.dotfull*⟩≡

```
"AssociationListAggregate(a:SetCategory,b:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALAGG"];
"AssociationListAggregate(a:SetCategory,b:SetCategory)" ->
    "TableAggregate(a:SetCategory,b:SetCategory)"
"AssociationListAggregate(a:SetCategory,b:SetCategory)" ->
    "ListAggregate(Record(a:SetCategory,b:SetCategory))"
```

⟨*ALAGG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"AssociationListAggregate(a:SetCategory,b:SetCategory)" [color=lightblue];
"AssociationListAggregate(a:SetCategory,b:SetCategory)" ->
    "TableAggregate(a:SetCategory,b:SetCategory)"
"AssociationListAggregate(a:SetCategory,b:SetCategory)" ->
    "ListAggregate(Record(a:SetCategory,b:SetCategory))"

"TableAggregate(a:SetCategory,b:SetCategory)" [color=lightblue];
"TableAggregate(a:SetCategory,b:SetCategory)" -> "KDAGG..."
"TableAggregate(a:SetCategory,b:SetCategory)" ->
    "IndexedAggregate(a:SetCategory,b:SetCategory)"

"IndexedAggregate(a:SetCategory,b:SetCategory)" [color=seagreen];
"IndexedAggregate(a:SetCategory,b:SetCategory)" -> "IXAGG..."

"ListAggregate(Record(a:SetCategory,b:SetCategory))" [color=seagreen];
"ListAggregate(Record(a:SetCategory,b:SetCategory))" ->
    "ListAggregate(a:Type)"

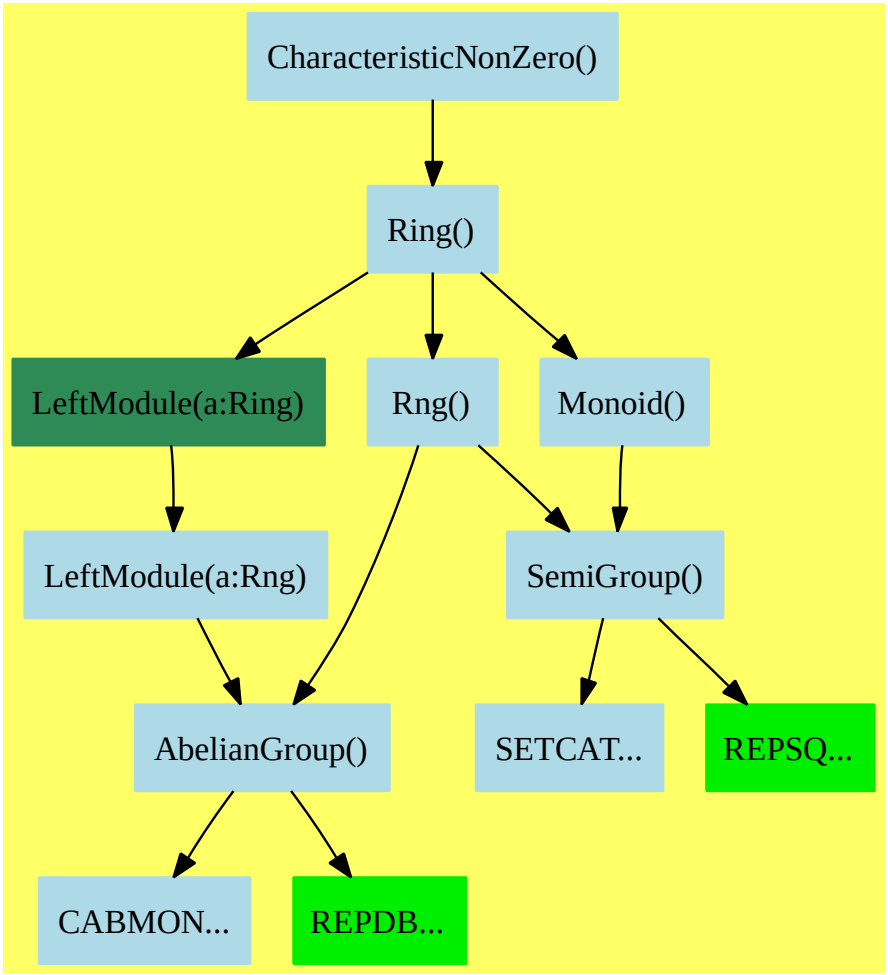
"ListAggregate(a:Type)" [color=lightblue];
"ListAggregate(a:Type)" -> "FiniteLinearAggregate(a:Type)"
"ListAggregate(a:Type)" -> "ExtensibleLinearAggregate(a:Type)"

"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LSAGG..."

"ExtensibleLinearAggregate(a:Type)" [color=lightblue];
"ExtensibleLinearAggregate(a:Type)" -> "LSAGG..."

"KDAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
"LSAGG..." [color=lightblue];
}
```

10.2 CharacteristicNonZero (CHARNZ)



See:

⇒ "FieldOfPrimeCharacteristic" (FPC) 17.3 on page 1084
⇒ "FiniteRankAlgebra" (FINRALG) 17.4 on page 1089
⇒ "FunctionSpace" (FS) 17.5 on page 1095
⇒ "QuotientFieldCategory" (QFCAT) 17.6 on page 1121
⇐ "Ring" (RING) 9.8 on page 628

Exports:

1	0	coerce	hash	latex
one?	recip	sample	zero?	characteristic
charthRoot	subtractIfCan	?~=?	?^?	?*?
?**?	?+?	?-?	-?	?=?

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are directly exported but not implemented:

```
charthRoot : % -> Union(%,"failed")
```

These exports come from (p628) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

⟨*category CHARNZ CharacteristicNonZero*⟩≡

```
)abbrev category CHARNZ CharacteristicNonZero
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
```

```
++ Description:
++ Rings of Characteristic Non Zero
CharacteristicNonZero():Category == Ring with
    charthRoot: % -> Union(%,"failed")
       ++ charthRoot(x) returns the pth root of x
       ++ where p is the characteristic of the ring.
```

⟨*CHARNZ.dotabb*⟩≡

```
"CHARNZ"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CHARNZ" -> "RING"
```

⟨*CHARNZ.dotfull*⟩≡

```
"CharacteristicNonZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CharacteristicNonZero()" -> "Ring()"
```

⟨*CHARNZ.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"CharacteristicNonZero()" [color=lightblue];
"CharacteristicNonZero()" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

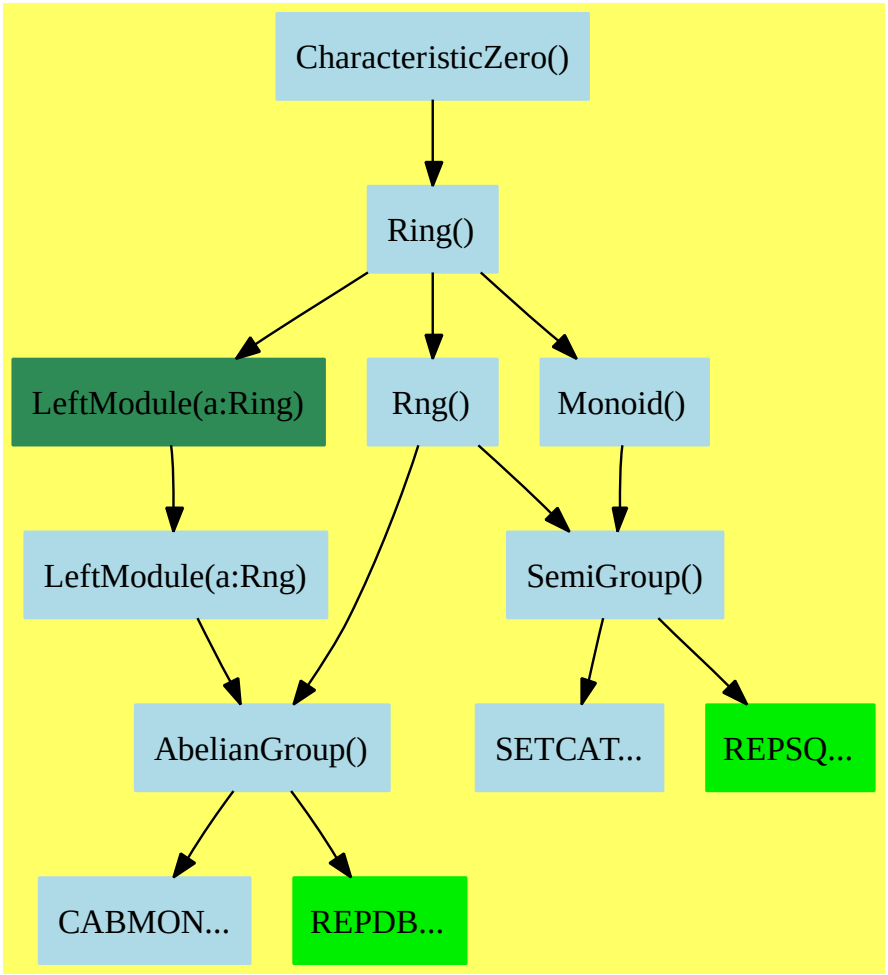
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

10.3 CharacteristicZero (CHARZ)



See:

⇒ “FiniteRankAlgebra” (FINRALG) 17.4 on page 1089
⇒ “FunctionSpace” (FS) 17.5 on page 1095
⇒ “IntegerNumberSystem” (INS) 16.2 on page 1011
⇒ “PAdicIntegerCategory” (PADICCT) 16.3 on page 1021
⇒ “QuotientFieldCategory” (QFCAT) 17.6 on page 1121
⇒ “RealClosedField” (RCFIELD) 17.7 on page 1132
⇒ “RealNumberSystem” (RNS) 17.8 on page 1141
⇐ “Ring” (RING) 9.8 on page 628

Exports:

1	0	coerce	hash	latex
one?	recip	sample	zero?	characteristic
subtractIfCan	?~=?	?^?	?*?	
?**?	?+?	?-?	-?	?=?

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These exports come from (p628) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

⟨*category CHARZ CharacteristicZero*⟩≡

```
)abbrev category CHARZ CharacteristicZero
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
```

```
  ++ Description:
  ++ Rings of Characteristic Zero.
  CharacteristicZero():Category == Ring
```

⟨*CHARZ.dotabb*⟩≡

```
  "CHARZ"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARZ"];
  "CHARZ" -> "RING"
```

⟨*CHARZ.dotfull*⟩≡

```
  "CharacteristicZero()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARZ"];
  "CharacteristicZero()" -> "Ring()"
```

⟨*CHARZ.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"CharacteristicZero()" [color=lightblue];
"CharacteristicZero()" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

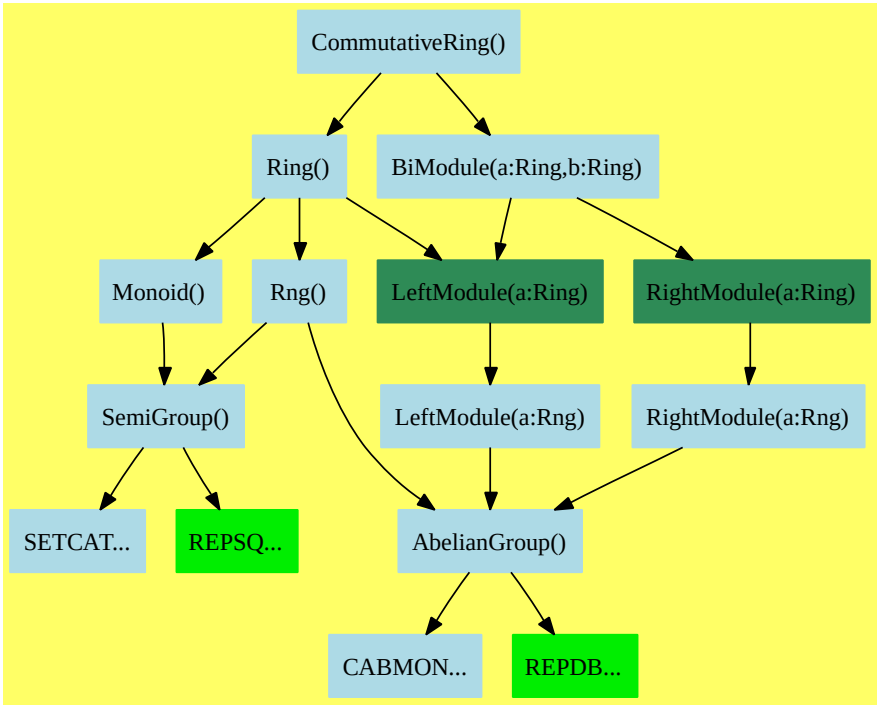
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

10.4 CommutativeRing (COMRING)



Commutative Rings are a subset of IntegralDomains.
⇒ "IntegralDomain" (INTDOM) 12.5 on page 857.

See:

⇒ "ComplexCategory" (COMPCAT) 20.1 on page 1315
⇒ "IntegralDomain" (INTDOM) 12.5 on page 857
⇒ "FunctionSpace" (FS) 17.5 on page 1095
⇒ "MonogenicAlgebra" (MONOGEN) 19.2 on page 1305
⇒ "RealClosedField" (RCFIELD) 17.7 on page 1132
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.12 on page 1204
⇐ "BiModule" (BMODULE) 9.1 on page 592
⇐ "Ring" (RING) 9.8 on page 628

Exports:

1	0	coerce	hash	latex
one?	recip	sample	zero?	characteristic
subtractIfCan	?~=?	?^?	?*?	?**?
?+?	?-?	-?	?=?	

Attributes exported:

- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$

which is commutative.

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p628) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

⟨*category COMRING CommutativeRing*⟩≡

```
)abbrev category COMRING CommutativeRing
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
```

```
  ++ The category of commutative rings with unity, i.e. rings where
  ++ \spadop{*} is commutative, and which have a multiplicative identity.
  ++ element.
  --CommutativeRing():Category == Join(Ring,BiModule(%:Ring,%:Ring)) with
  CommutativeRing():Category == Join(Ring,BiModule(%,%)) with
      commutative("*")  ++ multiplication is commutative.
```

⟨*COMRING.dotabb*⟩≡

```
  "COMRING"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=COMRING"];
  "COMRING" -> "RING"
  "COMRING" -> "BMODULE"
```

⟨*COMRING.dotfull*⟩≡

```
  "CommutativeRing()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=COMRING"];
  "CommutativeRing()" -> "Ring()"
  "CommutativeRing()" -> "BiModule(a:Ring,b:Ring)"
```

⟨*COMRING.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "Ring()"
"CommutativeRing()" -> "BiModule(a:Ring,b:Ring)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "AbelianGroup()"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

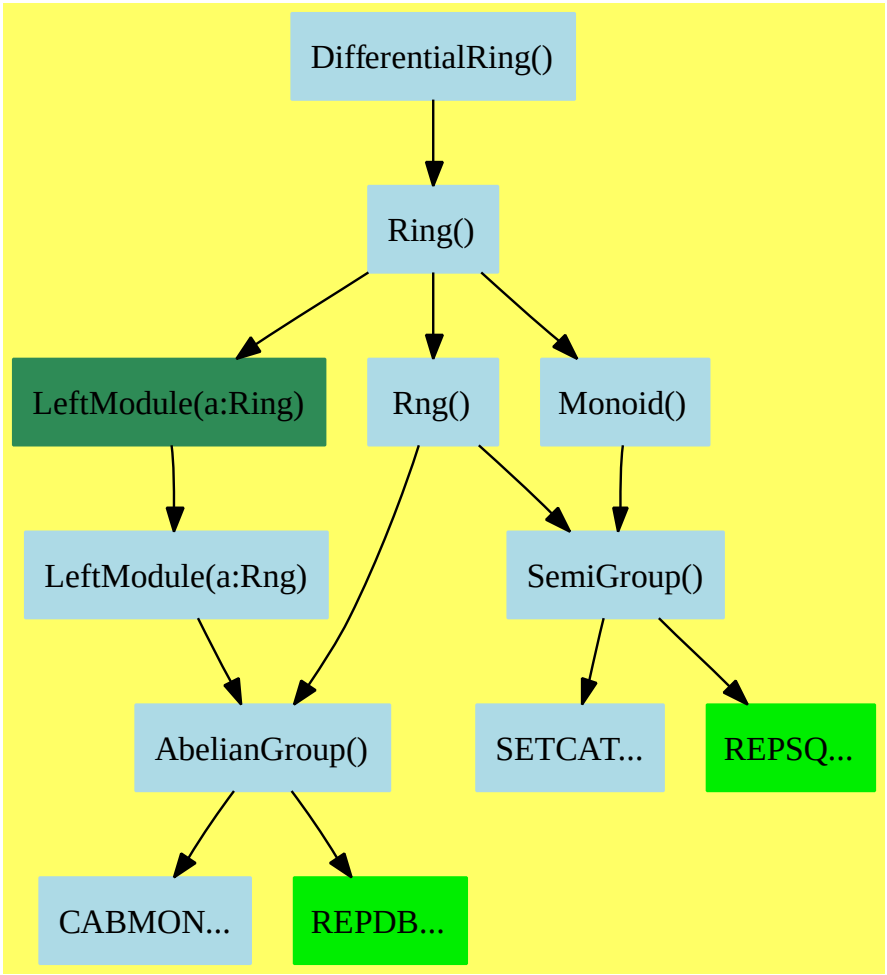
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."
```

```
"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

10.5 DifferentialRing (DIFRING)



See:

⇒ "IntegerNumberSystem" (INS) 16.2 on page 1011
⇒ "FiniteFieldCategory" (FFIELDC) 18.3 on page 1244
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.12 on page 1204
⇐ "Ring" (RING) 9.8 on page 628

Exports:

1	0	characteristic	coerce	D
differentiate	hash	latex	one?	recip
sample	subtractIfCan	zero?	?~=?	?**?
?^?	?*?	?+?	?-?	-?
?=?				

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are directly exported but not implemented:

```
differentiate : % -> %
```

These are implemented by this category:

```
D : % -> %
D : (%,NonNegativeInteger) -> %
differentiate : (%,NonNegativeInteger) -> %
```

These exports come from (p628) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

⟨*category DIFRING DifferentialRing*⟩≡

```
)abbrev category DIFRING DifferentialRing
++ Author:
++ Date Created:
++ Date Last Updated:
```

```
  ++ Basic Functions:
  ++ Related Constructors:
  ++ Also See:
  ++ AMS Classifications:
  ++ Keywords:
  ++ References:
  ++ Description:
  ++ An ordinary differential ring, that is, a ring with an operation
  ++ \spadfun{differentiate}.
  ++
  ++ Axioms:
  ++   \spad{differentiate(x+y) = differentiate(x)+differentiate(y)}
  ++   \spad{differentiate(x*y) = x*differentiate(y) + differentiate(x)*y}

  DifferentialRing(): Category == Ring with
      differentiate: % -> %
           ++ differentiate(x) returns the derivative of x.
           ++ This function is a simple differential operator
           ++ where no variable needs to be specified.
      D: % -> %
           ++ D(x) returns the derivative of x.
           ++ This function is a simple differential operator
           ++ where no variable needs to be specified.
      differentiate: (%, NonNegativeInteger) -> %
           ++ differentiate(x, n) returns the n-th derivative of x.
      D: (%, NonNegativeInteger) -> %
           ++ D(x, n) returns the n-th derivative of x.
    add
      D r == differentiate r
      differentiate(r, n) ==
        for i in 1..n repeat r := differentiate r
        r
      D(r,n) == differentiate(r,n)
```

⟨*DIFRING.dotabb*⟩≡

```
  "DIFRING"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=DIFRING"];
  "DIFRING" -> "RING"
```

⟨*DIFRING.dotfull*⟩≡

```
  "DifferentialRing()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=DIFRING"];
  "DifferentialRing()" -> "Ring()"
```

⟨*DIFRING.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DifferentialRing()" [color=lightblue];
"DifferentialRing()" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

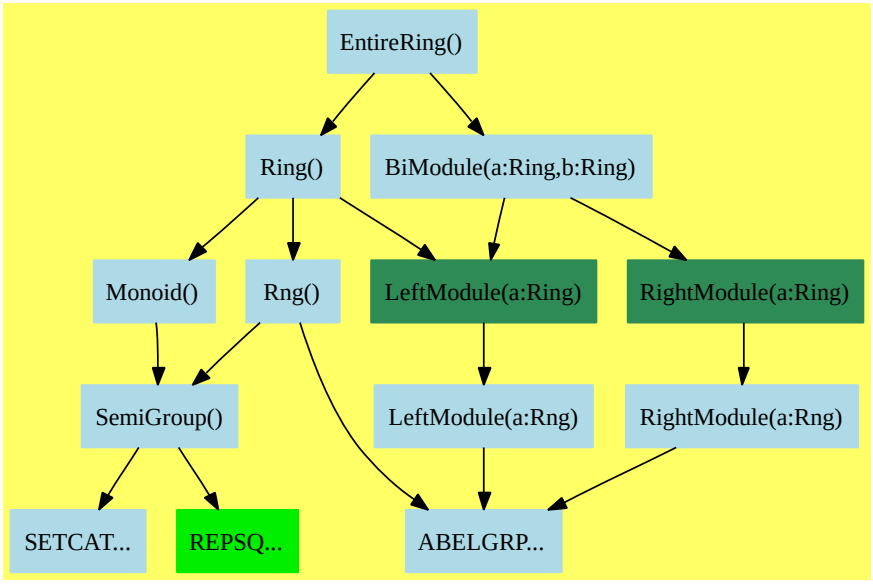
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

10.6 EntireRing (ENTIRER)



See:

⇒ "DivisionRing" (DIVRING) 12.2 on page 825
⇒ "IntegralDomain" (INTDOM) 12.5 on page 857
⇐ "BiModule" (BMODULE) 9.1 on page 592
⇐ "Ring" (RING) 9.8 on page 628

Exports:

1	0	characteristic	coerce	hash
latex	one?	recip	sample	subtractIfCan
zero?	?^?	?~=?	?*?	?**?
?+?	?-?	-?	?=?	

Attributes Exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p628) Ring():

```
0 : () -> %
```

```
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

⟨*category ENTIRER EntireRing*⟩≡

```
)abbrev category ENTIRER EntireRing
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Entire Rings (non-commutative Integral Domains), i.e. a ring
++ not necessarily commutative which has no zero divisors.
++
++ Axioms:
++   \spad{ab=0 => a=0 or b=0}  -- known as noZeroDivisors
++   \spad{not(1=0)}
--EntireRing():Category == Join(Ring,BiModule(%:Ring,%:Ring)) with
EntireRing():Category == Join(Ring,BiModule(%,%)) with
      noZeroDivisors  ++ if a product is zero then one of the factors
                      ++ must be zero.
```

⟨*ENTIRER.dotabb*⟩≡
```
  "ENTIRER"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ENTIRER"];
  "ENTIRER" -> "RING"
  "ENTIRER" -> "BMODULE"
```

⟨*ENTIRER.dotfull*⟩≡
```
  "EntireRing()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ENTIRER"];
  "EntireRing()" -> "Ring()"
  "EntireRing()" -> "BiModule(a:Ring,b:Ring)"
```

⟨*ENTIRER.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"EntireRing()" [color=lightblue];
"EntireRing()" -> "Ring()"
"EntireRing()" -> "BiModule(a:Ring,b:Ring)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

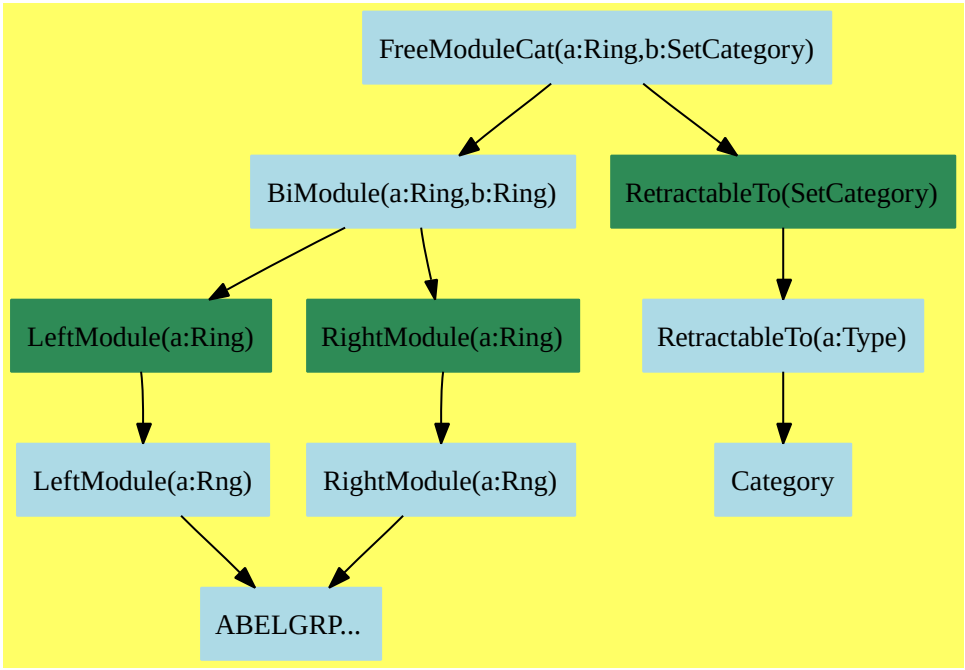
"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

10.7 FreeModuleCat (FMCAT)



See:

⇐ "BiModule" (BMODULE) 9.1 on page 592
⇐ "RetractableTo" (RETRACT) 2.18 on page 44

Exports:

0	coefficient	coefficients	coerce
hash	latex	leadingCoefficient	leadingMonomial
leadingTerm	ListOfTerms	map	monom
monomial?	monomials	numberOfMonomials	reductum
retract	retractIfCan	sample	subtractIfCan
zero?	?~=?	?*?	?+?
?-?	-?	?=?	

Attributes Exported:

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coefficient : (%,Basis) -> R
coefficients : % -> List R
```

```
leadingCoefficient : % -> R
leadingMonomial : % -> Basis
leadingTerm : % -> Record(k: Basis,c: R)
ListOfTerms : % -> List Record(k: Basis,c: R)
map : ((R -> R),%) -> %
monom : (Basis,R) -> %
monomial? : % -> Boolean
monomials : % -> List %
numberOfMonomials : % -> NonNegativeInteger
reductum : % -> %
?*? : (R,Basis) -> %
```

These exports come from (p592) BiModule(R:Ring,R:Ring):

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (R,%) -> %
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?-? : (%,%) -> %
-? : % -> %
?*? : (%,R) -> %
```

These exports come from (p44) RetractableTo(Basis:SetCategory):

```
coerce : Basis -> %
retract : % -> Basis
retractIfCan : % -> Union(Basis,"failed")
```

⟨*category FMCAT FreeModuleCat*⟩≡

```
)abbrev category FMCAT FreeModuleCat
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
```

```
++ References:
++ Description:
++   A domain of this category
++   implements formal linear combinations
++   of elements from a domain \spad{Basis} with coefficients
++   in a domain \spad{R}. The domain \spad{Basis} needs only
++   to belong to the category \spadtype{SetCategory} and \spad{R}
++   to the category \spadtype{Ring}. Thus the coefficient ring
++   may be non-commutative.
++   See the \spadtype{XDistributedPolynomial} constructor
++   for examples of domains built with the \spadtype{FreeModuleCat}
++   category constructor.
++   Author: Michel Petitot (petitot@lifl.fr)

FreeModuleCat(R, Basis):Category == Exports where
   R: Ring
   Basis: SetCategory
   TERM ==> Record(k: Basis, c: R)

   Exports == Join(BiModule(R,R), RetractableTo Basis) with
     "*"                : (R, Basis) -> %
       ++ \spad{r*b} returns the product of \spad{r} by \spad{b}.
     coefficient        : (%, Basis) -> R
       ++ \spad{coefficient(x,b)} returns the coefficient
       ++ of \spad{b} in \spad{x}.
     map                : (R -> R, %) -> %
       ++ \spad{map(fn,u)} maps function \spad{fn} onto the coefficients
       ++  of the non-zero monomials of \spad{u}.
     monom              : (Basis, R) -> %
       ++ \spad{monom(b,r)} returns the element with the single monomial
       ++  \spad{b} and coefficient \spad{r}.
     monomial?          : % -> Boolean
       ++ \spad{monomial?(x)} returns true if \spad{x} contains a single
       ++ monomial.
     ListOfTerms        : % -> List TERM
       ++ \spad{ListOfTerms(x)} returns a list \spad{lt} of terms with type
       ++ \spad{Record(k: Basis, c: R)} such that \spad{x} equals
       ++ \spad{reduce(+, map(x +-> monom(x.k, x.c), lt))}.
     coefficients       : % -> List R
       ++ \spad{coefficients(x)} returns the list of coefficients of \spad{x}
     monomials          : % -> List %
       ++ \spad{monomials(x)} returns the list of \spad{r_i*b_i}
       ++ whose sum is \spad{x}.
     numberOfMonomials  : % -> NonNegativeInteger
       ++ \spad{numberOfMonomials(x)} returns the number of monomials
       ++ of \spad{x}.
```

```
leadingMonomial    : % -> Basis
  ++ \spad{leadingMonomial(x)} returns the first element from
  ++ \spad{Basis} which appears in \spad{ListOfTerms(x)}.
leadingCoefficient : % -> R
  ++ \spad{leadingCoefficient(x)} returns the first coefficient
  ++ which appears in \spad{ListOfTerms(x)}.
leadingTerm        : % -> TERM
  ++ \spad{leadingTerm(x)} returns the first term which
  ++ appears in \spad{ListOfTerms(x)}.
reductum           : % -> %
  ++ \spad{reductum(x)} returns \spad{x} minus its leading term.

 -- attributs
if R has CommutativeRing then Module(R)
```

⟨*FMCAT.dotabb*⟩≡

```
"FMCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMCAT"];
"FMCAT" -> "BMODULE"
"FMCAT" -> "RETRACT"
```

⟨*FMCAT.dotfull*⟩≡

```
"FreeModuleCat(a:Ring,b:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMCAT"];
"FreeModuleCat(a:Ring,b:SetCategory)" -> "BiModule(a:Ring,b:Ring)"
"FreeModuleCat(a:Ring,b:SetCategory)" -> "RetractableTo(SetCategory)"
```

⟨*FMCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FreeModuleCat(a:Ring,b:SetCategory)" [color=lightblue];
"FreeModuleCat(a:Ring,b:SetCategory)" -> "BiModule(a:Ring,b:Ring)"
"FreeModuleCat(a:Ring,b:SetCategory)" -> "RetractableTo(SetCategory)"

"RetractableTo(SetCategory)" [color=seagreen];
"RetractableTo(SetCategory)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"Category" [color=lightblue];

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

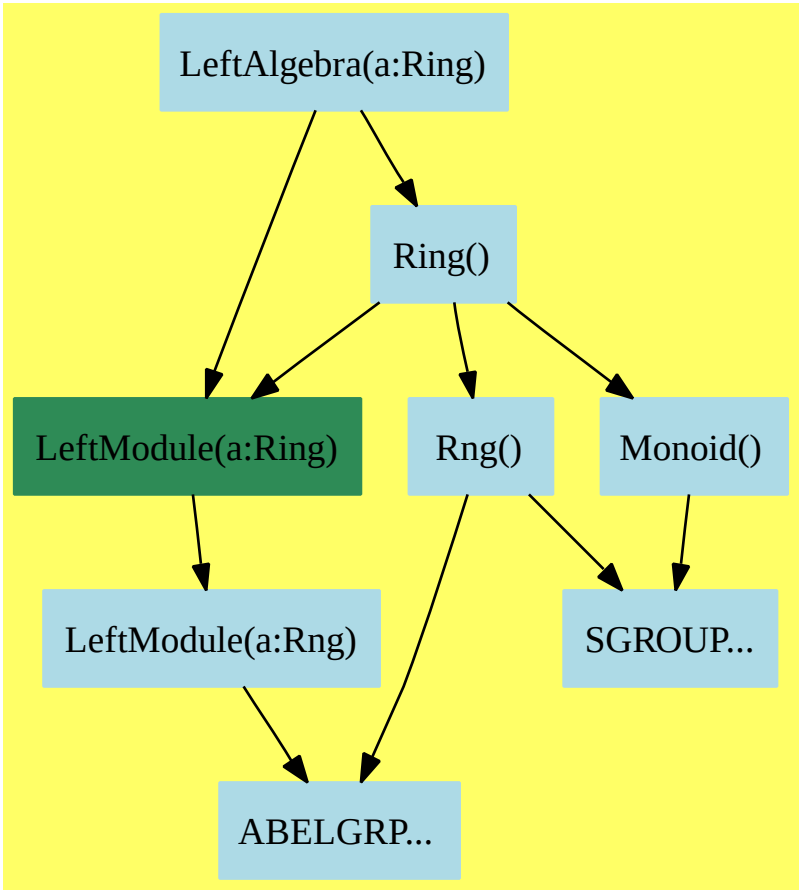
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"ABELGRP..." [color=lightblue];
}
```

10.8 LeftAlgebra (LALG)



See:

⇐ "LeftModule" (LMODULE) 8.5 on page 533
⇐ "Ring" (RING) 9.8 on page 628

Exports:

0	1	coerce	hash	latex
one?	recip	sample	zero?	characteristic
subtractIfCan	?*?	?+?	?-?	-?
?=?	?~=?	?**?	?^?	

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are implemented by this category:

```
coerce : R -> %
```

These exports come from (p628) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> OutputForm
coerce : Integer -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p533) LeftModule(R:Type):

```
?*? : (R,%) -> %
```

⟨*category LALG LeftAlgebra*⟩≡

```
)abbrev category LALG LeftAlgebra
++ Author: Larry A. Lambe
++ Date  : 03/01/89; revised 03/17/89; revised 12/02/90.
++ Description: The category of all left algebras over an arbitrary
++ ring.
LeftAlgebra(R:Ring): Category == Join(Ring, LeftModule R) with
      coerce: R -> %
        ++ coerce(r) returns r * 1 where 1 is the identity of the
        ++ left algebra.
    add
      coerce(x:R):% == x * 1$%
```

⟨*LALG.dotabb*⟩≡
```
"LALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LALG"];
"LALG" -> "LMODULE"
"LALG" -> "RING"
```

⟨*LALG.dotfull*⟩≡
```
"LeftAlgebra(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LALG"];
"LeftAlgebra(a:Ring)" -> "LeftModule(a:Ring)"
"LeftAlgebra(a:Ring)" -> "Ring()"
```

⟨*LALG.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"LeftAlgebra(a:Ring)" [color=lightblue];
"LeftAlgebra(a:Ring)" -> "LeftModule(a:Ring)"
"LeftAlgebra(a:Ring)" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SGROUP..."

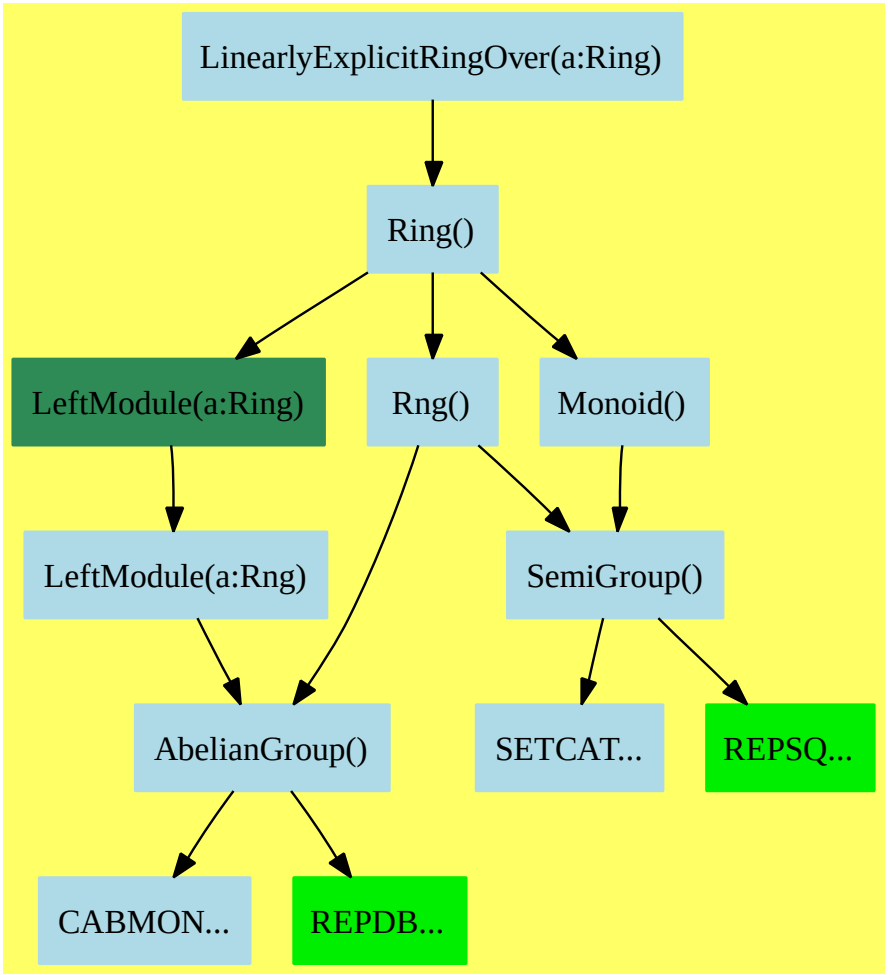
"Monoid()" [color=lightblue];
"Monoid()" -> "SGROUP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"SGROUP..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

10.9 LinearlyExplicitRingOver (LINEXP)



See:

⇒ "FullyLinearlyExplicitRingOver" (FLINEXP) 11.3 on page 782
⇒ "IntegerNumberSystem" (INS) 16.2 on page 1011
⇒ "MonogenicAlgebra" (MONOGEN) 19.2 on page 1305
⇐ "Ring" (RING) 9.8 on page 628

Exports:

0	1	characteristic	coerce	hash
latex	one?	recip	reducedSystem	subtractIfCan
sample	zero?	?*?	?**?	?+?
?-?	-?	?=?	?^?	?~=?

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return

These are directly exported but not implemented:

```
reducedSystem : (Matrix %,Vector %) ->
    Record(mat: Matrix R,vec: Vector R)
reducedSystem : Matrix % -> Matrix R
```

These exports come from (p628) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

⟨*category LINEXP LinearlyExplicitRingOver*⟩≡

```
)abbrev category LINEXP LinearlyExplicitRingOver
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
```

```
  ++ References:
  ++ Description:
  ++ An extension ring with an explicit linear dependence test.
  LinearlyExplicitRingOver(R:Ring): Category == Ring with
    reducedSystem: Matrix % -> Matrix R
      ++ reducedSystem(A) returns a matrix B such that \spad{A x = 0}
      ++ and \spad{B x = 0} have the same solutions in R.
    reducedSystem: (Matrix %,Vector %) -> Record(mat:Matrix R,vec:Vector R)
      ++ reducedSystem(A, v) returns a matrix B and a vector w such that
      ++ \spad{A x = v} and \spad{B x = w} have the same solutions in R.
```

⟨*LINEXP.dotabb*⟩≡

```
  "LINEXP"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=LINEXP"];
  "LINEXP" -> "RING"
```

⟨*LINEXP.dotfull*⟩≡

```
  "LinearlyExplicitRingOver(a:Ring)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=LINEXP"];
  "LinearlyExplicitRingOver(a:Ring)" -> "Ring()"

  "LinearlyExplicitRingOver(Integer)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=LINEXP"];
  "LinearlyExplicitRingOver(Integer)" -> "LinearlyExplicitRingOver(a:Ring)"

  "LinearlyExplicitRingOver(Fraction(Integer))"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=LINEXP"];
  "LinearlyExplicitRingOver(Fraction(Integer))" ->
      "LinearlyExplicitRingOver(a:Ring)"
```

⟨*LINEXP.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"LinearlyExplicitRingOver(a:Ring)" [color=lightblue];
"LinearlyExplicitRingOver(a:Ring)" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

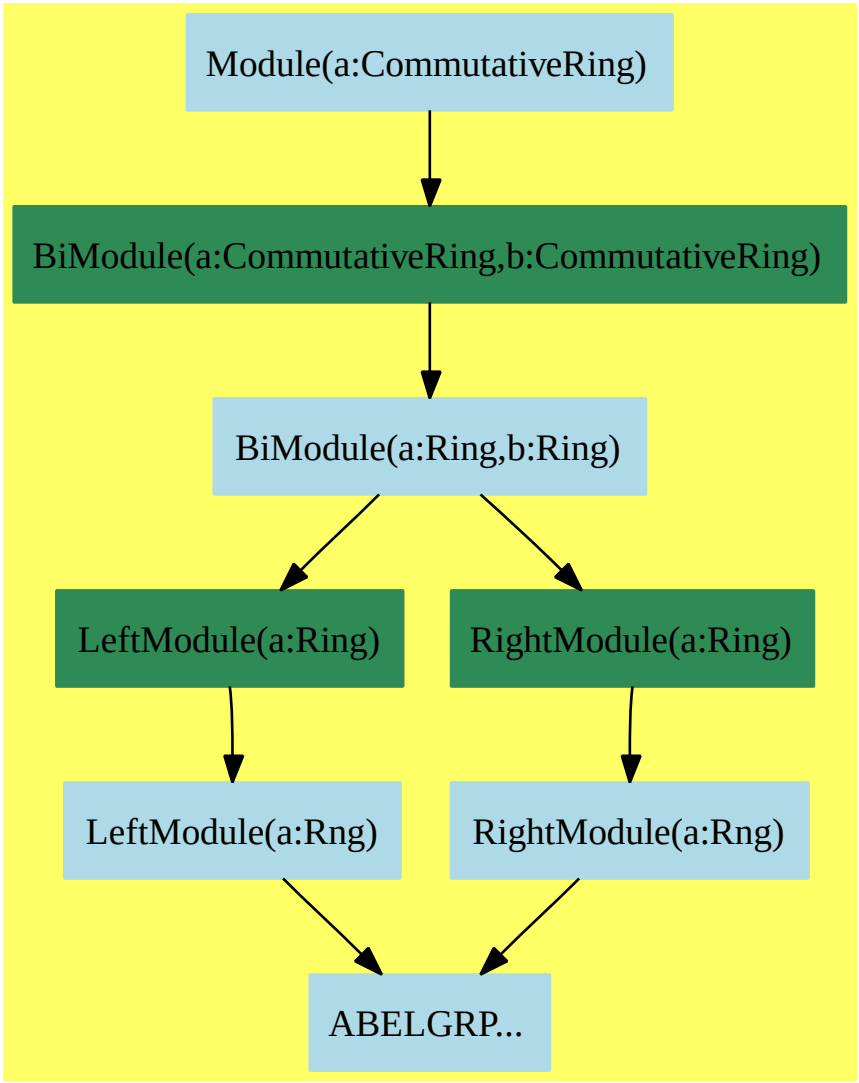
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

10.10 Module (MODULE)



See:

⇒ “Algebra” (ALGEBRA) 11.1 on page 771
⇒ “LieAlgebra” (LIECAT) 11.4 on page 787
⇒ “NonAssociativeAlgebra” (NAALG) 11.6 on page 798
⇒ “RectangularMatrixCategory” (RMATCAT) 10.14 on page 732
⇒ “VectorSpace” (VSPACE) 11.7 on page 803
⇐ “BiModule” (BMODULE) 9.1 on page 592

Exports:

0	coerce	hash	latex	sample
subtractIfCan	zero?	?~=?	?*?	?+?
?-?	-?	?=?		

Attributes exported:

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are implemented by this category:

```
?*? : (%,R) -> %
```

These exports come from (p592) BiModule(a:Ring,b:Ring):

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

⟨*category MODULE Module*⟩≡

```
)abbrev category MODULE Module
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of modules over a commutative ring.
++
++ Axioms:
```

```
++   \spad{1*x = x}
++   \spad{(a*b)*x = a*(b*x)}
++   \spad{(a+b)*x = (a*x)+(b*x)}
++   \spad{a*(x+y) = (a*x)+(a*y)}
Module(R:CommutativeRing): Category == BiModule(R,R)
  add
    if not(R is %) then x:%*r:R == r*x
```

⟨*MODULE.dotabb*⟩≡

```
"MODULE"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MODULE"];
"MODULE" -> "BMODULE"
```

⟨*MODULE.dotfull*⟩≡

```
"Module(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MODULE"];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"Module(Field)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=MODULE"];
"Module(Field)" -> "Module(a:CommutativeRing)"
```

⟨*MODULE.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

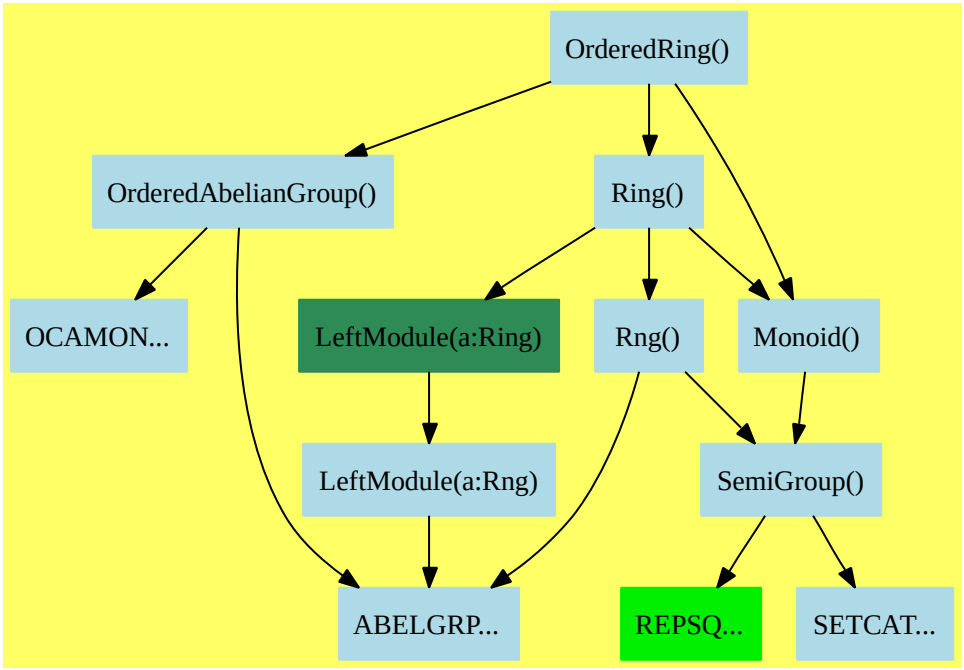
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"ABELGRP..." [color=lightblue];
}
```

10.11 OrderedRing (ORDRING)



See:

⇒ "DirectProductCategory" (DIRPCAT) 12.1 on page 815
⇒ "OrderedIntegralDomain" (OINTDOM) 13.5 on page 937
⇒ "RealClosedField" (RCFIELD) 17.7 on page 1132
⇒ "RealNumberSystem" (RNS) 17.8 on page 1141
⇐ "Monoid" (MONOID) 5.10 on page 256
⇐ "OrderedAbelianGroup" (OAGROUP) 9.5 on page 617
⇐ "Ring" (RING) 9.8 on page 628

Exports:

1	0	abs	characteristic	coerce
hash	latex	max	min	negative?
one?	positive?	recip	sample	sign
subtractIfCan	zero?	?^?	?~=?	?*?
?**?	?+?	-?	?-?	?<?
?<=?	?=?	?>?	?>=?	

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are implemented by this category:

```
abs : % -> %
negative? : % -> Boolean
positive? : % -> Boolean
sign : % -> Integer
```

These exports come from (p617) OrderedAbelianGroup():

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
max : (%,%) -> %
min : (%,%) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?+? : (%,%) -> %
-? : % -> %
?-? : (%,%) -> %
```

These exports come from (p628) Ring():

```
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
one? : % -> Boolean
recip : % -> Union(%,"failed")
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

⟨*category ORDRING OrderedRing*⟩≡

```
)abbrev category ORDRING OrderedRing
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
```

```
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Ordered sets which are also rings, that is, domains where the ring
++ operations are compatible with the ordering.
++
++ Axiom:
++   \spad{0<a and b<c => ab< ac}
OrderedRing(): Category == Join(OrderedAbelianGroup,Ring,Monoid) with
     positive?: % -> Boolean
       ++ positive?(x) tests whether x is strictly greater than 0.
     negative?: % -> Boolean
       ++ negative?(x) tests whether x is strictly less than 0.
     sign     : % -> Integer
       ++ sign(x) is 1 if x is positive, -1 if x is negative,
       ++ 0 if x equals 0.
     abs      : % -> %
       ++ abs(x) returns the absolute value of x.
  add
     positive? x == x>0
     negative? x == x<0
     sign x ==
       positive? x => 1
       negative? x => -1
       zero? x => 0
       error "x satisfies neither positive?, negative? or zero?"
     abs x ==
       positive? x => x
       negative? x => -x
       zero? x => 0
       error "x satisfies neither positive?, negative? or zero?"
```

⟨*ORDRING.dotabb*⟩≡

```
"ORDRING"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDRING"];
"ORDRING" -> "OAGROUP"
"ORDRING" -> "RING"
"ORDRING" -> "MONOID"
```

⟨*ORDRING.dotfull*⟩≡

```
"OrderedRing()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDRING"];
"OrderedRing()" -> "OrderedAbelianGroup()"
"OrderedRing()" -> "Ring()"
"OrderedRing()" -> "Monoid()"
```

⟨*ORDRING.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedRing()" [color=lightblue];
"OrderedRing()" -> "OrderedAbelianGroup()"
"OrderedRing()" -> "Ring()"
"OrderedRing()" -> "Monoid()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"OrderedAbelianGroup()" [color=lightblue];
"OrderedAbelianGroup()" -> "OCAMON..."
"OrderedAbelianGroup()" -> "ABELGRP..."

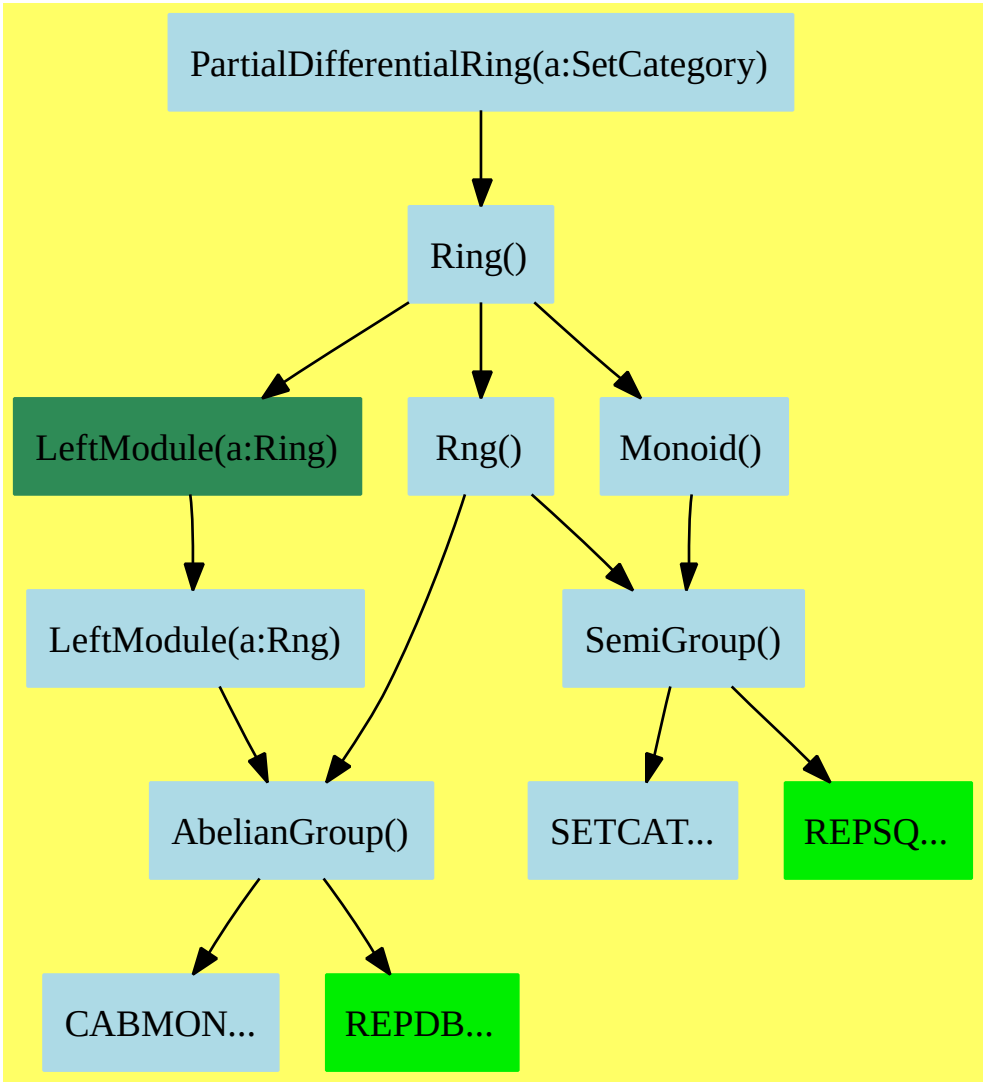
"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPSQ..." [color="#00EE00"];
"OCAMON..." [color=lightblue];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

10.12 PartialDifferentialRing (PDRING)



See:

⇒ “DifferentialExtension” (DIFEXT) 11.2 on page 777
⇒ “FunctionSpace” (FS) 17.5 on page 1095
⇒ “MultivariateTaylorSeriesCategory” (MTSCAT) 15.2 on page 983
⇒ “PolynomialCategory” (POLYCAT) 16.4 on page 1027
⇐ “Ring” (RING) 9.8 on page 628

Exports:

1	0	characteristic	coerce	D
differentiate	hash	latex	one?	recip
sample	subtractIfCan	zero?	?^?	?*?
?~=?	?**?	?+?	?-?	-?
?=?				

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are directly exported but not implemented:

```
differentiate : (%,S) -> %
```

These are implemented by this category:

```
D : (%,S) -> %
D : (%,List S) -> %
D : (%,S,NonNegativeInteger) -> %
D : (%,List S,List NonNegativeInteger) -> %
differentiate : (%,List S) -> %
differentiate : (%,S,NonNegativeInteger) -> %
differentiate : (%,List S,List NonNegativeInteger) -> %
```

These exports come from (p628) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

```
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

⟨*category PDRING PartialDifferentialRing*⟩≡

```
)abbrev category PDRING PartialDifferentialRing
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A partial differential ring with differentiations indexed by a
++ parameter type S.
++
++ Axioms:
++  \spad{differentiate(x+y,e) = differentiate(x,e)+differentiate(y,e)}
++  \spad{differentiate(x*y,e) = x*differentiate(y,e)+differentiate(x,e)*y}

PartialDifferentialRing(S:SetCategory): Category == Ring with
    differentiate: (%, S) -> %
        ++ differentiate(x,v) computes the partial derivative of x
        ++ with respect to v.
    differentiate: (%, List S) -> %
        ++ differentiate(x,[s1,...sn]) computes successive partial
        ++ derivatives,
        ++ i.e. \spad{differentiate(...differentiate(x, s1)..., sn)}.
    differentiate: (%, S, NonNegativeInteger) -> %
        ++ differentiate(x, s, n) computes multiple partial derivatives, i.e.
        ++ n-th derivative of x with respect to s.
    differentiate: (%, List S, List NonNegativeInteger) -> %
        ++ differentiate(x, [s1,...,sn], [n1,...,nn]) computes
        ++ multiple partial derivatives, i.e.
    D: (%, S) -> %
        ++ D(x,v) computes the partial derivative of x
        ++ with respect to v.
    D: (%, List S) -> %
        ++ D(x,[s1,...sn]) computes successive partial derivatives,
        ++ i.e. \spad{D(...D(x, s1)..., sn)}.
    D: (%, S, NonNegativeInteger) -> %
        ++ D(x, s, n) computes multiple partial derivatives, i.e.
        ++ n-th derivative of x with respect to s.
```

```
      D: (%, List S, List NonNegativeInteger) -> %
          ++ D(x, [s1,...,sn], [n1,...,nn]) computes
          ++ multiple partial derivatives, i.e.
          ++ \spad{D(...D(x, s1, n1)..., sn, nn)}.
    add
      differentiate(r:%, l:List S) ==
        for s in l repeat r := differentiate(r, s)
        r

      differentiate(r:%, s:S, n:NonNegativeInteger) ==
        for i in 1..n repeat r := differentiate(r, s)
        r

      differentiate(r:%, ls:List S, ln:List NonNegativeInteger) ==
        for s in ls for n in ln repeat r := differentiate(r, s, n)
        r

      D(r:%, v:S) == differentiate(r,v)
      D(r:%, lv:List S) == differentiate(r,lv)
      D(r:%, v:S, n:NonNegativeInteger) == differentiate(r,v,n)
      D(r:%, lv:List S, ln:List NonNegativeInteger) == differentiate(r, lv, ln)
```

⟨*PDRING.dotabb*⟩≡

```
"PDRING"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PDRING"];
"PDRING" -> "RING"
```

⟨*PDRING.dotfull*⟩≡

```
"PartialDifferentialRing(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PDRING"];
"PartialDifferentialRing(a:SetCategory)" -> "Ring()"

"PartialDifferentialRing(a:OrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PDRING"];
"PartialDifferentialRing(a:OrderedSet)" ->
    "PartialDifferentialRing(a:SetCategory)"

"PartialDifferentialRing(Symbol)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PDRING"];
"PartialDifferentialRing(Symbol)" ->
    "PartialDifferentialRing(a:SetCategory)"
```

⟨*PDRING.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PartialDifferentialRing(a:SetCategory)" [color=lightblue];
"PartialDifferentialRing(a:SetCategory)" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

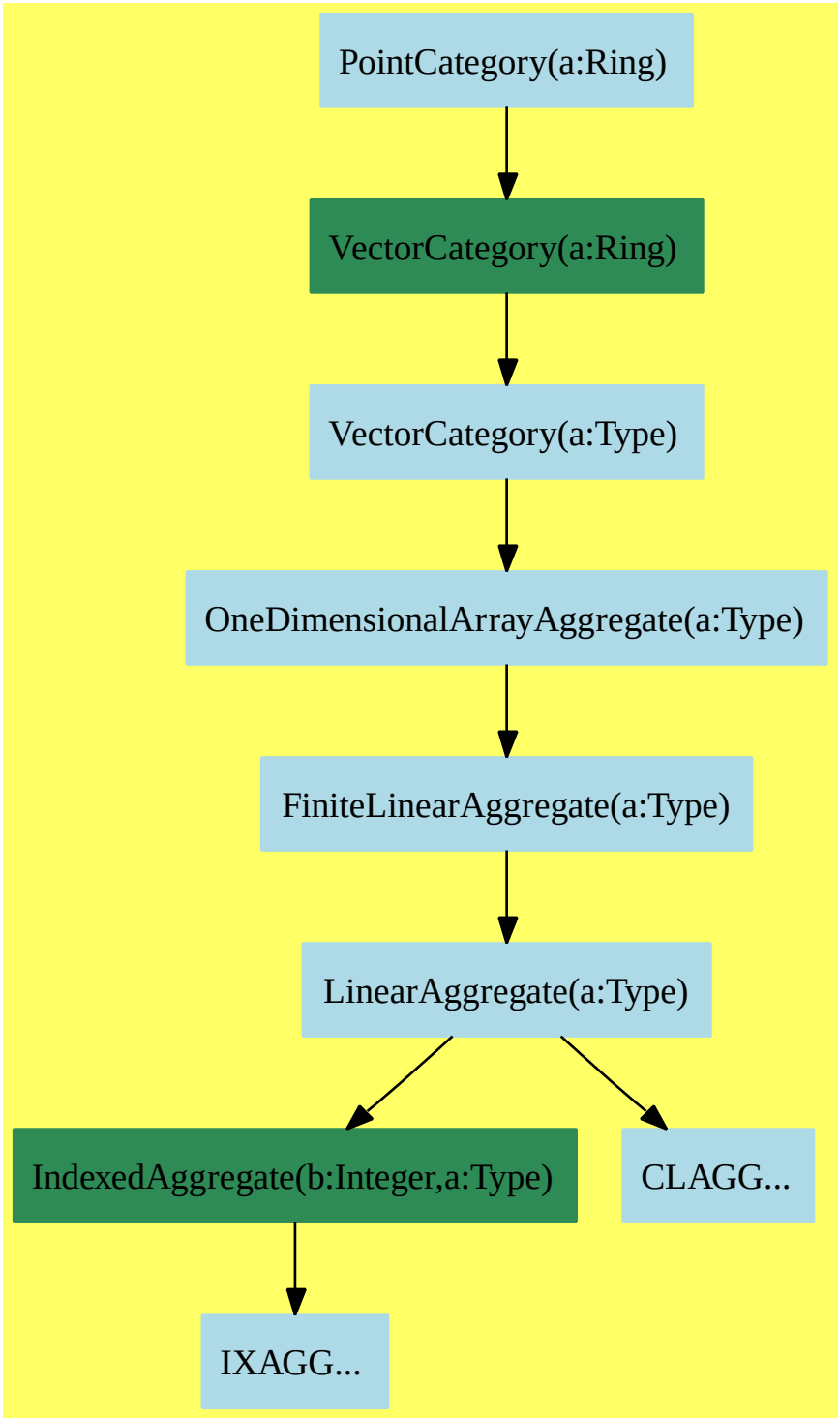
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

10.13 PointCategory (PTCAT)



See:

⇐ "VectorCategory" (VECTCAT) 9.12 on page 659

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	cross	count	delete
dimension	dot	elt	empty	empty?
entry?	entries	eq?	eval	every?
extend	fill!	find	first	hash
index?	indices	insert	latex	length
less?	magnitude	map	map!	max
maxIndex	member?	members	merge	min
minIndex	more?	new	outerProduct	parts
point	position	qelt	qsetelt!	reduce
remove	removeDuplicates	reverse	reverse!	sample
select	setelt	size?	sort	sort!
sorted?	swap!	zero	#?	?.?
?~=?	-?	?*?	?+?	?-?
?<?	?<=?	?=?	?>?	?>=?

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
convert : List R -> %
cross : (%,%) -> %
dimension : % -> PositiveInteger
extend : (%,List R) -> %
point : List R -> %
```

These exports come from (p659) VectorCategory(R:Ring):

```
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
coerce : % -> OutputForm if R has SETCAT
concat : List % -> %
concat : (%,%) -> %
concat : (R,%) -> %
concat : (%,R) -> %
construct : List R -> %
```

```
convert : % -> InputForm if R has KONVERT INFORM
copy : % -> %
copyInto! : (%,%,Integer) -> % if $ has shallowlyMutable
count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
delete : (%,Integer) -> %
delete : (%,UniversalSegment Integer) -> %
dot : (%,%) -> R if R has RING
elt : (%,Integer,R) -> R
empty : () -> %
empty? : % -> Boolean
entry? : (R,%) -> Boolean if $ has finiteAggregate and R has SETCAT
entries : % -> List R
eq? : (%,%) -> Boolean
eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
fill! : (%,R) -> % if $ has shallowlyMutable
find : ((R -> Boolean),%) -> Union(R,"failed")
first : % -> R if Integer has ORDSET
hash : % -> SingleInteger if R has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (%,%,Integer) -> %
insert : (R,%,Integer) -> %
latex : % -> String if R has SETCAT
length : % -> R if R has RING and R has RADCAT
less? : (%,NonNegativeInteger) -> Boolean
magnitude : % -> R if R has RING and R has RADCAT
map : (((R,R) -> R),%,%) -> %
map : ((R -> R),%) -> %
map! : ((R -> R),%) -> % if $ has shallowlyMutable
max : (%,%) -> % if R has ORDSET
maxIndex : % -> Integer if Integer has ORDSET
member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
members : % -> List R if $ has finiteAggregate
merge : (%,%) -> % if R has ORDSET
merge : (((R,R) -> Boolean),%,%) -> %
min : (%,%) -> % if R has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,R) -> %
outerProduct : (%,%) -> Matrix R if R has RING
parts : % -> List R if $ has finiteAggregate
position : (R,%,Integer) -> Integer if R has SETCAT
position : (R,%) -> Integer if R has SETCAT
position : ((R -> Boolean),%) -> Integer
qelt : (%,Integer) -> R
```

```
qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
reduce : (((R,R) -> R),%) -> R if $ has finiteAggregate
reduce : (((R,R) -> R),%,R) -> R if $ has finiteAggregate
reduce : (((R,R) -> R),%,R,R) -> R if R has SETCAT and $ has finiteAggregate
remove : ((R -> Boolean),%) -> % if $ has finiteAggregate
remove : (R,%) -> % if R has SETCAT and $ has finiteAggregate
removeDuplicates : % -> % if R has SETCAT and $ has finiteAggregate
reverse : % -> %
reverse! : % -> % if $ has shallowlyMutable
sample : () -> %
select : ((R -> Boolean),%) -> % if $ has finiteAggregate
setelt : (%,UniversalSegment Integer,R) -> R if $ has shallowlyMutable
setelt : (%,Integer,R) -> R if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
sort : % -> % if R has ORDSET
sort : (((R,R) -> Boolean),%) -> %
sort! : % -> % if R has ORDSET and $ has shallowlyMutable
sort! : (((R,R) -> Boolean),%) -> % if $ has shallowlyMutable
sorted? : % -> Boolean if R has ORDSET
sorted? : (((R,R) -> Boolean),%) -> Boolean
swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
zero : NonNegativeInteger -> % if R has ABELMON
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.? : (%,Integer) -> R
?.? : (%,UniversalSegment Integer) -> %
?~=? : (%,%) -> Boolean if R has SETCAT
?<? : (%,%) -> Boolean if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?=? : (%,%) -> Boolean if R has SETCAT
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
?*? : (Integer,%) -> % if R has ABELGRP
?*? : (%,R) -> % if R has MONOID
?*? : (R,%) -> % if R has MONOID
?-? : (%,%) -> % if R has ABELGRP
-? : % -> % if R has ABELGRP
?+? : (%,%) -> % if R has ABELSG
```

⟨*category PTCAT PointCategory*⟩≡

```
)abbrev category PTCAT PointCategory
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Operations: point, elt, setelt, copy, dimension, minIndex, maxIndex,
++ convert
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
```

```
++ References:
++ Description: PointCategory is the category of points in space which
++ may be plotted via the graphics facilities.  Functions are provided for
++ defining points and handling elements of points.

PointCategory(R:Ring) : Category == VectorCategory(R) with
  point: List R -> %
    ++ point(l) returns a point category defined by a list l of elements from
    ++ the domain R.
  dimension: % -> PositiveInteger
    ++ dimension(s) returns the dimension of the point category s.
  convert: List R -> %
    ++ convert(l) takes a list of elements, l, from the domain Ring and
    ++ returns the form of point category.
  cross: (%,%) -> %
      ++ cross(p,q) computes the cross product of the two points \spad{p}
      ++ and \spad{q}. Error if the p and q are not 3 dimensional
  extend : (%,List R) -> %
        ++ extend(x,l,r) \undocumented
```

⟨*PTCAT.dotabb*⟩≡

```
"PTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PTCAT"];
"PTCAT" -> "VECTCAT"
```

⟨*PTCAT.dotfull*⟩≡

```
"PointCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PTCAT"];
"PointCategory(a:Ring)" -> "VectorCategory(a:Ring)"
```

⟨*PTCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PointCategory(a:Ring)" [color=lightblue];
"PointCategory(a:Ring)" -> "VectorCategory(a:Ring)"

"VectorCategory(a:Ring)" [color=seagreen];
"VectorCategory(a:Ring)" -> "VectorCategory(a:Type)"

"VectorCategory(a:Type)" [color=lightblue];
"VectorCategory(a:Type)" -> "OneDimensionalArrayAggregate(a:Type)"

"OneDimensionalArrayAggregate(a:Type)" [color=lightblue];
"OneDimensionalArrayAggregate(a:Type)" ->
    "FiniteLinearAggregate(a:Type)"

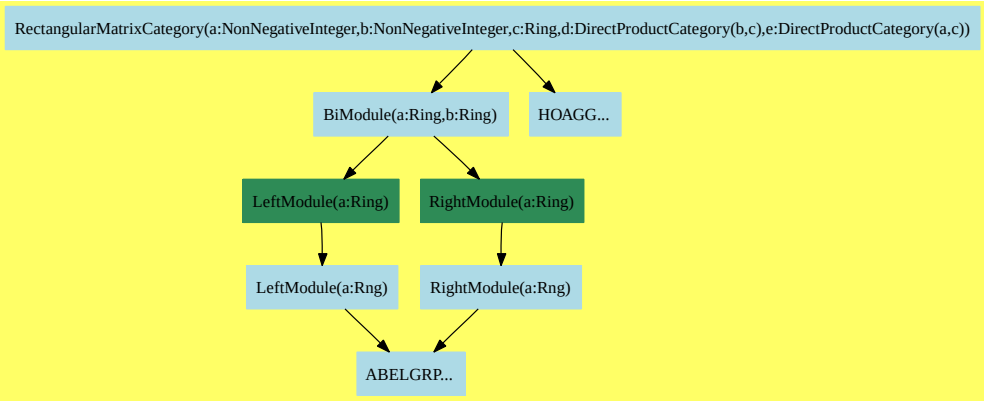
"FiniteLinearAggregate(a:Type)" [color=lightblue];
"FiniteLinearAggregate(a:Type)" -> "LinearAggregate(a:Type)"

"LinearAggregate(a:Type)" [color=lightblue];
"LinearAggregate(a:Type)" -> "IndexedAggregate(b:Integer,a:Type)"
"LinearAggregate(a:Type)" -> "CLAGG..."

"IndexedAggregate(b:Integer,a:Type)" [color=seagreen];
"IndexedAggregate(b:Integer,a:Type)" -> "IXAGG..."

"CLAGG..." [color=lightblue];
"IXAGG..." [color=lightblue];
}
```

10.14 RectangularMatrixCategory (RMATCAT)



We define three categories for matrices

- MatrixCategory is the category of all matrices
- RectangularMatrixCategory is the category of all matrices of a given dimension
- SquareMatrixCategory inherits from RectangularMatrixCategory

RectangularMatrixCategory does not automatically inherit MatrixCategory. Note that domains in DirectProductCategory(n,R), which are expected as parameters of RectangularMatrixCategory do not satisfy FiniteLinearAggregate(R) as required in MatrixCategory.

The RectangularMatrix domain is matrices of fixed dimension. **See:**

⇒ “SquareMatrixCategory” (SMATCAT) 12.9 on page 887
⇐ “BiModule” (BMODULE) 9.1 on page 592
⇐ “HomogeneousAggregate” (HOAGG) 4.12 on page 139
⇐ “MatrixCategory” (MATCAT) 6.7 on page 315

Exports:

0	antisymmetric?	any?	coerce	column
copy	count	diagonal?	elt	empty
empty?	eq?	eval	every?	exquo
hash	latex	less?	listOfLists	map
map!	matrix	maxColIndex	maxRowIndex	member?
members	minColIndex	minRowIndex	more?	ncols
nrows	nullSpace	nullity	parts	qelt
rank	row	rowEchelon	sample	size?
square?	subtractIfCan	symmetric?	zero?	#?
?*?	?/?	?+?	?-?	-?
?=?	?~=?			

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **nil**

These are directly exported but not implemented:

```
column : (%,Integer) -> Col
elt : (%,Integer,Integer) -> R
elt : (%,Integer,Integer,R) -> R
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
listOfLists : % -> List List R
map : (((R,R) -> R),%,%) -> %
map : ((R -> R),%) -> %
maxColIndex : % -> Integer
maxRowIndex : % -> Integer
matrix : List List R -> %
minColIndex : % -> Integer
minRowIndex : % -> Integer
nullity : % -> NonNegativeInteger if R has INTDOM
nullSpace : % -> List Col if R has INTDOM
qelt : (%,Integer,Integer) -> R
rank : % -> NonNegativeInteger if R has INTDOM
row : (%,Integer) -> Row
rowEchelon : % -> % if R has EUCDOM
?/? : (%,R) -> % if R has FIELD
```

These are implemented by this category:

```
antisymmetric? : % -> Boolean
diagonal? : % -> Boolean
ncols : % -> NonNegativeInteger
nrows : % -> NonNegativeInteger
square? : % -> Boolean
symmetric? : % -> Boolean
```

These exports come from (p592) BiModule(a:Ring,b:Ring)

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
```

```
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (R,%) -> %
?*? : (%,R) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

These exports come from (p139) HomogeneousAggregate(Ring)"

```
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
copy : % -> %
count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
less? : (%,NonNegativeInteger) -> Boolean
map! : ((R -> R),%) -> % if $ has shallowlyMutable
members : % -> List R if $ has finiteAggregate
member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List R if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
```

⟨*category RMATCAT RectangularMatrixCategory*⟩≡

```
)abbrev category RMATCAT RectangularMatrixCategory
++ Authors: Grabmeier, Gschnitzer, Williamson
++ Date Created: 1987
++ Date Last Updated: July 1990
++ Basic Operations:
++ Related Domains: RectangularMatrix(m,n,R)
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
```

```
++   \spadtype{RectangularMatrixCategory} is a category of matrices of fixed
++   dimensions.  The dimensions of the matrix will be parameters of the
++   domain.  Domains in this category will be R-modules and will be
++   non-mutable.
RectangularMatrixCategory(m,n,R,Row,Col): Category == Definition where
  m,n : NonNegativeInteger
  R   : Ring
  Row : DirectProductCategory(n,R)
  Col : DirectProductCategory(m,R)

  Definition ==> Join(BiModule(R,R),HomogeneousAggregate(R)) with

    finiteAggregate
      ++ matrices are finite

    if R has CommutativeRing then Module(R)

--% Matrix creation

    matrix: List List R -> %
      ++ \spad{matrix(l)} converts the list of lists l to a matrix, where the
      ++ list of lists is viewed as a list of the rows of the matrix.

--% Predicates

    square?  : % -> Boolean
      ++ \spad{square?(m)} returns true if m is a square matrix (i.e. if m
      ++ has the same number of rows as columns) and false otherwise.
    diagonal?: % -> Boolean
      ++ \spad{diagonal?(m)} returns true if the matrix m is square and
      ++ diagonal (i.e. all entries of m not on the diagonal are zero) and
      ++ false otherwise.
    symmetric?: % -> Boolean
      ++ \spad{symmetric?(m)} returns true if the matrix m is square and
      ++ symmetric (i.e. \spad{m[i,j] = m[j,i]} for all \spad{i} and j) and
      ++ false otherwise.
    antisymmetric?: % -> Boolean
      ++ \spad{antisymmetric?(m)} returns true if the matrix m is square and
      ++ antisymmetric (i.e. \spad{m[i,j] = -m[j,i]} for all \spad{i} and j)
      ++ and false otherwise.

--% Size inquiries

    minRowIndex : % -> Integer
      ++ \spad{minRowIndex(m)} returns the index of the 'first' row of the
      ++ matrix m.
```

```
    maxRowIndex : % -> Integer
      ++ \spad{maxRowIndex(m)} returns the index of the 'last' row of the
      ++ matrix m.
    minColIndex : % -> Integer
      ++ \spad{minColIndex(m)} returns the index of the 'first' column of the
      ++ matrix m.
    maxColIndex : % -> Integer
      ++ \spad{maxColIndex(m)} returns the index of the 'last' column of the
      ++ matrix m.
    nrows : % -> NonNegativeInteger
      ++ \spad{nrows(m)} returns the number of rows in the matrix m.
    ncols : % -> NonNegativeInteger
      ++ \spad{ncols(m)} returns the number of columns in the matrix m.

  --% Part extractions

    listOfLists: % -> List List R
      ++ \spad{listOfLists(m)} returns the rows of the matrix m as a list
      ++ of lists.
    elt: (%,Integer,Integer) -> R
      ++ \spad{elt(m,i,j)} returns the element in the \spad{i}th row and
      ++ \spad{j}th column of the matrix m.
      ++ Error: if indices are outside the proper
      ++ ranges.
    qelt: (%,Integer,Integer) -> R
      ++ \spad{qelt(m,i,j)} returns the element in the \spad{i}th row and
      ++ \spad{j}th column of
      ++ the matrix m. Note: there is NO error check to determine if indices
      ++ are in the proper ranges.
    elt: (%,Integer,Integer,R) -> R
      ++ \spad{elt(m,i,j,r)} returns the element in the \spad{i}th row and
      ++ \spad{j}th column of the matrix m, if m has an \spad{i}th row and a
      ++ \spad{j}th column, and returns r otherwise.
    row: (%,Integer) -> Row
      ++ \spad{row(m,i)} returns the \spad{i}th row of the matrix m.
      ++ Error: if the index is outside the proper range.
    column: (%,Integer) -> Col
      ++ \spad{column(m,j)} returns the \spad{j}th column of the matrix m.
      ++ Error: if the index outside the proper range.

  --% Map and Zip

    map: (R -> R,%) -> %
      ++ \spad{map(f,a)} returns b, where \spad{b(i,j) = a(i,j)} for all i, j.
    map:((R,R) -> R,%,%) -> %
      ++ \spad{map(f,a,b)} returns c, where c is such that
```

```
      ++ \spad{c(i,j) = f(a(i,j),b(i,j))} for all \spad{i}, j.

  --% Arithmetic

    if R has IntegralDomain then
      "exquo": (%,R) -> Union(%,"failed")
        ++ \spad{exquo(m,r)} computes the exact quotient of the elements
        ++ of m by r, returning \axiom{"failed"} if this is not possible.
    if R has Field then
      "/": (%,R) -> %
        ++ \spad{m/r} divides the elements of m by r. Error: if \spad{r = 0}.

  --% Linear algebra

    if R has EuclideanDomain then
      rowEchelon: % -> %
        ++ \spad{rowEchelon(m)} returns the row echelon form of the matrix m.
    if R has IntegralDomain then
      rank: % -> NonNegativeInteger
        ++ \spad{rank(m)} returns the rank of the matrix m.
      nullity: % -> NonNegativeInteger
        ++ \spad{nullity(m)} returns the nullity of the matrix m. This is
        ++ the dimension of the null space of the matrix m.
      nullSpace: % -> List Col
        ++ \spad{nullSpace(m)}+ returns a basis for the null space of
        ++ the matrix m.
   add
     nrows x == m

     ncols x == n

     square? x == m = n

     diagonal? x ==
       not square? x => false
       for i in minRowIndex x .. maxRowIndex x repeat
         for j in minColIndex x .. maxColIndex x
           | (j - minColIndex x) ^= (i - minRowIndex x) repeat
             not zero? qelt(x, i, j) => return false
       true

     symmetric? x ==
       m ^= n => false
       mr := minRowIndex x; mc := minColIndex x
       for i in 0..(n - 1) repeat
         for j in (i + 1)..(n - 1) repeat
```

```
            qelt(x,mr + i,mc + j) ^= qelt(x,mr + j,mc + i) => return false
        true

      antisymmetric? x ==
        m ^= n => false
        mr := minRowIndex x; mc := minColIndex x
        for i in 0..(n - 1) repeat
          for j in i..(n - 1) repeat
            qelt(x,mr + i,mc + j) ^= -qelt(x,mr + j,mc + i) =>
              return false
        true
```

⟨*RMATCAT.dotabb*⟩≡

```
"RMATCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RMATCAT"];
"RMATCAT" -> "BMODULE"
"RMATCAT" -> "HOAGG"
```

⟨*RMATCAT.dotfull*⟩≡

```
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:Dir
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RMATCAT"];
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:Dir
  -> "BiModule(a:Ring,b:Ring)"
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:Dir
  -> "HomogeneousAggregate(Ring)"
```

⟨*RMATCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductC
 [color=lightblue];
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductC
  -> "BiModule(a:Ring,b:Ring)"
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductC
  -> "HOAGG..."

"HOAGG..." [color=lightblue];
"ABELGRP..." [color=lightblue];

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

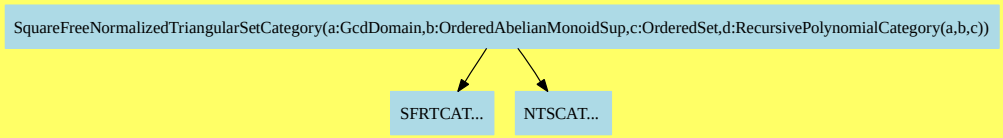
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

}
```

10.15 SquareFreeNormalizedTriangularSetCategory (SNTSCAT)



See:

⇐ "NormalizedTriangularSetCategory" (NTSCAT) 9.4 on page 608
⇐ "SquareFreeRegularTriangularSetCategory" (SFRTCAT) 9.9 on page 632

Exports:

algebraic?	algebraicCoefficients?
algebraicVariables	any?
augment	autoReduced?
basicSet	coerce
coHeight	collect
collectQuasiMonic	collectUnder
collectUpper	construct
convert	copy
count	degree
empty	empty?
eq?	eval
every?	extend
extendIfCan	find
first	hash
headReduce	headReduced?
headRemainder	infRittWu?
initiallyReduce	initiallyReduced?
initials	internalAugment
intersect	invertible?
invertibleElseSplit?	invertibleSet
last	lastSubResultant
lastSubResultantElseSplit	latex
less?	mainVariable?
mainVariables	map
map!	member?
members	more?
mvar	normalized?
parts	purelyAlgebraic?
purelyAlgebraicLeadingMonomial?	purelyTranscendental?
quasiComponent	reduce
reduced?	reduceByQuasiMonic
remainder	remove
removeDuplicates	removeZero
rest	retract
retractIfCan	rewriteIdealWithHeadRemainder
rewriteIdealWithRemainder	rewriteSetWithReduction
roughBase?	roughEqualIdeals?
roughSubIdeal?	roughUnitIdeal?
sample	select
size?	sort
squareFreePart	stronglyReduce
stronglyReduced?	triangular?
trivialIdeal?	variables
zeroSetSplit	zeroSetSplitIntoTriangularSystems
#?	?=?
?~=?	

Attributes Exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These exports come from
(p632) SquareFreeRegularTriangularSetCategory(R,E,V,P)
where R:GcdDomain, E:OrderedAbelianMonoidSup, V:OrderedSet,
P:RecursivePolynomialCategory(R,E,V)):

```
algebraic? : (V,%) -> Boolean
algebraicCoefficients? : (P,%) -> Boolean
algebraicVariables : % -> List V
any? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
augment : (List P,List %) -> List %
augment : (List P,%) -> List %
augment : (P,List %) -> List %
augment : (P,%) -> List %
autoReduced? : (%,((P,List P) -> Boolean)) -> Boolean
basicSet :
  (List P,(P -> Boolean),((P,P) -> Boolean)) ->
     Union(Record(bas: %,top: List P),"failed")
basicSet :
  (List P,((P,P) -> Boolean)) ->
     Union(Record(bas: %,top: List P),"failed")
coerce : % -> List P
coerce : % -> OutputForm
coHeight : % -> NonNegativeInteger if V has FINITE
collect : (%,V) -> %
collectQuasiMonic : % -> %
collectUnder : (%,V) -> %
collectUpper : (%,V) -> %
construct : List P -> %
convert : % -> InputForm if P has KONVERT INFORM
copy : % -> %
count : ((P -> Boolean),%) -> NonNegativeInteger
  if $ has finiteAggregate
count : (P,%) -> NonNegativeInteger
  if P has SETCAT
  and $ has finiteAggregate
degree : % -> NonNegativeInteger
empty : () -> %
empty? : % -> Boolean
```

```
eq? : (%,%) -> Boolean
eval : (%,List Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,Equation P) -> % if P has EVALAB P and P has SETCAT
eval : (%,P,P) -> % if P has EVALAB P and P has SETCAT
eval : (%,List P,List P) -> % if P has EVALAB P and P has SETCAT
every? : ((P -> Boolean),%) -> Boolean if $ has finiteAggregate
extend : (List P,List %) -> List %
extend : (List P,%) -> List %
extend : (P,List %) -> List %
extend : (P,%) -> List %
extend : (%,P) -> %
extendIfCan : (%,P) -> Union(%,"failed")
find : ((P -> Boolean),%) -> Union(P,"failed")
first : % -> Union(P,"failed")
hash : % -> SingleInteger
headReduce : (P,%) -> P
headReduced? : % -> Boolean
headReduced? : (P,%) -> Boolean
headRemainder : (P,%) -> Record(num: P,den: R) if R has INTDOM
infRittWu? : (%,%) -> Boolean
initiallyReduce : (P,%) -> P
initiallyReduced? : % -> Boolean
initiallyReduced? : (P,%) -> Boolean
initials : % -> List P
internalAugment : (P,%) -> %
internalAugment : (List P,%) -> %
intersect : (List P,List %) -> List %
intersect : (P,List %) -> List %
intersect : (List P,%) -> List %
intersect : (P,%) -> List %
invertible? : (P,%) -> Boolean
invertible? : (P,%) -> List Record(val: Boolean,tower: %)
invertibleElseSplit? : (P,%) -> Union(Boolean,List %)
invertibleSet : (P,%) -> List %
last : % -> Union(P,"failed")
lastSubResultant : (P,P,%) -> List Record(val: P,tower: %)
lastSubResultantElseSplit : (P,P,%) -> Union(P,List %)
latex : % -> String
less? : (%,NonNegativeInteger) -> Boolean
mainVariable? : (V,%) -> Boolean
mainVariables : % -> List V
map : ((P -> P),%) -> %
map! : ((P -> P),%) -> % if $ has shallowlyMutable
member? : (P,%) -> Boolean if P has SETCAT and $ has finiteAggregate
members : % -> List P if $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
mvar : % -> V
normalized? : % -> Boolean
normalized? : (P,%) -> Boolean
parts : % -> List P if $ has finiteAggregate
```

```
purelyAlgebraic? : % -> Boolean
purelyAlgebraic? : (P,%) -> Boolean
purelyAlgebraicLeadingMonomial? : (P,%) -> Boolean
purelyTranscendental? : (P,%) -> Boolean
quasiComponent : % -> Record(close: List P,open: List P)
reduce : (P,%,((P,P) -> P),((P,P) -> Boolean)) -> P
reduce : (((P,P) -> P),%) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P) -> P if $ has finiteAggregate
reduce : (((P,P) -> P),%,P,P) -> P
  if P has SETCAT
  and $ has finiteAggregate
reduced? : (P,%,((P,P) -> Boolean)) -> Boolean
reduceByQuasiMonic : (P,%) -> P
remainder : (P,%) -> Record(rnum: R,polnum: P,den: R) if R has INTDOM
remove : ((P -> Boolean),%) -> % if $ has finiteAggregate
remove : (P,%) -> % if P has SETCAT and $ has finiteAggregate
removeDuplicates : % -> % if P has SETCAT and $ has finiteAggregate
removeZero : (P,%) -> P
rest : % -> Union(%,"failed")
retract : List P -> %
retractIfCan : List P -> Union(%,"failed")
rewriteIdealWithHeadRemainder : (List P,%) -> List P if R has INTDOM
rewriteIdealWithRemainder : (List P,%) -> List P if R has INTDOM
rewriteSetWithReduction :
   (List P,%,((P,P) -> P),((P,P) -> Boolean)) -> List P
roughBase? : % -> Boolean if R has INTDOM
roughEqualIdeals? : (%,%) -> Boolean if R has INTDOM
roughSubIdeal? : (%,%) -> Boolean if R has INTDOM
roughUnitIdeal? : % -> Boolean if R has INTDOM
sample : () -> %
select : (%,V) -> Union(P,"failed")
select : ((P -> Boolean),%) -> % if $ has finiteAggregate
size? : (%,NonNegativeInteger) -> Boolean
sort : (%,V) -> Record(under: %,floor: %,upper: %)
squareFreePart : (P,%) -> List Record(val: P,tower: %)
stronglyReduce : (P,%) -> P
stronglyReduced? : (P,%) -> Boolean
stronglyReduced? : % -> Boolean
triangular? : % -> Boolean if R has INTDOM
trivialIdeal? : % -> Boolean
variables : % -> List V
zeroSetSplit : List P -> List %
zeroSetSplit : (List P,Boolean) -> List %
zeroSetSplitIntoTriangularSystems :
   List P -> List Record(close: %,open: List P)
#? : % -> NonNegativeInteger if $ has finiteAggregate
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

⟨*category SNTSCAT SquareFreeNormalizedTriangularSetCategory*⟩≡

```
)abbrev category SNTSCAT SquareFreeNormalizedTriangularSetCategory
++ Author: Marc Moreno Maza
++ Date Created: 10/07/1998
++ Date Last Updated: 12/16/1998
++ Basic Functions:
++ Related Constructors:
++ Also See: essai Graphisme
++ AMS Classifications:
++ Keywords: polynomial, multivariate, ordered variables set
++ Description:
++ The category of square-free and normalized triangular sets.
++ Thus, up to the primitivity axiom of [1], these sets are Lazard
++ triangular sets.\newline
++ References :
++  [1] D. LAZARD "A new method for solving algebraic systems of
++      positive dimension" Discr. App. Math. 33:147-160,1991
SquareFreeNormalizedTriangularSetCategory(R:GcdDomain,_
                                          E:OrderedAbelianMonoidSup,_
                                          V:OrderedSet,_
                                         P:RecursivePolynomialCategory(R,E,V)):
          Category ==
    Join(SquareFreeRegularTriangularSetCategory(R,E,V,P), _
         NormalizedTriangularSetCategory(R,E,V,P))
```

⟨*SNTSCAT.dotabb*⟩≡

```
"SNTSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SNTSCAT"];
"SNTSCAT" -> "NTSCAT"
"SNTSCAT" -> "SFRTCAT"
```

⟨*SNTSCAT.dotfull*⟩≡

```
"SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSe
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SNTSCAT"];
"SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSe
 ->
"SquareFreeRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,c

"SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSe
 ->
"NormalizedRegularTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,c:OrderedSet,c
```

⟨*SNTSCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

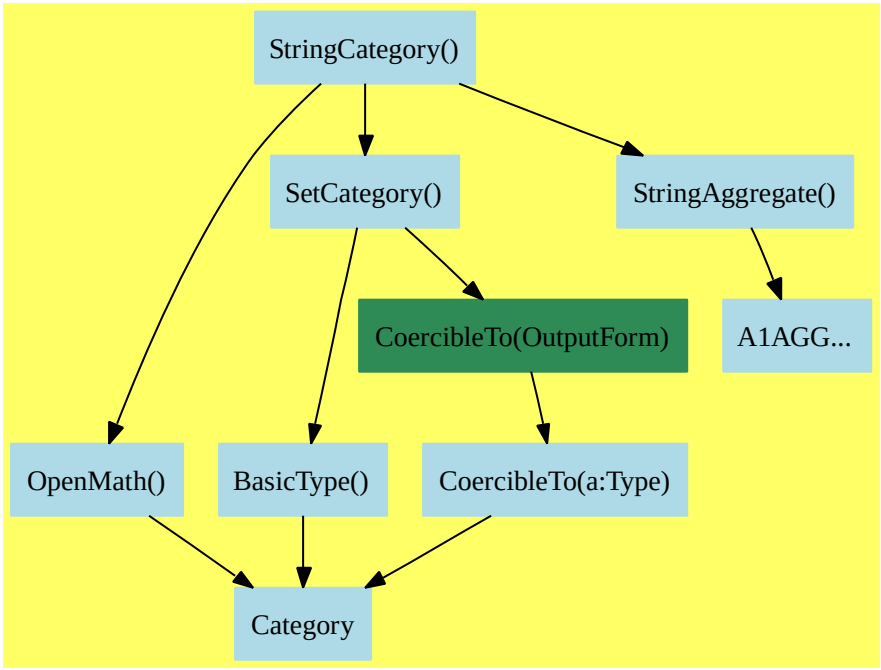
"SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,
 [color=lightblue];
"SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,
 -> "SFRTCAT..."

"SquareFreeNormalizedTriangularSetCategory(a:GcdDomain,b:OrderedAbelianMonoidSup,
 -> "NTSCAT..."

"SFRTCAT..."  [color=lightblue];
"NTSCAT..."  [color=lightblue];

}
```

10.16 StringCategory (STRICAT)



See:

⇐ "OpenMath" (OM) 2.13 on page 32
⇐ "SetCategory" (SETCAT) 3.13 on page 91
⇐ "StringAggregate" (SRAGG) 9.10 on page 640

Exports:

any?	coerce	concat	construct	convert
copy	copyInto!	count	delete	elt
empty	empty?	entry?	entries	eq?
eval	every?	fill!	find	first
hash	index?	indices	insert	latex
leftTrim	less?	lowerCase	lowerCase!	map
map!	match	match?	max	maxIndex
member?	members	merge	min	minIndex
more?	new	OMwrite	parts	position
prefix?	qelt	qsetelt!	reduce	remove
removeDuplicates	replace	reverse	reverse!	rightTrim
sample	select	setelt	size?	sort
sort!	sorted?	split	string	substring?
suffix?	swap!	trim	upperCase	upperCase!
#?	?<?	?<=?	?>?	?>=?
?=?	?.?	?~=?		

Attributes exported:

- **shallowlyMutable** is true if its values have immediate components that are updateable (mutable). Note: the properties of any component domain are irrevelant to the shallowlyMutable proper.
- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **nil**

These are directly exported but not implemented:

```
string : Integer -> %
```

These exports come from (p640) StringAggregate():

```
any? : ((Character -> Boolean),%) -> Boolean
    if $ has finiteAggregate
coerce : % -> OutputForm
coerce : Character -> %
concat : List % -> %
concat : (%,%) -> %
concat : (Character,%) -> %
concat : (%,Character) -> %
construct : List Character -> %
convert : % -> InputForm
    if Character has KONVERT INFORM
copy : % -> %
copyInto! : (%,%,Integer) -> %
    if $ has shallowlyMutable
count : (Character,%) -> NonNegativeInteger
```

```
    if Character has SETCAT
    and $ has finiteAggregate
count : ((Character -> Boolean),%) -> NonNegativeInteger
    if $ has finiteAggregate
delete : (%,UniversalSegment Integer) -> %
delete : (%,Integer) -> %
elt : (%,Integer,Character) -> Character
empty : () -> %
empty? : % -> Boolean
entry? : (Character,%) -> Boolean
    if $ has finiteAggregate
    and Character has SETCAT
entries : % -> List Character
eq? : (%,%) -> Boolean
eval : (%,List Character,List Character) -> %
    if Character has EVALAB CHAR
    and Character has SETCAT
eval : (%,Character,Character) -> %
    if Character has EVALAB CHAR
    and Character has SETCAT
eval : (%,Equation Character) -> %
    if Character has EVALAB CHAR
    and Character has SETCAT
eval : (%,List Equation Character) -> %
    if Character has EVALAB CHAR
    and Character has SETCAT
every? : ((Character -> Boolean),%) -> Boolean
    if $ has finiteAggregate
fill! : (%,Character) -> %
    if $ has shallowlyMutable
find : ((Character -> Boolean),%) -> Union(Character,"failed")
first : % -> Character
    if Integer has ORDSET
hash : % -> SingleInteger
index? : (Integer,%) -> Boolean
indices : % -> List Integer
insert : (%,%,Integer) -> %
insert : (Character,%,Integer) -> %
latex : % -> String
leftTrim : (%,Character) -> %
leftTrim : (%,CharacterClass) -> %
less? : (%,NonNegativeInteger) -> Boolean
lowerCase : % -> %
lowerCase! : % -> %
map : (((Character,Character) -> Character),%,%) -> %
map : ((Character -> Character),%) -> %
map! : ((Character -> Character),%) -> %
    if $ has shallowlyMutable
match : (%,%,Character) -> NonNegativeInteger
match? : (%,%,Character) -> Boolean
```

```
max : (%,%) -> % if Character has ORDSET
maxIndex : % -> Integer if Integer has ORDSET
member? : (Character,%) -> Boolean
    if Character has SETCAT
    and $ has finiteAggregate
members : % -> List Character
    if $ has finiteAggregate
merge : (%,%) -> % if Character has ORDSET
merge : (((Character,Character) -> Boolean),%,%) -> %
min : (%,%) -> % if Character has ORDSET
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
new : (NonNegativeInteger,Character) -> %
parts : % -> List Character if $ has finiteAggregate
position : (Character,%) -> Integer
    if Character has SETCAT
position : ((Character -> Boolean),%) -> Integer
position : (Character,%,Integer) -> Integer
    if Character has SETCAT
position : (CharacterClass,%,Integer) -> Integer
position : (%,%,Integer) -> Integer
prefix? : (%,%) -> Boolean
qelt : (%,Integer) -> Character
qsetelt! : (%,Integer,Character) -> Character
    if $ has shallowlyMutable
reduce : (((Character,Character) -> Character),%)
   -> Character
    if $ has finiteAggregate
reduce : (((Character,Character) -> Character),%,Character)
   -> Character
    if $ has finiteAggregate
reduce :
 (((Character,Character) -> Character),%,Character,Character)
   -> Character
    if Character has SETCAT
    and $ has finiteAggregate
remove : ((Character -> Boolean),%) -> %
    if $ has finiteAggregate
remove : (Character,%) -> %
    if Character has SETCAT
    and $ has finiteAggregate
removeDuplicates : % -> %
    if Character has SETCAT
    and $ has finiteAggregate
replace : (%,UniversalSegment Integer,%) -> %
reverse : % -> %
reverse! : % -> % if $ has shallowlyMutable
rightTrim : (%,CharacterClass) -> %
rightTrim : (%,Character) -> %
sample : () -> %
```

```
select : ((Character -> Boolean),%) -> %
    if $ has finiteAggregate
setelt :
  (%,UniversalSegment Integer,Character) -> Character
    if $ has shallowlyMutable
setelt : (%,Integer,Character) -> Character
    if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
sort : % -> % if Character has ORDSET
sort : (((Character,Character) -> Boolean),%) -> %
sort! : % -> %
    if Character has ORDSET
    and $ has shallowlyMutable
sort! : (((Character,Character) -> Boolean),%) -> %
    if $ has shallowlyMutable
sorted? : (((Character,Character) -> Boolean),%) -> Boolean
sorted? : % -> Boolean if Character has ORDSET
split : (%,CharacterClass) -> List %
split : (%,Character) -> List %
substring? : (%,%,Integer) -> Boolean
suffix? : (%,%) -> Boolean
swap! : (%,Integer,Integer) -> Void
    if $ has shallowlyMutable
trim : (%,CharacterClass) -> %
trim : (%,Character) -> %
upperCase : % -> %
upperCase! : % -> %
#? : % -> NonNegativeInteger if $ has finiteAggregate
?<? : (%,%) -> Boolean if Character has ORDSET
?<=? : (%,%) -> Boolean if Character has ORDSET
?>? : (%,%) -> Boolean if Character has ORDSET
?>=? : (%,%) -> Boolean if Character has ORDSET
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?.? : (%,UniversalSegment Integer) -> %
?.? : (%,Integer) -> Character
?.? : (%,%) -> %
```

These exports come from (p91) SetCategory():

These exports come from (p32) OpenMath():

```
OMwrite : (%,Boolean) -> String
OMwrite : % -> String
OMwrite : (OpenMathDevice,%,Boolean) -> Void
OMwrite : (OpenMathDevice,%) -> Void
```

⟨*category STRICAT StringCategory*⟩≡

```
)abbrev category STRICAT StringCategory
-- Note that StringCategory is built into the old compiler
-- redundant SetCategory added to help A# compiler
++ Description:
++ A category for string-like objects

StringCategory():Category == _
     Join(StringAggregate(), SetCategory, OpenMath) with
  string: Integer -> %
    ++ string(i) returns the decimal representation of i in a string
```

⟨*STRICAT.dotabb*⟩≡

```
"STRICAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=STRICAT"];
"STRICAT" -> "OM"
"STRICAT" -> "SETCAT"
"STRICAT" -> "SRAGG"
```

⟨*STRICAT.dotfull*⟩≡

```
"StringCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=STRICAT"];
"StringCategory()" -> "OpenMath()"
"StringCategory()" -> "SetCategory()"
"StringCategory()" -> "StringAggregate()"
```

⟨*STRICAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"StringCategory()" [color=lightblue];
"StringCategory()" -> "OpenMath()"
"StringCategory()" -> "SetCategory()"
"StringCategory()" -> "StringAggregate()"

"OpenMath()" [color=lightblue];
"OpenMath()" -> "Category"

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

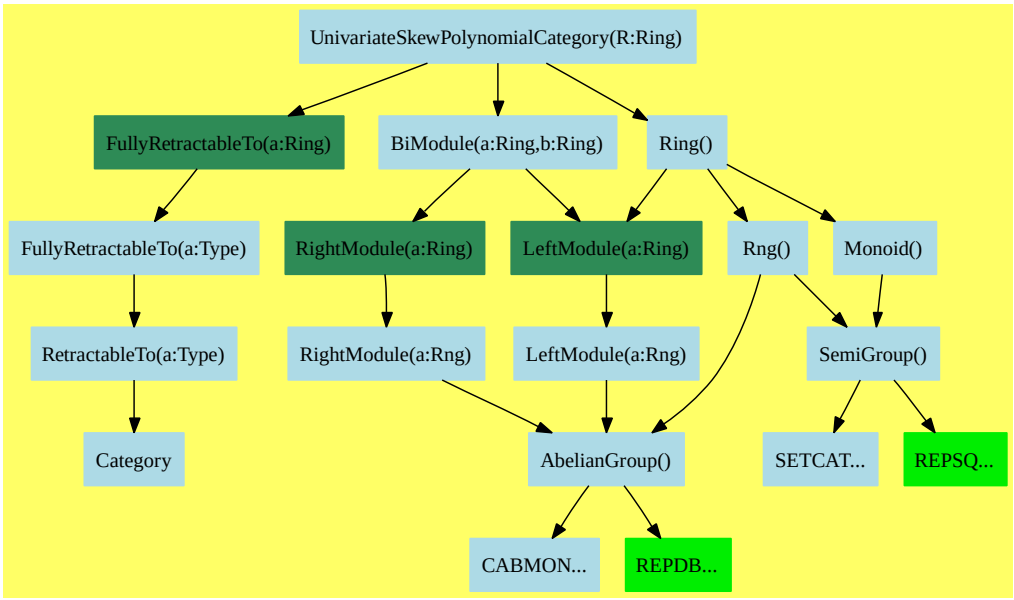
"StringAggregate()" [color=lightblue];
"StringAggregate()" -> "A1AGG..."

"A1AGG..." [color=lightblue];

"Category" [color=lightblue];

}
```

10.17 UnivariateSkewPolynomialCategory (OREPCAT)



See:

⇒ "LinearOrdinaryDifferentialOperatorCategory" (LODOCAT) 11.5 on page 791

⇐ "BiModule" (BMODULE) 9.1 on page 592
⇐ "FullyRetractableTo" (FRETRCT) 3.6 on page 71
⇐ "Ring" (RING) 9.8 on page 628

Exports:

0	1	apply	characteristic
coefficient	coefficients	coerce	content
degree	exquo	hash	latex
leadingCoefficient	leftDivide	leftExactQuotient	leftExtendedGcd
leftGcd	leftLcm	leftQuotient	leftRemainder
minimumDegree	monicLeftDivide	monicRightDivide	monomial
one?	primitivePart	recip	reductum
retract	retractIfCan	rightDivide	rightExactQuotient
rightExtendedGcd	rightGcd	rightLcm	rightQuotient
rightRemainder	sample	subtractIfCan	zero?
?*?	?**?	?+?	?-?
-?	?=?	?^?	?~=?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
apply : (%,R,R) -> R
coefficient : (%,NonNegativeInteger) -> R
degree : % -> NonNegativeInteger
leadingCoefficient : % -> R
leftDivide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
minimumDegree : % -> NonNegativeInteger
monicLeftDivide : (%,%) -> Record(quotient: %,remainder: %) if R has INTDOM
monicRightDivide : (%,%) -> Record(quotient: %,remainder: %) if R has INTDOM
monomial : (R,NonNegativeInteger) -> %
reductum : % -> %
rightDivide : (%,%) -> Record(quotient: %,remainder: %) if R has FIELD
```

These are implemented by this category:

```
coefficients : % -> List R
coerce : R -> %
content : % -> R if R has GCDDOM
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
leftExactQuotient : (%,%) -> Union(%,"failed") if R has FIELD
leftExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
leftGcd : (%,%) -> % if R has FIELD
leftLcm : (%,%) -> % if R has FIELD
leftQuotient : (%,%) -> % if R has FIELD
leftRemainder : (%,%) -> % if R has FIELD
primitivePart : % -> % if R has GCDDOM
retractIfCan : % -> Union(R,"failed")
rightExactQuotient : (%,%) -> Union(%,"failed") if R has FIELD
rightExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has FIELD
rightGcd : (%,%) -> % if R has FIELD
rightLcm : (%,%) -> % if R has FIELD
rightQuotient : (%,%) -> % if R has FIELD
rightRemainder : (%,%) -> % if R has FIELD
?*? : (R,%) -> %
```

These exports come from (p628) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
```

```
coerce : % -> OutputForm
coerce : Integer -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p592) BiModule(R:Ring,R:Ring):

```
?*? : (%,R) -> %
```

These exports come from (p71) FullyRetractableTo(R:Ring):

```
coerce : Fraction Integer -> % if R has RETRACT FRAC INT
retract : % -> R
retract : % -> Fraction Integer if R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
```

⟨*category OREPCAT UnivariateSkewPolynomialCategory*⟩≡

```
)abbrev category OREPCAT UnivariateSkewPolynomialCategory
++ Author: Manuel Bronstein, Jean Della Dora, Stephen M. Watt
++ Date Created: 19 October 1993
++ Date Last Updated: 1 February 1994
++ Description:
++   This is the category of univariate skew polynomials over an Ore
++   coefficient ring.
++   The multiplication is given by \spad{x a = \sigma(a) x + \delta a}.
++   This category is an evolution of the types
++     MonogenicLinearOperator, OppositeMonogenicLinearOperator, and
++     NonCommutativeOperatorDivision
++   developped by Jean Della Dora and Stephen M. Watt.
```

```
UnivariateSkewPolynomialCategory(R:Ring):
  Category == Join(Ring, BiModule(R, R), FullyRetractableTo R) with
      degree: $ -> NonNegativeInteger
          ++ degree(l) is \spad{n} if
          ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
      minimumDegree: $ -> NonNegativeInteger
          ++ minimumDegree(l) is the smallest \spad{k} such that
          ++ \spad{a(k) ^= 0} if
          ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
      leadingCoefficient: $ -> R
          ++ leadingCoefficient(l) is \spad{a(n)} if
          ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
      reductum: $ -> $
          ++ reductum(l) is \spad{l - monomial(a(n),n)} if
          ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
      coefficient: ($, NonNegativeInteger) -> R
          ++ coefficient(l,k) is \spad{a(k)} if
          ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
      monomial: (R, NonNegativeInteger) -> $
          ++ monomial(c,k) produces c times the k-th power of
          ++ the generating operator, \spad{monomial(1,1)}.
      coefficients: % -> List R
          ++ coefficients(l) returns the list of all the nonzero
          ++ coefficients of l.
      apply: (%, R, R) -> R
        ++ apply(p, c, m) returns \spad{p(m)} where the action is
        ++ given by \spad{x m = c sigma(m) + delta(m)}.
      if R has CommutativeRing then Algebra R
      if R has IntegralDomain then
        "exquo": (%, R) -> Union(%, "failed")
          ++ exquo(l, a) returns the exact quotient of l by a,
          ++ returning \axiom{"failed"} if this is not possible.
        monicLeftDivide:   (%, %) -> Record(quotient: %, remainder: %)
          ++ monicLeftDivide(a,b) returns the pair \spad{[q,r]} such that
          ++ \spad{a = b*q + r} and the degree of \spad{r} is
          ++ less than the degree of \spad{b}.
          ++ \spad{b} must be monic.
          ++ This process is called ``left division''.
        monicRightDivide:   (%, %) -> Record(quotient: %, remainder: %)
          ++ monicRightDivide(a,b) returns the pair \spad{[q,r]} such that
          ++ \spad{a = q*b + r} and the degree of \spad{r} is
          ++ less than the degree of \spad{b}.
          ++ \spad{b} must be monic.
          ++ This process is called ``right division''.
      if R has GcdDomain then
        content: % -> R
```

```
    ++ content(l) returns the gcd of all the coefficients of l.
  primitivePart: % -> %
    ++ primitivePart(l) returns l0 such that \spad{l = a * l0}
    ++ for some a in R, and \spad{content(l0) = 1}.
if R has Field then
  leftDivide:   (%, %) -> Record(quotient: %, remainder: %)
    ++ leftDivide(a,b) returns the pair \spad{[q,r]} such that
    ++ \spad{a = b*q + r} and the degree of \spad{r} is
    ++ less than the degree of \spad{b}.
    ++ This process is called ``left division''.
  leftQuotient:  (%, %) -> %
    ++ leftQuotient(a,b) computes the pair \spad{[q,r]} such that
    ++ \spad{a = b*q + r} and the degree of \spad{r} is
    ++ less than the degree of \spad{b}.
    ++ The value \spad{q} is returned.
  leftRemainder:  (%, %) -> %
    ++ leftRemainder(a,b) computes the pair \spad{[q,r]} such that
    ++ \spad{a = b*q + r} and the degree of \spad{r} is
    ++ less than the degree of \spad{b}.
    ++ The value \spad{r} is returned.
  leftExactQuotient:(%, %) -> Union(%, "failed")
    ++ leftExactQuotient(a,b) computes the value \spad{q}, if it exists,
    ++  such that \spad{a = b*q}.
  leftGcd:   (%, %) -> %
    ++ leftGcd(a,b) computes the value \spad{g} of highest degree
    ++ such that
    ++    \spad{a = g*aa}
    ++    \spad{b = g*bb}
    ++ for some values \spad{aa} and \spad{bb}.
    ++ The value \spad{g} is computed using left-division.
  leftExtendedGcd:   (%, %) -> Record(coef1:%, coef2:%, generator:%)
    ++ leftExtendedGcd(a,b) returns \spad{[c,d]} such that
    ++ \spad{g = a * c + b * d = leftGcd(a, b)}.
  rightLcm:   (%, %) -> %
    ++ rightLcm(a,b) computes the value \spad{m} of lowest degree
    ++ such that \spad{m = a*aa = b*bb} for some values
    ++ \spad{aa} and \spad{bb}.  The value \spad{m} is
    ++ computed using left-division.
  rightDivide:   (%, %) -> Record(quotient: %, remainder: %)
    ++ rightDivide(a,b) returns the pair \spad{[q,r]} such that
    ++ \spad{a = q*b + r} and the degree of \spad{r} is
    ++ less than the degree of \spad{b}.
    ++ This process is called ``right division''.
  rightQuotient:  (%, %) -> %
    ++ rightQuotient(a,b) computes the pair \spad{[q,r]} such that
    ++ \spad{a = q*b + r} and the degree of \spad{r} is
```

```
      ++ less than the degree of \spad{b}.
      ++ The value \spad{q} is returned.
    rightRemainder:  (%, %) -> %
      ++ rightRemainder(a,b) computes the pair \spad{[q,r]} such that
      ++ \spad{a = q*b + r} and the degree of \spad{r} is
      ++ less than the degree of \spad{b}.
      ++ The value \spad{r} is returned.
    rightExactQuotient:(%, %) -> Union(%, "failed")
      ++ rightExactQuotient(a,b) computes the value \spad{q}, if it exists
      ++ such that \spad{a = q*b}.
    rightGcd:   (%, %) -> %
      ++ rightGcd(a,b) computes the value \spad{g} of highest degree
      ++ such that
      ++    \spad{a = aa*g}
      ++    \spad{b = bb*g}
      ++ for some values \spad{aa} and \spad{bb}.
      ++ The value \spad{g} is computed using right-division.
    rightExtendedGcd:   (%, %) -> Record(coef1:%, coef2:%, generator:%)
      ++ rightExtendedGcd(a,b) returns \spad{[c,d]} such that
      ++ \spad{g = c * a + d * b = rightGcd(a, b)}.
    leftLcm:   (%, %) -> %
      ++ leftLcm(a,b) computes the value \spad{m} of lowest degree
      ++ such that \spad{m = aa*a = bb*b} for some values
      ++ \spad{aa} and \spad{bb}.  The value \spad{m} is
      ++ computed using right-division.

  add
    coerce(x:R):% == monomial(x, 0)

    coefficients l ==
      ans:List(R) := empty()
      while l ^= 0 repeat
        ans := concat(leadingCoefficient l, ans)
        l   := reductum l
      ans

    a:R * y:% ==
      z:% := 0
      while y ^= 0 repeat
        z := z + monomial(a * leadingCoefficient y, degree y)
        y := reductum y
      z

    retractIfCan(x:%):Union(R, "failed") ==
      zero? x or zero? degree x => leadingCoefficient x
      "failed"
```

```
if R has IntegralDomain then
  l exquo a ==
    ans:% := 0
    while l ^= 0 repeat
      (u := (leadingCoefficient(l) exquo a)) case "failed" =>
         return "failed"
      ans := ans + monomial(u::R, degree l)
      l   := reductum l
    ans

if R has GcdDomain then
  content l       == gcd coefficients l

  primitivePart l == (l exquo content l)::%

if R has Field then
  leftEEA:  (%, %) -> Record(gcd:%, coef1:%, coef2:%, lcm:%)
  rightEEA: (%, %) -> Record(gcd:%, coef1:%, coef2:%, lcm:%)
  ncgcd:    (%, %, (%, %) -> %) -> %
  nclcm:  (%, %, (%, %) -> Record(gcd:%, coef1:%, coef2:%, lcm:%)) -> %
  exactQuotient: Record(quotient:%, remainder:%) -> Union(%, "failed")
  extended: (%, %, (%, %) -> Record(gcd:%, coef1:%, coef2:%, lcm:%)) ->
                                  Record(coef1:%, coef2:%, generator:%)

  leftQuotient(a, b)         == leftDivide(a,b).quotient

  leftRemainder(a, b)        == leftDivide(a,b).remainder

  leftExtendedGcd(a, b)      == extended(a, b, leftEEA)

  rightLcm(a, b)             == nclcm(a, b, leftEEA)

  rightQuotient(a, b)        == rightDivide(a,b).quotient

  rightRemainder(a, b)       == rightDivide(a,b).remainder

  rightExtendedGcd(a, b)     == extended(a, b, rightEEA)

  leftLcm(a, b)              == nclcm(a, b, rightEEA)

  leftExactQuotient(a, b)    == exactQuotient leftDivide(a, b)

  rightExactQuotient(a, b)   == exactQuotient rightDivide(a, b)

  rightGcd(a, b)             == ncgcd(a, b, rightRemainder)
```

```
leftGcd(a, b)              == ncgcd(a, b, leftRemainder)

exactQuotient qr  == (zero?(qr.remainder) => qr.quotient; "failed")

-- returns [g = leftGcd(a, b), c, d, l = rightLcm(a, b)]
-- such that g := a c + b d
leftEEA(a, b) ==
  a0 := a
  u0:% := v:% := 1
  v0:% := u:% := 0
  while b ^= 0 repeat
    qr     := leftDivide(a, b)
    (a, b) := (b, qr.remainder)
    (u0, u):= (u, u0 - u * qr.quotient)
    (v0, v):= (v, v0 - v * qr.quotient)
  [a, u0, v0, a0 * u]

ncgcd(a, b, ncrem) ==
  zero? a => b
  zero? b => a
  degree a < degree b => ncgcd(b, a, ncrem)
  while b ^= 0 repeat (a, b) := (b, ncrem(a, b))
  a

extended(a, b, eea) ==
  zero? a => [0, 1, b]
  zero? b => [1, 0, a]
  degree a < degree b =>
    rec := eea(b, a)
    [rec.coef2, rec.coef1, rec.gcd]
  rec := eea(a, b)
  [rec.coef1, rec.coef2, rec.gcd]

nclcm(a, b, eea) ==
  zero? a or zero? b => 0
  degree a < degree b => nclcm(b, a, eea)
  rec := eea(a, b)
  rec.lcm

-- returns [g = rightGcd(a, b), c, d, l = leftLcm(a, b)]
-- such that g := a c + b d
rightEEA(a, b) ==
  a0 := a
  u0:% := v:% := 1
  v0:% := u:% := 0
```

```
            while b ^= 0 repeat
              qr     := rightDivide(a, b)
              (a, b) := (b, qr.remainder)
              (u0, u):= (u, u0 - qr.quotient * u)
              (v0, v):= (v, v0 - qr.quotient * v)
            [a, u0, v0, u * a0]
```

⟨*OREPCAT.dotabb*⟩≡

```
"OREPCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OREPCAT"];
"OREPCAT" -> "BMODULE"
"OREPCAT" -> "FRETRCT"
"OREPCAT" -> "RING"
```

⟨*OREPCAT.dotfull*⟩≡

```
"UnivariateSkewPolynomialCategory(R:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OREPCAT"];
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "BiModule(a:Ring,b:Ring)"
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "FullyRetractableTo(a:Ring)"
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "Ring()"
```

⟨*OREPCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UnivariateSkewPolynomialCategory(R:Ring)" [color=lightblue];
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "BiModule(a:Ring,b:Ring)"
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "FullyRetractableTo(a:Ring)"
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "Ring()"

"FullyRetractableTo(a:Ring)" [color=seagreen];
"FullyRetractableTo(a:Ring)" -> "FullyRetractableTo(a:Type)"

"FullyRetractableTo(a:Type)" [color=lightblue];
"FullyRetractableTo(a:Type)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"Category" [color=lightblue];

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "AbelianGroup()"
```

```
"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

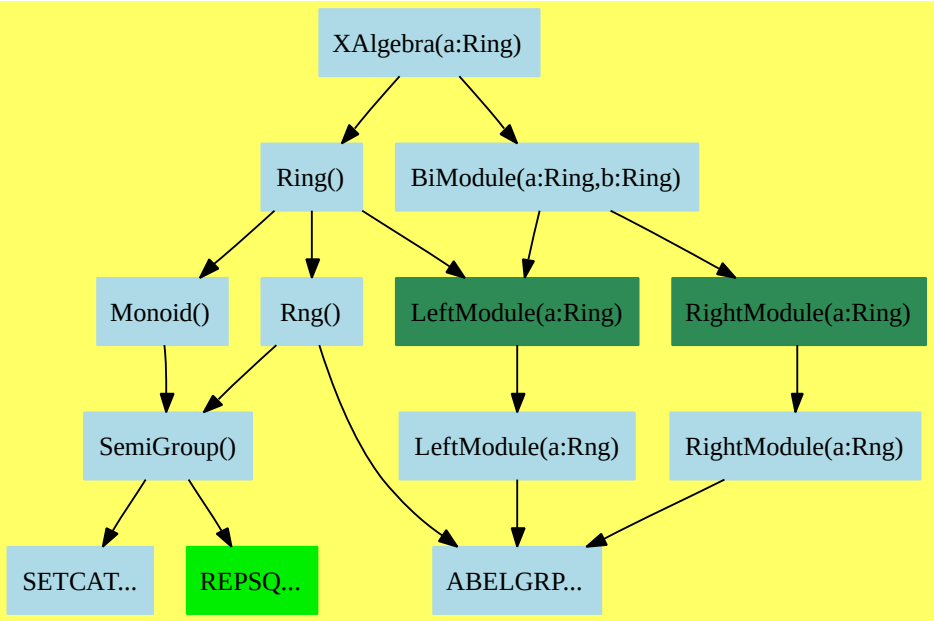
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

10.18 XAlgebra (XALG)



See:

⇒ "XFreeAlgebra" (XFALG) 11.8 on page 807
⇐ "BiModule" (BMODULE) 9.1 on page 592
⇐ "Ring" (RING) 9.8 on page 628

Exports:

0	1	characteristic	coerce	hash
latex	one?	recip	sample	subtractIfCan
zero?	?^?	?~=?	?*?	?**?
?+?	?-?	-?	?=?	

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coerce : R -> %
```

These exports come from (p628) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> OutputForm
coerce : Integer -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p592) BiModule(R:Ring,R:Ring):

```
?*? : (R,%) -> %
?*? : (%,R) -> %
```

⟨*category XALG XAlgebra*⟩≡

```
)abbrev category XALG XAlgebra
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++   This is the category of algebras over non-commutative rings.
++   It is used by constructors of non-commutative algebras such as:
++       \spadtype{XPolynomialRing}.
```

```
++       \spadtype{XFreeAlgebra}
++   Author: Michel Petitot (petitot@lifl.fr)

XAlgebra(R: Ring): Category ==
  Join(Ring, BiModule(R,R)) with
    coerce: R -> %
      ++ \spad{coerce(r)} equals  \spad{r*1}.
    if R has CommutativeRing then Algebra(R)
```

⟨*XALG.dotabb*⟩≡

```
"XALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XALG"];
"XALG" -> "BMODULE"
"XALG" -> "RING"
```

⟨*XALG.dotfull*⟩≡

```
"XAlgebra(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XALG"];
"XAlgebra(a:Ring)" -> "Ring()"
"XAlgebra(a:Ring)" -> "BiModule(a:Ring,b:Ring)"
```

⟨*XALG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"XAlgebra(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XALG"];
"XAlgebra(a:Ring)" -> "Ring()"
"XAlgebra(a:Ring)" -> "BiModule(a:Ring,b:Ring)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

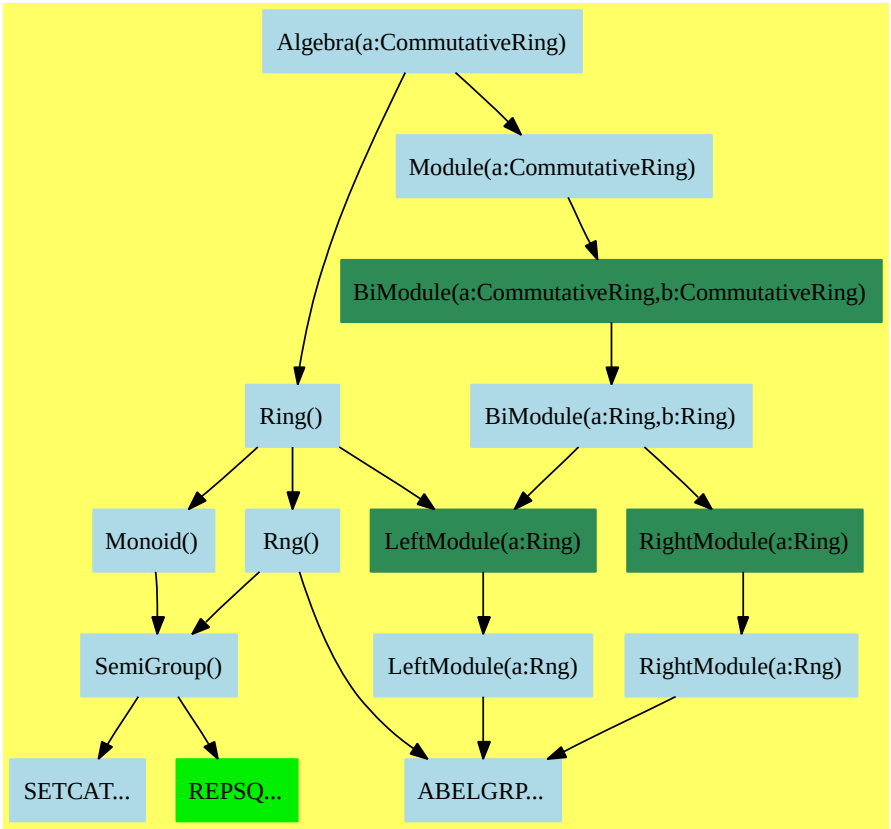
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
```

```
}
```

Chapter 11

Category Layer 10

11.1 Algebra (ALGEBRA)



See:

⇒ "DivisionRing" (DIVRING) 12.2 on page 825
⇒ "FiniteRankAlgebra" (FINRALG) 17.4 on page 1089
⇒ "FunctionSpace" (FS) 17.5 on page 1095
⇒ "IntegralDomain" (INTDOM) 12.5 on page 857
⇒ "MonogenicLinearOperator" (MLO) 12.6 on page 862
⇒ "OctonionCategory" (OC) 12.7 on page 868
⇒ "QuotientFieldCategory" (QFCAT) 17.6 on page 1121
⇒ "QuaternionCategory" (QUATCAT) 12.8 on page 878
⇒ "RealClosedField" (RCFIELD) 17.7 on page 1132
⇐ "Module" (MODULE) 10.10 on page 711
⇐ "Ring" (RING) 9.8 on page 628

Exports:

1	0	characteristic	coerce	hash
latex	one?	recip	sample	subtractIfCan
zero?	?*?	?+?	?-?	-?
?=?	?~=?	?**?	?^?	

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are implemented by this category:

```
coerce : R -> %
```

These exports come from (p628) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
```

```
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p711) Module(R:CommutativeRing):

```
?*? : (R,%) -> %
?*? : (%,R) -> %
```

⟨*category ALGEBRA Algebra*⟩≡

```
)abbrev category ALGEBRA Algebra
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of associative algebras (modules which are themselves rings).
++
++ Axioms:
++   \spad{(b+c)::% = (b::%) + (c::%)}
++   \spad{(b*c)::% = (b::%) * (c::%)}
++   \spad{(1::R)::% = 1::%}
++   \spad{b*x = (b::%)*x}
++   \spad{r*(a*b) = (r*a)*b = a*(r*b)}
Algebra(R:CommutativeRing): Category ==
  Join(Ring, Module R) with
      coerce: R -> %
          ++ coerce(r) maps the ring element r to a member of the algebra.
 add
  coerce(x:R):% == x * 1$%
```

⟨*ALGEBRA.dotabb*⟩≡
```
  "ALGEBRA"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
  "ALGEBRA" -> "RING"
  "ALGEBRA" -> "MODULE"
```

⟨*ALGEBRA.dotfull*⟩≡
```
  "Algebra(a:CommutativeRing)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
  "Algebra(a:CommutativeRing)" -> "Ring()"
  "Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

  "Algebra(a:Field)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
  "Algebra(a:Field)" -> "Algebra(a:CommutativeRing)"

  "Algebra(a:CommutativeRing)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
  "Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

  "Algebra(Fraction(Integer))"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
  "Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

  "Algebra(Integer)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
  "Algebra(Integer)" -> "Algebra(a:CommutativeRing)"

  "Algebra(IntegralDomain)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
  "Algebra(IntegralDomain)" -> "Algebra(a:CommutativeRing)"
```

⟨*ALGEBRA.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "Ring()"
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SemiGroup()"

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

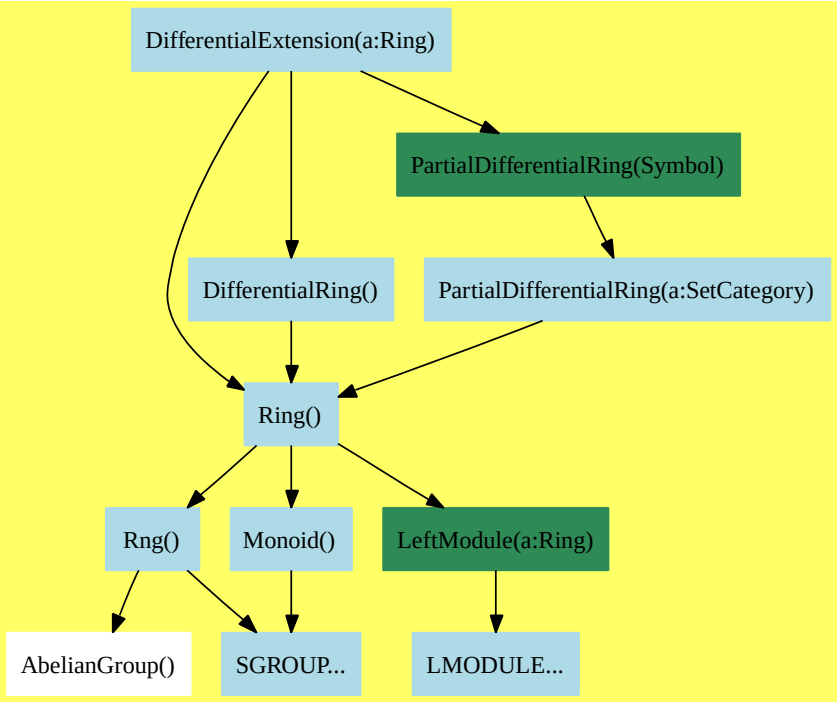
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"
```

```
  "LeftModule(a:Rng)" [color=lightblue];
  "LeftModule(a:Rng)" -> "ABELGRP..."

  "ABELGRP..." [color=lightblue];
  "REPSQ..." [color="#00EE00"];
  "SETCAT..." [color=lightblue];
  }
```

11.2 DifferentialExtension (DIFEXT)



See:

⇒ "ComplexCategory" (COMPCAT) 20.1 on page 1315
⇒ "DifferentialPolynomialCategory" (DPOLCAT) 17.2 on page 1069
⇒ "DirectProductCategory" (DIRPCAT) 12.1 on page 815
⇒ "QuaternionCategory" (QUATCAT) 12.8 on page 878
⇒ "QuotientFieldCategory" (QFCAT) 17.6 on page 1121
⇒ "SquareMatrixCategory" (SMATCAT) 12.9 on page 887
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.12 on page 1204
⇐ "PartialDifferentialRing" (PDRING) 10.12 on page 720
⇐ "Ring" (RING) 9.8 on page 628

Exports:

1	0	characteristic	coerce	D
differentiate	hash	latex	one?	recip
sample	subtractIfCan	zero?	?*?	?~=?
?**?	?+?	?-?	-?	?=?
?^?				

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1)

has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are directly exported but not implemented:

```
differentiate : (%,(R -> R)) -> %
```

These are implemented by this category:

```
D : (%,(R -> R)) -> %
D : (%,(R -> R),NonNegativeInteger) -> %
differentiate : % -> % if R has DIFRING
differentiate : (%,(R -> R),NonNegativeInteger) -> %
differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
```

These exports come from (p628) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p690) DifferentialRing():

```
D : % -> % if R has DIFRING
D : (%,NonNegativeInteger) -> % if R has DIFRING
differentiate : (%,NonNegativeInteger) -> %
    if R has DIFRING
```

These exports come from (p720) PartialDifferentialRing(Symbol):

```
D : (%,Symbol) -> % if R has PDRING SYMBOL
D : (%,List Symbol) -> % if R has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> %
    if R has PDRING SYMBOL
D : (%,List Symbol,List NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : (%,List Symbol) -> %
    if R has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if R has PDRING SYMBOL
```

⟨*category DIFEXT DifferentialExtension*⟩≡

```
)abbrev category DIFEXT DifferentialExtension
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Differential extensions of a ring R.
++ Given a differentiation on R, extend it to a differentiation on %.

DifferentialExtension(R:Ring): Category == Ring with
    differentiate: (%, R -> R) -> %
       ++ differentiate(x, deriv) differentiates x extending
       ++ the derivation deriv on R.
    differentiate: (%, R -> R, NonNegativeInteger) -> %
       ++ differentiate(x, deriv, n) differentiate x n times
       ++ using a derivation which extends deriv on R.
    D: (%, R -> R) -> %
       ++ D(x, deriv) differentiates x extending
       ++ the derivation deriv on R.
    D: (%, R -> R, NonNegativeInteger) -> %
       ++ D(x, deriv, n) differentiate x n times
       ++ using a derivation which extends deriv on R.
    if R has DifferentialRing then DifferentialRing
    if R has PartialDifferentialRing(Symbol) then
              PartialDifferentialRing(Symbol)
  add
    differentiate(x:%, derivation: R -> R, n:NonNegativeInteger):% ==
```

```
      for i in 1..n repeat x := differentiate(x, derivation)
      x
    D(x:%, derivation: R -> R) == differentiate(x, derivation)
    D(x:%, derivation: R -> R, n:NonNegativeInteger) ==
           differentiate(x, derivation, n)

    if R has DifferentialRing then
      differentiate x == differentiate(x, differentiate$R)

    if R has PartialDifferentialRing Symbol then
      differentiate(x:%, v:Symbol):% ==
        differentiate(x, s +-> differentiate(s, v)$R)
```

⟨*DIFEXT.dotabb*⟩≡

```
"DIFEXT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIFEXT"];
"DIFEXT" -> "RING"
"DIFEXT" -> "DIFRING"
"DIFEXT" -> "PDRING"
```

⟨*DIFEXT.dotfull*⟩≡

```
"DifferentialExtension(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIFEXT"];
"DifferentialExtension(a:Ring)" -> "Ring()"
"DifferentialExtension(a:Ring)" -> "DifferentialRing()"
"DifferentialExtension(a:Ring)" -> "PartialDifferentialRing(Symbol)"

"DifferentialExtension(IntegralDomain)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=DIFEXT"];
"DifferentialExtension(IntegralDomain)" ->
  "DifferentialExtension(a:Ring)"

"DifferentialExtension(CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=DIFEXT"];
"DifferentialExtension(CommutativeRing)" ->
  "DifferentialExtension(a:Ring)"
```

⟨*DIFEXT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DifferentialExtension(a:Ring)" [color=lightblue];
"DifferentialExtension(a:Ring)" -> "Ring()"
"DifferentialExtension(a:Ring)" -> "DifferentialRing()"
"DifferentialExtension(a:Ring)" -> "PartialDifferentialRing(Symbol)"

"PartialDifferentialRing(Symbol)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PDRING"];
"PartialDifferentialRing(Symbol)" ->
    "PartialDifferentialRing(a:SetCategory)"

"PartialDifferentialRing(a:SetCategory)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PDRING"];
"PartialDifferentialRing(a:SetCategory)" -> "Ring()"

"DifferentialRing()" [color=lightblue];
"DifferentialRing()" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

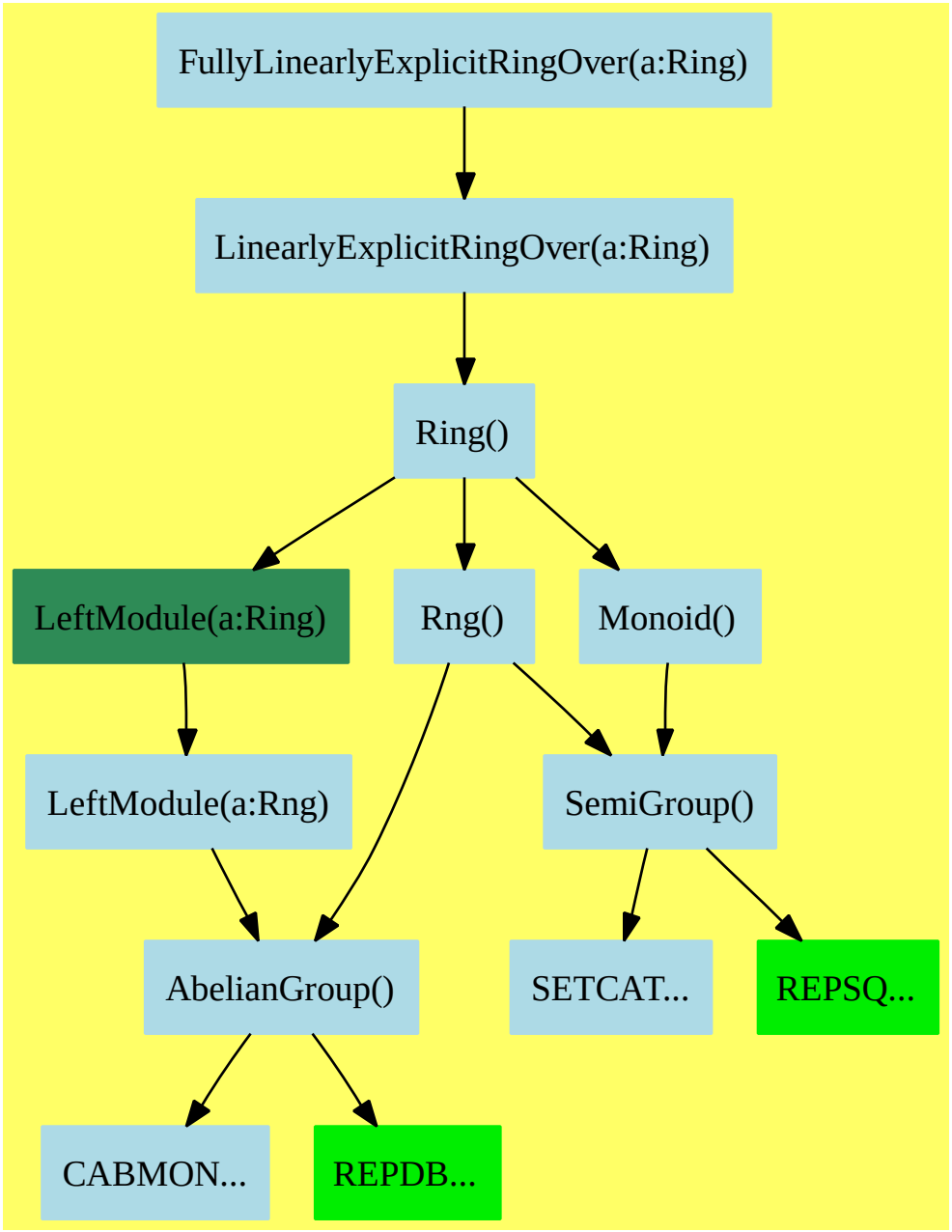
"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SGROUP..."

"Monoid()" [color=lightblue];
"Monoid()" -> "SGROUP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LMODULE..."

"SGROUP..." [color=lightblue];
"LMODULE..." [color=lightblue];
}
```

11.3 FullyLinearlyExplicitRingOver (FLINEXP)



See:

⇒ “ComplexCategory” (COMPCAT) 20.1 on page 1315

⇒ "DirectProductCategory" (DIRPCAT) 12.1 on page 815
⇒ "FunctionSpace" (FS) 17.5 on page 1095
⇒ "MonogenicAlgebra" (MONOGEN) 19.2 on page 1305
⇒ "PolynomialCategory" (POLYCAT) 16.4 on page 1027
⇒ "QuaternionCategory" (QUATCAT) 12.8 on page 878
⇒ "QuotientFieldCategory" (QFCAT) 17.6 on page 1121
⇒ "SquareMatrixCategory" (SMATCAT) 12.9 on page 887
⇐ "LinearlyExplicitRingOver" (LINEXP) 10.9 on page 707

Exports:

1	0	characteristic	coerce	hash
latex	one?	recip	reducedSystem	sample
subtractIfCan	zero?	?*?	?**?	?+?
?-?	-?	?=?	?^?	?~=?

Attributes exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

These are implemented by this category:

```
reducedSystem : Matrix % ->
    Matrix Integer if R has LINEXP INT
reducedSystem : (Matrix %,Vector %) ->
    Record(mat: Matrix Integer,vec: Vector Integer)
         if R has LINEXP INT
```

These exports come from (p707) LinearlyExplicitRingOver(a:Ring):

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
reducedSystem : (Matrix %,Vector %) ->
    Record(mat: Matrix R,vec: Vector R)
reducedSystem : Matrix % -> Matrix R
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
```

```
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

⟨*category FLINEXP FullyLinearlyExplicitRingOver*⟩≡

```
)abbrev category FLINEXP FullyLinearlyExplicitRingOver
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ S is \spadtype{FullyLinearlyExplicitRingOver R} means that S is a
++ \spadtype{LinearlyExplicitRingOver R} and, in addition, if R is a
++ \spadtype{LinearlyExplicitRingOver Integer}, then so is S
FullyLinearlyExplicitRingOver(R:Ring):Category ==
  LinearlyExplicitRingOver R with
    if (R has LinearlyExplicitRingOver Integer) then
            LinearlyExplicitRingOver Integer
 add
  if not(R is Integer) then
    if (R has LinearlyExplicitRingOver Integer) then
      reducedSystem(m:Matrix %):Matrix(Integer) ==
        reducedSystem(reducedSystem(m)@Matrix(R))

      reducedSystem(m:Matrix %, v:Vector %):
        Record(mat:Matrix(Integer), vec:Vector(Integer)) ==
          rec := reducedSystem(m, v)@Record(mat:Matrix R, vec:Vector R)
          reducedSystem(rec.mat, rec.vec)
```

⟨*FLINEXP.dotabb*⟩≡

```
"FLINEXP"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FLINEXP"];
"FLINEXP" -> "LINEXP"
```

⟨*FLINEXP.dotfull*⟩≡

```
"FullyLinearlyExplicitRingOver(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FLINEXP"];
"FullyLinearlyExplicitRingOver(a:Ring)" ->
     "LinearlyExplicitRingOver(a:Ring)"

"FullyLinearlyExplicitRingOver(a:CommutativeRing)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FLINEXP"];
"FullyLinearlyExplicitRingOver(a:CommutativeRing)" ->
    "FullyLinearlyExplicitRingOver(a:Ring)"

"FullyLinearlyExplicitRingOver(IntegralDomain)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FLINEXP"];
"FullyLinearlyExplicitRingOver(IntegralDomain)" ->
    "FullyLinearlyExplicitRingOver(a:Ring)"
```

⟨*FLINEXP.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FullyLinearlyExplicitRingOver(a:Ring)" [color=lightblue];
"FullyLinearlyExplicitRingOver(a:Ring)" ->
     "LinearlyExplicitRingOver(a:Ring)"

"LinearlyExplicitRingOver(a:Ring)" [color=lightblue];
"LinearlyExplicitRingOver(a:Ring)" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

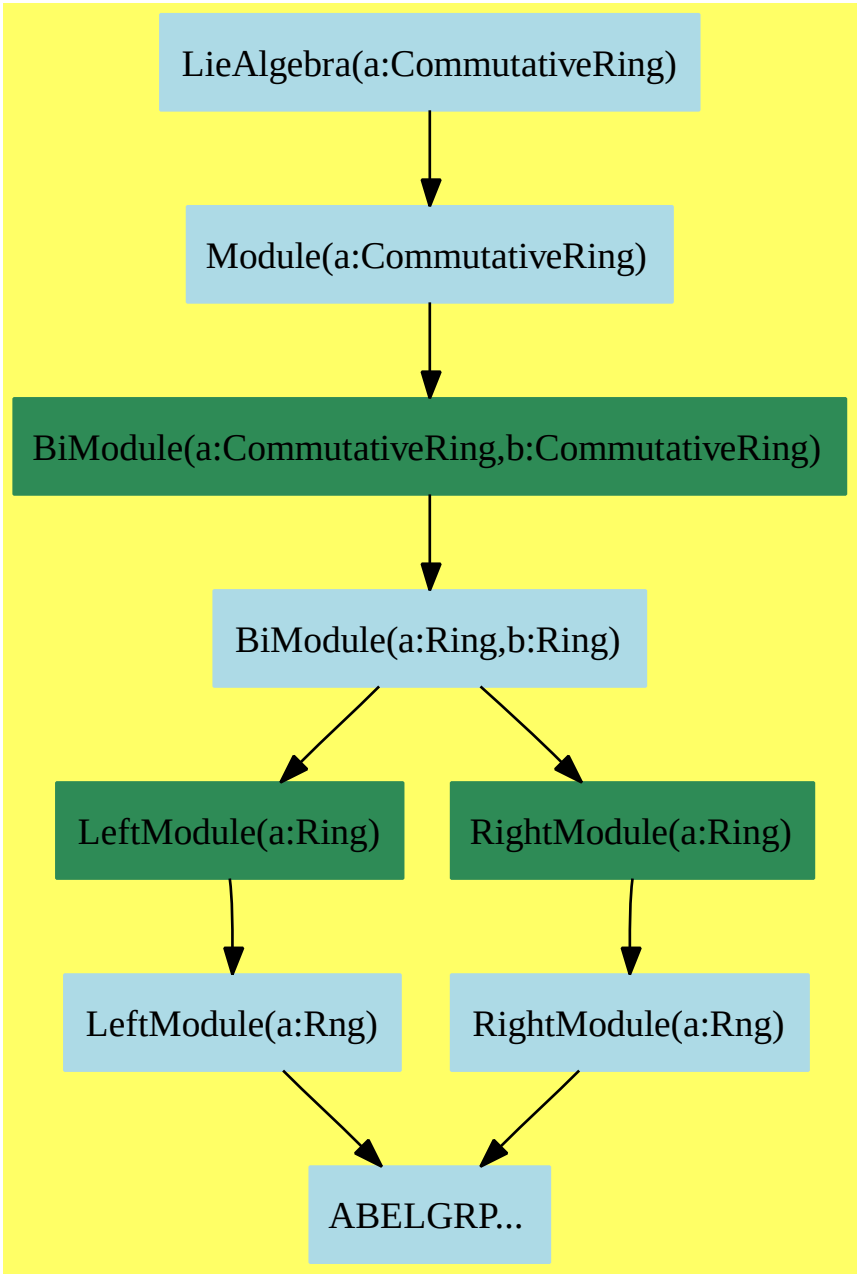
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

11.4 LieAlgebra (LIECAT)



See:

⇒ "FreeLieAlgebra" (FLALG) 12.4 on page 852

⇐ "Module" (MODULE) 10.10 on page 711

Exports:

0	coerce	construct	hash	latex
sample	subtractIfCan	zero?	?~=?	?/?
?*?	?+?	?-?	-?	?=?

Attributes Exported:

- **NullSquare** means that $[x, x] = 0$ holds. See `LieAlgebra`.
- **JacobiIdentity** means that $[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$ holds. See `LieAlgebra`.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
construct : (%,%) -> %
```

These are implemented by this category:

```
?/? : (%,R) -> % if R has FIELD
```

These exports come from (p711) Module(R:Ring):

```
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

⟨*category LIECAT LieAlgebra*⟩≡

```
)abbrev category LIECAT LieAlgebra
++ Author: Michel Petitot (petitot@lifl.fr).
++ Date Created: 91
```

```
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of Lie Algebras.
++ It is used by the following domains of non-commutative algebra:
++ \axiomType{LiePolynomial} and
++ \axiomType{XPBWPolynomial}. \newline
++ Author : Michel Petitot (petitot@lifl.fr).
LieAlgebra(R: CommutativeRing): Category ==  Module(R) with
    NullSquare
      ++ \axiom{NullSquare} means that \axiom{[x,x] = 0} holds.
    JacobiIdentity
      ++ \axiom{JacobiIdentity} means that
      ++ \axiom{[x,[y,z]]+[y,[z,x]]+[z,[x,y]] = 0} holds.
    construct:  ($,$) -> $
      ++ \axiom{construct(x,y)} returns the Lie bracket of \axiom{x}
      ++ and \axiom{y}.
    if R has Field then
       "/"   :  ($,R) -> $
         ++ \axiom{x/r} returns the division of \axiom{x} by \axiom{r}.
  add
    if R has Field then x / r == inv(r)$R * x
```

⟨*LIECAT.dotabb*⟩≡

```
"LIECAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LIECAT"];
"LIECAT" -> "MODULE"
```

⟨*LIECAT.dotfull*⟩≡

```
"LieAlgebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LIECAT"];
"LieAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"
```

⟨*LIECAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"LieAlgebra(a:CommutativeRing)" [color=lightblue];
"LieAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

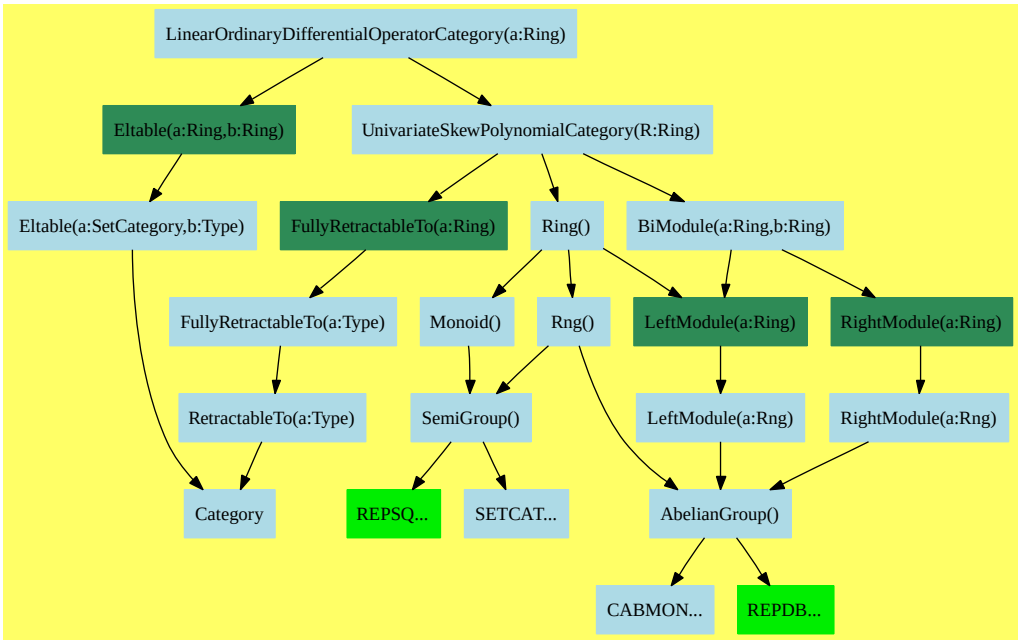
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"ABELGRP..." [color=lightblue];
}
```

11.5 LinearOrdinaryDifferentialOperatorCategory (LODOCAT)



See:

⇐ "Eltable" (ELTAB) 2.10 on page 25
⇐ "UnivariateSkewPolynomialCategory" (OREPCAT) 10.17 on page 754

Exports:

0	1	adjoint
apply	characteristic	coefficient
coefficients	coerce	content
D	degree	directSum
exquo	hash	latex
leadingCoefficient	leftDivide	leftExactQuotient
leftExtendedGcd	leftGcd	leftLcm
leftQuotient	leftRemainder	minimumDegree
monicLeftDivide	monicRightDivide	monomial
one?	primitivePart	recip
reductum	retract	retractIfCan
rightDivide	rightExactQuotient	rightExtendedGcd
rightGcd	rightLcm	rightQuotient
rightRemainder	sample	subtractIfCan
symmetricPower	symmetricProduct	symmetricSquare
zero?	?^?	?.?
?~=?	?*?	?**?
?+?	?-?	-?
?=?		

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
directSum : (%,%) -> % if A has FIELD
symmetricPower : (%,NonNegativeInteger) -> % if A has FIELD
symmetricProduct : (%,%) -> % if A has FIELD
```

These are implemented by this category:

```
adjoint : % -> %
D : () -> %
symmetricSquare : % -> % if A has FIELD
```

These exports come from (p754) UnivariateSkewPolynomialCategory(A:Ring):

```
0 : () -> %
1 : () -> %
apply : (%,A,A) -> A
characteristic : () -> NonNegativeInteger
```

```
coefficient : (%,NonNegativeInteger) -> A
coefficients : % -> List A
coerce : % -> OutputForm
coerce : Integer -> %
coerce : A -> %
coerce : Fraction Integer -> % if A has RETRACT FRAC INT
content : % -> A if A has GCDDOM
degree : % -> NonNegativeInteger
exquo : (%,A) -> Union(%,"failed") if A has INTDOM
hash : % -> SingleInteger
latex : % -> String
leadingCoefficient : % -> A
leftDivide : (%,%) -> Record(quotient: %,remainder: %)
   if A has FIELD
leftExactQuotient : (%,%) -> Union(%,"failed") if A has FIELD
leftExtendedGcd : (%,%) -> Record(coef1: %,coef2: %,generator: %)
   if A has FIELD
leftGcd : (%,%) -> % if A has FIELD
leftLcm : (%,%) -> % if A has FIELD
leftQuotient : (%,%) -> % if A has FIELD
leftRemainder : (%,%) -> % if A has FIELD
minimumDegree : % -> NonNegativeInteger
monicLeftDivide : (%,%) -> Record(quotient: %,remainder: %)
   if A has INTDOM
monicRightDivide : (%,%) -> Record(quotient: %,remainder: %)
   if A has INTDOM
monomial : (A,NonNegativeInteger) -> %
one? : % -> Boolean
primitivePart : % -> % if A has GCDDOM
recip : % -> Union(%,"failed")
reductum : % -> %
retract : % -> A
retract : % -> Fraction Integer if A has RETRACT FRAC INT
retract : % -> Integer if A has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
   if A has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
   if A has RETRACT INT
retractIfCan : % -> Union(A,"failed")
rightDivide : (%,%) -> Record(quotient: %,remainder: %)
   if A has FIELD
rightExactQuotient : (%,%) -> Union(%,"failed") if A has FIELD
rightExtendedGcd :
  (%,%) -> Record(coef1: %,coef2: %,generator: %) if A has FIELD
rightGcd : (%,%) -> % if A has FIELD
rightLcm : (%,%) -> % if A has FIELD
rightQuotient : (%,%) -> % if A has FIELD
rightRemainder : (%,%) -> % if A has FIELD
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
```

```
zero? : % -> Boolean
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?*? : (%,A) -> %
?*? : (A,%) -> %
?~=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p25) Eltable(A:Ring,A:Ring):

```
?.? : (%,A) -> A
```

⟨*category LODOCAT LinearOrdinaryDifferentialOperatorCategory*⟩≡

```
)abbrev category LODOCAT LinearOrdinaryDifferentialOperatorCategory
++ Author: Manuel Bronstein
++ Date Created: 9 December 1993
++ Date Last Updated: 15 April 1994
++ Keywords: differential operator
++ Description:
++   \spad{LinearOrdinaryDifferentialOperatorCategory} is the category
++   of differential operators with coefficients in a ring A with a given
++   derivation.
++   Multiplication of operators corresponds to functional composition:
++       \spad{(L1 * L2).(f) = L1 L2 f}
LinearOrdinaryDifferentialOperatorCategory(A:Ring): Category ==
  Join(UnivariateSkewPolynomialCategory A, Eltable(A, A)) with
        D: () -> %
            ++ D() provides the operator corresponding to a derivation
            ++ in the ring \spad{A}.
        adjoint: % -> %
            ++ adjoint(a) returns the adjoint operator of a.
        if A has Field then
          symmetricProduct: (%, %) -> %
            ++ symmetricProduct(a,b) computes an operator \spad{c} of
            ++ minimal order such that the nullspace of \spad{c} is
            ++ generated by all the products of a solution of \spad{a} by
            ++ a solution of \spad{b}.
          symmetricPower  : (%, NonNegativeInteger) -> %
            ++ symmetricPower(a,n) computes an operator \spad{c} of
```

```
            ++ minimal order such that the nullspace of \spad{c} is
            ++ generated by all the products of \spad{n} solutions
            ++ of \spad{a}.
          symmetricSquare : % -> %
            ++ symmetricSquare(a) computes \spad{symmetricProduct(a,a)}
            ++ using a more efficient method.
          directSum: (%, %) -> %
            ++ directSum(a,b) computes an operator \spad{c} of
            ++ minimal order such that the nullspace of \spad{c} is
            ++ generated by all the sums of a solution of \spad{a} by
            ++ a solution of \spad{b}.
    add
        m1monom: NonNegativeInteger -> %

        D() == monomial(1, 1)

        m1monom n ==
          a:A := (odd? n => -1; 1)
          monomial(a, n)

        adjoint a ==
          ans:% := 0
          while a ^= 0 repeat
            ans := ans + m1monom(degree a) * leadingCoefficient(a)::%
            a   := reductum a
          ans

        if A has Field then symmetricSquare l == symmetricPower(l, 2)
```

⟨*LODOCAT.dotabb*⟩≡

```
"LODOCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LODOCAT"];
"LODOCAT" -> "ELTAB"
"LODOCAT" -> "OREPCAT"
```

⟨*LODOCAT.dotfull*⟩≡

```
"LinearOrdinaryDifferentialOperatorCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=LODOCAT"];
"LinearOrdinaryDifferentialOperatorCategory(a:Ring)"
  -> "Eltable(a:Ring,b:Ring)"
"LinearOrdinaryDifferentialOperatorCategory(a:Ring)"
  -> "UnivariateSkewPolynomialCategory(R:Ring)"
```

⟨*LODOCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"LinearOrdinaryDifferentialOperatorCategory(a:Ring)" [color=lightblue];
"LinearOrdinaryDifferentialOperatorCategory(a:Ring)"
  -> "Eltable(a:Ring,b:Ring)"
"LinearOrdinaryDifferentialOperatorCategory(a:Ring)"
  -> "UnivariateSkewPolynomialCategory(R:Ring)"

"Eltable(a:Ring,b:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ELTAB"];
"Eltable(a:Ring,b:Ring)" ->
   "Eltable(a:SetCategory,b:Type)"

"Eltable(a:SetCategory,b:Type)" [color=lightblue];
"Eltable(a:SetCategory,b:Type)" -> "Category"

"UnivariateSkewPolynomialCategory(R:Ring)" [color=lightblue];
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "BiModule(a:Ring,b:Ring)"
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "FullyRetractableTo(a:Ring)"
"UnivariateSkewPolynomialCategory(R:Ring)"
  -> "Ring()"

"FullyRetractableTo(a:Ring)" [color=seagreen];
"FullyRetractableTo(a:Ring)" -> "FullyRetractableTo(a:Type)"

"FullyRetractableTo(a:Type)" [color=lightblue];
"FullyRetractableTo(a:Type)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"Category" [color=lightblue];

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
```

```
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "AbelianGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

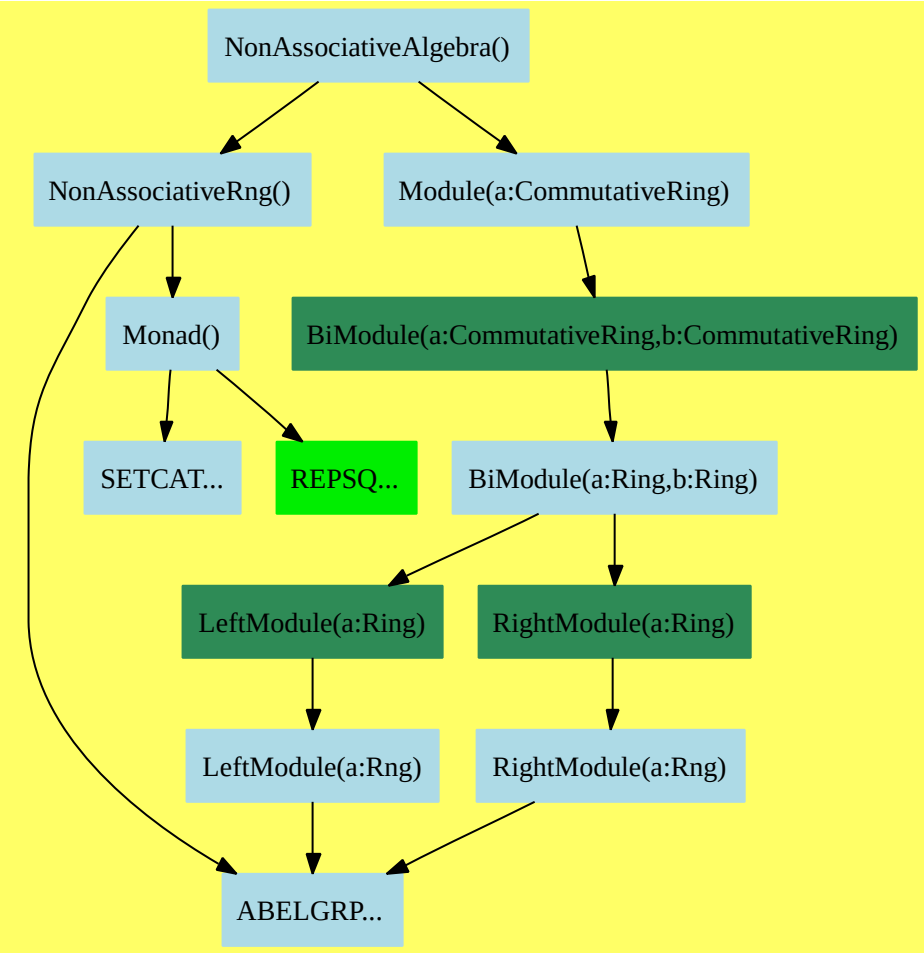
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

11.6 NonAssociativeAlgebra (NAALG)



See:

⇒ "FiniteRankNonAssociativeAlgebra" (FINAALG) 12.3 on page 830
⇐ "Module" (MODULE) 10.10 on page 711
⇐ "NonAssociativeRng" (NARNG) 8.8 on page 552

Exports:

0	antiCommutator	associator	coerce	commutator
hash	latex	leftPower	plenaryPower	rightPower
sample	subtractIfCan	zero?	?~=?	?*?
?**?	?+?	?-?	-?	?=?

Attributes exported:

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are implemented by this category:

```
plenaryPower : (%,PositiveInteger) -> %
```

These exports come from (p552) NonAssociativeRng():

```
0 : () -> %
antiCommutator : (%,%) -> %
associator : (%,%,%) -> %
coerce : % -> OutputForm
commutator : (%,%) -> %
hash : % -> SingleInteger
latex : % -> String
leftPower : (%,PositiveInteger) -> %
rightPower : (%,PositiveInteger) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?*? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p711) Module(R:CommutativeRing):

```
?*? : (R,%) -> %
?*? : (%,R) -> %
```

⟨*category NAALG NonAssociativeAlgebra*⟩≡

```
)abbrev category NAALG NonAssociativeAlgebra
++ Author: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 11 June 1991
++ Basic Operations: +, -, *, **
++ Related Constructors: Algebra
++ Also See:
++ AMS Classifications:
++ Keywords: nonassociative algebra
++ Reference:
```

```
++  R.D. Schafer: An Introduction to Nonassociative Algebras
++  Academic Press, New York, 1966
++ Description:
++   NonAssociativeAlgebra is the category of non associative algebras
++   (modules which are themselves non associative rngs).
++   Axioms
++      r*(a*b) = (r*a)*b = a*(r*b)
NonAssociativeAlgebra(R:CommutativeRing): Category == _
  Join(NonAssociativeRng, Module R) with
    plenaryPower : (%,PositiveInteger) -> %
      ++ plenaryPower(a,n) is recursively defined to be
      ++ \spad{plenaryPower(a,n-1)*plenaryPower(a,n-1)} for \spad{n>1}
      ++ and \spad{a} for \spad{n=1}.
  add
    plenaryPower(a,n) ==
--      one? n => a
      ( n = 1 ) => a
      n1 : PositiveInteger := (n-1)::NonNegativeInteger::PositiveInteger
      plenaryPower(a,n1) * plenaryPower(a,n1)
```

⟨*NAALG.dotabb*⟩≡

```
"NAALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NAALG"];
"NAALG" -> "NARNG"
"NAALG" -> "MODULE"
```

⟨*NAALG.dotfull*⟩≡

```
"NonAssociativeAlgebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=NAALG"];
"NonAssociativeAlgebra(a:CommutativeRing)" -> "NonAssociativeRng()"
"NonAssociativeAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"
```

⟨*NAALG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"NonAssociativeAlgebra(a:CommutativeRing)" [color=lightblue];
"NonAssociativeAlgebra(a:CommutativeRing)" -> "NonAssociativeRng()"
"NonAssociativeAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"NonAssociativeRng()" [color=lightblue];
"NonAssociativeRng()" -> "ABELGRP..."
"NonAssociativeRng()" -> "Monad()"

"Monad()" [color=lightblue];
"Monad()" -> "SETCAT..."
"Monad()" -> "REPSQ..."

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

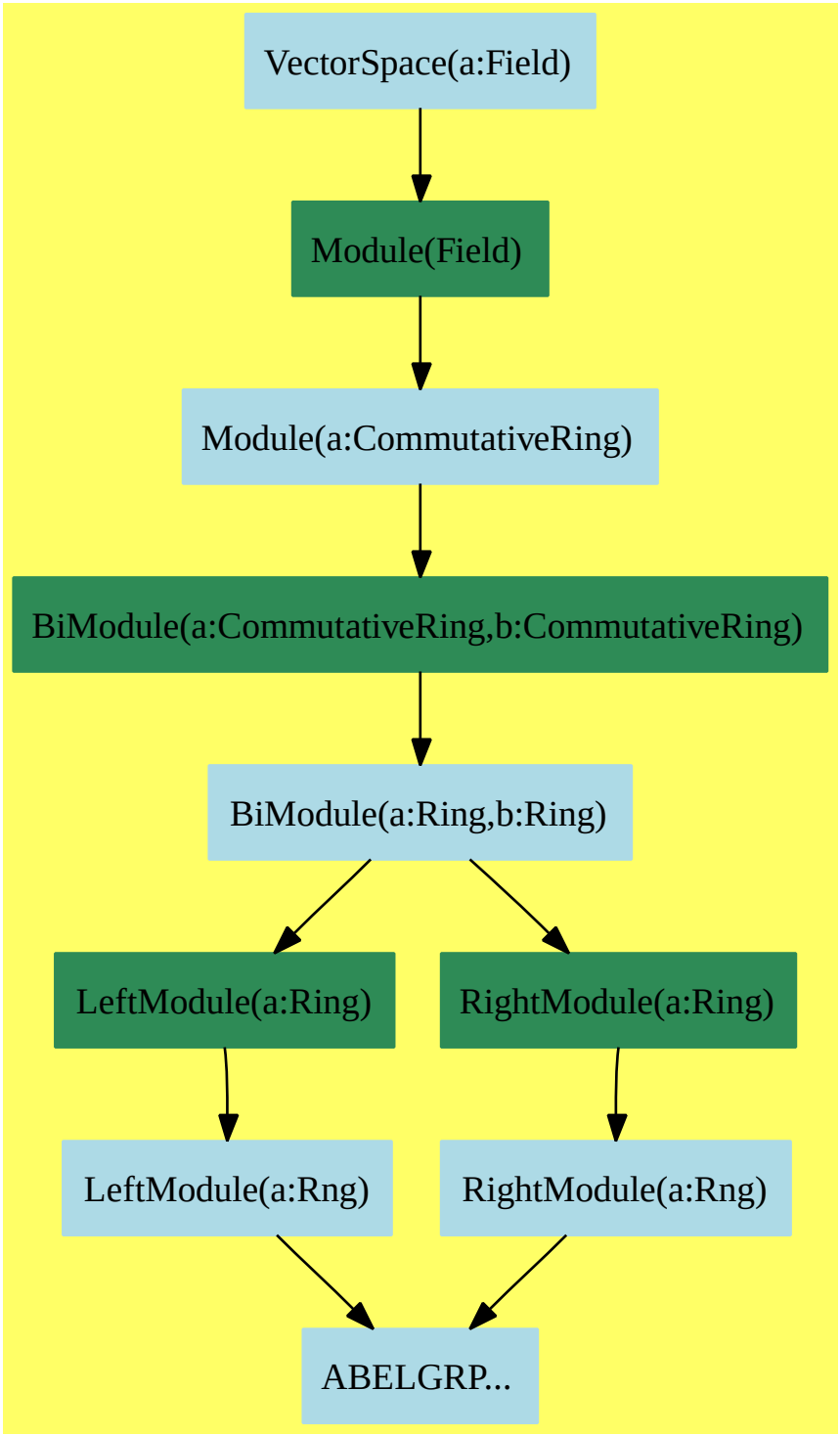
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

11.7 VectorSpace (VSPACE)



See:

⇒ "ExtensionField" (XF) 18.2 on page 1237
⇐ "Module" (MODULE) 10.10 on page 711

Exports:

0	coerce	dimension	hash	latex
sample	subtractIfCan	zero?	?~=?	?*?
?+?	?-?	-?	?/?	?=?

Attributes exported:

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
dimension : () -> CardinalNumber
```

These are implemented by this category:

```
?/? : (%,S) -> %
```

These exports come from (p711) Module():

```
?*? : (%,S) -> %
0 : () -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (S,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

⟨*category VSPACE VectorSpace*⟩≡

```
)abbrev category VSPACE VectorSpace
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
```

```
  ++ Related Constructors:
  ++ Also See:
  ++ AMS Classifications:
  ++ Keywords:
  ++ References:
  ++ Description:
  ++ Vector Spaces (not necessarily finite dimensional) over a field.

  VectorSpace(S:Field): Category ==  Module(S) with
      "/"       : (%, S) -> %
        ++ x/y divides the vector x by the scalar y.
      dimension: () -> CardinalNumber
        ++ dimension() returns the dimensionality of the vector space.
    add
      (v:% / s:S):% == inv(s) * v
```

⟨*VSPACE.dotabb*⟩≡

```
  "VSPACE"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=VSPACE"];
  "VSPACE" -> "MODULE"
```

⟨*VSPACE.dotfull*⟩≡

```
  "VectorSpace(a:Field)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=VSPACE"];
  "VectorSpace(a:Field)" -> "Module(Field)"
```

⟨ *VSPACE.dotpic* ⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"VectorSpace(a:Field)" [color=lightblue];
"VectorSpace(a:Field)" -> "Module(Field)"

"Module(Field)" [color=seagreen];
"Module(Field)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

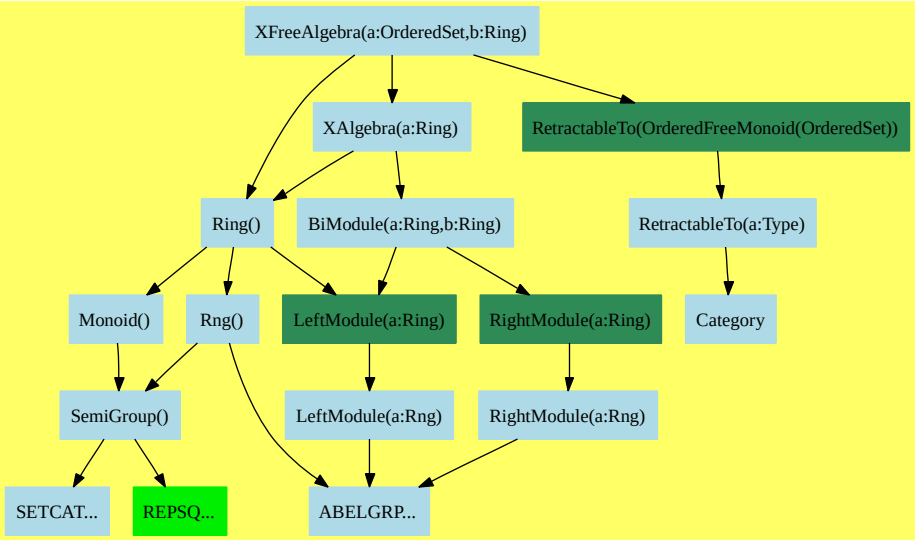
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"ABELGRP..." [color=lightblue];
}
```

11.8 XFreeAlgebra (XFALG)



See:

⇒ "XPolynomialsCat" (XPOLYC) 12.10 on page 898
⇐ "RetractableTo" (RETRACT) 2.18 on page 44
⇐ "Ring" (RING) 9.8 on page 628
⇐ "XAlgebra" (XALG) 10.18 on page 765

Exports:

0	1	characteristic	coef	coerce
constant	constant?	hash	latex	lquo
map	mindeg	mindegTerm	mirror	monom
monomial?	one?	quasiRegular	quasiRegular?	recip
retract	retractIfCan	rquo	sample	sh
subtractIfCan	varList	zero?	?*?	?**?
?+?	?-?	-?	?=?	?^?
?~=?				

Attributes Exported:

- if Ring has noZeroDivisors then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coef : (%,OrderedFreeMonoid vl) -> R
coef : (%,%) -> R
coerce : vl -> %
constant : % -> R
constant? : % -> Boolean
lquo : (%,vl) -> %
lquo : (%,%) -> %
lquo : (%,OrderedFreeMonoid vl) -> %
map : ((R -> R),%) -> %
mindeg : % -> OrderedFreeMonoid vl
mindegTerm : % -> Record(k: OrderedFreeMonoid vl,c: R)
mirror : % -> %
monom : (OrderedFreeMonoid vl,R) -> %
monomial? : % -> Boolean
quasiRegular : % -> %
quasiRegular? : % -> Boolean
rquo : (%,OrderedFreeMonoid vl) -> %
rquo : (%,%) -> %
rquo : (%,vl) -> %
sh : (%,NonNegativeInteger) -> % if R has COMRING
sh : (%,%) -> % if R has COMRING
varList : % -> List vl
?*? : (vl,%) -> %
?*? : (%,R) -> %
```

These exports come from (p628) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> OutputForm
coerce : Integer -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
```

```
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p765) XAlgebra(a:Ring):

```
coerce : R -> %
?*? : (R,%) -> %
```

These exports come from (p44) RetractableTo(WORD)
where WORD:OrderedFreeMonoid(OrderedSet))

```
coerce : OrderedFreeMonoid vl -> %
retract : % -> OrderedFreeMonoid vl
retractIfCan : % -> Union(OrderedFreeMonoid vl,"failed")
```

⟨*category XFALG XFreeAlgebra*⟩≡

```
)abbrev category XFALG XFreeAlgebra
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++    This category specifies opeations for  polynomials
++    and formal series with non-commutative variables.
++ Author: Michel Petitot (petitot@lifl.fr)

XFreeAlgebra(vl:OrderedSet,R:Ring):Category == Catdef where
   WORD   ==> OrderedFreeMonoid(vl)          -- monoide libre
   NNI    ==> NonNegativeInteger
   I      ==> Integer
   TERM   ==> Record(k: WORD, c: R)

   Catdef == Join(Ring, XAlgebra(R), RetractableTo WORD)
     with
       "*": (vl,%) -> %
         ++ \spad{v * x} returns the product of a variable \spad{x}
```

```
  ++ by \spad{x}.
"*": (%, R) -> %
  ++ \spad{x * r} returns the product of \spad{x} by \spad{r}.
  ++ Usefull if \spad{R} is a non-commutative Ring.
mindeg: % -> WORD
  ++ \spad{mindeg(x)} returns the little word which appears
  ++ in \spad{x}. Error if \spad{x=0}.
mindegTerm: % -> TERM
  ++ \spad{mindegTerm(x)} returns the term whose word is
  ++ \spad{mindeg(x)}.
coef  : (%,WORD) -> R
  ++ \spad{coef(x,w)} returns the coefficient of the word \spad{w}
  ++ in \spad{x}.
coef  : (%,%) -> R
  ++ \spad{coef(x,y)} returns scalar product of \spad{x} by \spad{y},
  ++ the set of words being regarded as an orthogonal basis.
lquo  : (%,vl) -> %
  ++ \spad{lquo(x,v)} returns the left simplification of \spad{x}
  ++ by the variable \spad{v}.
lquo  : (%,WORD) -> %
  ++ \spad{lquo(x,w)} returns the left simplification of \spad{x}
  ++ by the word \spad{w}.
lquo  : (%,%) -> %
  ++ \spad{lquo(x,y)} returns the left simplification of \spad{x}
  ++ by \spad{y}.
rquo  : (%,vl) -> %
  ++ \spad{rquo(x,v)} returns the right simplification of \spad{x}
  ++ by the variable \spad{v}.
rquo  : (%,WORD) -> %
  ++ \spad{rquo(x,w)} returns the right simplification of \spad{x}
  ++ by \spad{w}.
rquo  : (%,%) -> %
  ++ \spad{rquo(x,y)} returns the right simplification of \spad{x}
  ++ by \spad{y}.
monom : (WORD , R) -> %
  ++ \spad{monom(w,r)} returns the product of the word \spad{w}
  ++ by the coefficient \spad{r}.
monomial? : % -> Boolean
  ++ \spad{monomial?(x)} returns true if \spad{x} is a monomial
mirror: % -> %
  ++ \spad{mirror(x)} returns \spad{Sum(r_i mirror(w_i))} if
  ++ \spad{x} writes \spad{Sum(r_i w_i)}.
coerce : vl -> %
  ++ \spad{coerce(v)} returns \spad{v}.
constant?:% -> Boolean
  ++ \spad{constant?(x)} returns true if \spad{x} is constant.
```

```
        constant: % -> R
          ++ \spad{constant(x)} returns the constant term of \spad{x}.
        quasiRegular? : % -> Boolean
          ++ \spad{quasiRegular?(x)} return true if \spad{constant(x)} is zero
        quasiRegular : % -> %
          ++ \spad{quasiRegular(x)} return \spad{x} minus its constant term.
        if R has CommutativeRing then
           sh :(%,%) -> %
              ++ \spad{sh(x,y)} returns the shuffle-product of \spad{x}
              ++ by \spad{y}.
              ++ This multiplication is associative and commutative.
           sh :(%,NNI) -> %
              ++ \spad{sh(x,n)} returns the shuffle power of \spad{x} to
              ++ the \spad{n}.
        map   : (R -> R, %) -> %
          ++ \spad{map(fn,x)} returns \spad{Sum(fn(r_i) w_i)} if \spad{x}
          ++ writes \spad{Sum(r_i w_i)}.
        varList: % -> List vl
          ++ \spad{varList(x)} returns the list of variables which
          ++ appear in \spad{x}.

      -- Attributs
        if R has noZeroDivisors then noZeroDivisors
```

⟨*XFALG.dotabb*⟩≡

```
"XFALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XFALG"];
"XFALG" -> "RETRACT"
"XFALG" -> "RING"
"XFALG" -> "XALG"
```

⟨*XFALG.dotfull*⟩≡

```
"XFreeAlgebra(a:OrderedSet,b:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XFALG"];
"XFreeAlgebra(a:OrderedSet,b:Ring)" -> "Ring()"
"XFreeAlgebra(a:OrderedSet,b:Ring)" -> "XAlgebra(a:Ring)"
"XFreeAlgebra(a:OrderedSet,b:Ring)" ->
    "RetractableTo(OrderedFreeMonoid(OrderedSet))"
```

⟨*XFALG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"XFreeAlgebra(a:OrderedSet,b:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XFALG"];
"XFreeAlgebra(a:OrderedSet,b:Ring)" -> "Ring()"
"XFreeAlgebra(a:OrderedSet,b:Ring)" -> "XAlgebra(a:Ring)"
"XFreeAlgebra(a:OrderedSet,b:Ring)" ->
    "RetractableTo(OrderedFreeMonoid(OrderedSet))"

"RetractableTo(OrderedFreeMonoid(OrderedSet))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(OrderedFreeMonoid(OrderedSet))" -> "RetractableTo(a:Type)"

"XAlgebra(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XALG"];
"XAlgebra(a:Ring)" -> "Ring()"
"XAlgebra(a:Ring)" -> "BiModule(a:Ring,b:Ring)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"
```

```
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

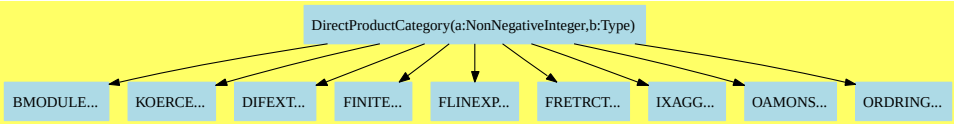
"Category" [color=lightblue];

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

Chapter 12

Category Layer 11

12.1 DirectProductCategory (DIRPCAT)



See:

⇐ “BiModule” (BMODULE) 9.1 on page 592
⇐ “CoercibleTo” (KOERCE) 2.6 on page 15
⇐ “DifferentialExtension” (DIFEXT) 11.2 on page 777
⇐ “IndexedAggregate” (IXAGG) 5.8 on page 246
⇐ “Finite” (FINITE) 4.9 on page 129
⇐ “FullyLinearlyExplicitRingOver” (FLINEXP) 11.3 on page 782
⇐ “FullyRetractableTo” (FRETRCT) 3.6 on page 71
⇐ “OrderedRing” (ORDRING) 10.11 on page 715
⇐ “OrderedAbelianMonoidSup” (OAMONS) 9.6 on page 620

Exports:

0	1	abs	any?	characteristic
coerce	copy	count	D	differentiate
dimension	directProduct	dot	elt	empty
empty?	entry?	entries	eq?	eval
every?	fill!	first	hash	index
index?	indices	latex	less?	lookup
map	map!	max	maxIndex	member?
members	min	minIndex	more?	negative?
one?	parts	positive?	qelt	qsetelt!
random	recip	reducedSystem	retract	retractIfCan
sample	setelt	sign	size	size?
subtractIfCan	sup	swap!	unitVector	zero?
?~=?	-?	?.?	#?	?*?
?**?	?+?	?-?	?/? ?<?	
?<=?	?=?	?>?	?>=?	?^?

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- if #2 has commutativeRing then commutative("*") where **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- if #2 has unitsKnown then unitsKnown where **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- if #2 has Ring then rightUnitary where **leftUnitary** is true if $1 * x = x$ for all x.
- if #2 has Ring then rightUnitary where **rightUnitary** is true if $x * 1 = x$ for all x.
- **nil**

These are directly exported but not implemented:

```
directProduct : Vector R -> %
dot : (%,%) -> R if R has RING
unitVector : PositiveInteger -> % if R has RING
?*? : (R,%) -> % if R has MONOID
?*? : (%,R) -> % if R has MONOID
```

These are implemented by this category:

```
characteristic : () -> NonNegativeInteger if R has RING
coerce : Integer -> %
```

```
  if and(has(R,RetractableTo Integer),
         has(R,SetCategory))
  or R has RING
differentiate : (%,(R -> R)) -> % if R has RING
dimension : () -> CardinalNumber if R has FIELD
reducedSystem : Matrix % -> Matrix R if R has RING
reducedSystem :
   (Matrix %,Vector %) ->
      Record(mat: Matrix R,vec: Vector R)
         if R has RING
size : () -> NonNegativeInteger if R has FINITE
?/? : (%,R) -> % if R has FIELD
```

These exports come from (p246) IndexedAggregate(a:SetCategory,R:Type):

```
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
coerce : % -> OutputForm if R has SETCAT
copy : % -> %
count : (R,%) -> NonNegativeInteger
  if R has SETCAT
  and $ has finiteAggregate
count : ((R -> Boolean),%) -> NonNegativeInteger
  if $ has finiteAggregate
elt : (%,Integer,R) -> R
empty : () -> %
empty? : % -> Boolean
entries : % -> List R
entry? : (R,%) -> Boolean
  if $ has finiteAggregate
  and R has SETCAT
eq? : (%,%) -> Boolean
eval : (%,List R,List R) -> %
  if R has EVALAB R
  and R has SETCAT
eval : (%,R,R) -> %
  if R has EVALAB R
  and R has SETCAT
eval : (%,Equation R) -> %
  if R has EVALAB R
  and R has SETCAT
eval : (%,List Equation R) -> %
  if R has EVALAB R
  and R has SETCAT
every? : ((R -> Boolean),%) -> Boolean
  if $ has finiteAggregate
fill! : (%,R) -> % if $ has shallowlyMutable
first : % -> R if Integer has ORDSET
hash : % -> SingleInteger if R has SETCAT
index? : (Integer,%) -> Boolean
indices : % -> List Integer
```

```
less? : (%,NonNegativeInteger) -> Boolean
latex : % -> String if R has SETCAT
map : ((R -> R),%) -> %
map! : ((R -> R),%) -> % if $ has shallowlyMutable
maxIndex : % -> Integer if Integer has ORDSET
member? : (R,%) -> Boolean
  if R has SETCAT
  and $ has finiteAggregate
members : % -> List R if $ has finiteAggregate
minIndex : % -> Integer if Integer has ORDSET
more? : (%,NonNegativeInteger) -> Boolean
parts : % -> List R if $ has finiteAggregate
qelt : (%,Integer) -> R
qsetelt! : (%,Integer,R) -> R if $ has shallowlyMutable
sample : () -> %
setelt : (%,Integer,R) -> R if $ has shallowlyMutable
size? : (%,NonNegativeInteger) -> Boolean
swap! : (%,Integer,Integer) -> Void if $ has shallowlyMutable
#? : % -> NonNegativeInteger if $ has finiteAggregate
?.? : (%,Integer) -> R
?=? : (%,%) -> Boolean if R has SETCAT
?~=? : (%,%) -> Boolean if R has SETCAT
```

These exports come from (p15) CoercibleTo(Vector(R:Type)):

```
coerce : % -> Vector R
```

These exports come from (p71) FullyRetractableTo(R:SetCategory):

```
coerce : R -> % if R has SETCAT
coerce : Fraction Integer -> %
  if and(has(R,RetractableTo Fraction Integer),
         has(R,SetCategory))
retract : % -> Integer
  if and(has(R,RetractableTo Integer),
         has(R,SetCategory))
retract : % -> R if R has SETCAT
retract : % -> Fraction Integer
  if and(has(R,RetractableTo Fraction Integer),
         has(R,SetCategory))
retractIfCan : % -> Union(Integer,"failed")
  if and(has(R,RetractableTo Integer),
         has(R,SetCategory))
retractIfCan : % -> Union(R,"failed") if R has SETCAT
retractIfCan : % -> Union(Fraction Integer,"failed")
  if and(has(R,RetractableTo Fraction Integer),
         has(R,SetCategory))
```

These exports come from (p592) BiModule(R:Ring,R:Ring):

```
0 : () -> % if R has CABMON
subtractIfCan : (%,%) -> Union(%,"failed") if R has CABMON
zero? : % -> Boolean if R has CABMON
?+? : (%,%) -> % if R has ABELSG
?*? : (PositiveInteger,%) -> % if R has ABELSG
?*? : (NonNegativeInteger,%) -> % if R has CABMON
?*? : (Integer,%) -> % if R has RING
?-? : (%,%) -> % if R has RING
-? : % -> % if R has RING
```

These exports come from (p777) DifferentialExtension(R:Ring):

```
1 : () -> % if R has RING
D : (%,(R -> R)) -> % if R has RING
D : (%,(R -> R),NonNegativeInteger) -> % if R has RING
D : % -> % if and(has(R,DifferentialRing),has(R,Ring))
D : (%,NonNegativeInteger) -> %
  if and(has(R,DifferentialRing),has(R,Ring))
differentiate : (%,NonNegativeInteger) -> %
  if and(has(R,DifferentialRing),has(R,Ring))
D : (%,List Symbol,List NonNegativeInteger) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
D : (%,Symbol,NonNegativeInteger) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
D : (%,List Symbol) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
D : (%,Symbol) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
differentiate : (%,Symbol,NonNegativeInteger) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
differentiate : (%,List Symbol) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
differentiate : % -> %
  if and(has(R,DifferentialRing),has(R,Ring))
differentiate : (%,(R -> R),NonNegativeInteger) -> %
  if R has RING
differentiate : (%,Symbol) -> %
  if and(has(R,PartialDifferentialRing Symbol),has(R,Ring))
one? : % -> Boolean if R has RING
recip : % -> Union(%,"failed") if R has RING
?*? : (%,%) -> % if R has RING
?**? : (%,PositiveInteger) -> % if R has RING
?**? : (%,NonNegativeInteger) -> % if R has RING
?^? : (%,PositiveInteger) -> % if R has RING
?^? : (%,NonNegativeInteger) -> % if R has RING
```

These exports come from (p782) FullyLinearlyExplicitRingOver(R:Ring):

```
reducedSystem :
  (Matrix %,Vector %) ->
    Record(mat: Matrix Integer,vec: Vector Integer)
      if and(has(R,LinearlyExplicitRingOver Integer),has(R,Ring))
reducedSystem : Matrix % -> Matrix Integer
  if and(has(R,LinearlyExplicitRingOver Integer),has(R,Ring))
```

These exports come from (p129) Finite():

```
index : PositiveInteger -> % if R has FINITE
lookup : % -> PositiveInteger if R has FINITE
random : () -> % if R has FINITE
```

These exports come from (p715) OrderedRing():

```
abs : % -> % if R has ORDRING
max : (%,%) -> % if R has ORDRING or R has OAMONS
min : (%,%) -> % if R has ORDRING or R has OAMONS
negative? : % -> Boolean if R has ORDRING
positive? : % -> Boolean if R has ORDRING
sign : % -> Integer if R has ORDRING
?<? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
?<=? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
?>? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
?>=? : (%,%) -> Boolean if R has ORDRING or R has OAMONS
```

These exports come from (p620) OrderedAbelianMonoidSup():

```
sup : (%,%) -> % if R has OAMONS
```

⟨*category DIRPCAT DirectProductCategory*⟩≡

```
)abbrev category DIRPCAT DirectProductCategory
-- all direct product category domains must be compiled
-- without subsumption, set SourceLevelSubset to EQUAL
--)bo $noSubsumption := true

--% DirectProductCategory

++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors: DirectProduct
++ Also See: VectorCategory
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
```

```
++   This category represents a finite cartesian product of a given type.
++ Many categorical properties are preserved under this construction.

DirectProductCategory(dim:NonNegativeInteger, R:Type): Category ==
  Join(IndexedAggregate(Integer, R), CoercibleTo Vector R) with
         finiteAggregate
           ++ attribute to indicate an aggregate of finite size
         directProduct: Vector R -> %
           ++ directProduct(v) converts the vector v to become
           ++ a direct product. Error: if the length of v is
           ++ different from dim.
         if R has SetCategory then FullyRetractableTo R
         if R has Ring then
           BiModule(R, R)
           DifferentialExtension R
           FullyLinearlyExplicitRingOver R
           unitVector: PositiveInteger -> %
             ++ unitVector(n) produces a vector with 1 in position n and
             ++ zero elsewhere.
           dot: (%, %) -> R
             ++ dot(x,y) computes the inner product of the vectors x and y.
         if R has AbelianSemiGroup then AbelianSemiGroup
         if R has CancellationAbelianMonoid then CancellationAbelianMonoid
         if R has Monoid then
            Monoid
           _* : (R, %) -> %
             ++ r * y multiplies the element r times each component of the
             ++ vector y.
           _* : (%, R) -> %
             ++ y*r multiplies each component of the vector y by the element r.
         if R has Finite then Finite
         if R has CommutativeRing then
           Algebra R
           CommutativeRing
         if R has unitsKnown then unitsKnown
         if R has OrderedRing then OrderedRing
         if R has OrderedAbelianMonoidSup then OrderedAbelianMonoidSup
         if R has Field then VectorSpace R
 add
      if R has Ring then
        equation2R: Vector % -> Matrix R

        coerce(n:Integer):%          == n::R::%

        characteristic()             == characteristic()$R
```

```
        differentiate(z:%, d:R -> R) == map(d, z)

        equation2R v ==
          ans:Matrix(R) := new(dim, #v, 0)
          for i in minRowIndex ans .. maxRowIndex ans repeat
            for j in minColIndex ans .. maxColIndex ans repeat
              qsetelt_!(ans, i, j, qelt(qelt(v, j), i))
          ans

        reducedSystem(m:Matrix %):Matrix(R) ==
          empty? m => new(0, 0, 0)
          reduce(vertConcat, [equation2R row(m, i)
                 for i in minRowIndex m .. maxRowIndex m])$List(Matrix R)

        reducedSystem(m:Matrix %, v:Vector %):
          Record(mat:Matrix R, vec:Vector R) ==
            vh:Vector(R) :=
              empty? v => empty()
              rh := reducedSystem(v::Matrix %)@Matrix(R)
              column(rh, minColIndex rh)
            [reducedSystem(m)@Matrix(R), vh]

      if R has Finite then size == size$R ** dim

      if R has Field then
        x / b       == x * inv b

        dimension() == dim::CardinalNumber
```

⟨*DIRPCAT.dotabb*⟩≡

```
  "DIRPCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=DIRPCAT"];
  "DIRPCAT" -> "IXAGG"
  "DIRPCAT" -> "KOERCE"
  "DIRPCAT" -> "FRETRCT"
  "DIRPCAT" -> "BMODULE"
  "DIRPCAT" -> "DIFEXT"
  "DIRPCAT" -> "FLINEXP"
  "DIRPCAT" -> "FINITE"
  "DIRPCAT" -> "ORDRING"
  "DIRPCAT" -> "OAMONS"
```

⟨*DIRPCAT.dotfull*⟩≡
```
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIRPCAT"];
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "IndexedAggregate(a:SetCategory,b:Type)"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "CoercibleTo(a:Type)"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "FullyRetractableTo(a:Type)"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "BiModule(a:Ring,b:Ring)"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "DifferentialExtension(a:Ring)"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "FullyLinearlyExplicitRingOver(a:Ring)"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "Finite()"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "OrderedRing()"
"DirectProductCategory(a:NonNegativeInteger,b:Type)"
  -> "OrderedAbelianMonoidSup()"
```

⟨*DIRPCAT.dotpic*⟩≡

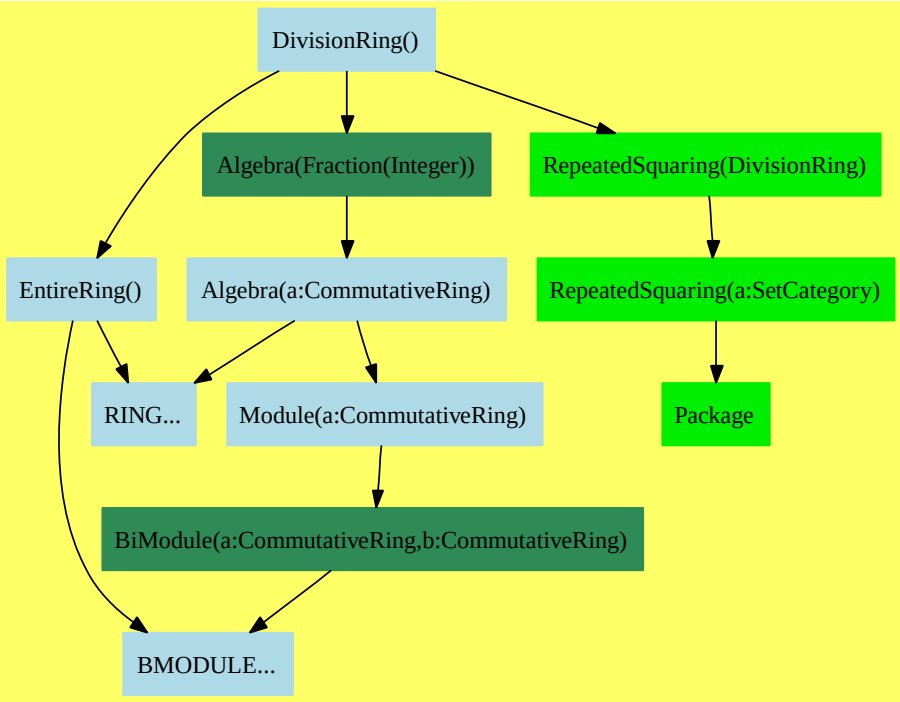
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DirectProductCategory(a:NonNegativeInteger,b:Type)" [color=lightblue];
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "BMODULE..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "KOERCE..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "DIFEXT..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "FINITE..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "FLINEXP..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "FRETRCT..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "IXAGG..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "OAMONS..."
"DirectProductCategory(a:NonNegativeInteger,b:Type)" -> "ORDRING..."

"BMODULE..." [color=lightblue];
"KOERCE..." [color=lightblue];
"DIFEXT..." [color=lightblue];
"FINITE..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"IXAGG..." [color=lightblue];
"OAMONS..." [color=lightblue];
"ORDRING..." [color=lightblue];

}
```

12.2 DivisionRing (DIVRING)



See:

⇒ "Field" (FIELD) 16.1 on page 1005
⇐ "Algebra" (ALGEBRA) 11.1 on page 771
⇐ "EntireRing" (ENTIRER) 10.6 on page 694

Exports:

1	0	characteristic	coerce	hash
inv	latex	one?	recip	sample
subtractIfCan	zero?	?^?	?~=?	?**?
?*?	?+?	?-?	-?	?=?

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

- **nil**

These are directly exported but not implemented:

```
inv : % -> %
```

These are implemented by this category:

```
?^? : (%,Integer) -> %
?**? : (%,Integer) -> %
```

These exports come from (p694) EntireRing():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p771) Algebra(Fraction(Integer)):

```
coerce : Fraction Integer -> %
?*? : (%,Fraction Integer) -> %
?*? : (Fraction Integer,%) -> %
```

⟨*category DIVRING DivisionRing*⟩≡

```
)abbrev category DIVRING DivisionRing
++ Author:
++ Date Created:
```

```
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A division ring (sometimes called a skew field),
++ i.e. a not necessarily commutative ring where
++ all non-zero elements have multiplicative inverses.

DivisionRing(): Category ==
 Join(EntireRing, Algebra Fraction Integer) with
      "**": (%,Integer) -> %
          ++ x**n returns x raised to the integer power n.
      "^" : (%,Integer) -> %
          ++ x^n returns x raised to the integer power n.
      inv : % -> %
          ++ inv x returns the multiplicative inverse of x.
          ++ Error: if x is 0.
-- Q-algebra is a lie, should be conditional on characteristic 0,
-- but knownInfo cannot handle the following commented
--    if % has CharacteristicZero then Algebra Fraction Integer
    add
      n: Integer
      x: %
      _^(x:%, n:Integer):% == x ** n
      import RepeatedSquaring(%)
      x ** n: Integer ==
         zero? n => 1
         zero? x =>
            n<0 => error "division by zero"
            x
         n<0 =>
            expt(inv x,(-n) pretend PositiveInteger)
         expt(x,n pretend PositiveInteger)
--    if % has CharacteristicZero() then
      q:Fraction(Integer) * x:% == numer(q) * inv(denom(q)::%) * x
```

⟨*DIVRING.dotabb*⟩≡
```
  "DIVRING"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=DIVRING"];
  "DIVRING" -> "ENTIRER"
  "DIVRING" -> "ALGEBRA"
```

⟨*DIVRING.dotfull*⟩≡
```
  "DivisionRing()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=DIVRING"];
  "DivisionRing()" -> "EntireRing()"
  "DivisionRing()" -> "Algebra(Fraction(Integer))"
  "DivisionRing()" -> "RepeatedSquaring(DivisionRing)"
```

⟨*DIVRING.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DivisionRing()" [color=lightblue];
"DivisionRing()" -> "EntireRing()"
"DivisionRing()" -> "Algebra(Fraction(Integer))"
"DivisionRing()" -> "RepeatedSquaring(DivisionRing)"

"RepeatedSquaring(DivisionRing)" [color="#00EE00"];
"RepeatedSquaring(DivisionRing)" -> "RepeatedSquaring(a:SetCategory)"

"RepeatedSquaring(a:SetCategory)" [color="#00EE00"];
"RepeatedSquaring(a:SetCategory)" -> "Package"

"Package" [color="#00EE00"];

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(Fraction(Integer))" [color=seagreen];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

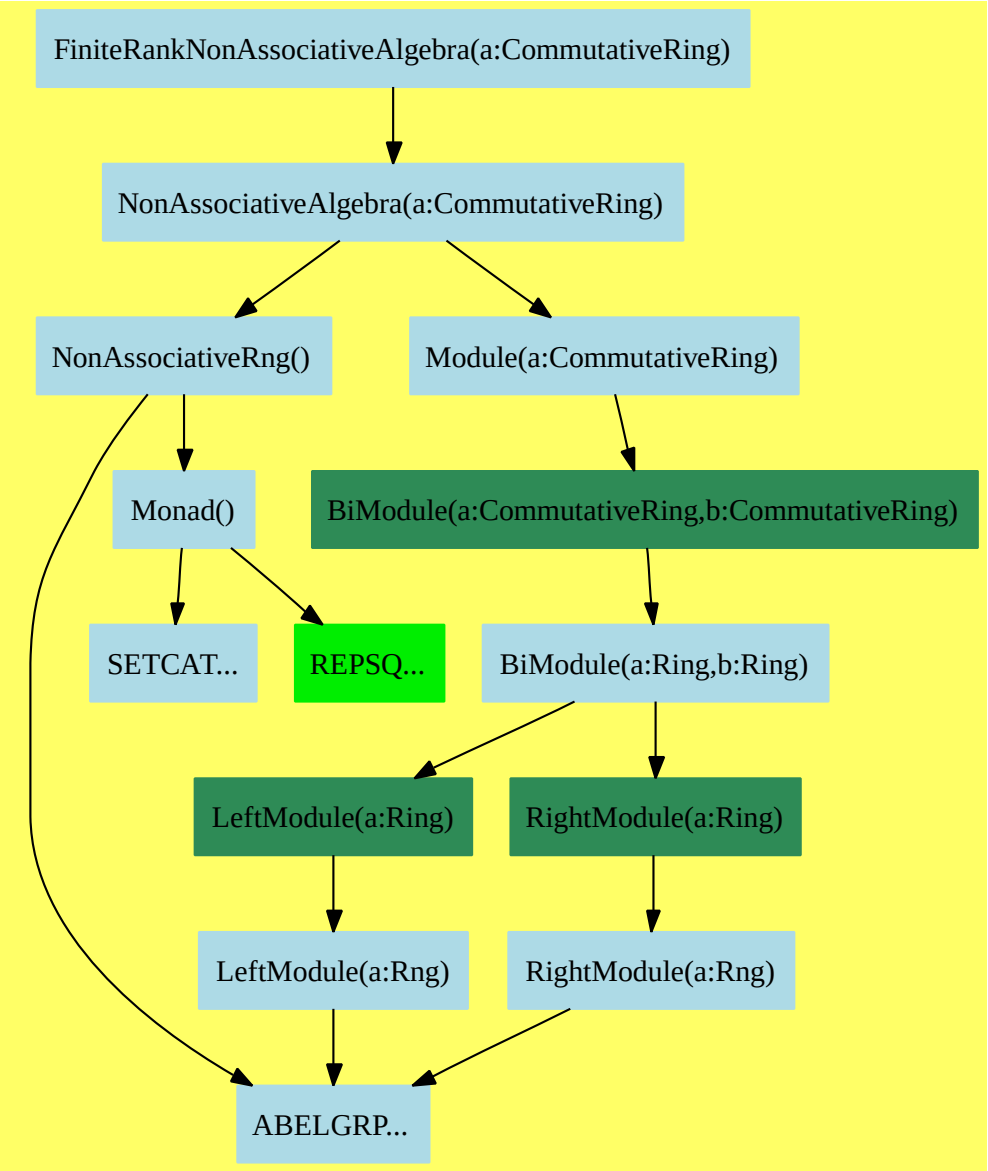
"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"RING..." [color=lightblue];
"BMODULE..." [color=lightblue];
}
```

12.3 FiniteRankNonAssociativeAlgebra (FINAALG)



See:

⇒ "FramedNonAssociativeAlgebra" (FRNAALG) 13.3 on page 918
⇐ "NonAssociativeAlgebra" (NAALG) 11.6 on page 798

Exports:

0	alternative?
antiAssociative?	antiCommutative?
antiCommutator	associative?
associator	associatorDependence
coerce	commutative?
commutator	conditionsForIdempotents
coordinates	flexible?
hash	jacobiIdentity?
jordanAdmissible?	jordanAlgebra?
latex	leftAlternative?
leftCharacteristicPolynomial	leftDiscriminant
leftMinimalPolynomial	leftNorm
leftPower	leftRecip
leftRegularRepresentation	leftTrace
leftTraceMatrix	leftUnit
leftUnits	lieAdmissible?
lieAlgebra?	noncommutativeJordanAlgebra?
plenaryPower	powerAssociative?
rank	recip
represents	rightAlternative?
rightCharacteristicPolynomial	rightDiscriminant
rightMinimalPolynomial	rightNorm
rightPower	rightRecip
rightRegularRepresentation	rightTrace
rightTraceMatrix	rightUnit
rightUnits	sample
someBasis	structuralConstants
subtractIfCan	unit
zero?	?*?
?**?	?+?
?-?	-?
?=?	?~=?

Attributes Exported:

- if R has IntegralDomain then unitsKnown where **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
conditionsForIdempotents : Vector % -> List Polynomial R
coordinates : (%,Vector %) -> Vector R
```

```
leftUnit : () -> Union(%,"failed") if R has INTDOM
leftUnits : () -> Union(Record(particular: %,basis: List %),"failed")
    if R has INTDOM
powerAssociative? : () -> Boolean
rank : () -> PositiveInteger
rightUnit : () -> Union(%,"failed") if R has INTDOM
rightUnits : () -> Union(Record(particular: %,basis: List %),"failed")
    if R has INTDOM
someBasis : () -> Vector %
unit : () -> Union(%,"failed") if R has INTDOM
```

These are implemented by this category:

```
alternative? : () -> Boolean
antiAssociative? : () -> Boolean
antiCommutative? : () -> Boolean
associative? : () -> Boolean
associatorDependence : () -> List Vector R if R has INTDOM
commutative? : () -> Boolean
coordinates : (Vector %,Vector %) -> Matrix R
flexible? : () -> Boolean
jacobiIdentity? : () -> Boolean
jordanAdmissible? : () -> Boolean
jordanAlgebra? : () -> Boolean
leftAlternative? : () -> Boolean
leftCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
leftDiscriminant : Vector % -> R
leftMinimalPolynomial : % -> SparseUnivariatePolynomial R if R has INTDOM
leftNorm : % -> R
leftRecip : % -> Union(%,"failed") if R has INTDOM
leftRegularRepresentation : (%,Vector %) -> Matrix R
leftTrace : % -> R
leftTraceMatrix : Vector % -> Matrix R
lieAdmissible? : () -> Boolean
lieAlgebra? : () -> Boolean
noncommutativeJordanAlgebra? : () -> Boolean
recip : % -> Union(%,"failed") if R has INTDOM
represents : (Vector R,Vector %) -> %
rightAlternative? : () -> Boolean
rightCharacteristicPolynomial : % -> SparseUnivariatePolynomial R
rightDiscriminant : Vector % -> R
rightMinimalPolynomial : % -> SparseUnivariatePolynomial R if R has INTDOM
rightNorm : % -> R
rightRecip : % -> Union(%,"failed") if R has INTDOM
rightRegularRepresentation : (%,Vector %) -> Matrix R
rightTrace : % -> R
rightTraceMatrix : Vector % -> Matrix R
structuralConstants : Vector % -> Vector Matrix R
```

These exports come from (p798) NonAssociativeAlgebra(R:CommutativeRing):

```
0 : () -> %
antiCommutator : (%,%) -> %
associator : (%,%,%) -> %
coerce : % -> OutputForm
commutator : (%,%) -> %
hash : % -> SingleInteger
latex : % -> String
leftPower : (%,PositiveInteger) -> %
plenaryPower : (%,PositiveInteger) -> %
rightPower : (%,PositiveInteger) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (R,%) -> %
?*? : (%,R) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
```

⟨*category FINAALG FiniteRankNonAssociativeAlgebra*⟩≡

```
)abbrev category FINAALG FiniteRankNonAssociativeAlgebra
++ Author: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 12 June 1991
++ Basic Operations: +,-,*,**, someBasis
++ Related Constructors: FramedNonAssociativeAlgebra, FramedAlgebra,
++   FiniteRankAssociativeAlgebra
++ Also See:
++ AMS Classifications:
++ Keywords: nonassociative algebra, basis
++ References:
++   R.D. Schafer: An Introduction to Nonassociative Algebras
++   Academic Press, New York, 1966
++
++   R. Wisbauer: Bimodule Structure of Algebra
++   Lecture Notes Univ. Duesseldorf 1991
++ Description:
++   A FiniteRankNonAssociativeAlgebra is a non associative algebra over
++   a commutative ring R which is a free \spad{R}-module of finite rank.
FiniteRankNonAssociativeAlgebra(R:CommutativeRing):
 Category == NonAssociativeAlgebra R with
```

```
someBasis : () -> Vector %
  ++ someBasis() returns some \spad{R}-module basis.
rank : () -> PositiveInteger
  ++ rank() returns the rank of the algebra as \spad{R}-module.
conditionsForIdempotents: Vector % -> List Polynomial R
  ++ conditionsForIdempotents([v1,...,vn]) determines a complete list
  ++ of polynomial equations for the coefficients of idempotents
  ++ with respect to the \spad{R}-module basis \spad{v1},...,\spad{vn}.
structuralConstants: Vector % -> Vector Matrix R
  ++ structuralConstants([v1,v2,...,vm]) calculates the structural
  ++ constants \spad{[(gammaijk) for k in 1..m]} defined by
  ++ \spad{vi * vj = gammaij1 * v1 + ... + gammaijm * vm},
  ++ where \spad{[v1,...,vm]} is an \spad{R}-module basis
  ++ of a subalgebra.
leftRegularRepresentation: (% , Vector %) -> Matrix R
  ++ leftRegularRepresentation(a,[v1,...,vn]) returns the matrix of
  ++ the linear map defined by left multiplication by \spad{a}
  ++ with respect to the \spad{R}-module basis \spad{[v1,...,vn]}.
rightRegularRepresentation: (% , Vector %) -> Matrix R
  ++ rightRegularRepresentation(a,[v1,...,vn]) returns the matrix of
  ++ the linear map defined by right multiplication by \spad{a}
  ++ with respect to the \spad{R}-module basis \spad{[v1,...,vn]}.
leftTrace: %  -> R
  ++ leftTrace(a) returns the trace of the left regular representation
  ++ of \spad{a}.
rightTrace: %  -> R
  ++ rightTrace(a) returns the trace of the right regular representation
  ++ of \spad{a}.
leftNorm: %  -> R
  ++ leftNorm(a) returns the determinant of the left regular
  ++ representation of \spad{a}.
rightNorm: %  -> R
  ++ rightNorm(a) returns the determinant of the right regular
  ++ representation of \spad{a}.
coordinates: (%, Vector %) -> Vector R
  ++ coordinates(a,[v1,...,vn]) returns the coordinates of \spad{a}
  ++ with respect to the \spad{R}-module basis \spad{v1},...,\spad{vn}.
coordinates: (Vector %, Vector %) -> Matrix R
  ++ coordinates([a1,...,am],[v1,...,vn]) returns a matrix whose
  ++ i-th row is formed by the coordinates of \spad{ai}
  ++ with respect to the \spad{R}-module basis \spad{v1},...,\spad{vn}.
represents: (Vector R, Vector %) -> %
  ++ represents([a1,...,am],[v1,...,vm]) returns the linear
  ++ combination \spad{a1*vm + ... + an*vm}.
leftDiscriminant: Vector % -> R
  ++ leftDiscriminant([v1,...,vn]) returns  the determinant of the
```

```
  ++ \spad{n}-by-\spad{n} matrix whose element at the \spad{i}-th row
  ++ and \spad{j}-th column is given by the left trace of the product
  ++ \spad{vi*vj}.
  ++ Note: the same as \spad{determinant(leftTraceMatrix([v1,...,vn]))}.
rightDiscriminant: Vector % -> R
  ++ rightDiscriminant([v1,...,vn]) returns  the determinant of the
  ++ \spad{n}-by-\spad{n} matrix whose element at the \spad{i}-th row
  ++ and \spad{j}-th column is given by the right trace of the product
  ++ \spad{vi*vj}.
  ++ Note: the same as \spad{determinant(rightTraceMatrix([v1,...,vn]))}.
leftTraceMatrix: Vector % -> Matrix R
  ++ leftTraceMatrix([v1,...,vn]) is the \spad{n}-by-\spad{n} matrix
  ++ whose element at the \spad{i}-th row and \spad{j}-th column is given
  ++ by the left trace of the product \spad{vi*vj}.
rightTraceMatrix: Vector % -> Matrix R
  ++ rightTraceMatrix([v1,...,vn]) is the \spad{n}-by-\spad{n} matrix
  ++ whose element at the \spad{i}-th row and \spad{j}-th column is given
  ++ by the right trace of the product \spad{vi*vj}.
leftCharacteristicPolynomial: % -> SparseUnivariatePolynomial R
  ++ leftCharacteristicPolynomial(a) returns the characteristic
  ++ polynomial of the left regular representation of \spad{a}
  ++ with respect to any basis.
rightCharacteristicPolynomial: % -> SparseUnivariatePolynomial R
  ++ rightCharacteristicPolynomial(a) returns the characteristic
  ++ polynomial of the right regular representation of \spad{a}
  ++ with respect to any basis.

--we not necessarily have a unit
--if R has CharacteristicZero then CharacteristicZero
--if R has CharacteristicNonZero then CharacteristicNonZero

commutative?:()-> Boolean
  ++ commutative?() tests if multiplication in the algebra
  ++ is commutative.
antiCommutative?:()-> Boolean
  ++ antiCommutative?() tests if \spad{a*a = 0}
  ++ for all \spad{a} in the algebra.
  ++ Note: this implies \spad{a*b + b*a = 0} for all
  ++ \spad{a} and \spad{b}.
associative?:()-> Boolean
  ++ associative?() tests if multiplication in algebra
  ++ is associative.
antiAssociative?:()-> Boolean
  ++ antiAssociative?() tests if multiplication in algebra
  ++ is anti-associative, i.e. \spad{(a*b)*c + a*(b*c) = 0}
  ++ for all \spad{a},b,c in the algebra.
```

```
leftAlternative?: ()-> Boolean
  ++ leftAlternative?() tests if \spad{2*associator(a,a,b) = 0}
  ++ for all \spad{a}, b in the algebra.
  ++ Note: we only can test this; in general we don't know
  ++ whether \spad{2*a=0} implies \spad{a=0}.
rightAlternative?: ()-> Boolean
  ++ rightAlternative?() tests if \spad{2*associator(a,b,b) = 0}
  ++ for all \spad{a}, b in the algebra.
  ++ Note: we only can test this; in general we don't know
  ++ whether \spad{2*a=0} implies \spad{a=0}.
flexible?: ()->  Boolean
  ++ flexible?() tests if \spad{2*associator(a,b,a) = 0}
  ++ for all \spad{a}, b in the algebra.
  ++ Note: we only can test this; in general we don't know
  ++ whether \spad{2*a=0} implies \spad{a=0}.
alternative?: ()-> Boolean
  ++ alternative?() tests if
  ++ \spad{2*associator(a,a,b) = 0 = 2*associator(a,b,b)}
  ++ for all \spad{a}, b in the algebra.
  ++ Note: we only can test this; in general we don't know
  ++ whether \spad{2*a=0} implies \spad{a=0}.
powerAssociative?:()-> Boolean
  ++ powerAssociative?() tests if all subalgebras
  ++ generated by a single element are associative.
jacobiIdentity?:() -> Boolean
  ++ jacobiIdentity?() tests if \spad{(a*b)*c + (b*c)*a + (c*a)*b = 0}
  ++ for all \spad{a},b,c in the algebra. For example, this holds
  ++ for crossed products of 3-dimensional vectors.
lieAdmissible?: () -> Boolean
  ++ lieAdmissible?() tests if the algebra defined by the commutators
  ++ is a Lie algebra, i.e. satisfies the Jacobi identity.
  ++ The property of anticommutativity follows from definition.
jordanAdmissible?: () -> Boolean
  ++ jordanAdmissible?() tests if 2 is invertible in the
  ++ coefficient domain and the multiplication defined by
  ++ \spad{(1/2)(a*b+b*a)} determines a
  ++ Jordan algebra, i.e. satisfies the Jordan identity.
  ++ The property of \spadatt{commutative("*")}
  ++ follows from by definition.
noncommutativeJordanAlgebra?: () -> Boolean
  ++ noncommutativeJordanAlgebra?() tests if the algebra
  ++ is flexible and Jordan admissible.
jordanAlgebra?:() -> Boolean
  ++ jordanAlgebra?() tests if the algebra is commutative,
  ++ characteristic is not 2, and \spad{(a*b)*a**2 - a*(b*a**2) = 0}
  ++ for all \spad{a},b,c in the algebra (Jordan identity).
```

```
    ++ Example:
    ++ for every associative algebra \spad{(A,+,@)} we can construct a
    ++ Jordan algebra \spad{(A,+,*)}, where \spad{a*b := (a@b+b@a)/2}.
  lieAlgebra?:() -> Boolean
    ++ lieAlgebra?() tests if the algebra is anticommutative
    ++ and \spad{(a*b)*c + (b*c)*a + (c*a)*b = 0}
    ++ for all \spad{a},b,c in the algebra (Jacobi identity).
    ++ Example:
    ++ for every associative algebra \spad{(A,+,@)} we can construct a
    ++ Lie algebra \spad{(A,+,*)}, where \spad{a*b := a@b-b@a}.

  if R has IntegralDomain then
    -- we not neccessarily have a unit, hence we don't inherit
    -- the next 3 functions anc hence copy them from MonadWithUnit:
    recip: % -> Union(%,"failed")
      ++ recip(a) returns an element, which is both a left and a right
      ++ inverse of \spad{a},
      ++ or \spad{"failed"} if there is no unit element, if such an
      ++ element doesn't exist or cannot be determined (see unitsKnown).
    leftRecip: % -> Union(%,"failed")
      ++ leftRecip(a) returns an element, which is a left inverse of
      ++ \spad{a}, or \spad{"failed"} if there is no unit element, if such
      ++ an element doesn't exist or cannot be determined (see unitsKnown).
    rightRecip: % -> Union(%,"failed")
      ++ rightRecip(a) returns an element, which is a right inverse of
      ++ \spad{a},
      ++ or \spad{"failed"} if there is no unit element, if such an
      ++ element doesn't exist or cannot be determined (see unitsKnown).
    associatorDependence:() -> List Vector R
      ++ associatorDependence() looks for the associator identities, i.e.
      ++ finds a basis of the solutions of the linear combinations of the
      ++ six permutations of \spad{associator(a,b,c)} which yield 0,
      ++ for all \spad{a},b,c in the algebra.
      ++ The order of the permutations is \spad{123 231 312 132 321 213}.
    leftMinimalPolynomial : % -> SparseUnivariatePolynomial R
      ++ leftMinimalPolynomial(a) returns the polynomial determined by the
      ++ smallest non-trivial linear combination of left powers of
      ++ \spad{a}. Note: the polynomial never has a constant term as in
      ++ general the algebra has no unit.
    rightMinimalPolynomial : % -> SparseUnivariatePolynomial R
      ++ rightMinimalPolynomial(a) returns the polynomial determined by the
      ++ smallest non-trivial linear
      ++ combination of right powers of \spad{a}.
      ++ Note: the polynomial never has a constant term as in general
      ++ the algebra has no unit.
    leftUnits:() -> Union(Record(particular: %, basis: List %), "failed")
```

```
      ++ leftUnits() returns the affine space of all left units of the
      ++ algebra, or \spad{"failed"} if there is none.
    rightUnits:() -> Union(Record(particular: %, basis: List %), "failed")
      ++ rightUnits() returns the affine space of all right units of the
      ++ algebra, or \spad{"failed"} if there is none.
    leftUnit:() -> Union(%, "failed")
      ++ leftUnit() returns a left unit of the algebra
      ++ (not necessarily unique), or \spad{"failed"} if there is none.
    rightUnit:() -> Union(%, "failed")
      ++ rightUnit() returns a right unit of the algebra
      ++ (not necessarily unique), or \spad{"failed"} if there is none.
    unit:() -> Union(%, "failed")
      ++ unit() returns a unit of the algebra (necessarily unique),
      ++ or \spad{"failed"} if there is none.
    -- we not necessarily have a unit, hence we can't say anything
    -- about characteristic
    -- if R has CharacteristicZero then CharacteristicZero
    -- if R has CharacteristicNonZero then CharacteristicNonZero
    unitsKnown
      ++ unitsKnown means that \spadfun{recip} truly yields reciprocal
      ++ or \spad{"failed"} if not a unit,
      ++ similarly for \spadfun{leftRecip} and
      ++ \spadfun{rightRecip}. The reason is that we use left, respectively
      ++ right, minimal polynomials to decide this question.
  add
    --n := rank()
    --b := someBasis()
    --gamma : Vector Matrix R := structuralConstants b
    -- here is a problem: there seems to be a problem having local
    -- variables in the capsule of a category, furthermore
    -- see the commented code of conditionsForIdempotents, where
    -- we call structuralConstants, which also doesn't work
    -- at runtime, i.e. is not properly inherited, hence for
    -- the moment we put the code for
    -- conditionsForIdempotents, structuralConstants, unit, leftUnit,
    -- rightUnit into the domain constructor ALGSC
    V  ==> Vector
    M  ==> Matrix
    REC  ==> Record(particular: Union(V R,"failed"),basis: List V R)
    LSMP ==> LinearSystemMatrixPackage(R,V R,V R, M R)


    SUP ==>  SparseUnivariatePolynomial
    NNI ==>  NonNegativeInteger
    -- next 2 functions: use a general characteristicPolynomial
    leftCharacteristicPolynomial a ==
```

```
    n := rank()$%
    ma : Matrix R := leftRegularRepresentation(a,someBasis()$%)
    mb : Matrix SUP R := zero(n,n)
    for i in 1..n repeat
      for j in 1..n repeat
        mb(i,j):=
          i=j => monomial(ma(i,j),0)$SUP(R) - monomial(1,1)$SUP(R)
          monomial(ma(i,j),1)$SUP(R)
    determinant mb

  rightCharacteristicPolynomial a ==
    n := rank()$%
    ma : Matrix R := rightRegularRepresentation(a,someBasis()$%)
    mb : Matrix SUP R := zero(n,n)
    for i in 1..n repeat
      for j in 1..n repeat
        mb(i,j):=
          i=j => monomial(ma(i,j),0)$SUP(R) - monomial(1,1)$SUP(R)
          monomial(ma(i,j),1)$SUP(R)
    determinant mb

  leftTrace a ==
    t : R := 0
    ma : Matrix R := leftRegularRepresentation(a,someBasis()$%)
    for i in 1..rank()$% repeat
      t := t + elt(ma,i,i)
    t

  rightTrace a ==
    t : R := 0
    ma : Matrix R := rightRegularRepresentation(a,someBasis()$%)
    for i in 1..rank()$% repeat
      t := t + elt(ma,i,i)
    t

  leftNorm a == determinant leftRegularRepresentation(a,someBasis()$%)

  rightNorm a == determinant rightRegularRepresentation(a,someBasis()$%)

  antiAssociative?() ==
    b := someBasis()
    n := rank()
    for i in 1..n repeat
      for j in 1..n repeat
        for k in 1..n repeat
          not zero? ( (b.i*b.j)*b.k + b.i*(b.j*b.k) )  =>
```

```
          messagePrint("algebra is not anti-associative")$OutputForm
          return false
  messagePrint("algebra is anti-associative")$OutputForm
  true

jordanAdmissible?() ==
  b := someBasis()
  n := rank()
  recip(2 * 1$R) case "failed" =>
    messagePrint("this algebra is not Jordan admissible, " _
     "as 2 is not invertible in the ground ring")$OutputForm
    false
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     for l in 1..n repeat
       not zero? ( _
         antiCommutator(antiCommutator(b.i,b.j),_
                        antiCommutator(b.l,b.k)) + _
         antiCommutator(antiCommutator(b.l,b.j),_
                        antiCommutator(b.k,b.i)) + _
         antiCommutator(antiCommutator(b.k,b.j),_
                        antiCommutator(b.i,b.l))   _
                ) =>
           messagePrint(_
                    "this algebra is not Jordan admissible")$OutputForm
           return false
  messagePrint("this algebra is Jordan admissible")$OutputForm
  true

lieAdmissible?() ==
  n := rank()
  b := someBasis()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
      not zero? (commutator(commutator(b.i,b.j),b.k) _
              + commutator(commutator(b.j,b.k),b.i) _
              + commutator(commutator(b.k,b.i),b.j))   =>
        messagePrint("this algebra is not Lie admissible")$OutputForm
        return false
  messagePrint("this algebra is Lie admissible")$OutputForm
  true

-- conditionsForIdempotents b  ==
--   n := rank()
```

```
--   gamma : Vector Matrix R := structuralConstants b
--   listOfNumbers : List String :=  [STRINGIMAGE(q)$Lisp for q in 1..n]
--   symbolsForCoef : Vector Symbol :=
--     [concat("%", concat("x", i))::Symbol  for i in listOfNumbers]
--   conditions : List Polynomial R := []
--  for k in 1..n repeat
--    xk := symbolsForCoef.k
--    p : Polynomial R :=  monomial( - 1$Polynomial(R), [xk], [1] )
--    for i in 1..n repeat
--      for j in 1..n repeat
--        xi := symbolsForCoef.i
--        xj := symbolsForCoef.j
--        p := p + monomial(_
--          elt((gamma.k),i,j) :: Polynomial(R), [xi,xj], [1,1])
--    conditions := cons(p,conditions)
--  conditions

structuralConstants b ==
  --n := rank()
  -- be careful with the possibility that b is not a basis
  m : NonNegativeInteger := (maxIndex b) :: NonNegativeInteger
  sC : Vector Matrix R := [new(m,m,0$R) for k in 1..m]
  for i in 1..m repeat
    for j in 1..m repeat
      covec : Vector R := coordinates(b.i * b.j, b)
      for k in 1..m repeat
         setelt( sC.k, i, j, covec.k )
  sC

if R has IntegralDomain then

  leftRecip x ==
    zero? x => "failed"
    lu := leftUnit()
    lu case "failed" => "failed"
    b := someBasis()
    xx : % := (lu :: %)
    k  : PositiveInteger := 1
    cond : Matrix R := coordinates(xx,b) :: Matrix(R)
    listOfPowers : List % := [xx]
    while rank(cond) = k repeat
      k := k+1
      xx := xx*x
      listOfPowers := cons(xx,listOfPowers)
      cond := horizConcat(cond, coordinates(xx,b) :: Matrix(R) )
    vectorOfCoef : Vector R := (nullSpace(cond)$Matrix(R)).first
```

```
  invC := recip vectorOfCoef.1
  invC case "failed" => "failed"
  invCR : R :=  - (invC :: R)
  reduce(_+,[(invCR*vectorOfCoef.i)*power for i in _
   2..maxIndex vectorOfCoef for power in reverse listOfPowers])

rightRecip x ==
  zero? x => "failed"
  ru := rightUnit()
  ru case "failed" => "failed"
  b := someBasis()
  xx : % := (ru :: %)
  k  : PositiveInteger := 1
  cond : Matrix R := coordinates(xx,b) :: Matrix(R)
  listOfPowers : List % := [xx]
  while rank(cond) = k repeat
    k := k+1
    xx := x*xx
    listOfPowers := cons(xx,listOfPowers)
    cond := horizConcat(cond, coordinates(xx,b) :: Matrix(R) )
  vectorOfCoef : Vector R := (nullSpace(cond)$Matrix(R)).first
  invC := recip vectorOfCoef.1
  invC case "failed" => "failed"
  invCR : R :=  - (invC :: R)
  reduce(_+,[(invCR*vectorOfCoef.i)*power for i in _
   2..maxIndex vectorOfCoef for power in reverse listOfPowers])

recip x ==
  lrx := leftRecip x
  lrx case "failed" => "failed"
  rrx := rightRecip x
  rrx case "failed" => "failed"
  (lrx :: %) ^= (rrx :: %)  => "failed"
  lrx :: %

leftMinimalPolynomial x ==
  zero? x =>  monomial(1$R,1)$(SparseUnivariatePolynomial R)
  b := someBasis()
  xx : % := x
  k  : PositiveInteger := 1
  cond : Matrix R := coordinates(xx,b) :: Matrix(R)
  while rank(cond) = k repeat
    k := k+1
    xx := x*xx
    cond := horizConcat(cond, coordinates(xx,b) :: Matrix(R) )
  vectorOfCoef : Vector R := (nullSpace(cond)$Matrix(R)).first
```

```
      res : SparseUnivariatePolynomial R := 0
      for i in 1..k repeat
        res:=res+monomial(vectorOfCoef.i,i)$(SparseUnivariatePolynomial R)
      res

    rightMinimalPolynomial x ==
      zero? x =>  monomial(1$R,1)$(SparseUnivariatePolynomial R)
      b := someBasis()
      xx : % := x
      k  : PositiveInteger := 1
      cond : Matrix R := coordinates(xx,b) :: Matrix(R)
      while rank(cond) = k repeat
        k := k+1
        xx := xx*x
        cond := horizConcat(cond, coordinates(xx,b) :: Matrix(R) )
      vectorOfCoef : Vector R := (nullSpace(cond)$Matrix(R)).first
      res : SparseUnivariatePolynomial R := 0
      for i in 1..k repeat
        res:=res+monomial(vectorOfCoef.i,i)$(SparseUnivariatePolynomial R)
      res

    associatorDependence() ==
      n := rank()
      b := someBasis()
      cond : Matrix(R) := new(n**4,6,0$R)$Matrix(R)
      z : Integer := 0
      for i in 1..n repeat
       for j in 1..n repeat
        for k in 1..n repeat
         a123 : Vector R := coordinates(associator(b.i,b.j,b.k),b)
         a231 : Vector R := coordinates(associator(b.j,b.k,b.i),b)
         a312 : Vector R := coordinates(associator(b.k,b.i,b.j),b)
         a132 : Vector R := coordinates(associator(b.i,b.k,b.j),b)
         a321 : Vector R := coordinates(associator(b.k,b.j,b.i),b)
         a213 : Vector R := coordinates(associator(b.j,b.i,b.k),b)
         for r in 1..n repeat
          z:= z+1
          setelt(cond,z,1,elt(a123,r))
          setelt(cond,z,2,elt(a231,r))
          setelt(cond,z,3,elt(a312,r))
          setelt(cond,z,4,elt(a132,r))
          setelt(cond,z,5,elt(a321,r))
          setelt(cond,z,6,elt(a213,r))
      nullSpace(cond)

  jacobiIdentity?()  ==
```

```
  n := rank()
  b := someBasis()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
      not zero? ((b.i*b.j)*b.k + (b.j*b.k)*b.i + (b.k*b.i)*b.j) =>
        messagePrint("Jacobi identity does not hold")$OutputForm
        return false
  messagePrint("Jacobi identity holds")$OutputForm
  true

lieAlgebra?()  ==
  not antiCommutative?() =>
    messagePrint("this is not a Lie algebra")$OutputForm
    false
  not jacobiIdentity?() =>
    messagePrint("this is not a Lie algebra")$OutputForm
    false
  messagePrint("this is a Lie algebra")$OutputForm
  true

jordanAlgebra?()  ==
  b := someBasis()
  n := rank()
  recip(2 * 1$R) case "failed" =>
    messagePrint("this is not a Jordan algebra, as 2 is not " _
     "invertible in the ground ring")$OutputForm
    false
  not commutative?() =>
    messagePrint("this is not a Jordan algebra")$OutputForm
    false
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     for l in 1..n repeat
       not zero? (associator(b.i,b.j,b.l*b.k)+_
           associator(b.l,b.j,b.k*b.i)+associator(b.k,b.j,b.i*b.l)) =>
         messagePrint("not a Jordan algebra")$OutputForm
         return false
  messagePrint("this is a Jordan algebra")$OutputForm
  true

noncommutativeJordanAlgebra?() ==
  b := someBasis()
  n := rank()
  recip(2 * 1$R) case "failed" =>
```

```
      messagePrint("this is not a noncommutative Jordan algebra,_
  as 2 is not invertible in the ground ring")$OutputForm
      false
     not flexible?()$% =>
      messagePrint("this is not a noncommutative Jordan algebra,_
  as it is not flexible")$OutputForm
      false
     not jordanAdmissible?()$% =>
      messagePrint("this is not a noncommutative Jordan algebra,_
  as it is not Jordan admissible")$OutputForm
      false
     messagePrint("this is a noncommutative Jordan algebra")$OutputForm
     true

   antiCommutative?() ==
     b := someBasis()
     n := rank()
     for i in 1..n repeat
       for j in i..n repeat
         not zero? (i=j => b.i*b.i; b.i*b.j + b.j*b.i) =>
           messagePrint("algebra is not anti-commutative")$OutputForm
           return false
     messagePrint("algebra is anti-commutative")$OutputForm
     true

   commutative?() ==
     b := someBasis()
     n := rank()
     for i in 1..n repeat
      for j in i+1..n repeat
        not zero? commutator(b.i,b.j) =>
          messagePrint("algebra is not commutative")$OutputForm
          return false
     messagePrint("algebra is commutative")$OutputForm
     true

   associative?() ==
     b := someBasis()
     n := rank()
     for i in 1..n repeat
      for j in 1..n repeat
       for k in 1..n repeat
        not zero? associator(b.i,b.j,b.k) =>
          messagePrint("algebra is not associative")$OutputForm
          return false
     messagePrint("algebra is associative")$OutputForm
```

```
  true

leftAlternative?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     not zero? (associator(b.i,b.j,b.k) + associator(b.j,b.i,b.k)) =>
       messagePrint("algebra is not left alternative")$OutputForm
       return false
  messagePrint("algebra satisfies 2*associator(a,a,b) = 0")$OutputForm
  true

rightAlternative?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     not zero? (associator(b.i,b.j,b.k) + associator(b.i,b.k,b.j)) =>
       messagePrint("algebra is not right alternative")$OutputForm
       return false
  messagePrint("algebra satisfies 2*associator(a,b,b) = 0")$OutputForm
  true

flexible?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     not zero? (associator(b.i,b.j,b.k) + associator(b.k,b.j,b.i)) =>
       messagePrint("algebra is not flexible")$OutputForm
       return false
  messagePrint("algebra satisfies 2*associator(a,b,a) = 0")$OutputForm
  true

alternative?() ==
  b := someBasis()
  n := rank()
  for i in 1..n repeat
   for j in 1..n repeat
    for k in 1..n repeat
     not zero? (associator(b.i,b.j,b.k) + associator(b.j,b.i,b.k)) =>
       messagePrint("algebra is not alternative")$OutputForm
```

```
        return false
      not zero? (associator(b.i,b.j,b.k) + associator(b.i,b.k,b.j)) =>
        messagePrint("algebra is not alternative")$OutputForm
        return false
    messagePrint("algebra satisfies 2*associator(a,b,b) = 0 " _
                 "=  2*associator(a,a,b) = 0")$OutputForm
    true

  leftDiscriminant v == determinant leftTraceMatrix v
  rightDiscriminant v == determinant rightTraceMatrix v

  coordinates(v:Vector %, b:Vector %) ==
    m := new(#v, #b, 0)$Matrix(R)
    for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
      setRow_!(m, j, coordinates(qelt(v, i), b))
    m

  represents(v, b) ==
    m := minIndex v - 1
    reduce(_+,[v(i+m) * b(i+m) for i in 1..maxIndex b])

  leftTraceMatrix v ==
    matrix [[leftTrace(v.i*v.j) for j in minIndex v..maxIndex v]$List(R)
             for i in minIndex v .. maxIndex v]$List(List R)

  rightTraceMatrix v ==
    matrix [[rightTrace(v.i*v.j) for j in minIndex v..maxIndex v]$List(R)
             for i in minIndex v .. maxIndex v]$List(List R)

  leftRegularRepresentation(x, b) ==
    m := minIndex b - 1
    matrix
     [parts coordinates(x*b(i+m),b) for i in 1..rank()]$List(List R)

  rightRegularRepresentation(x, b) ==
    m := minIndex b - 1
    matrix
     [parts coordinates(b(i+m)*x,b) for i in 1..rank()]$List(List R)
```

⟨*FINAALG.dotabb*⟩≡

```
"FINAALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FINAALG"];
"FINAALG" -> "NAALG"
```

⟨*FINAALG.dotfull*⟩≡

```
  "FiniteRankNonAssociativeAlgebra(a:CommutativeRing)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FINAALG"];
  "FiniteRankNonAssociativeAlgebra(a:CommutativeRing)" ->
      "NonAssociativeAlgebra(a:CommutativeRing)"
```

⟨*FINAALG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FiniteRankNonAssociativeAlgebra(a:CommutativeRing)" [color=lightblue];
"FiniteRankNonAssociativeAlgebra(a:CommutativeRing)" ->
    "NonAssociativeAlgebra(a:CommutativeRing)"

"NonAssociativeAlgebra(a:CommutativeRing)" [color=lightblue];
"NonAssociativeAlgebra(a:CommutativeRing)" -> "NonAssociativeRng()"
"NonAssociativeAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"NonAssociativeRng()" [color=lightblue];
"NonAssociativeRng()" -> "ABELGRP..."
"NonAssociativeRng()" -> "Monad()"

"Monad()" [color=lightblue];
"Monad()" -> "SETCAT..."
"Monad()" -> "REPSQ..."

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

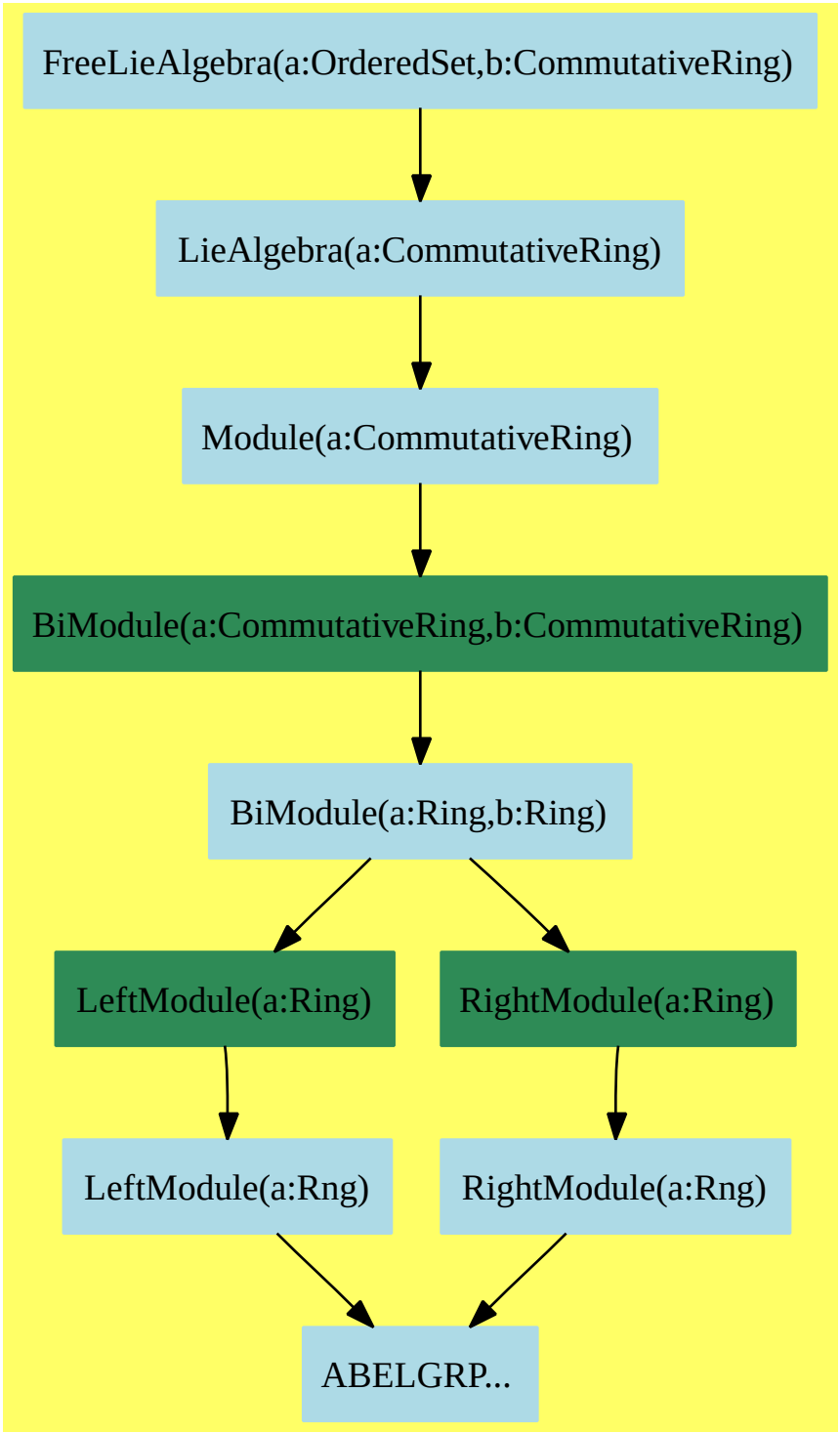
"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"REPSQ..." [color="#00EE00"];
```

```
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

12.4 FreeLieAlgebra (FLALG)



See:

⇐ "LieAlgebra" (LIECAT) 11.4 on page 787

Exports:

0	coef	coerce	construct	degree
eval	hash	latex	LiePoly	lquo
mirror	rquo	sample	subtractIfCan	trunc
varList	zero?	?~=?	?*?	?/?
?+?	?-?	-?	?=?	

Attributes Exported:

- **NullSquare** means that $[x, x] = 0$ holds. See `LieAlgebra`.
- **JacobiIdentity** means that $[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$ holds. See `LieAlgebra`.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coef : (XRecursivePolynomial(VarSet,R),%) -> R
coerce : VarSet -> %
coerce : % -> XRecursivePolynomial(VarSet,R)
coerce : % -> XDistributedPolynomial(VarSet,R)
degree : % -> NonNegativeInteger
eval : (%,List VarSet,List %) -> %
eval : (%,VarSet,%) -> %
LiePoly : LyndonWord VarSet -> %
lquo : (XRecursivePolynomial(VarSet,R),%) -> XRecursivePolynomial(VarSet,R)
mirror : % -> %
rquo : (XRecursivePolynomial(VarSet,R),%) -> XRecursivePolynomial(VarSet,R)
trunc : (%,NonNegativeInteger) -> %
varList : % -> List VarSet
```

These exports come from (p787) LieAlgebra(CommutativeRing):

```
0 : () -> %
coerce : % -> OutputForm
construct : (%,%) -> %
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?/? : (%,R) -> % if R has FIELD
?~=? : (%,%) -> Boolean
```

```
?*? : (NonNegativeInteger,%) -> %
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

⟨*category FLALG FreeLieAlgebra*⟩≡

```
)abbrev category FLALG FreeLieAlgebra
++ Author: Michel Petitot (petitot@lifl.fr)
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of free Lie algebras.
++ It is used by domains of non-commutative algebra:
++ \spadtype{LiePolynomial} and
++ \spadtype{XPBWPolynomial}. \newline Author: Michel Petitot (petitot@lifl.fr)

FreeLieAlgebra(VarSet:OrderedSet, R:CommutativeRing) :Category == _
 CatDef where
   XRPOLY  ==> XRecursivePolynomial(VarSet,R)
   XDPOLY  ==> XDistributedPolynomial(VarSet,R)
   RN      ==> Fraction Integer
   LWORD   ==> LyndonWord(VarSet)

   CatDef ==  Join(LieAlgebra(R)) with
     coef      : (XRPOLY , $) -> R
       ++ \axiom{coef(x,y)} returns the scalar product of \axiom{x} by
       ++ \axiom{y}, the set of words being regarded as an orthogonal basis.
     coerce    : VarSet -> $
       ++ \axiom{coerce(x)} returns \axiom{x} as a Lie polynomial.
     coerce    : $ -> XDPOLY
       ++ \axiom{coerce(x)} returns \axiom{x} as distributed polynomial.
     coerce    : $ -> XRPOLY
       ++ \axiom{coerce(x)} returns \axiom{x} as a recursive polynomial.
     degree    : $ -> NonNegativeInteger
       ++ \axiom{degree(x)} returns the greatest length of a word in the
```

```
      ++ support of \axiom{x}.
    --if R has Module(RN) then
    --  Hausdorff : ($,$,PositiveInteger) -> $
    lquo      : (XRPOLY , $) -> XRPOLY
      ++ \axiom{lquo(x,y)} returns the left simplification of \axiom{x}
      ++ by \axiom{y}.
    rquo      : (XRPOLY , $) -> XRPOLY
      ++ \axiom{rquo(x,y)} returns the right simplification of \axiom{x}
      ++ by \axiom{y}.
    LiePoly   : LWORD -> $
      ++ \axiom{LiePoly(l)} returns the bracketed form of \axiom{l} as
      ++ a Lie polynomial.
    mirror    : $ -> $
      ++ \axiom{mirror(x)} returns \axiom{Sum(r_i mirror(w_i))}
      ++ if \axiom{x} is \axiom{Sum(r_i w_i)}.
    trunc     : ($, NonNegativeInteger) -> $
      ++ \axiom{trunc(p,n)} returns the polynomial \axiom{p}
      ++ truncated at order \axiom{n}.
    varList   : $ -> List VarSet
      ++ \axiom{varList(x)} returns the list of distinct entries
      ++ of \axiom{x}.
    eval      : ($, VarSet, $) -> $
      ++ \axiom{eval(p, x, v)} replaces \axiom{x} by \axiom{v}
      ++ in \axiom{p}.
    eval      : ($, List VarSet, List $) -> $
      ++ \axiom{eval(p, [x1,...,xn], [v1,...,vn])} replaces \axiom{xi}
      ++ by \axiom{vi} in \axiom{p}.
```

⟨*FLALG.dotabb*⟩≡

```
"FLALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FLALG"];
"FLALG" -> "LIECAT"
```

⟨*FLALG.dotfull*⟩≡

```
"FreeLieAlgebra(a:OrderedSet,b:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FLALG"];
"FreeLieAlgebra(a:OrderedSet,b:CommutativeRing)" ->
   "LieAlgebra(a:CommutativeRing)"
```

⟨*FLALG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FreeLieAlgebra(a:OrderedSet,b:CommutativeRing)" [color=lightblue];
"FreeLieAlgebra(a:OrderedSet,b:CommutativeRing)" ->
   "LieAlgebra(a:CommutativeRing)"

"LieAlgebra(a:CommutativeRing)" [color=lightblue];
"LieAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

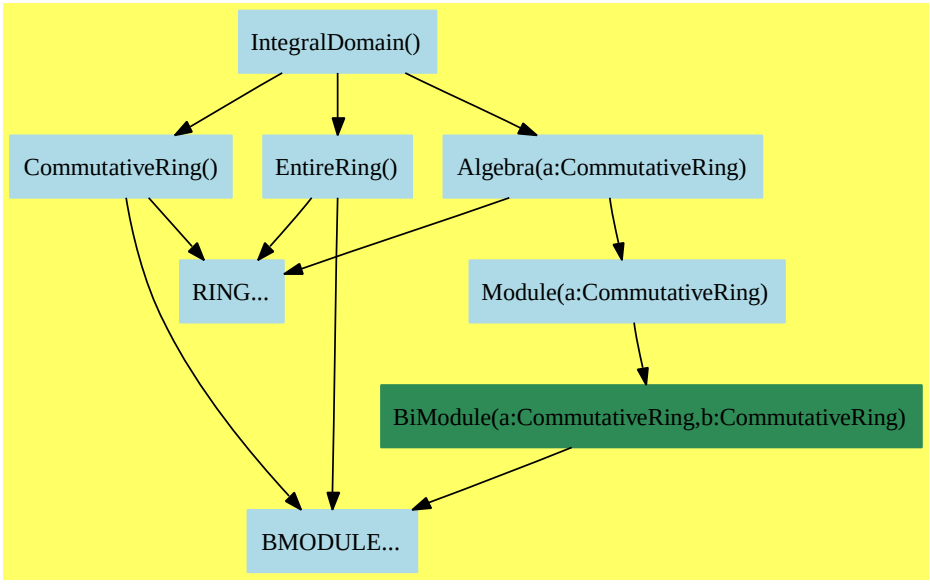
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"ABELGRP..." [color=lightblue];
}
```

12.5 IntegralDomain (INTDOM)



All Commutative Rings are Integral Domains.
⇐ "CommutativeRing" (COMRING) 10.4 on page 685 Integral Domains are a subset of Unique Factorization domains.
⇒ "UniqueFactorizationDomain" (UFD) 14.5 on page 969.

See:

⇒ "FortranMachineTypeCategory" (FMTC) 13.2 on page 913
⇒ "FunctionSpace" (FS) 17.5 on page 1095
⇒ "GcdDomain" (GCDDOM) 13.4 on page 932
⇒ "OrderedIntegralDomain" (OINTDOM) 13.5 on page 937
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.12 on page 1204
⇐ "Algebra" (ALGEBRA) 11.1 on page 771
⇐ "CommutativeRing" (COMRING) 10.4 on page 685
⇐ "EntireRing" (ENTIRER) 10.6 on page 694

Exports:

0	1	associates?	characteristic	coerce
exquo	hash	latex	one?	recip
sample	subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?~=?	?*?	?**?	?^?
?+?	?-?	-?	?=?	

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.

- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

Attributes Used:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.

These are directly exported but not implemented:

```
exquo : (%,%) -> Union(%,"failed")
```

These are implemented by this category:

```
associates? : (%,%) -> Boolean
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
```

These exports come from (p685) CommutativeRing():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
```

```
?-? : (%,%) -> %
-? : % -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

TPDHERE: Should we construct this coercion?

These exports come from (p771) Algebra(a:IntegralDomain):

```
coerce : % -> %
```

⟨*category INTDOM IntegralDomain*⟩≡

```
)abbrev category INTDOM IntegralDomain
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References: Davenport & Trager I
++ Description:
++ The category of commutative integral domains, i.e. commutative
++ rings with no zero divisors.
++
++ Conditional attributes:
++   canonicalUnitNormal\tab{20}the canonical field is the same for
++   all associates canonicalsClosed\tab{20}the product of two
++   canonicals is itself canonical

IntegralDomain(): Category ==
  Join(CommutativeRing, Algebra(%), EntireRing) with
    "exquo": (%,%) -> Union(%,"failed")
        ++ exquo(a,b) either returns an element c such that
        ++ \spad{c*b=a} or "failed" if no such element can be found.
    unitNormal: % -> Record(unit:%,canonical:%,associate:%)
        ++ unitNormal(x) tries to choose a canonical element
        ++ from the associate class of x.
        ++ The attribute canonicalUnitNormal, if asserted, means that
        ++ the "canonical" element is the same across all associates of x
        ++ if \spad{unitNormal(x) = [u,c,a]} then
        ++ \spad{u*c = x}, \spad{a*u = 1}.
    unitCanonical: % -> %
        ++ \spad{unitCanonical(x)} returns \spad{unitNormal(x).canonical}.
    associates?: (%,%) -> Boolean
```

```
        ++ associates?(x,y) tests whether x and y are associates, i.e.
        ++ differ by a unit factor.
     unit?: % -> Boolean
        ++ unit?(x) tests whether x is a unit, i.e. is invertible.
  add
       x,y: %

       UCA ==> Record(unit:%,canonical:%,associate:%)
       if not (% has Field) then
         unitNormal(x) == [1$%,x,1$%]$UCA -- the non-canonical definition
       unitCanonical(x) == unitNormal(x).canonical -- always true
       recip(x) == if zero? x then "failed" else _exquo(1$%,x)
       unit?(x) == (recip x case "failed" => false; true)
       if % has canonicalUnitNormal then
          associates?(x,y) ==
            (unitNormal x).canonical = (unitNormal y).canonical
        else
          associates?(x,y) ==
            zero? x => zero? y
            zero? y => false
            x exquo y case "failed" => false
            y exquo x case "failed" => false
            true
```

⟨*INTDOM.dotabb*⟩≡

```
"INTDOM"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INTDOM"];
"INTDOM" -> "COMRING"
"INTDOM" -> "ALGEBRA"
"INTDOM" -> "ENTIRER"
```

⟨*INTDOM.dotfull*⟩≡

```
"IntegralDomain()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INTDOM"];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"
```

⟨*INTDOM.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

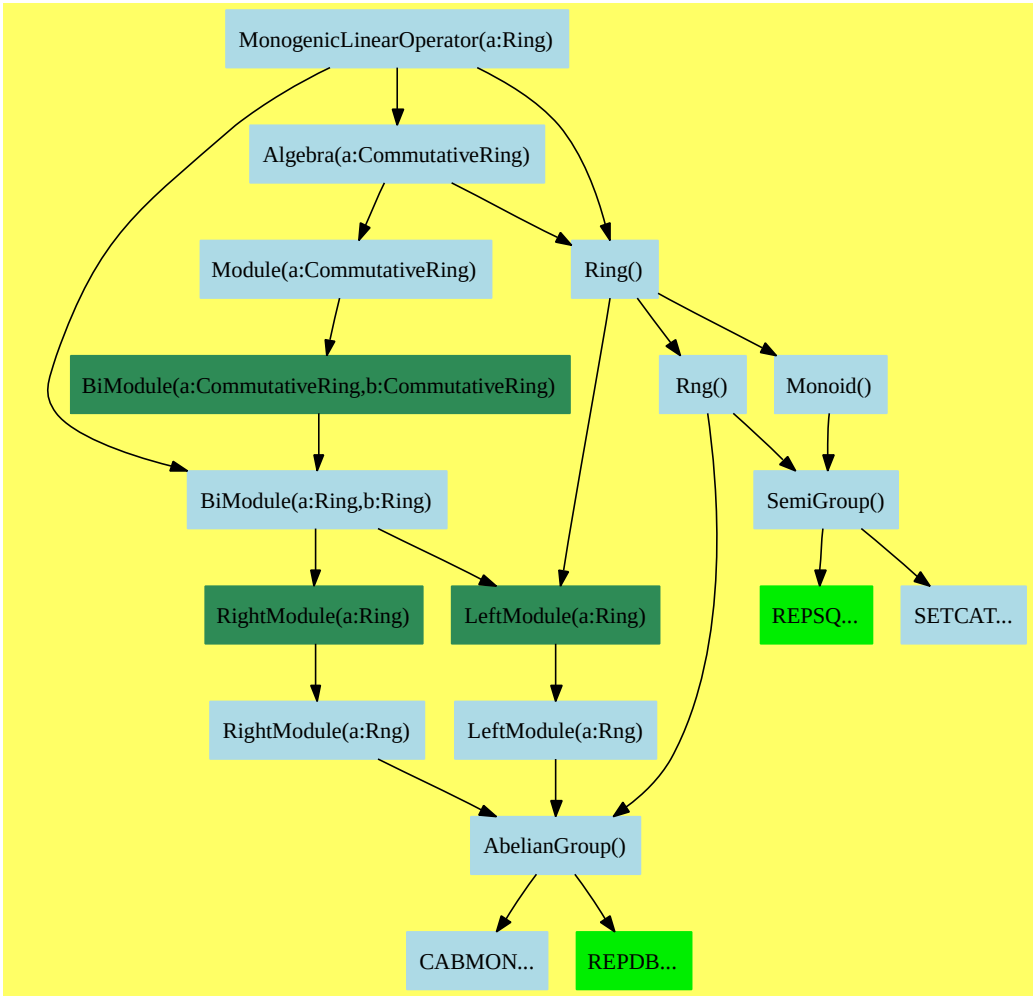
"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

12.6 MonogenicLinearOperator (MLO)



See:

⇐ “Algebra” (ALGEBRA) 11.1 on page 771
⇐ “BiModule” (BMODULE) 9.1 on page 592
⇐ “Ring” (RING) 9.8 on page 628

Exports:

0	1	characteristic	coefficient
coerce	degree	hash	latex
leadingCoefficient	minimumDegree	monomial	one?
recip	reductum	sample	subtractIfCan
zero?	?*?	?**?	?+?
?-?	-?	?=?	?^?
?~=?			

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coefficient : (%,NonNegativeInteger) -> R
degree : % -> NonNegativeInteger
leadingCoefficient : % -> R
minimumDegree : % -> NonNegativeInteger
monomial : (R,NonNegativeInteger) -> %
reductum : % -> %
```

These exports come from (p628) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> OutputForm
coerce : Integer -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

```
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

These exports come from (p592) BiModule(R:Ring,R:Ring):

```
?*? : (R,%) -> %
?*? : (%,R) -> %
```

These exports come from (p771) Algebra(R:CommutativeRing):

```
coerce : R -> % if R has COMRING
```

⟨*category MLO MonogenicLinearOperator*⟩≡

```
)abbrev category MLO MonogenicLinearOperator
++ Author: Stephen M. Watt
++ Date Created: 1986
++ Date Last Updated: May 30, 1991
++ Basic Operations:
++ Related Domains:  NonCommutativeOperatorDivision
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++   This is the category of linear operator rings with one generator.
++   The generator is not named by the category but can always be
++   constructed as \spad{monomial(1,1)}.
++
++   For convenience, call the generator \spad{G}.
++   Then each value is equal to
++       \spad{sum(a(i)*G**i, i = 0..n)}
++   for some unique \spad{n} and \spad{a(i)} in \spad{R}.
++
++   Note that multiplication is not necessarily commutative.
++   In fact,  if \spad{a} is in \spad{R}, it is quite normal
++   to have \spad{a*G \^= G*a}.

MonogenicLinearOperator(R): Category == Defn where
    E ==> NonNegativeInteger
    R: Ring
    Defn == Join(Ring, BiModule(R,R)) with
        if R has CommutativeRing then Algebra(R)
        degree: $ -> E
            ++ degree(l) is \spad{n} if
```

```
        ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
    minimumDegree: $ -> E
        ++ minimumDegree(l) is the smallest \spad{k} such that
        ++ \spad{a(k) \^= 0} if
        ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
    leadingCoefficient: $ -> R
        ++ leadingCoefficient(l) is \spad{a(n)} if
        ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
    reductum: $ -> $
        ++ reductum(l) is \spad{l - monomial(a(n),n)} if
        ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
    coefficient: ($, E) -> R
        ++ coefficient(l,k) is \spad{a(k)} if
        ++   \spad{l = sum(monomial(a(i),i), i = 0..n)}.
    monomial: (R, E) -> $
        ++ monomial(c,k) produces c times the k-th power of
        ++ the generating operator, \spad{monomial(1,1)}.
```

⟨*MLO.dotabb*⟩≡

```
"MLO"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MLO"];
"MLO" -> "BMODULE"
"MLO" -> "RING"
"MLO" -> "ALGEBRA"
```

⟨*MLO.dotfull*⟩≡

```
"MonogenicLinearOperator(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MLO"];
"MonogenicLinearOperator(a:Ring)" -> "Ring()"
"MonogenicLinearOperator(a:Ring)" -> "BiModule(a:Ring,b:Ring)"
"MonogenicLinearOperator(a:Ring)" -> "Algebra(a:CommutativeRing)"
```

⟨*MLO.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"MonogenicLinearOperator(a:Ring)" [color=lightblue];
"MonogenicLinearOperator(a:Ring)" -> "Ring()"
"MonogenicLinearOperator(a:Ring)" -> "BiModule(a:Ring,b:Ring)"
"MonogenicLinearOperator(a:Ring)" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "Ring()"
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "AbelianGroup()"
"Rng()" -> "SemiGroup()"

"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "AbelianGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"
```

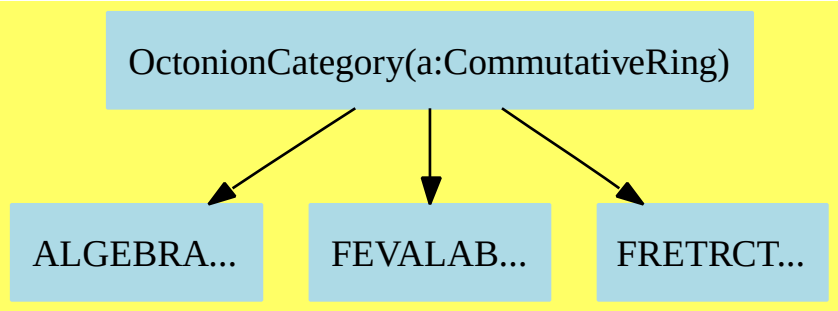
```
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "AbelianGroup()"

"AbelianGroup()" [color=lightblue];
"AbelianGroup()" -> "CABMON..."
"AbelianGroup()" -> "REPDB..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"REPDB..." [color="#00EE00"];
"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"CABMON..." [color=lightblue];
}
```

12.7 OctonionCategory (OC)



See:

⇐ "Algebra" (ALGEBRA) 11.1 on page 771
⇐ "FullyEvalableOver" (FEVALAB) 4.7 on page 121
⇐ "FullyRetractableTo" (FRETRCT) 3.6 on page 71

Exports:

0	1	abs	characteristic	charthRoot
coerce	conjugate	convert	eval	hash
imagE	imagI	imagJ	imagK	imagi
imagj	imagk	index	inv	latex
lookup	map	max	min	norm
octon	one?	random	rational	rational?
rationalIfCan	real	recip	retract	retractIfCan
sample	size	subtractIfCan	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?~=?	?<?	?<=?	?>?
?>=?	?.?			

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
imagi : % -> R
imagj : % -> R
imagk : % -> R
imagE : % -> R
imagI : % -> R
```

```
imagJ : % -> R
imagK : % -> R
octon : (R,R,R,R,R,R,R,R) -> %
real : % -> R
```

These are implemented by this category:

```
abs : % -> R if R has RNS
characteristic : () -> NonNegativeInteger
coerce : R -> %
coerce : Integer -> %
coerce : % -> OutputForm
conjugate : % -> %
convert : % -> InputForm if R has KONVERT INFORM
inv : % -> % if R has FIELD
map : ((R -> R),%) -> %
norm : % -> R
rational : % -> Fraction Integer if R has INS
rational? : % -> Boolean if R has INS
rationalIfCan : % -> Union(Fraction Integer,"failed") if R has INS
retract : % -> R
retractIfCan : % -> Union(R,"failed")
zero? : % -> Boolean
?<? : (%,%) -> Boolean if R has ORDSET
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
-? : % -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
```

These exports come from (p771) Algebra(R:CommutativeRing):

```
0 : () -> %
1 : () -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?*? : (%,R) -> %
```

These exports come from (p71) FullyRetractableTo(R:CommutativeRing):

```
coerce : Fraction Integer -> % if R has RETRACT FRAC INT
retract : % -> Fraction Integer if R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
```

These exports come from (p121) FullyEvalableOver(R:CommutativeRing):

```
eval : (%,Equation R) -> % if R has EVALAB R
eval : (%,List Symbol,List R) -> % if R has IEVALAB(SYMBOL,R)
eval : (%,List Equation R) -> % if R has EVALAB R
eval : (%,R,R) -> % if R has EVALAB R
eval : (%,List R,List R) -> % if R has EVALAB R
eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
?.? : (%,R) -> % if R has ELTAB(R,R)
```

These exports come from (p129) Finite():

```
index : PositiveInteger -> % if R has FINITE
lookup : % -> PositiveInteger if R has FINITE
random : () -> % if R has FINITE
size : () -> NonNegativeInteger if R has FINITE
```

These exports come from (p168) OrderedSet():

```
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p677) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed") if R has CHARNZ
```

⟨*category OC OctonionCategory*⟩≡

```
)abbrev category OC OctonionCategory
++ Author: R. Wisbauer, J. Grabmeier
++ Date Created: 05 September 1990
++ Date Last Updated: 19 September 1990
++ Basic Operations: _+, _*, octon, real, imagi, imagj, imagk,
++  imagE, imagI, imagJ, imagK
++ Related Constructors: QuaternionCategory
++ Also See:
++ AMS Classifications:
++ Keywords: octonion, non-associative algebra, Cayley-Dixon
```

```
++ References: e.g. I.L Kantor, A.S. Solodovnikov:
++  Hypercomplex Numbers, Springer Verlag Heidelberg, 1989,
++  ISBN 0-387-96980-2
++ Description:
++  OctonionCategory gives the categorial frame for the
++  octonions, and eight-dimensional non-associative algebra,
++  doubling the the quaternions in the same way as doubling
++  the Complex numbers to get the quaternions.
-- Examples: octonion.input

OctonionCategory(R: CommutativeRing): Category ==
  -- we are cheating a little bit, algebras in \Language{}
  -- are mainly considered to be associative, but that's not
  -- an attribute and we can't guarantee that there is no piece
  -- of code which implicitly
  -- uses this. In a later version we shall properly combine
  -- all this code in the context of general, non-associative
  -- algebras, which are meanwhile implemented in \Language{}
  Join(Algebra R, FullyRetractableTo R, FullyEvalableOver R) with
     conjugate: % -> %
       ++ conjugate(o) negates the imaginary parts i,j,k,E,I,J,K of octonian o.
     real:    % -> R
       ++ real(o) extracts real part of octonion o.
     imagi:   % -> R
       ++ imagi(o) extracts the i part of octonion o.
     imagj:   % -> R
       ++ imagj(o) extracts the j part of octonion o.
     imagk:   % -> R
       ++ imagk(o) extracts the k part of octonion o.
     imagE:   % -> R
       ++ imagE(o) extracts the imaginary E part of octonion o.
     imagI:   % -> R
       ++ imagI(o) extracts the imaginary I part of octonion o.
     imagJ:   % -> R
       ++ imagJ(o) extracts the imaginary J part of octonion o.
     imagK:   % -> R
       ++ imagK(o) extracts the imaginary K part of octonion o.
     norm:    % -> R
       ++ norm(o) returns the norm of an octonion, equal to
       ++ the sum of the squares
       ++ of its coefficients.
     octon: (R,R,R,R,R,R,R,R) -> %
       ++ octon(re,ri,rj,rk,rE,rI,rJ,rK) constructs an octonion
       ++ from scalars.
     if R has Finite then Finite
     if R has OrderedSet then OrderedSet
```

```
    if R has ConvertibleTo InputForm then ConvertibleTo InputForm
    if R has CharacteristicZero then CharacteristicZero
    if R has CharacteristicNonZero then CharacteristicNonZero
    if R has RealNumberSystem then
      abs:   % -> R
        ++ abs(o) computes the absolute value of an octonion, equal to
        ++ the square root of the \spadfunFrom{norm}{Octonion}.
    if R has IntegerNumberSystem then
      rational?    : % -> Boolean
        ++ rational?(o) tests if o is rational, i.e. that all seven
        ++ imaginary parts are 0.
      rational     : % -> Fraction Integer
        ++ rational(o) returns the real part if all seven
        ++ imaginary parts are 0.
        ++ Error: if o is not rational.
      rationalIfCan: % -> Union(Fraction Integer, "failed")
        ++ rationalIfCan(o) returns the real part if
        ++ all seven imaginary parts are 0, and "failed" otherwise.
    if R has Field then
      inv : % -> %
        ++ inv(o) returns the inverse of o if it exists.
  add
    characteristic() ==
      characteristic()$R

    conjugate x ==
      octon(real x, - imagi x, - imagj x, - imagk x, - imagE x,_
      - imagI x, - imagJ x, - imagK x)

    map(fn, x)       ==
      octon(fn real x,fn imagi x,fn imagj x,fn imagk x, fn imagE x,_
      fn imagI x, fn imagJ x,fn imagK x)

    norm x ==
      real x * real x + imagi x * imagi x + _
      imagj x * imagj x + imagk x * imagk x + _
      imagE x * imagE x + imagI x * imagI x + _
      imagJ x * imagJ x + imagK x * imagK x

    x = y            ==
      (real x = real y) and (imagi x = imagi y) and _
      (imagj x = imagj y) and (imagk x = imagk y) and _
      (imagE x = imagE y) and (imagI x = imagI y) and _
      (imagJ x = imagJ y) and (imagK x = imagK y)

    x + y            ==
```

```
    octon(real x + real y, imagi x + imagi y,_
    imagj x + imagj y, imagk x + imagk y,_
    imagE x + imagE y, imagI x + imagI y,_
    imagJ x + imagJ y, imagK x + imagK y)

  - x              ==
    octon(- real x, - imagi x, - imagj x, - imagk x,_
    - imagE x, - imagI x, - imagJ x, - imagK x)

  r:R * x:%        ==
    octon(r * real x, r * imagi x, r * imagj x, r * imagk x,_
    r * imagE x, r * imagI x, r * imagJ x, r * imagK x)

  n:Integer * x:%  ==
    octon(n * real x, n * imagi x, n * imagj x, n * imagk x,_
    n * imagE x, n * imagI x, n * imagJ x, n * imagK x)

  coerce(r:R)      ==
    octon(r,0$R,0$R,0$R,0$R,0$R,0$R,0$R)

  coerce(n:Integer)      ==
    octon(n :: R,0$R,0$R,0$R,0$R,0$R,0$R,0$R)

  zero? x ==
    zero? real x and zero? imagi x and _
    zero? imagj x and zero? imagk x and _
    zero? imagE x and zero? imagI x and _
    zero? imagJ x and zero? imagK x

  retract(x):R ==
    not (zero? imagi x and zero? imagj x and zero? imagk x and _
    zero? imagE x and zero? imagI x and zero? imagJ x and zero? imagK x)=>
      error "Cannot retract octonion."
    real x

  retractIfCan(x):Union(R,"failed") ==
    not (zero? imagi x and zero? imagj x and zero? imagk x and _
    zero? imagE x and zero? imagI x and zero? imagJ x and zero? imagK x)=>
      "failed"
    real x

  coerce(x:%):OutputForm ==
      part,z : OutputForm
      y : %
      zero? x => (0$R) :: OutputForm
      not zero?(real x) =>
```

```
          y := octon(0$R,imagi(x),imagj(x),imagk(x),imagE(x),
            imagI(x),imagJ(x),imagK(x))
          zero? y => real(x) :: OutputForm
          (real(x) :: OutputForm) + (y :: OutputForm)
        -- we know that the real part is 0
        not zero?(imagi(x)) =>
          y := octon(0$R,0$R,imagj(x),imagk(x),imagE(x),
            imagI(x),imagJ(x),imagK(x))
          z :=
            part := "i"::Symbol::OutputForm
--            one? imagi(x) => part
            (imagi(x) = 1) => part
            (imagi(x) :: OutputForm) * part
          zero? y => z
          z + (y :: OutputForm)
        -- we know that the real part and i part are 0
        not zero?(imagj(x)) =>
          y := octon(0$R,0$R,0$R,imagk(x),imagE(x),
            imagI(x),imagJ(x),imagK(x))
          z :=
            part := "j"::Symbol::OutputForm
--            one? imagj(x) => part
            (imagj(x) = 1) => part
            (imagj(x) :: OutputForm) * part
          zero? y => z
          z + (y :: OutputForm)
        -- we know that the real part and i and j parts are 0
        not zero?(imagk(x)) =>
          y := octon(0$R,0$R,0$R,0$R,imagE(x),
            imagI(x),imagJ(x),imagK(x))
          z :=
            part := "k"::Symbol::OutputForm
--            one? imagk(x) => part
            (imagk(x) = 1) => part
            (imagk(x) :: OutputForm) * part
          zero? y => z
          z + (y :: OutputForm)
        -- we know that the real part,i,j,k parts are 0
        not zero?(imagE(x)) =>
          y := octon(0$R,0$R,0$R,0$R,0$R,
            imagI(x),imagJ(x),imagK(x))
          z :=
            part := "E"::Symbol::OutputForm
--            one? imagE(x) => part
            (imagE(x) = 1) => part
            (imagE(x) :: OutputForm) * part
```

```
            zero? y => z
            z + (y :: OutputForm)
          -- we know that the real part,i,j,k,E parts are 0
          not zero?(imagI(x)) =>
            y := octon(0$R,0$R,0$R,0$R,0$R,0$R,imagJ(x),imagK(x))
            z :=
              part := "I"::Symbol::OutputForm
--              one? imagI(x) => part
              (imagI(x) = 1) => part
              (imagI(x) :: OutputForm) * part
            zero? y => z
            z + (y :: OutputForm)
          -- we know that the real part,i,j,k,E,I parts are 0
          not zero?(imagJ(x)) =>
            y := octon(0$R,0$R,0$R,0$R,0$R,0$R,0$R,imagK(x))
            z :=
              part := "J"::Symbol::OutputForm
--              one? imagJ(x) => part
              (imagJ(x) = 1) => part
              (imagJ(x) :: OutputForm) * part
            zero? y => z
            z + (y :: OutputForm)
          -- we know that the real part,i,j,k,E,I,J parts are 0
          part := "K"::Symbol::OutputForm
--          one? imagK(x) => part
          (imagK(x) = 1) => part
          (imagK(x) :: OutputForm) * part

      if R has Field then
        inv x ==
          (norm x) = 0 => error "This octonion is not invertible."
          (inv norm x) * conjugate x

      if R has ConvertibleTo InputForm then
        convert(x:%):InputForm ==
          l : List InputForm := [convert("octon" :: Symbol),
            convert(real x)$R, convert(imagi x)$R, convert(imagj x)$R,_
              convert(imagk x)$R, convert(imagE x)$R,_
              convert(imagI x)$R, convert(imagJ x)$R,_
              convert(imagK x)$R]
          convert(l)$InputForm

      if R has OrderedSet then
        x < y ==
          real x = real y =>
           imagi x = imagi y =>
```

```
            imagj x = imagj y =>
             imagk x = imagk y =>
              imagE x = imagE y =>
               imagI x = imagI y =>
                imagJ x = imagJ y =>
                 imagK x < imagK y
                imagJ x < imagJ y
               imagI x < imagI y
              imagE x < imagE y
             imagk x < imagk y
            imagj x < imagj y
           imagi x < imagi y
          real x < real y

      if R has RealNumberSystem then
        abs x == sqrt norm x

      if R has IntegerNumberSystem then
        rational? x ==
          (zero? imagi x) and (zero? imagj x) and (zero? imagk x) and _
          (zero? imagE x) and (zero? imagI x) and (zero? imagJ x) and _
          (zero? imagK x)

        rational  x ==
          rational? x => rational real x
          error "Not a rational number"

        rationalIfCan x ==
          rational? x => rational real x
          "failed"
```

⟨*OC.dotabb*⟩≡

```
"OC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OC"];
"OC" -> "ALGEBRA"
"OC" -> "FEVALAB"
"OC" -> "FRETRCT"
```

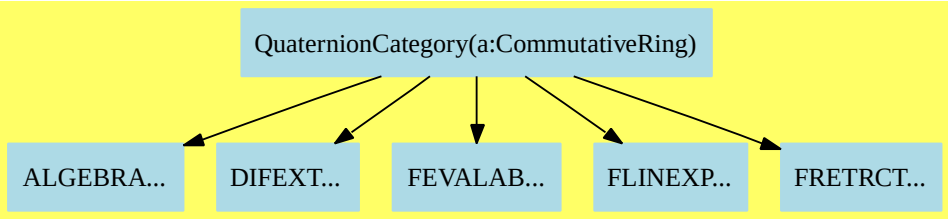
⟨*OC.dotfull*⟩≡
```
"OctonionCategory(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OC"];
"OctonionCategory(a:CommutativeRing)" -> "Algebra(a:CommutativeRing)"
"OctonionCategory(a:CommutativeRing)" -> "FullyEvalableOver(CommutativeRing)"
"OctonionCategory(a:CommutativeRing)" ->
   "FullyRetractableTo(a:CommutativeRing)"
```

⟨*OC.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OctonionCategory(a:CommutativeRing)" [color=lightblue];
"OctonionCategory(a:CommutativeRing)" -> "ALGEBRA..."
"OctonionCategory(a:CommutativeRing)" -> "FEVALAB..."
"OctonionCategory(a:CommutativeRing)" -> "FRETRCT..."

"ALGEBRA..." [color=lightblue];
"FEVALAB..." [color=lightblue];
"FRETRCT..." [color=lightblue];
}
```

12.8 QuaternionCategory (QUATCAT)



See:

⇐ "Algebra" (ALGEBRA) 11.1 on page 771
⇐ "DifferentialExtension" (DIFEXT) 11.2 on page 777
⇐ "FullyEvalableOver" (FEVALAB) 4.7 on page 121
⇐ "FullyLinearlyExplicitRingOver" (FLINEXP) 11.3 on page 782
⇐ "FullyRetractableTo" (FRETRCT) 3.6 on page 71

Exports:

0	1	abs	characteristic
charthRoot	coerce	conjugate	convert
D	differentiate	eval	hash
imagI	imagJ	imagK	inv
latex	map	max	min
norm	one?	quatern	rational
rational?	rationalIfCan	real	recip
reducedSystem	retract	retractIfCan	sample
subtractIfCan	zero?	?*?	?**?
?+?	?-?	-?	?=?
?^?	?~=?	?<?	?<=?
?>?	?>=?	?.?	

Attributes Exported:

- if #1 has EntireRing then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
imagI : % -> R
imagJ : % -> R
```

```
imagK : % -> R
quatern : (R,R,R,R) -> %
real : % -> R
```

These are implemented by this category:

```
abs : % -> R if R has RNS
characteristic : () -> NonNegativeInteger
coerce : R -> %
coerce : Integer -> %
coerce : % -> OutputForm
conjugate : % -> %
convert : % -> InputForm if R has KONVERT INFORM
differentiate : (%,(R -> R)) -> %
inv : % -> % if R has FIELD
map : ((R -> R),%) -> %
norm : % -> R
one? : % -> Boolean
rational : % -> Fraction Integer if R has INS
rational? : % -> Boolean if R has INS
rationalIfCan : % -> Union(Fraction Integer,"failed") if R has INS
retract : % -> R
retractIfCan : % -> Union(R,"failed")
zero? : % -> Boolean
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
?<? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p771) Algebra(R:CommutativeRing):

```
0 : () -> %
1 : () -> %
hash : % -> SingleInteger
latex : % -> String
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?*? : (%,R) -> %
```

These exports come from (p71) FullyRetractableTo(R:CommutativeRing):

```
coerce : Fraction Integer -> % if R has FIELD or R has RETRACT FRAC INT
retract : % -> Fraction Integer if R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
  if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
```

These exports come from (p777) DifferentialExtension(R:CommutativeRing):

```
D : % -> % if R has DIFRING
D : (%,NonNegativeInteger) -> % if R has DIFRING
D : (%,(R -> R)) -> %
D : (%,(R -> R),NonNegativeInteger) -> %
D : (%,Symbol) -> % if R has PDRING SYMBOL
D : (%,List Symbol) -> % if R has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if R has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
differentiate : (%,NonNegativeInteger) -> % if R has DIFRING
differentiate : % -> % if R has DIFRING
differentiate : (%,(R -> R),NonNegativeInteger) -> %
differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
?*? : (%,%) -> %
```

These exports come from (p121) FullyEvalableOver(R:CommutativeRing):

```
eval : (%,Equation R) -> % if R has EVALAB R
eval : (%,List Symbol,List R) -> % if R has IEVALAB(SYMBOL,R)
eval : (%,List Equation R) -> % if R has EVALAB R
eval : (%,R,R) -> % if R has EVALAB R
eval : (%,List R,List R) -> % if R has EVALAB R
eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
?.? : (%,R) -> % if R has ELTAB(R,R)
```

These exports come from (p782) FullyLinearlyExplicitRingOver(R)
where R:CommutativeRing:

```
recip : % -> Union(%,"failed")
reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
reducedSystem :
  (Matrix %,Vector %) ->
     Record(mat: Matrix Integer,vec: Vector Integer)
       if R has LINEXP INT
reducedSystem : Matrix % -> Matrix R
reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
```

These exports come from (p168) OrderedSet():

```
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p825) DivisionRing():

```
?**? : (%,Integer) -> % if R has FIELD
?^? : (%,Integer) -> % if R has FIELD
?*? : (Fraction Integer,%) -> % if R has FIELD
?*? : (%,Fraction Integer) -> % if R has FIELD
```

These exports come from (p677) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed") if R has CHARNZ
```

⟨*category QUATCAT QuaternionCategory*⟩≡

```
)abbrev category QUATCAT QuaternionCategory
++ Author: Robert S. Sutor
++ Date Created: 23 May 1990
++ Change History:
++   10 September 1990
++ Basic Operations: (Algebra)
++   abs, conjugate, imagI, imagJ, imagK, norm, quatern, rational,
++   rational?, real
++ Related Constructors: Quaternion, QuaternionCategoryFunctions2
++ Also See: DivisionRing
++ AMS Classifications: 11R52
++ Keywords: quaternions, division ring, algebra
++ Description:
++   \spadtype{QuaternionCategory} describes the category of quaternions
++   and implements functions that are not representation specific.

QuaternionCategory(R: CommutativeRing): Category ==
  Join(Algebra R, FullyRetractableTo R, DifferentialExtension R,
   FullyEvalableOver R, FullyLinearlyExplicitRingOver R) with

     conjugate: $ -> $
       ++ conjugate(q) negates the imaginary parts of quaternion \spad{q}.
     imagI:   $ -> R
       ++ imagI(q) extracts the imaginary i part of quaternion \spad{q}.
     imagJ:   $ -> R
       ++ imagJ(q) extracts the imaginary j part of quaternion \spad{q}.
     imagK:   $ -> R
       ++ imagK(q) extracts the imaginary k part of quaternion \spad{q}.
     norm:    $ -> R
```

```
      ++ norm(q) computes the norm of \spad{q} (the sum of the
      ++ squares of the components).
    quatern: (R,R,R,R) -> $
      ++ quatern(r,i,j,k) constructs a quaternion from scalars.
    real:    $ -> R
      ++ real(q) extracts the real part of quaternion \spad{q}.

    if R has EntireRing then EntireRing
    if R has OrderedSet then OrderedSet
    if R has Field then DivisionRing
    if R has ConvertibleTo InputForm then ConvertibleTo InputForm
    if R has CharacteristicZero then CharacteristicZero
    if R has CharacteristicNonZero then CharacteristicNonZero
    if R has RealNumberSystem then
      abs    : $ -> R
        ++ abs(q) computes the absolute value of quaternion \spad{q}
        ++ (sqrt of norm).
    if R has IntegerNumberSystem then
      rational?    : $ -> Boolean
        ++ rational?(q) returns {\it true} if all the imaginary
        ++ parts of \spad{q} are zero and the real part can be
        ++ converted into a rational number, and {\it false}
        ++ otherwise.
      rational     : $ -> Fraction Integer
        ++ rational(q) tries to convert \spad{q} into a
        ++ rational number. Error: if this is not
        ++ possible. If \spad{rational?(q)} is true, the
        ++ conversion will be done and the rational number returned.
      rationalIfCan: $ -> Union(Fraction Integer, "failed")
        ++ rationalIfCan(q) returns \spad{q} as a rational number,
        ++ or "failed" if this is not possible.
        ++ Note: if \spad{rational?(q)} is true, the conversion
        ++ can be done and the rational number will be returned.

  add

      characteristic() ==
        characteristic()$R

      conjugate x      ==
        quatern(real x, - imagI x, - imagJ x, - imagK x)

      map(fn, x)       ==
        quatern(fn real x, fn imagI x, fn imagJ x, fn imagK x)

      norm x ==
```

```
      real x * real x + imagI x * imagI x +
        imagJ x * imagJ x + imagK x * imagK x

    x = y             ==
      (real x = real y) and (imagI x = imagI y) and
        (imagJ x = imagJ y) and (imagK x = imagK y)

    x + y             ==
      quatern(real x + real y, imagI x + imagI y,
        imagJ x + imagJ y, imagK x + imagK y)

    x - y             ==
      quatern(real x - real y, imagI x - imagI y,
        imagJ x - imagJ y, imagK x - imagK y)

    - x               ==
      quatern(- real x, - imagI x, - imagJ x, - imagK x)

    r:R * x:$         ==
      quatern(r * real x, r * imagI x, r * imagJ x, r * imagK x)

    n:Integer * x:$   ==
      quatern(n * real x, n * imagI x, n * imagJ x, n * imagK x)

    differentiate(x:$, d:R -> R) ==
      quatern(d real x, d imagI x, d imagJ x, d imagK x)

    coerce(r:R)       ==
      quatern(r,0$R,0$R,0$R)

    coerce(n:Integer)       ==
      quatern(n :: R,0$R,0$R,0$R)

    one? x ==
--        one? real x and zero? imagI x and
      (real x) = 1 and zero? imagI x and
        zero? imagJ x and zero? imagK x

    zero? x ==
      zero? real x and zero? imagI x and
        zero? imagJ x and zero? imagK x

    retract(x):R ==
      not (zero? imagI x and zero? imagJ x and zero? imagK x) =>
        error "Cannot retract quaternion."
      real x
```

```
        retractIfCan(x):Union(R,"failed") ==
          not (zero? imagI x and zero? imagJ x and zero? imagK x) =>
            "failed"
          real x

        coerce(x:$):OutputForm ==
          part,z : OutputForm
          y : $
          zero? x => (0$R) :: OutputForm
          not zero?(real x) =>
            y := quatern(0$R,imagI(x),imagJ(x),imagK(x))
            zero? y => real(x) :: OutputForm
            (real(x) :: OutputForm) + (y :: OutputForm)
          -- we know that the real part is 0
          not zero?(imagI(x)) =>
            y := quatern(0$R,0$R,imagJ(x),imagK(x))
            z :=
              part := "i"::Symbol::OutputForm
--              one? imagI(x) => part
              (imagI(x) = 1) => part
              (imagI(x) :: OutputForm) * part
            zero? y => z
            z + (y :: OutputForm)
          -- we know that the real part and i part are 0
          not zero?(imagJ(x)) =>
            y := quatern(0$R,0$R,0$R,imagK(x))
            z :=
              part := "j"::Symbol::OutputForm
--              one? imagJ(x) => part
              (imagJ(x) = 1) => part
              (imagJ(x) :: OutputForm) * part
            zero? y => z
            z + (y :: OutputForm)
          -- we know that the real part and i and j parts are 0
          part := "k"::Symbol::OutputForm
--          one? imagK(x) => part
          (imagK(x) = 1) => part
          (imagK(x) :: OutputForm) * part

        if R has Field then
          inv x ==
            norm x = 0 => error "This quaternion is not invertible."
            (inv norm x) * conjugate x

        if R has ConvertibleTo InputForm then
```

```
      convert(x:$):InputForm ==
        l : List InputForm := [convert("quatern" :: Symbol),
          convert(real x)$R, convert(imagI x)$R, convert(imagJ x)$R,
            convert(imagK x)$R]
        convert(l)$InputForm

    if R has OrderedSet then
      x < y ==
        real x = real y =>
          imagI x = imagI y =>
            imagJ x = imagJ y =>
              imagK x < imagK y
            imagJ x < imagJ y
          imagI x < imagI y
        real x < real y

    if R has RealNumberSystem then
      abs x == sqrt norm x

    if R has IntegerNumberSystem then
      rational? x ==
        (zero? imagI x) and (zero? imagJ x) and (zero? imagK x)

      rational  x ==
        rational? x => rational real x
        error "Not a rational number"

      rationalIfCan x ==
        rational? x => rational real x
        "failed"
```

⟨*QUATCAT.dotabb*⟩≡

```
"QUATCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=QUATCAT"];
"QUATCAT" -> "ALGEBRA"
"QUATCAT" -> "DIFEXT"
"QUATCAT" -> "FEVALAB"
"QUATCAT" -> "FLINEXP"
"QUATCAT" -> "FRETRCT"
```

⟨*QUATCAT.dotfull*⟩≡

```
"QuaternionCategory(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=QUATCAT"];
"QuaternionCategory(a:CommutativeRing)" ->
   "Algebra(a:CommutativeRing)"
"QuaternionCategory(a:CommutativeRing)" ->
   "DifferentialExtension(CommutativeRing)"
"QuaternionCategory(a:CommutativeRing)" ->
   "FullyEvalableOver(CommutativeRing)"
"QuaternionCategory(a:CommutativeRing)" ->
   "FullyLinearlyExplicitRingOver(a:CommutativeRing)"
"QuaternionCategory(a:CommutativeRing)" ->
   "FullyRetractableTo(a:CommutativeRing)"
```

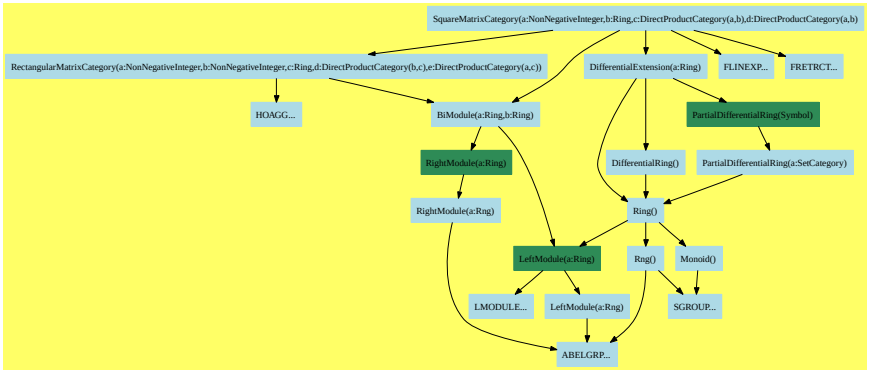
⟨*QUATCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"QuaternionCategory(a:CommutativeRing)" [color=lightblue];
"QuaternionCategory(a:CommutativeRing)" -> "ALGEBRA..."
"QuaternionCategory(a:CommutativeRing)" -> "DIFEXT..."
"QuaternionCategory(a:CommutativeRing)" -> "FEVALAB..."
"QuaternionCategory(a:CommutativeRing)" -> "FLINEXP..."
"QuaternionCategory(a:CommutativeRing)" -> "FRETRCT..."

"ALGEBRA..." [color=lightblue];
"DIFEXT..." [color=lightblue];
"FEVALAB..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"FRETRCT..." [color=lightblue];
}
```

12.9 SquareMatrixCategory (SMATCAT)



We define three categories for matrices

- MatrixCategory is the category of all matrices
- RectangularMatrixCategory is the category of all matrices of a given dimension
- SquareMatrixCategory inherits from RectangularMatrixCategory

The SquareMatrix domain is for square matrices of fixed dimension.

See:

⇐ "BiModule" (BMODULE) 9.1 on page 592
⇐ "DifferentialExtension" (DIFEXT) 11.2 on page 777
⇐ "FullyLinearlyExplicitRingOver" (FLINEXP) 11.3 on page 782
⇐ "FullyRetractableTo" (FRETRCT) 3.6 on page 71
⇐ "MatrixCategory" (MATCAT) 6.7 on page 315
⇐ "RectangularMatrixCategory" (RMATCAT) 10.14 on page 732

Exports:

0	1	antisymmetric?
any?	characteristic	coerce
column	copy	count
D	determinant	differentiate
diagonal	diagonal?	diagonalMatrix
diagonalProduct	elt	empty
empty?	eq?	eval
every?	exquo	hash
inverse	latex	less?
listOfLists	map	map!
matrix	maxColIndex	maxRowIndex
member?	members	minColIndex
minordet	minRowIndex	more?
ncols	nrows	nullSpace
nullity	one?	parts
qelt	rank	recip
reducedSystem	retract	retractIfCan
row	rowEchelon	sample
scalarMatrix	size?	square?
subtractIfCan	symmetric?	trace
zero?	#?	?^?
?*?	?**?	?+?
?-?	-?	?=?
?~=?	?/?	

Attributes Exported:

- **finiteAggregate** is true if it is an aggregate with a finite number of elements.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **nil**

TPDHERE: How did MATCAT get in the type tower?

```
determinant : % -> R if R has commutative *
inverse : % -> Union(%,"failed") if R has FIELD
```

These are directly exported but not implemented:

```
diagonalMatrix : List R -> %
```

```
minordet : % -> R if R has commutative *
scalarMatrix : R -> %
?*? : (Row,%) -> Row
?*? : (%,Col) -> Col
```

These are implemented by this category:

```
coerce : R -> %
diagonal : % -> Row
diagonalProduct : % -> R
differentiate : (%,(R -> R)) -> %
reducedSystem : Matrix % -> Matrix R
reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
retract : % -> R
retractIfCan : % -> Union(R,"failed")
trace : % -> R
?**? : (%,Integer) -> % if R has FIELD
?**? : (%,NonNegativeInteger) -> %
```

These exports come from (p777) DifferentialExtension(R:Ring):

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
D : % -> % if R has DIFRING
D : (%,NonNegativeInteger) -> % if R has DIFRING
D : (%,(R -> R)) -> %
D : (%,(R -> R),NonNegativeInteger) -> %
D : (%,Symbol) -> % if R has PDRING SYMBOL
D : (%,List Symbol) -> % if R has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,NonNegativeInteger) -> % if R has DIFRING
differentiate : % -> % if R has DIFRING
differentiate : (%,(R -> R),NonNegativeInteger) -> %
differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
```

```
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p592) BiModule(R:Ring,R:Ring):

```
?*? : (R,%) -> %
?*? : (%,R) -> %
```

These exports come from
(p732) RectangularMatrixCategory(ndim,ndim,R,Row,Col)
where ndim:NonNegativeInteger, R:Ring, Row:DirectProductCategory(ndim,R)
Col:DirectProductCategory(ndim,R):

```
antisymmetric? : % -> Boolean
any? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
column : (%,Integer) -> Col
copy : % -> %
count : (R,%) -> NonNegativeInteger if R has SETCAT and $ has finiteAggregate
count : ((R -> Boolean),%) -> NonNegativeInteger if $ has finiteAggregate
diagonal? : % -> Boolean
elt : (%,Integer,Integer) -> R
elt : (%,Integer,Integer,R) -> R
empty : () -> %
empty? : % -> Boolean
eq? : (%,%) -> Boolean
eval : (%,List R,List R) -> % if R has EVALAB R and R has SETCAT
eval : (%,R,R) -> % if R has EVALAB R and R has SETCAT
eval : (%,Equation R) -> % if R has EVALAB R and R has SETCAT
eval : (%,List Equation R) -> % if R has EVALAB R and R has SETCAT
every? : ((R -> Boolean),%) -> Boolean if $ has finiteAggregate
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
less? : (%,NonNegativeInteger) -> Boolean
listOfLists : % -> List List R
map : ((R -> R),%) -> %
map : (((R,R) -> R),%,%) -> %
map! : ((R -> R),%) -> % if $ has shallowlyMutable
matrix : List List R -> %
maxColIndex : % -> Integer
maxRowIndex : % -> Integer
minColIndex : % -> Integer
minRowIndex : % -> Integer
```

```
members : % -> List R if $ has finiteAggregate
member? : (R,%) -> Boolean if R has SETCAT and $ has finiteAggregate
more? : (%,NonNegativeInteger) -> Boolean
ncols : % -> NonNegativeInteger
nrows : % -> NonNegativeInteger
nullity : % -> NonNegativeInteger if R has INTDOM
nullSpace : % -> List Col if R has INTDOM
parts : % -> List R if $ has finiteAggregate
qelt : (%,Integer,Integer) -> R
rank : % -> NonNegativeInteger if R has INTDOM
row : (%,Integer) -> Row
rowEchelon : % -> % if R has EUCDOM
size? : (%,NonNegativeInteger) -> Boolean
square? : % -> Boolean
symmetric? : % -> Boolean
#? : % -> NonNegativeInteger if $ has finiteAggregate
?/? : (%,R) -> % if R has FIELD
```

These exports come from (p71) FullyRetractableTo(R:Ring):

```
coerce : Fraction Integer -> % if R has RETRACT FRAC INT
retract : % -> Fraction Integer if R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
```

These exports come from (p782) FullyLinearlyExplicitRingOver(R:Ring):

```
reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R has LINEXP
reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
```

⟨*category SMATCAT SquareMatrixCategory*⟩≡

```
)abbrev category SMATCAT SquareMatrixCategory
++ Authors: Grabmeier, Gschnitzer, Williamson
++ Date Created: 1987
++ Date Last Updated: July 1990
++ Basic Operations:
++ Related Domains: SquareMatrix(ndim,R)
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++   \spadtype{SquareMatrixCategory} is a general square matrix category which
++   allows different representations and indexing schemes.  Rows and
++   columns may be extracted with rows returned as objects of
++   type Row and colums returned as objects of type Col.
SquareMatrixCategory(ndim,R,Row,Col): Category == Definition where
```

```
    ndim : NonNegativeInteger
    R    : Ring
    Row  : DirectProductCategory(ndim,R)
    Col  : DirectProductCategory(ndim,R)
    I ==> Integer

    Definition ==> Join(DifferentialExtension R, BiModule(R, R),_
                        RectangularMatrixCategory(ndim,ndim,R,Row,Col),_
                        FullyRetractableTo R,_
                        FullyLinearlyExplicitRingOver R) with
      if R has CommutativeRing then Module(R)
      scalarMatrix: R -> %
        ++ \spad{scalarMatrix(r)} returns an n-by-n matrix with r's on the
        ++ diagonal and zeroes elsewhere.
      diagonalMatrix: List R -> %
        ++ \spad{diagonalMatrix(l)} returns a diagonal matrix with the elements
        ++ of l on the diagonal.
      diagonal: % -> Row
        ++ \spad{diagonal(m)} returns a row consisting of the elements on the
        ++ diagonal of the matrix m.
      trace: % -> R
        ++ \spad{trace(m)} returns the trace of the matrix m. this is the sum
        ++ of the elements on the diagonal of the matrix m.
      diagonalProduct: % -> R
        ++ \spad{diagonalProduct(m)} returns the product of the elements on the
        ++ diagonal of the matrix m.
      "*": (%,Col) -> Col
        ++ \spad{x * c} is the product of the matrix x and the column vector c.
        ++ Error: if the dimensions are incompatible.
      "*": (Row,%) -> Row
        ++ \spad{r * x} is the product of the row vector r and the matrix x.
        ++ Error: if the dimensions are incompatible.

  --% Linear algebra

      if R has commutative("*") then
        Algebra R
        determinant: % -> R
          ++ \spad{determinant(m)} returns the determinant of the matrix m.
        minordet: % -> R
          ++ \spad{minordet(m)} computes the determinant of the matrix m
          ++ using minors.
      if R has Field then
        inverse: % -> Union(%,"failed")
          ++ \spad{inverse(m)} returns the inverse of the matrix m, if that
          ++ matrix is invertible and returns "failed" otherwise.
```

```
      "**": (%,Integer) -> %
        ++ \spad{m**n} computes an integral power of the matrix m.
        ++ Error: if the matrix is not invertible.

    add
     minr ==> minRowIndex
     maxr ==> maxRowIndex
     minc ==> minColIndex
     maxc ==> maxColIndex
     mini ==> minIndex
     maxi ==> maxIndex

     positivePower:(%,Integer) -> %
     positivePower(x,n) ==
--       one? n => x
       (n = 1) => x
       odd? n => x * positivePower(x,n - 1)
       y := positivePower(x,n quo 2)
       y * y

     x:% ** n:NonNegativeInteger ==
       zero? n => scalarMatrix 1
       positivePower(x,n)

     coerce(r:R) == scalarMatrix r

     equation2R: Vector % -> Matrix R

     differentiate(x:%,d:R -> R) == map(d,x)

     diagonal x ==
       v:Vector(R) := new(ndim,0)
       for i in minr x .. maxr x
         for j in minc x .. maxc x
           for k in minIndex v .. maxIndex v repeat
             qsetelt_!(v, k, qelt(x, i, j))
       directProduct v

     retract(x:%):R ==
       diagonal? x => retract diagonal x
       error "Not retractable"

     retractIfCan(x:%):Union(R, "failed") ==
       diagonal? x => retractIfCan diagonal x
       "failed"
```

```
equation2R v ==
  ans:Matrix(Col) := new(ndim,#v,0)
  for i in minr ans .. maxr ans repeat
    for j in minc ans .. maxc ans repeat
      qsetelt_!(ans, i, j, column(qelt(v, j), i))
  reducedSystem ans

reducedSystem(x:Matrix %):Matrix(R) ==
  empty? x => new(0,0,0)
  reduce(vertConcat, [equation2R row(x, i)
                          for i in minr x .. maxr x])$List(Matrix R)

reducedSystem(m:Matrix %, v:Vector %):
 Record(mat:Matrix R, vec:Vector R) ==
  vh:Vector(R) :=
    empty? v => new(0,0)
    rh := reducedSystem(v::Matrix %)@Matrix(R)
    column(rh, minColIndex rh)
  [reducedSystem(m)@Matrix(R), vh]

trace x ==
  tr : R := 0
  for i in minr(x)..maxr(x) for j in minc(x)..maxc(x) repeat
    tr := tr + x(i,j)
  tr

diagonalProduct x ==
  pr : R := 1
  for i in minr(x)..maxr(x) for j in minc(x)..maxc(x) repeat
    pr := pr * x(i,j)
  pr

if R has Field then

  x:% ** n:Integer ==
    zero? n => scalarMatrix 1
    positive? n => positivePower(x,n)
    (xInv := inverse x) case "failed" =>
      error "**: matrix must be invertible"
    positivePower(xInv :: %,-n)
```

⟨*SMATCAT.dotabb*⟩≡
```
"SMATCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SMATCAT"];
"SMATCAT" -> "BMODULE"
"SMATCAT" -> "DIFEXT"
"SMATCAT" -> "FLINEXP"
"SMATCAT" -> "FRETRCT"
"SMATCAT" -> "RMATCAT"
```

⟨*SMATCAT.dotfull*⟩≡
```
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProdu
 [color=lightblue,href="bookvol10.2.pdf#nameddest=SMATCAT"];
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProdu
 -> "BiModule(a:Ring,b:Ring)"
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProdu
 -> "DifferentialExtension(a:Ring)"
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProdu
 -> "FullyLinearlyExplicitRingOver(a:Ring)"
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProdu
 -> "FullyRetractableTo(a:Ring)"
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:DirectProdu
 -> "RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProc
```

⟨*SMATCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:
 [color=lightblue];
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:
 -> "BiModule(a:Ring,b:Ring)"
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:
 -> "DifferentialExtension(a:Ring)"
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:
 -> "FLINEXP..."
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:
 -> "FRETRCT..."
"SquareMatrixCategory(a:NonNegativeInteger,b:Ring,c:DirectProductCategory(a,b),d:
 -> "RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d

"DifferentialExtension(a:Ring)" [color=lightblue];
"DifferentialExtension(a:Ring)" -> "Ring()"
"DifferentialExtension(a:Ring)" -> "DifferentialRing()"
"DifferentialExtension(a:Ring)" -> "PartialDifferentialRing(Symbol)"

"PartialDifferentialRing(Symbol)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PDRING"];
"PartialDifferentialRing(Symbol)" ->
    "PartialDifferentialRing(a:SetCategory)"

"PartialDifferentialRing(a:SetCategory)" [color=lightblue];
"PartialDifferentialRing(a:SetCategory)" -> "Ring()"

"DifferentialRing()" [color=lightblue];
"DifferentialRing()" -> "Ring()"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SGROUP..."

"Monoid()" [color=lightblue];
"Monoid()" -> "SGROUP..."
```

```
"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LMODULE..."

"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductC
 [color=lightblue];
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductC
  -> "BiModule(a:Ring,b:Ring)"
"RectangularMatrixCategory(a:NonNegativeInteger,b:NonNegativeInteger,c:Ring,d:DirectProductC
  -> "HOAGG..."

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

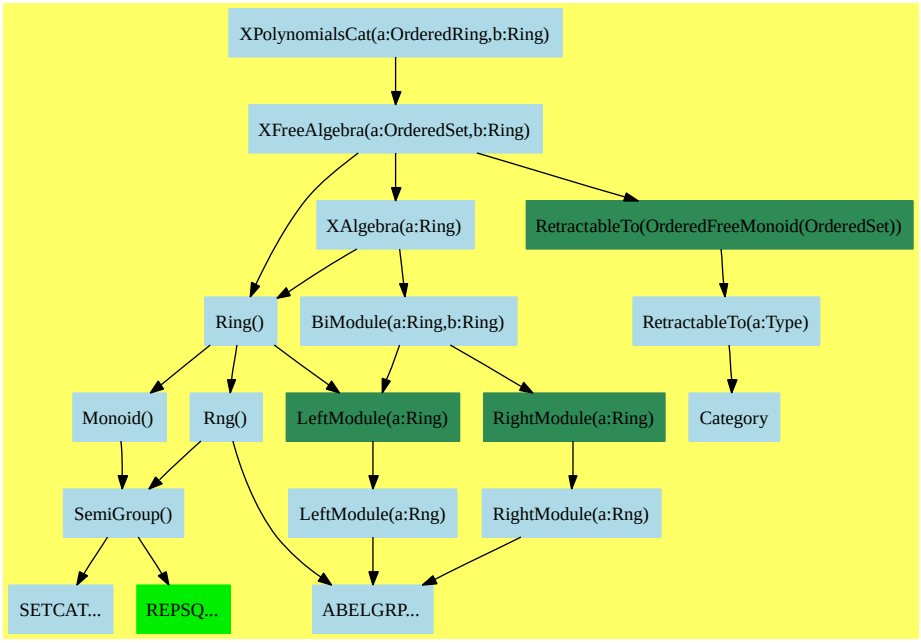
"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"FRETRCT..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"SGROUP..." [color=lightblue];
"LMODULE..." [color=lightblue];
"HOAGG..." [color=lightblue];
"ABELGRP..." [color=lightblue];

}
```

12.10 XPolynomialsCat (XPOLYC)



See:

⇐ "XFreeAlgebra" (XFALG) 11.8 on page 807

Exports:

0	1	characteristic	coef	coerce
constant	constant?	degree	hash	latex
lquo	map	maxdeg	mindeg	mindegTerm
mirror	monom	monomial?	one?	quasiRegular
quasiRegular?	recip	retract	retractIfCan	rquo
sample	sh	subtractIfCan	trunc	varList
zero?	?*?	?**?	?+?	?-?
-?	?=?	?^?	?~=?	

Attributes Exported:

- if Ring has noZeroDivisors then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
degree : % -> NonNegativeInteger
maxdeg : % -> OrderedFreeMonoid vl
trunc : (%,NonNegativeInteger) -> %
```

These exports come from (p55) Aggregate():

These exports come from (p807) XFreeAlgebra(vl:OrderedSet,R:Ring):

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coef : (%,OrderedFreeMonoid vl) -> R
coef : (%,%) -> R
coerce : % -> OutputForm
coerce : R -> %
coerce : OrderedFreeMonoid vl -> %
coerce : Integer -> %
coerce : vl -> %
constant : % -> R
constant? : % -> Boolean
hash : % -> SingleInteger
latex : % -> String
lquo : (%,OrderedFreeMonoid vl) -> %
lquo : (%,%) -> %
lquo : (%,vl) -> %
map : ((R -> R),%) -> %
mindeg : % -> OrderedFreeMonoid vl
mindegTerm : % -> Record(k: OrderedFreeMonoid vl,c: R)
mirror : % -> %
monom : (OrderedFreeMonoid vl,R) -> %
monomial? : % -> Boolean
one? : % -> Boolean
quasiRegular : % -> %
quasiRegular? : % -> Boolean
recip : % -> Union(%,"failed")
retract : % -> OrderedFreeMonoid vl
retractIfCan : % -> Union(OrderedFreeMonoid vl,"failed")
rquo : (%,OrderedFreeMonoid vl) -> %
rquo : (%,%) -> %
rquo : (%,vl) -> %
sample : () -> %
sh : (%,NonNegativeInteger) -> % if R has COMRING
sh : (%,%) -> % if R has COMRING
subtractIfCan : (%,%) -> Union(%,"failed")
varList : % -> List vl
```

```
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?*? : (R,%) -> %
?*? : (%,R) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (vl,%) -> %
```

These exports come from (p91) SetCategory():

⟨*category XPOLYC XPolynomialsCat*⟩≡

```
)abbrev category XPOLYC XPolynomialsCat
++ Author: Michel Petitot petitot@lifl.fr
++ Date Created: 91
++ Date Last Updated: 7 Juillet 92
++ Fix History: compilation v 2.1 le 13 dec 98
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++   The Category of polynomial rings with non-commutative variables.
++   The coefficient ring may be non-commutative too.
++   However coefficients commute with vaiables.
++ Author: Michel Petitot (petitot@lifl.fr)

XPolynomialsCat(vl:OrderedSet,R:Ring):Category == Export where
  WORD ==> OrderedFreeMonoid(vl)

  Export == XFreeAlgebra(vl,R) with
    maxdeg: % -> WORD
      ++ \spad{maxdeg(p)} returns the greatest leading word in the
      ++ support of \spad{p}.
    degree: % -> NonNegativeInteger
```

```
        ++ \spad{degree(p)} returns the degree of \spad{p}.
        ++  Note that the degree of a word is its length.
      trunc : (% , NonNegativeInteger) -> %
        ++  \spad{trunc(p,n)} returns the polynomial \spad{p} truncated
        ++ at order \spad{n}.
```

⟨*XPOLYC.dotabb*⟩≡

```
"XPOLYC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XPOLYC"];
"XPOLYC" -> "XFALG"
```

⟨*XPOLYC.dotfull*⟩≡

```
"XPolynomialsCat(a:OrderedRing,b:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XPOLYC"];
"XPolynomialsCat(a:OrderedRing,b:Ring)" ->
   "XFreeAlgebra(a:OrderedSet,b:Ring)"
```

⟨*XPOLYC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"XPolynomialsCat(a:OrderedRing,b:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XPOLYC"];
"XPolynomialsCat(a:OrderedRing,b:Ring)" ->
   "XFreeAlgebra(a:OrderedSet,b:Ring)"

"XFreeAlgebra(a:OrderedSet,b:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XFALG"];
"XFreeAlgebra(a:OrderedSet,b:Ring)" -> "Ring()"
"XFreeAlgebra(a:OrderedSet,b:Ring)" -> "XAlgebra(a:Ring)"
"XFreeAlgebra(a:OrderedSet,b:Ring)" ->
    "RetractableTo(OrderedFreeMonoid(OrderedSet))"

"RetractableTo(OrderedFreeMonoid(OrderedSet))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(OrderedFreeMonoid(OrderedSet))" -> "RetractableTo(a:Type)"

"XAlgebra(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XALG"];
"XAlgebra(a:Ring)" -> "Ring()"
"XAlgebra(a:Ring)" -> "BiModule(a:Ring,b:Ring)"

"Ring()" [color=lightblue];
"Ring()" -> "Rng()"
"Ring()" -> "Monoid()"
"Ring()" -> "LeftModule(a:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"Rng()" [color=lightblue];
"Rng()" -> "ABELGRP..."
"Rng()" -> "SemiGroup()"
```

```
"Monoid()" [color=lightblue];
"Monoid()" -> "SemiGroup()"

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"

"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"SemiGroup()" [color=lightblue];
"SemiGroup()" -> "SETCAT..."
"SemiGroup()" -> "REPSQ..."

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

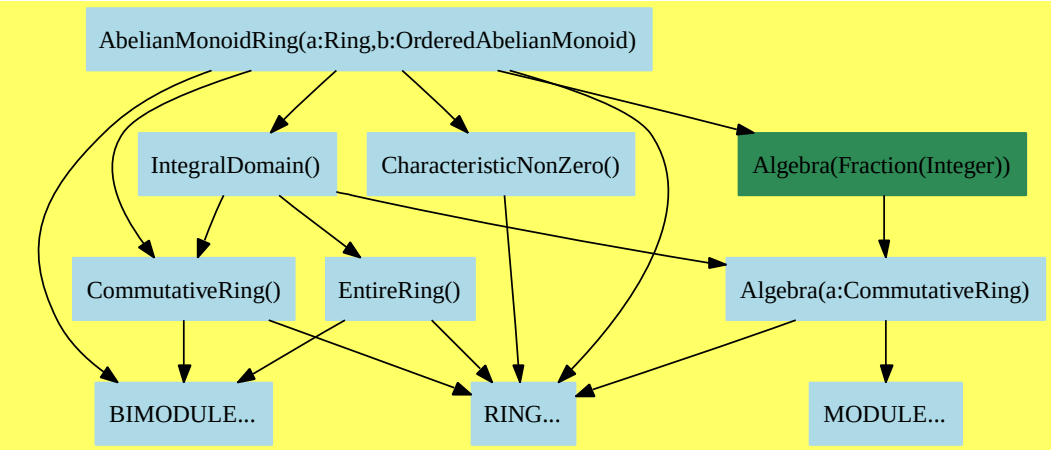
"Category" [color=lightblue];

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

Chapter 13

Category Layer 12

13.1 AbelianMonoidRing (AMR)



See:

⇒ "FiniteAbelianMonoidRing" (FAMR) 14.1 on page 941
⇒ "PowerSeriesCategory" (PSCAT) 14.3 on page 958
⇐ "BiModule" (BMODULE) 9.1 on page 592
⇐ "Ring" (RING) 9.8 on page 628

Exports:

0	1	associates?	characteristic
charthRoot	coefficient	coerce	degree
exquo	hash	latex	leadingCoefficient
leadingMonomial	map	monomial	monomial?
one?	recip	reductum	sample
subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?*?	?**?	?+?
?-?	-?	?=?	?^?
?~=?	?/?		

Attributes exported:

- if $ has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- if $ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coefficient : (%,E) -> R
degree : % -> E
leadingCoefficient : % -> R
leadingMonomial : % -> %
monomial : (R,E) -> %
reductum : % -> %
?/? : (%,R) -> % if R has FIELD
```

These are implemented by this category:

```
map : ((R -> R),%) -> %
monomial? : % -> Boolean
?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
```

These exports come from (p628) Ring():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> OutputForm
coerce : Integer -> %
```

```
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p592) BiModule(R:Ring,R:Ring):

```
?*? : (R,%) -> %
?*? : (%,R) -> %
```

These exports come from (p857) IntegralDomain():

```
associates? : (%,%) -> Boolean if R has INTDOM
coerce : % -> % if R has INTDOM
exquo : (%,%) -> Union(%,"failed") if R has INTDOM
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM
```

These exports come from (p677) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed") if R has CHARNZ
```

These exports come from (p685) CommutativeRing():

```
coerce : R -> % if R has COMRING
```

These exports come from (p771) Algebra(Fraction(Integer)):

```
coerce : Fraction Integer -> % if R has ALGEBRA FRAC INT
?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
```

⟨*category AMR AbelianMonoidRing*⟩≡

```
)abbrev category AMR AbelianMonoidRing
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ Abelian monoid ring elements (not necessarily of finite support)
++ of this ring are of the form formal SUM (r_i * e_i)
++ where the r_i are coefficents and the e_i, elements of the
++ ordered abelian monoid, are thought of as exponents or monomials.
++ The monomials commute with each other, and with
++ the coefficients (which themselves may or may not be commutative).
++ See \spadtype{FiniteAbelianMonoidRing} for the case of finite support
++ a useful common model for polynomials and power series.
++ Conceptually at least, only the non-zero terms are ever operated on.
AbelianMonoidRing(R:Ring, E:OrderedAbelianMonoid): Category ==
     Join(Ring,BiModule(R,R)) with
  leadingCoefficient: % -> R
    ++ leadingCoefficient(p) returns the coefficient highest
    ++ degree term of p.
  leadingMonomial: % -> %
    ++ leadingMonomial(p) returns the monomial of p with the highest degree.
  degree: % -> E
    ++ degree(p) returns the maximum of the exponents of the terms of p.
  map: (R -> R, %) -> %
    ++ map(fn,u) maps function fn onto the coefficients
    ++ of the non-zero monomials of u.
  monomial?: % -> Boolean
    ++ monomial?(p) tests if p is a single monomial.
  monomial: (R,E) -> %
    ++ monomial(r,e) makes a term from a coefficient r and an exponent e.
  reductum: % -> %
    ++ reductum(u) returns u minus its leading monomial
    ++ returns zero if handed the zero element.
  coefficient: (%,E) -> R
    ++ coefficient(p,e) extracts the coefficient of the monomial with
    ++ exponent e from polynomial p, or returns zero if exponent
    ++ is not present.
  if R has Field then "/": (%,R) -> %
    ++ p/c divides p by the coefficient c.
```

```
    if R has CommutativeRing then
       CommutativeRing
       Algebra R
    if R has CharacteristicZero then CharacteristicZero
    if R has CharacteristicNonZero then CharacteristicNonZero
    if R has IntegralDomain then IntegralDomain
    if R has Algebra Fraction Integer then Algebra Fraction Integer
   add
    monomial? x == zero? reductum x

    map(fn:R -> R, x: %) ==
          -- this default definition assumes that reductum is cheap
       zero? x => 0
       r:=fn leadingCoefficient x
       zero? r => map(fn,reductum x)
       monomial(r, degree x) + map(fn,reductum x)

    if R has Algebra Fraction Integer then
      q:Fraction(Integer) * p:% == map(x1 +-> q * x1, p)
```

⟨*AMR.dotabb*⟩≡

```
  "AMR"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=AMR"];
  "AMR" -> "RING"
  "AMR" -> "BMODULE"
  "AMR" -> "INTDOM"
  "AMR" -> "CHARNZ"
  "AMR" -> "COMRING"
  "AMR" -> "ALGEBRA"
```

⟨*AMR.dotfull*⟩≡

```
  "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=AMR"];
  "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" -> "Ring()"
  "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
      "BiModule(a:Ring,b:OrderedAbelianMonoid)"
  "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
      "IntegralDomain()"
  "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
      "CharacteristicNonZero()"
  "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
      "CommutativeRing()"
  "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
      "Algebra(Fraction(Integer))"
```

⟨*AMR.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" [color=lightblue];
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" -> "RING..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "BIMODULE..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "IntegralDomain()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CharacteristicNonZero()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CommutativeRing()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "Algebra(Fraction(Integer))"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BIMODULE..."

"CharacteristicNonZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CharacteristicNonZero()" -> "RING..."

"Algebra(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

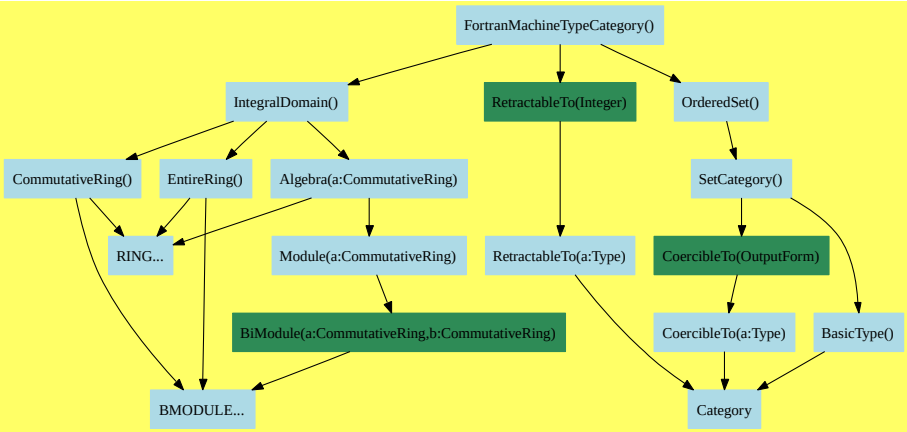
"Algebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BIMODULE..."

"BIMODULE..." [color=lightblue];
```

```
  "RING..." [color=lightblue];
  "MODULE..." [color=lightblue];
  }
```

13.2 FortranMachineTypeCategory (FMTC)



See:

⇐ "IntegralDomain" (INTDOM) 12.5 on page 857
⇐ "OrderedSet" (ORDSET) 4.19 on page 168
⇐ "RetractableTo" (RETRACT) 2.18 on page 44

Exports:

0	1	associates?	characteristic	coerce
exquo	hash	latex	max	min
one?	recip	retract	retractIfCan	sample
subtractIfCan	unit?	unitCanonical	unitNormal	zero?
?~=?	?^?	?*?	?**?	?+?
?-?	-?	?<?	?<=?	?=?
?>?	?>=?			

Attributes Exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p857) IntegralDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
coerce : % -> %
exquo : (%,%) -> Union(%,"failed")
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p168) OrderedSet():

```
max : (%,%) -> %
min : (%,%) -> %
?<? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

These exports come from (p44) RetractableTo(Integer):

```
coerce : Integer -> %
retract : % -> Integer
retractIfCan : % -> Union(Integer,"failed")
```

⟨*category FMTC FortranMachineTypeCategory*⟩≡

```
)abbrev category FMTC FortranMachineTypeCategory
++ Author: Mike Dewar
```

```
++ Date Created:  December 1993
++ Date Last Updated:
++ Basic Operations:
++ Related Domains:
++ Also See: FortranExpression, MachineInteger, MachineFloat, MachineComplex
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description: A category of domains which model machine arithmetic
++ used by machines in the AXIOM-NAG link.
FortranMachineTypeCategory():Category == Join(IntegralDomain,OrderedSet,
                                              RetractableTo(Integer) )
```

⟨*FMTC.dotabb*⟩≡

```
"FMTC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMTC"];
"FMTC" -> "INTDOM"
"FMTC" -> "ORDSET"
"FMTC" -> "RETRACT"
```

⟨*FMTC.dotfull*⟩≡

```
"FortranMachineTypeCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMTC"];
"FortranMachineTypeCategory()" -> "IntegralDomain()"
"FortranMachineTypeCategory()" -> "OrderedSet()"
"FortranMachineTypeCategory()" -> "RetractableTo(Integer)"
```

⟨*FMTC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FortranMachineTypeCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FMTC"];
"FortranMachineTypeCategory()" -> "IntegralDomain()"
"FortranMachineTypeCategory()" -> "OrderedSet()"
"FortranMachineTypeCategory()" -> "RetractableTo(Integer)"

"RetractableTo(Integer)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(Integer)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(a:Type)" -> "Category"

"OrderedSet()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ORDSET"];
"OrderedSet()" -> "SetCategory()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
```

```
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];

"SetCategory()" [color=lightblue];
"SetCategory()" -> "BasicType()"
"SetCategory()" -> "CoercibleTo(OutputForm)"

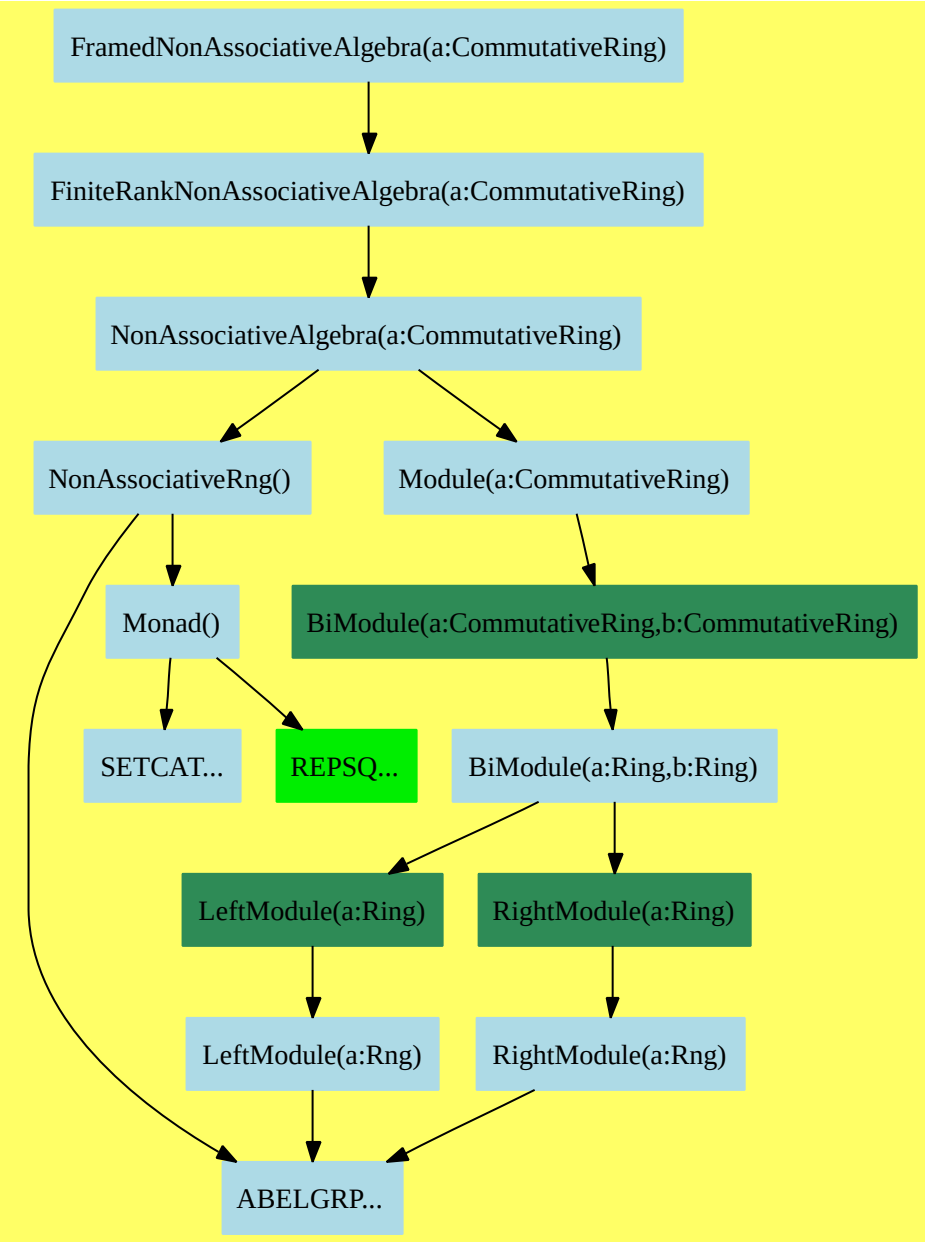
"BasicType()" [color=lightblue];
"BasicType()" -> "Category"

"CoercibleTo(OutputForm)" [color=seagreen];
"CoercibleTo(OutputForm)" -> "CoercibleTo(a:Type)"

"CoercibleTo(a:Type)" [color=lightblue];
"CoercibleTo(a:Type)" -> "Category"

"Category" [color=lightblue];
}
```

13.3 FramedNonAssociativeAlgebra (FRNAALG)



See:

⇐ "FiniteRankNonAssociativeAlgebra" (FINAALG) 12.3 on page 830

Exports:

0	alternative?
antiAssociative?	antiCommutative?
antiCommutator	apply
associative?	associator
associatorDependence	basis
coerce	commutative?
commutator	conditionsForIdempotents
convert	coordinates
flexible?	hash
jacobiIdentity?	jordanAdmissible?
jordanAlgebra?	latex
leftAlternative?	leftCharacteristicPolynomial
leftDiscriminant	leftMinimalPolynomial
leftNorm	leftPower
leftRankPolynomial	leftRecip
leftRegularRepresentation	leftTrace
leftTraceMatrix	leftUnit
leftUnits	lieAdmissible?
lieAlgebra?	noncommutativeJordanAlgebra?
plenaryPower	powerAssociative?
rank	recip
represents	rightAlternative?
rightCharacteristicPolynomial	rightDiscriminant
rightMinimalPolynomial	rightNorm
rightPower	rightRankPolynomial
rightRecip	rightRegularRepresentation
rightTrace	rightTraceMatrix
rightUnit	rightUnits
sample	someBasis
structuralConstants	subtractIfCan
unit	zero?
?*?	?**?
?+?	?-?
-?	?=?
?.?	?~=?

Attributes exported:

- if $ has IntegralDomain then unitsKnown where **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
apply : (Matrix R,%) -> %
basis : () -> Vector %
convert : Vector R -> %
?.? : (%,Integer) -> R
```

These are implemented by this category:

```
conditionsForIdempotents : () -> List Polynomial R
convert : % -> Vector R
coordinates : % -> Vector R
coordinates : Vector % -> Matrix R
leftDiscriminant : () -> R
leftRankPolynomial : () ->
  SparseUnivariatePolynomial Polynomial R
    if R has FIELD
leftRegularRepresentation : % -> Matrix R
leftTraceMatrix : () -> Matrix R
leftUnit : () -> Union(%,"failed") if R has INTDOM
leftUnits : () ->
  Union(Record(particular: %,basis: List %),"failed")
    if R has INTDOM
represents : Vector R -> %
rightDiscriminant : () -> R
rightRankPolynomial : () ->
  SparseUnivariatePolynomial Polynomial R
    if R has FIELD
rightRegularRepresentation : % -> Matrix R
rightTraceMatrix : () -> Matrix R
rightUnit : () -> Union(%,"failed") if R has INTDOM
rightUnits : () ->
  Union(Record(particular: %,basis: List %),"failed")
    if R has INTDOM
structuralConstants : () -> Vector Matrix R
unit : () -> Union(%,"failed") if R has INTDOM
```

These exports come from (p830) FiniteRankNonAssociativeAlgebra(R) where R:CommutativeRing:

```
0 : () -> %
alternative? : () -> Boolean
antiAssociative? : () -> Boolean
antiCommutative? : () -> Boolean
antiCommutator : (%,%) -> %
associative? : () -> Boolean
associator : (%,%,%) -> %
associatorDependence : () -> List Vector R
    if R has INTDOM
coerce : % -> OutputForm
commutative? : () -> Boolean
commutator : (%,%) -> %
```

```
conditionsForIdempotents : Vector % -> List Polynomial R
coordinates : (Vector %,Vector %) -> Matrix R
coordinates : (%,Vector %) -> Vector R
flexible? : () -> Boolean
hash : % -> SingleInteger
jacobiIdentity? : () -> Boolean
jordanAdmissible? : () -> Boolean
jordanAlgebra? : () -> Boolean
latex : % -> String
leftAlternative? : () -> Boolean
leftCharacteristicPolynomial : % ->
  SparseUnivariatePolynomial R
leftDiscriminant : Vector % -> R
leftMinimalPolynomial : % ->
  SparseUnivariatePolynomial R
    if R has INTDOM
leftNorm : % -> R
leftPower : (%,PositiveInteger) -> %
leftRecip : % -> Union(%,"failed") if R has INTDOM
leftRegularRepresentation : (%,Vector %) -> Matrix R
leftTrace : % -> R
leftTraceMatrix : Vector % -> Matrix R
lieAdmissible? : () -> Boolean
lieAlgebra? : () -> Boolean
noncommutativeJordanAlgebra? : () -> Boolean
plenaryPower : (%,PositiveInteger) -> %
powerAssociative? : () -> Boolean
rank : () -> PositiveInteger
recip : % -> Union(%,"failed") if R has INTDOM
represents : (Vector R,Vector %) -> %
rightAlternative? : () -> Boolean
rightCharacteristicPolynomial : % ->
  SparseUnivariatePolynomial R
rightDiscriminant : Vector % -> R
rightMinimalPolynomial : % ->
  SparseUnivariatePolynomial R
    if R has INTDOM
rightNorm : % -> R
rightPower : (%,PositiveInteger) -> %
rightRecip : % -> Union(%,"failed") if R has INTDOM
rightRegularRepresentation : (%,Vector %) -> Matrix R
rightTrace : % -> R
rightTraceMatrix : Vector % -> Matrix R
sample : () -> %
someBasis : () -> Vector %
structuralConstants : Vector % -> Vector Matrix R
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?*? : (PositiveInteger,%) -> %
```

```
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (R,%) -> %
?*? : (%,R) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
```

⟨*category FRNAALG FramedNonAssociativeAlgebra*⟩≡

```
)abbrev category FRNAALG FramedNonAssociativeAlgebra
++ Author: J. Grabmeier, R. Wisbauer
++ Date Created: 01 March 1991
++ Date Last Updated: 11 June 1991
++ Basic Operations: +,-,*,**,basis
++ Related Constructors: FiniteRankNonAssociativeAlgebra, FramedAlgebra,
++   FiniteRankAssociativeAlgebra
++ Also See:
++ AMS Classifications:
++ Keywords: nonassociative algebra, basis
++ Reference:
++  R.D. Schafer: An Introduction to Nonassociative Algebras
++  Academic Press, New York, 1966
++ Description:
++   FramedNonAssociativeAlgebra(R) is a
++   \spadtype{FiniteRankNonAssociativeAlgebra} (i.e. a non associative
++   algebra over R which is a free \spad{R}-module of finite rank)
++   over a commutative ring R together with a fixed \spad{R}-module basis.
FramedNonAssociativeAlgebra(R:CommutativeRing):
        Category == FiniteRankNonAssociativeAlgebra(R) with
    basis: () -> Vector %
      ++ basis() returns the fixed \spad{R}-module basis.
    coordinates: % -> Vector R
      ++ coordinates(a) returns the coordinates of \spad{a}
      ++ with respect to the
      ++ fixed \spad{R}-module basis.
    coordinates: Vector % -> Matrix R
      ++ coordinates([a1,...,am]) returns a matrix whose i-th row
      ++ is formed by the coordinates of \spad{ai} with respect to the
      ++ fixed \spad{R}-module basis.
    elt : (%,Integer) -> R
      ++ elt(a,i) returns the i-th coefficient of \spad{a} with respect
      ++ to the fixed \spad{R}-module basis.
    structuralConstants:() -> Vector Matrix R
      ++ structuralConstants() calculates the structural constants
```

```
    ++ \spad{[(gammaijk) for k in 1..rank()]} defined by
    ++ \spad{vi * vj = gammaij1 * v1 + ... + gammaijn * vn},
    ++ where \spad{v1},...,\spad{vn} is the fixed \spad{R}-module basis.
  conditionsForIdempotents: () -> List Polynomial R
    ++ conditionsForIdempotents() determines a complete list
    ++ of polynomial equations for the coefficients of idempotents
    ++ with respect to the fixed \spad{R}-module basis.
  represents: Vector R -> %
    ++ represents([a1,...,an]) returns \spad{a1*v1 + ... + an*vn},
    ++ where \spad{v1}, ..., \spad{vn} are the elements of the
    ++ fixed \spad{R}-module basis.
  convert: % -> Vector R
    ++ convert(a) returns the coordinates of \spad{a} with respect to the
    ++ fixed \spad{R}-module basis.
  convert: Vector R -> %
    ++ convert([a1,...,an]) returns \spad{a1*v1 + ... + an*vn},
    ++ where \spad{v1}, ..., \spad{vn} are the elements of the
    ++ fixed \spad{R}-module basis.
  leftDiscriminant : () -> R
    ++ leftDiscriminant() returns the
    ++ determinant of the \spad{n}-by-\spad{n}
    ++ matrix whose element at the \spad{i}-th row and \spad{j}-th column
    ++ is given by the left trace of the product \spad{vi*vj}, where
    ++ \spad{v1},...,\spad{vn} are the
    ++ elements of the fixed \spad{R}-module basis.
    ++ Note: the same as \spad{determinant(leftTraceMatrix())}.
  rightDiscriminant : () -> R
    ++ rightDiscriminant() returns the determinant of the
    ++ \spad{n}-by-\spad{n} matrix whose element at the \spad{i}-th row
    ++ and \spad{j}-th column is
    ++ given by the right trace of the product \spad{vi*vj}, where
    ++ \spad{v1},...,\spad{vn} are the elements of
    ++ the fixed \spad{R}-module basis.
    ++ Note: the same as \spad{determinant(rightTraceMatrix())}.
  leftTraceMatrix : () -> Matrix R
    ++ leftTraceMatrix() is the \spad{n}-by-\spad{n}
    ++ matrix whose element at the \spad{i}-th row and \spad{j}-th column
    ++ is given by left trace of the product \spad{vi*vj},
    ++ where \spad{v1},...,\spad{vn} are the
    ++ elements of the fixed \spad{R}-module basis.
  rightTraceMatrix : () -> Matrix R
    ++ rightTraceMatrix() is the \spad{n}-by-\spad{n}
    ++ matrix whose element at the \spad{i}-th row and \spad{j}-th column
    ++ is given by the right trace of the product \spad{vi*vj}, where
    ++ \spad{v1},...,\spad{vn} are the elements
    ++ of the fixed \spad{R}-module basis.
```

```
  leftRegularRepresentation : % -> Matrix R
    ++ leftRegularRepresentation(a) returns the matrix of the linear
    ++ map defined by left multiplication by \spad{a} with respect
    ++ to the fixed \spad{R}-module basis.
  rightRegularRepresentation : % -> Matrix R
    ++ rightRegularRepresentation(a) returns the matrix of the linear
    ++ map defined by right multiplication by \spad{a} with respect
    ++ to the fixed \spad{R}-module basis.
  if R has Field then
    leftRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R
      ++ leftRankPolynomial() calculates the left minimal polynomial
      ++ of the generic element in the algebra,
      ++ defined by the same structural
      ++ constants over the polynomial ring in symbolic coefficients with
      ++ respect to the fixed basis.
    rightRankPolynomial : () -> SparseUnivariatePolynomial Polynomial R
      ++ rightRankPolynomial() calculates the right minimal polynomial
      ++ of the generic element in the algebra,
      ++ defined by the same structural
      ++ constants over the polynomial ring in symbolic coefficients with
      ++ respect to the fixed basis.
  apply: (Matrix R, %) -> %
    ++ apply(m,a) defines a left operation of n by n matrices
    ++ where n is the rank of the algebra in terms of matrix-vector
    ++ multiplication, this is a substitute for a left module structure.
    ++ Error: if shape of matrix doesn't fit.
  --attributes
    --separable <=> discriminant() ^= 0
add

  V  ==> Vector
  M  ==> Matrix
  P  ==> Polynomial
  F  ==> Fraction
  REC  ==> Record(particular: Union(V R,"failed"),basis: List V R)
  LSMP ==> LinearSystemMatrixPackage(R,V R,V R, M R)
  CVMP ==> CoerceVectorMatrixPackage(R)

  --GA ==> GenericNonAssociativeAlgebra(R,rank()$%,_
  -- [random()$Character :: String :: Symbol for i in 1..rank()$%], _
  -- structuralConstants()$%)
  --y : GA := generic()
  if R has Field then
    leftRankPolynomial() ==
      n := rank()
      b := basis()
```

```
    gamma : Vector Matrix R := structuralConstants b
    listOfNumbers : List String :=  [STRINGIMAGE(q)$Lisp for q in 1..n]
    symbolsForCoef : Vector Symbol :=
      [concat("%", concat("x", i))::Symbol  for i in listOfNumbers]
    xx : M P R
    mo : P R
    x : M P R := new(1,n,0)
    for i in 1..n repeat
      mo := monomial(1, [symbolsForCoef.i], [1])$(P R)
      qsetelt_!(x,1,i,mo)
    y : M P R := copy x
    k  : PositiveInteger := 1
    cond : M P R := copy x
    -- multiplication in the generic algebra means using
    -- the structural matrices as bilinear forms.
    -- left multiplication by x, we prepare for that:
    genGamma : V M P R :=  coerceP$CVMP gamma
    x := reduce(horizConcat,[x*genGamma(i) for i in 1..#genGamma])
    while rank(cond) = k repeat
      k := k+1
      for i in 1..n repeat
        setelt(xx,[1],[i],x*transpose y)
      y := copy xx
      cond := horizConcat(cond, xx)
    vectorOfCoef : Vector P R := (nullSpace(cond)$Matrix(P R)).first
    res : SparseUnivariatePolynomial P R := 0
    for i in 1..k repeat
     res:=res+monomial(vectorOfCoef.i,i)$(SparseUnivariatePolynomial P R)
    res

  rightRankPolynomial() ==
    n := rank()
    b := basis()
    gamma : Vector Matrix R := structuralConstants b
    listOfNumbers : List String :=  [STRINGIMAGE(q)$Lisp for q in 1..n]
    symbolsForCoef : Vector Symbol :=
      [concat("%", concat("x", i))::Symbol  for i in listOfNumbers]
    xx : M P R
    mo : P R
    x : M P R := new(1,n,0)
    for i in 1..n repeat
      mo := monomial(1, [symbolsForCoef.i], [1])$(P R)
      qsetelt_!(x,1,i,mo)
    y : M P R := copy x
    k  : PositiveInteger := 1
    cond : M P R := copy x
```

```
  -- multiplication in the generic algebra means using
  -- the structural matrices as bilinear forms.
  -- left multiplication by x, we prepare for that:
  genGamma : V M P R :=  coerceP$CVMP gamma
  x := _
   reduce(horizConcat,[genGamma(i)*transpose x for i in 1..#genGamma])
  while rank(cond) = k repeat
    k := k+1
    for i in 1..n repeat
      setelt(xx,[1],[i],y * transpose x)
    y := copy xx
    cond := horizConcat(cond, xx)
  vectorOfCoef : Vector P R := (nullSpace(cond)$Matrix(P R)).first
  res : SparseUnivariatePolynomial P R := 0
  for i in 1..k repeat
   res := _
    res+monomial(vectorOfCoef.i,i)$(SparseUnivariatePolynomial  P R)
  res

leftUnitsInternal : () -> REC
leftUnitsInternal() ==
  n := rank()
  b := basis()
  gamma : Vector Matrix R := structuralConstants b
  cond : Matrix(R) := new(n**2,n,0$R)$Matrix(R)
  rhs : Vector(R) := new(n**2,0$R)$Vector(R)
  z : Integer := 0
  addOn : R := 0
  for k in 1..n repeat
   for i in 1..n repeat
     z := z+1   -- index for the rows
     addOn :=
       k=i => 1
       0
     setelt(rhs,z,addOn)$Vector(R)
     for j in 1..n repeat  -- index for the columns
       setelt(cond,z,j,elt(gamma.k,j,i))$Matrix(R)
  solve(cond,rhs)$LSMP


leftUnit() ==
  res : REC := leftUnitsInternal()
  res.particular case "failed" =>
    messagePrint("this algebra has no left unit")$OutputForm
    "failed"
  represents (res.particular :: V R)
```

```
leftUnits() ==
  res : REC := leftUnitsInternal()
  res.particular case "failed" =>
    messagePrint("this algebra has no left unit")$OutputForm
    "failed"
  [represents(res.particular :: V R)$%, _
    map(represents, res.basis)$ListFunctions2(Vector R, %) ]

rightUnitsInternal : () -> REC
rightUnitsInternal() ==
  n := rank()
  b := basis()
  gamma : Vector Matrix R := structuralConstants b
  condo : Matrix(R) := new(n**2,n,0$R)$Matrix(R)
  rhs : Vector(R) := new(n**2,0$R)$Vector(R)
  z : Integer := 0
  addOn : R := 0
  for k in 1..n repeat
   for i in 1..n repeat
     z := z+1   -- index for the rows
     addOn :=
       k=i => 1
       0
     setelt(rhs,z,addOn)$Vector(R)
     for j in 1..n repeat  -- index for the columns
       setelt(condo,z,j,elt(gamma.k,i,j))$Matrix(R)
  solve(condo,rhs)$LSMP

rightUnit() ==
  res : REC := rightUnitsInternal()
  res.particular case "failed" =>
    messagePrint("this algebra has no right unit")$OutputForm
    "failed"
  represents (res.particular :: V R)

rightUnits() ==
  res : REC := rightUnitsInternal()
  res.particular case "failed" =>
    messagePrint("this algebra has no right unit")$OutputForm
    "failed"
  [represents(res.particular :: V R)$%, _
    map(represents, res.basis)$ListFunctions2(Vector R, %) ]

unit() ==
  n := rank()
```

```
    b := basis()
    gamma : Vector Matrix R := structuralConstants b
    cond : Matrix(R) := new(2*n**2,n,0$R)$Matrix(R)
    rhs : Vector(R) := new(2*n**2,0$R)$Vector(R)
    z : Integer := 0
    u : Integer := n*n
    addOn : R := 0
    for k in 1..n repeat
     for i in 1..n repeat
       z := z+1   -- index for the rows
       addOn :=
         k=i => 1
         0
       setelt(rhs,z,addOn)$Vector(R)
       setelt(rhs,u,addOn)$Vector(R)
       for j in 1..n repeat  -- index for the columns
         setelt(cond,z,j,elt(gamma.k,j,i))$Matrix(R)
         setelt(cond,u,j,elt(gamma.k,i,j))$Matrix(R)
    res : REC := solve(cond,rhs)$LSMP
    res.particular case "failed" =>
      messagePrint("this algebra has no unit")$OutputForm
      "failed"
    represents (res.particular :: V R)
apply(m:Matrix(R),a:%) ==
  v : Vector R := coordinates(a)
  v := m *$Matrix(R) v
  convert v


structuralConstants()   == structuralConstants basis()
conditionsForIdempotents() == conditionsForIdempotents basis()
convert(x:%):Vector(R)  == coordinates(x, basis())
convert(v:Vector R):%   == represents(v, basis())
leftTraceMatrix()       == leftTraceMatrix basis()
rightTraceMatrix()      == rightTraceMatrix basis()
leftDiscriminant()      == leftDiscriminant basis()
rightDiscriminant()     == rightDiscriminant basis()
leftRegularRepresentation x == leftRegularRepresentation(x, basis())
rightRegularRepresentation x == rightRegularRepresentation(x, basis())
coordinates x           == coordinates(x, basis())
represents(v:Vector R):%== represents(v, basis())

coordinates(v:Vector %) ==
  m := new(#v, rank(), 0)$Matrix(R)
  for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
    setRow_!(m, j, coordinates qelt(v, i))
```

```
        m
```

⟨*FRNAALG.dotabb*⟩≡

```
  "FRNAALG"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FRNAALG"];
  "FRNAALG" -> "FINAALG"
```

⟨*FRNAALG.dotfull*⟩≡

```
  "FramedNonAssociativeAlgebra(a:CommutativeRing)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FRNAALG"];
  "FramedNonAssociativeAlgebra(a:CommutativeRing)" ->
      "FiniteRankNonAssociativeAlgebra(a:CommutativeRing)"
```

⟨*FRNAALG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FramedNonAssociativeAlgebra(a:CommutativeRing)" [color=lightblue];
"FramedNonAssociativeAlgebra(a:CommutativeRing)" ->
    "FiniteRankNonAssociativeAlgebra(a:CommutativeRing)"

"FiniteRankNonAssociativeAlgebra(a:CommutativeRing)" [color=lightblue];
"FiniteRankNonAssociativeAlgebra(a:CommutativeRing)" ->
    "NonAssociativeAlgebra(a:CommutativeRing)"

"NonAssociativeAlgebra(a:CommutativeRing)" [color=lightblue];
"NonAssociativeAlgebra(a:CommutativeRing)" -> "NonAssociativeRng()"
"NonAssociativeAlgebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"NonAssociativeRng()" [color=lightblue];
"NonAssociativeRng()" -> "ABELGRP..."
"NonAssociativeRng()" -> "Monad()"

"Monad()" [color=lightblue];
"Monad()" -> "SETCAT..."
"Monad()" -> "REPSQ..."

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BiModule(a:Ring,b:Ring)"

"BiModule(a:Ring,b:Ring)" [color=lightblue];
"BiModule(a:Ring,b:Ring)" -> "LeftModule(a:Ring)"
"BiModule(a:Ring,b:Ring)" -> "RightModule(a:Ring)"

"RightModule(a:Ring)" [color=seagreen];
"RightModule(a:Ring)" -> "RightModule(a:Rng)"

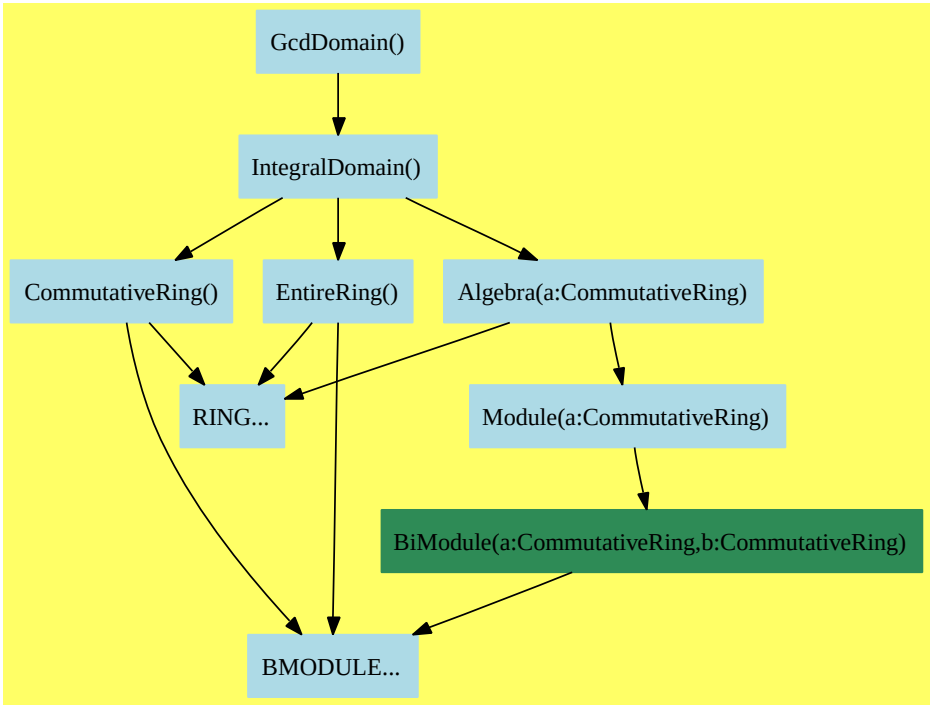
"RightModule(a:Rng)" [color=lightblue];
"RightModule(a:Rng)" -> "ABELGRP..."

"LeftModule(a:Ring)" [color=seagreen];
"LeftModule(a:Ring)" -> "LeftModule(a:Rng)"
```

```
"LeftModule(a:Rng)" [color=lightblue];
"LeftModule(a:Rng)" -> "ABELGRP..."

"REPSQ..." [color="#00EE00"];
"SETCAT..." [color=lightblue];
"ABELGRP..." [color=lightblue];
}
```

13.4 GcdDomain (GCDDOM)



See:

⇒ "IntervalCategory" (INTCAT) 14.2 on page 950
⇒ "PolynomialCategory" (POLYCAT) 16.4 on page 1027
⇒ "PrincipalIdealDomain" (PID) 14.4 on page 965
⇒ "UniqueFactorizationDomain" (UFD) 14.5 on page 969
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.12 on page 1204
⇐ "IntegralDomain" (INTDOM) 12.5 on page 857

Exports:

0	1	associates?	characteristic	coerce
exquo	gcd	gcdPolynomial	hash	latex
lcm	one?	recip	sample	subtractIfCan
unit?	unitCanonical	unitNormal	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?~=?			

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
gcd : (%,%) -> %
```

These are implemented by this category:

```
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
lcm : (%,%) -> %
lcm : List % -> %
```

These exports come from (p857) IntegralDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
exquo : (%,%) -> Union(%,"failed")
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
```

```
 -? : % -> %
 ?**? : (%,PositiveInteger) -> %
 ?**? : (%,NonNegativeInteger) -> %
 ?^? : (%,PositiveInteger) -> %
 ?^? : (%,NonNegativeInteger) -> %
```

⟨*category GCDDOM GcdDomain*⟩≡

```
 )abbrev category GCDDOM GcdDomain
 ++ Author:
 ++ Date Created:
 ++ Date Last Updated:
 ++ Basic Functions:
 ++ Related Constructors:
 ++ Also See:
 ++ AMS Classifications:
 ++ Keywords:
 ++ References: Davenport & Trager 1
 ++ Description:
 ++ This category describes domains where
 ++ \spadfun{gcd} can be computed but where there is no guarantee
 ++ of the existence of \spadfun{factor} operation for factorisation
 ++ into irreducibles. However, if such a \spadfun{factor} operation exist,
 ++ factorization will be unique up to order and units.

 GcdDomain(): Category == IntegralDomain with
     gcd: (%,%) -> %
       ++ gcd(x,y) returns the greatest common divisor of x and y.
       -- gcd(x,y) = gcd(y,x) in the presence of canonicalUnitNormal,
       -- but not necessarily elsewhere
     gcd: List(%) -> %
       ++ gcd(l) returns the common gcd of the elements in the list l.
     lcm: (%,%) -> %
       ++ lcm(x,y) returns the least common multiple of x and y.
       -- lcm(x,y) = lcm(y,x) in the presence of canonicalUnitNormal,
       -- but not necessarily elsewhere
     lcm: List(%) -> %
       ++ lcm(l) returns the least common multiple of the elements of
       ++ the list l.
     gcdPolynomial: (SparseUnivariatePolynomial %, _
                     SparseUnivariatePolynomial %) -> _
                        SparseUnivariatePolynomial %
       ++ gcdPolynomial(p,q) returns the greatest common divisor (gcd) of
       ++ univariate polynomials over the domain
   add
    lcm(x: %,y: %) ==
      y = 0 => 0
      x = 0 => 0
```

```
      LCM : Union(%,"failed") := y exquo gcd(x,y)
      LCM case % =>  x * LCM
      error "bad gcd in lcm computation"
    lcm(l:List %) == reduce(lcm,l,1,0)
    gcd(l:List %) == reduce(gcd,l,0,1)
    SUP ==> SparseUnivariatePolynomial
    gcdPolynomial(p1,p2) ==
      zero? p1 => unitCanonical p2
      zero? p2 => unitCanonical p1
      c1:= content(p1); c2:= content(p2)
      p1:= (p1 exquo c1)::SUP %
      p2:= (p2 exquo c2)::SUP %
      if (e1:=minimumDegree p1) > 0 then p1:=(p1 exquo monomial(1,e1))::SUP %
      if (e2:=minimumDegree p2) > 0 then p2:=(p2 exquo monomial(1,e2))::SUP %
      e1:=min(e1,e2); c1:=gcd(c1,c2)
      p1:=
         degree p1 = 0 or degree p2 = 0 => monomial(c1,0)
         p:= subResultantGcd(p1,p2)
         degree p = 0 => monomial(c1,0)
         c2:= gcd(leadingCoefficient p1,leadingCoefficient p2)
         unitCanonical(_
           c1 * primitivePart(((c2*p) exquo leadingCoefficient p)::SUP %))
      zero? e1 => p1
      monomial(1,e1)*p1
```

⟨*GCDDOM.dotabb*⟩≡

```
"GCDDOM"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=GCDDOM"];
"GCDDOM" -> "INTDOM"
```

⟨*GCDDOM.dotfull*⟩≡

```
"GcdDomain()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=GCDDOM"];
"GcdDomain()" -> "IntegralDomain()"
```

⟨*GCDDOM.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

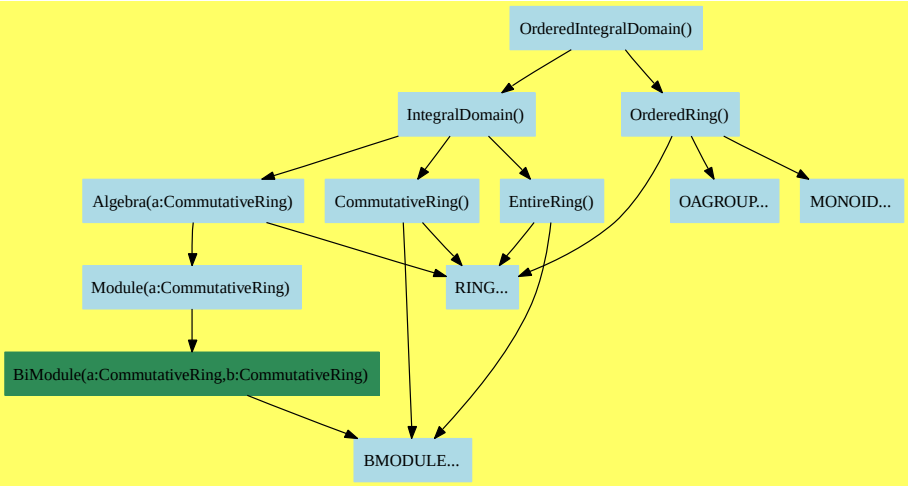
"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

13.5 OrderedIntegralDomain (OINTDOM)



See:

⇒ "IntegerNumberSystem" (INS) 16.2 on page 1011
⇐ "IntegralDomain" (INTDOM) 12.5 on page 857
⇐ "QuotientFieldCategory" (QFCAT) 17.6 on page 1121
⇐ "OrderedRing" (ORDRING) 10.11 on page 715

Exports:

1	0	abs	associates?	characteristic
coerce	exquo	hash	latex	max
min	negative?	one?	positive?	recip
sample	sign	subtractIfCan	unit?	unitCanonical
unitNormal	zero?	?*?	?**?	?+?
?-?	-?	?<?	?<=?	?=?
?>?	?>=?	?^?	?~=?	

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These exports come from (p857) IntegralDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
exquo : (%,%) -> Union(%,"failed")
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?<? : (%,%) -> Boolean
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p715) OrderedRing():

```
abs : % -> %
max : (%,%) -> %
min : (%,%) -> %
negative? : % -> Boolean
positive? : % -> Boolean
sign : % -> Integer
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

⟨*category OINTDOM OrderedIntegralDomain*⟩≡

```
)abbrev category OINTDOM OrderedIntegralDomain
++ Author: JH Davenport (after L Gonzalez-Vega)
```

```
  ++ Date Created: 30.1.96
  ++ Date Last Updated:
  ++ Basic Functions:
  ++ Related Constructors:
  ++ Also See:
  ++ AMS Classifications:
  ++ Keywords:
  ++ Description:
  ++ The category of ordered commutative integral domains, where ordering
  ++ and the arithmetic operations are compatible
  ++

  OrderedIntegralDomain(): Category ==
    Join(IntegralDomain, OrderedRing)
```

⟨*OINTDOM.dotabb*⟩≡

```
  "OINTDOM"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=OINTDOM"];
  "OINTDOM" -> "INTDOM"
  "OINTDOM" -> "ORDRING"
```

⟨*OINTDOM.dotfull*⟩≡

```
  "OrderedIntegralDomain()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=OINTDOM"];
  "OrderedIntegralDomain()" -> "IntegralDomain()"
  "OrderedIntegralDomain()" -> "OrderedRing()"
```

⟨*OINTDOM.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"OrderedIntegralDomain()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=OINTDOM"];
"OrderedIntegralDomain()" -> "IntegralDomain()"
"OrderedIntegralDomain()" -> "OrderedRing()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"OrderedRing()" [color=lightblue];
"OrderedRing()" -> "OAGROUP..."
"OrderedRing()" -> "RING..."
"OrderedRing()" -> "MONOID..."

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

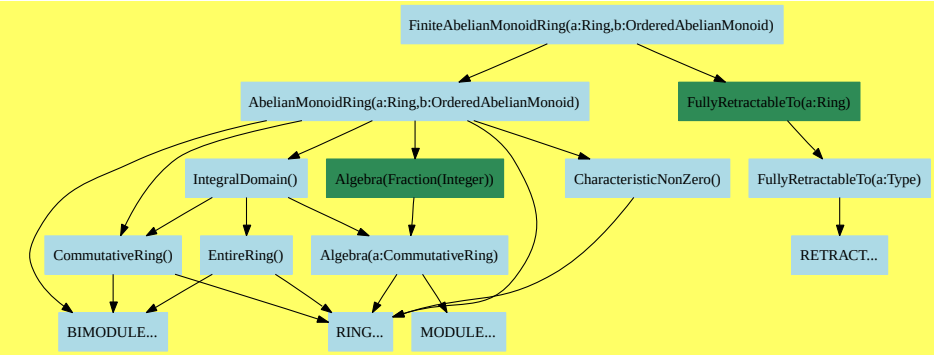
"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"OAGROUP..." [color=lightblue];
"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
"MONOID..." [color=lightblue];
}
```

Chapter 14

Category Layer 13

14.1 FiniteAbelianMonoidRing (FAMR)



See:

⇒ “PolynomialCategory” (POLYCAT) 16.4 on page 1027
⇐ “AbelianMonoidRing” (AMR) 13.1 on page 905
⇐ “FullyRetractableTo” (FRETRCT) 3.6 on page 71

Exports:

0	1	associates?	binomThmExpt
characteristic	charthRoot	coefficient	coefficients
coerce	content	degree	exquo
ground	ground?	hash	latex
leadingCoefficient	leadingMonomial	map	mapExponents
minimumDegree	monomial	monomial?	numberOfMonomials
one?	pomopo!	primitivePart	recip
reductum	retract	retractIfCan	sample
subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?*?	?**?	?+?
?-?	-?	?=?	?^?
?~=?	?/?		

Attributes exported:

- if $ has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- if $ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
minimumDegree : % -> E
numberOfMonomials : % -> NonNegativeInteger
```

These are implemented by this category:

```
binomThmExpt : (%,%,NonNegativeInteger) -> %
    if R has COMRING
coefficients : % -> List R
content : % -> R if R has GCDDOM
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
ground : % -> R
ground? : % -> Boolean
mapExponents : ((E -> E),%) -> %
pomopo! : (%,R,E,%) -> %
primitivePart : % -> % if R has GCDDOM
?/? : (%,R) -> % if R has FIELD
```

These exports come from (p905) AbelianMonoidRing(R,E)
where R:Ring and E:OrderedAbelianMonoid:

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean if R has INTDOM
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if R has CHARNZ
coefficient : (%,E) -> R
coerce : R -> %
coerce : Fraction Integer -> %
    if R has RETRACT FRAC INT
    or R has ALGEBRA FRAC INT
coerce : % -> % if R has INTDOM
coerce : % -> OutputForm
coerce : Integer -> %
degree : % -> E
exquo : (%,%) -> Union(%,"failed") if R has INTDOM
hash : % -> SingleInteger
latex : % -> String
leadingCoefficient : % -> R
leadingMonomial : % -> %
map : ((R -> R),%) -> %
monomial : (R,E) -> %
monomial? : % -> Boolean
one? : % -> Boolean
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal :
   % -> Record(unit: %,canonical: %,associate: %)
   if R has INTDOM
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (R,%) -> %
?*? : (%,R) -> %
?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (%,%) -> %
?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p71) FullyRetractableTo(R:Ring):

```
retract : % -> Fraction Integer
    if R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retract : % -> R
retractIfCan : % -> Union(R,"failed")
retractIfCan : % -> Union(Integer,"failed")
    if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if R has RETRACT FRAC INT
```

⟨*category FAMR FiniteAbelianMonoidRing*⟩≡

```
 )abbrev category FAMR FiniteAbelianMonoidRing
 ++ Author:
 ++ Date Created:
 ++ Date Last Updated: 14.08.2000 Exported pomopo! and binomThmExpt [MMM]
 ++ Basic Functions:
 ++ Related Constructors:
 ++ Also See:
 ++ AMS Classifications:
 ++ Keywords:
 ++ References:
 ++ Description: This category is
 ++ similar to AbelianMonoidRing, except that the sum is assumed to be finite.
 ++ It is a useful model for polynomials,
 ++ but is somewhat more general.

 FiniteAbelianMonoidRing(R:Ring, E:OrderedAbelianMonoid): Category ==
    Join(AbelianMonoidRing(R,E),FullyRetractableTo R) with
     ground?: % -> Boolean
       ++ ground?(p) tests if polynomial p is a member of the
       ++ coefficient ring.
       -- can't be defined earlier, since a power series
       -- might not know if there were other terms or not
     ground: % -> R
       ++ ground(p) retracts polynomial p to the coefficient ring.
     coefficients: % -> List R
       ++ coefficients(p) gives the list of non-zero coefficients
       ++ of polynomial p.
     numberOfMonomials: % -> NonNegativeInteger
       ++ numberOfMonomials(p) gives the number of non-zero monomials
       ++ in polynomial p.
     minimumDegree: % -> E
       ++ minimumDegree(p) gives the least exponent of a non-zero term
       ++ of polynomial p. Error: if applied to 0.
     mapExponents: (E -> E, %) -> %
```

```
      ++ mapExponents(fn,u) maps function fn onto the exponents
      ++ of the non-zero monomials of polynomial u.
    pomopo!: (%,R,E,%) -> %
      ++ \spad{pomopo!(p1,r,e,p2)} returns \spad{p1 + monomial(e,r) * p2}
      ++ and may use \spad{p1} as workspace. The constaant \spad{r} is
      ++ assumed to be nonzero.
    if R has CommutativeRing then
       binomThmExpt: (%,%,NonNegativeInteger) -> %
         ++ \spad{binomThmExpt(p,q,n)} returns \spad{(x+y)^n}
         ++ by means of the binomial theorem trick.
    if R has IntegralDomain then
       "exquo": (%,R) -> Union(%,"failed")
       ++ exquo(p,r) returns the exact quotient of polynomial p by r,
       ++ or "failed" if none exists.
    if R has GcdDomain then
       content: % -> R
         ++ content(p) gives the gcd of the coefficients of polynomial p.
       primitivePart: % -> %
         ++ primitivePart(p) returns the unit normalized form of polynomial p
         ++ divided by the content of p.
  add
    pomopo!(p1,r,e,p2) == p1 + r * mapExponents(x1+->x1+e,p2)

    if R has CommutativeRing then
       binomThmExpt(x,y,nn) ==
               nn = 0 => 1$%
               ans,xn,yn: %
               bincoef: Integer
               powl: List(%):= [x]
               for i in 2..nn repeat powl:=[x * powl.first, :powl]
               yn:=y; ans:=powl.first; i:=1; bincoef:=nn
               for xn in powl.rest repeat
                  ans:= bincoef * xn * yn + ans
                  bincoef:= (nn-i) * bincoef quo (i+1);  i:= i+1
                  -- last I and BINCOEF unused
                  yn:= y * yn
               ans + yn
    ground? x ==
      retractIfCan(x)@Union(R,"failed") case "failed" => false
      true

    ground x == retract(x)@R

    mapExponents (fn:E -> E, x: %) ==
         -- this default definition assumes that reductum is cheap
       zero? x => 0
```

```
        monomial(leadingCoefficient x,fn degree x)+mapExponents(fn,reductum x)

      coefficients x ==
        zero? x => empty()
        concat(leadingCoefficient x, coefficients reductum x)

      if R has Field then
         x/r == map(x1+->x1/r,x)

      if R has IntegralDomain then
         x exquo r ==
            -- probably not a very good definition in most special cases
            zero? x => 0
            ans:% :=0
            t:=leadingCoefficient x exquo r
            while not (t case "failed") and not zero? x repeat
              ans:=ans+monomial(t::R,degree x)
              x:=reductum x
              if not zero? x then t:=leadingCoefficient x exquo r
            t case "failed" => "failed"
            ans

      if R has GcdDomain then
         content x ==       -- this assumes  reductum is cheap
            zero? x => 0
            r:=leadingCoefficient x
            x:=reductum x
  --          while not zero? x and not one? r repeat
            while not zero? x and not (r = 1) repeat
              r:=gcd(r,leadingCoefficient x)
              x:=reductum x
            r

         primitivePart x ==
            zero? x => x
            c := content x
            unitCanonical((x exquo c)::%)
```

⟨*FAMR.dotabb*⟩≡

```
  "FAMR"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FAMR"];
  "FAMR" -> "AMR"
  "FAMR" -> "FRETRCT"
```

⟨*FAMR.dotfull*⟩≡

```
  "FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FAMR"];
  "FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
      "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"
  "FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
      "FullyRetractableTo(a:Ring)"

  "FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoidSup)"
   [color=seagreen,href="bookvol10.2.pdf#nameddest=FAMR"];
  "FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoidSup)" ->
      "FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"
```

⟨*FAMR.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" [color=lightblue];
"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"
"FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "FullyRetractableTo(a:Ring)"

"FullyRetractableTo(a:Ring)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(a:Ring)" -> "FullyRetractableTo(a:Type)"

"FullyRetractableTo(a:Type)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRETRCT"];
"FullyRetractableTo(a:Type)" -> "RETRACT..."

"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" [color=lightblue];
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" -> "RING..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "BIMODULE..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "IntegralDomain()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CharacteristicNonZero()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CommutativeRing()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "Algebra(Fraction(Integer))"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BIMODULE..."

"CharacteristicNonZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CharacteristicNonZero()" -> "RING..."
```

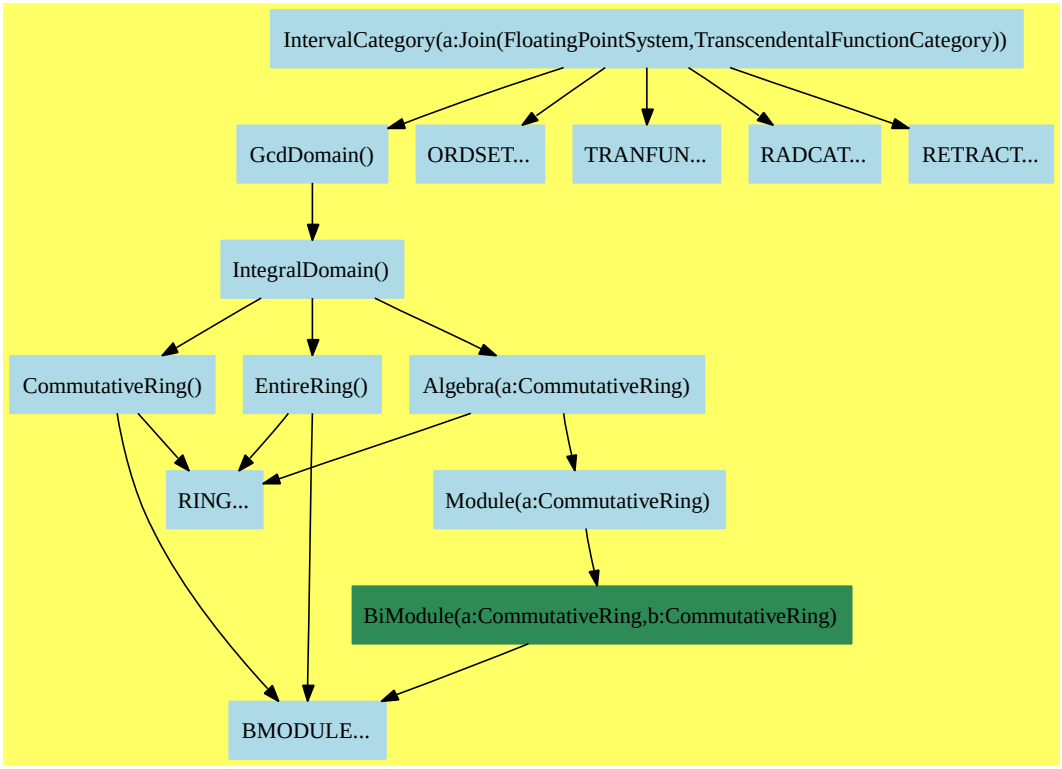
```
"Algebra(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BIMODULE..."

"RETRACT..." [color=lightblue];
"BIMODULE..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

14.2 IntervalCategory (INTCAT)



See:

⇐ “GcdDomain” (GCDDOM) 13.4 on page 932
⇐ “OrderedSet” (ORDSET) 4.19 on page 168
⇐ “RadicalCategory” (RADCAT) 2.17 on page 42
⇐ “RetractableTo” (RETRACT) 2.18 on page 44
⇐ “TranscendentalFunctionCategory” (TRANFUN) 3.14 on page 94

Exports:

0	1	acos	acosh	acot
acoth	acsc	acsch	asec	asech
asin	asinh	associates?	atan	atanh
characteristic	coerce	contains?	cos	cosh
cot	coth	csc	csch	exp
exquo	gcd	gcdPolynomial	hash	inf
interval	latex	lcm	log	max
min	negative?	nthRoot	one?	pi
positive?	qinterval	recip	retract	retractIfCan
sample	sec	sech	sin	sinh
sqrt	subtractIfCan	sup	tan	tanh
unit?	unitCanonical	unitNormal	width	zero?
?*?	?**?	?+?	?-?	-?
?<?	?<=?	?=?	?>?	?>=?
?^?	?~=?			

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **approximate** means "is an approximation to the real numbers".

TPDHERE: Note that the signature `coerce : Integer -> %` shows up twice.

These are directly exported but not implemented:

```
contains? : (%,R) -> Boolean
inf : % -> R
interval : (R,R) -> %
interval : R -> %
interval : Fraction Integer -> %
negative? : % -> Boolean
positive? : % -> Boolean
qinterval : (R,R) -> %
sup : % -> R
width : % -> R
```

These exports come from (p932) GcdDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
exquo : (%,%) -> Union(%,"failed")
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial :
 (SparseUnivariatePolynomial %,
  SparseUnivariatePolynomial %) ->
    SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

These exports come from (p168) OrderedSet():

```
max : (%,%) -> %
min : (%,%) -> %
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
```

These exports come from (p94) TranscendentalFunctionCategory():

```
acos : % -> %
acosh : % -> %
acot : % -> %
acoth : % -> %
acsc : % -> %
acsch : % -> %
asec : % -> %
asech : % -> %
asin : % -> %
asinh : % -> %
atan : % -> %
atanh : % -> %
cos : % -> %
cosh : % -> %
cot : % -> %
coth : % -> %
csc : % -> %
csch : % -> %
exp : % -> %
log : % -> %
pi : () -> %
sec : % -> %
sech : % -> %
sin : % -> %
sinh : % -> %
tan : % -> %
tanh : % -> %
?**? : (%,%) -> %
```

These exports come from (p42) RadicalCategory():

```
nthRoot : (%,Integer) -> %
sqrt : % -> %
?**? : (%,Fraction Integer) -> %
```

These exports come from (p44) RetractableTo(Integer):

```
coerce : Integer -> %
retract : % -> Integer
retractIfCan : % -> Union(Integer,"failed")
```

⟨*category INTCAT IntervalCategory*⟩≡

```
)abbrev category INTCAT IntervalCategory
+++ Author: Mike Dewar
+++ Date Created: November 1996
+++ Date Last Updated:
+++ Basic Functions:
+++ Related Constructors:
+++ Also See:
```

```
+++ AMS Classifications:
+++ Keywords:
+++ References:
+++ Description:
+++ This category implements of interval arithmetic and transcendental
+++ functions over intervals.
IntervalCategory(R: Join(FloatingPointSystem,TranscendentalFunctionCategory)):
 Category == _
  Join(GcdDomain, OrderedSet, TranscendentalFunctionCategory, _
       RadicalCategory, RetractableTo(Integer)) with
  approximate
  interval : (R,R) -> %
    ++ interval(inf,sup) creates a new interval, either \axiom{[inf,sup]} if
    ++ \axiom{inf <= sup} or \axiom{[sup,in]} otherwise.
  qinterval : (R,R) -> %
    ++ qinterval(inf,sup) creates a new interval \axiom{[inf,sup]}, without
    ++ checking the ordering on the elements.
  interval : R -> %
    ++ interval(f) creates a new interval around f.
  interval : Fraction Integer -> %
    ++ interval(f) creates a new interval around f.
  inf : % -> R
    ++ inf(u) returns the infinum of \axiom{u}.
  sup : % -> R
    ++ sup(u) returns the supremum of \axiom{u}.
  width : % -> R
    ++ width(u) returns \axiom{sup(u) - inf(u)}.
  positive? : % -> Boolean
    ++ positive?(u) returns \axiom{true} if every element of u is positive,
    ++ \axiom{false} otherwise.
  negative? : % -> Boolean
    ++ negative?(u) returns \axiom{true} if every element of u is negative,
    ++ \axiom{false} otherwise.
  contains? : (%,R) -> Boolean
    ++ contains?(i,f) returns true if \axiom{f} is contained within the
    ++ interval \axiom{i}, false otherwise.
```

⟨*INTCAT.dotabb*⟩≡
```
"INTCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INTCAT"];
"INTCAT" -> "GCDDOM"
"INTCAT" -> "ORDSET"
"INTCAT" -> "RADCAT"
"INTCAT" -> "RETRACT"
"INTCAT" -> "TRANFUN"
```

⟨*INTCAT.dotfull*⟩≡
```
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INTCAT"];
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "GcdDomain()"
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "OrderedSet()"
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "TranscendentalFunctionCategory()"
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "RadicalCategory()"
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "RetractableTo(Integer)"
```

⟨*INTCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
 [color=lightblue];
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "GcdDomain()"
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "ORDSET..."
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "TRANFUN..."
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "RADCAT..."
"IntervalCategory(a:Join(FloatingPointSystem,TranscendentalFunctionCategory))"
  -> "RETRACT..."

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

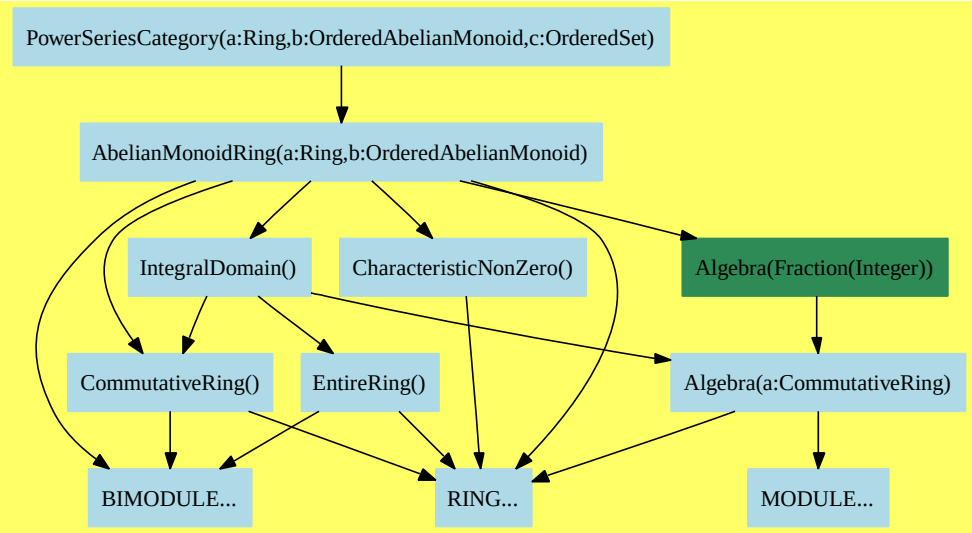
"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."
```

```
"RETRACT..." [color=lightblue];
"RADCAT..." [color=lightblue];
"TRANFUN..." [color=lightblue];
"ORDSET..." [color=lightblue];
"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

14.3 PowerSeriesCategory (PSCAT)



See:

⇒ "MultivariateTaylorSeriesCategory" (MTSCAT) 15.2 on page 983
⇒ "UnivariatePowerSeriesCategory" (UPSCAT) 15.4 on page 996
⇐ "AbelianMonoidRing" (AMR) 13.1 on page 905

Exports:

0	1	associates?	characteristic	charthRoot
coefficient	coerce	complete	degree	exquo
hash	latex	leadingCoefficient	leadingMonomial	map
monomial	monomial?	one?	pole?	recip
reductum	sample	subtractIfCan	variables	unit?
unitCanonical	unitNormal	zero?	?*?	?**?
?+?	?-?	-?	?=?	?~=?
?/?	?^?			

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.

- if #1 has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.

These are directly exported but not implemented:

```
complete : % -> %
degree : % -> Expon
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
monomial : (%,List Var,List Expon) -> %
monomial : (%,Var,Expon) -> %
pole? : % -> Boolean
variables : % -> List Var
```

These are implemented by this category:

```
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
-? : % -> %
```

These exports come from (p905) AbelianMonoidRing(Coef,Expon)
where Coef:Ring and Expon:OrderedAbelianMonoid:

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean if Coef has INTDOM
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,Expon) -> Coef
coerce : Coef -> % if Coef has COMRING
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : % -> % if Coef has INTDOM
coerce : % -> OutputForm
coerce : Integer -> %
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
hash : % -> SingleInteger
latex : % -> String
map : ((Coef -> Coef),%) -> %
monomial : (Coef,Expon) -> %
monomial? : % -> Boolean
one? : % -> Boolean
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
```

```
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if Coef has INTDOM
zero? : % -> Boolean
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
```

⟨*category PSCAT PowerSeriesCategory*⟩≡

```
)abbrev category PSCAT PowerSeriesCategory
++ Author: Clifton J. Williamson
++ Date Created: 21 December 1989
++ Date Last Updated: 25 February 1990
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: power series
++ Examples:
++ References:
++ Description:
++   \spadtype{PowerSeriesCategory} is the most general power series
++   category with exponents in an ordered abelian monoid.
PowerSeriesCategory(Coef,Expon,Var): Category == Definition where
  Coef  : Ring
  Expon : OrderedAbelianMonoid
  Var   : OrderedSet
  I   ==> Integer
  RN  ==> Fraction Integer

  Definition ==> AbelianMonoidRing(Coef,Expon) with
    monomial: (%,Var,Expon) -> %
      ++ \spad{monomial(a,x,n)} computes \spad{a*x**n}.
    monomial: (%,List Var,List Expon) -> %
      ++ \spad{monomial(a,[x1,..,xk],[n1,..,nk])} computes
      ++ \spad{a * x1**n1 * .. * xk**nk}.
    leadingMonomial: % -> %
      ++ leadingMonomial(f) returns the monomial of \spad{f} of lowest order.
```

```
    leadingCoefficient: % -> Coef
      ++ leadingCoefficient(f) returns the coefficient of the lowest order
      ++ term of \spad{f}
    degree : % -> Expon
      ++ degree(f) returns the exponent of the lowest order term of \spad{f}.
    variables: % -> List Var
      ++ \spad{variables(f)} returns a list of the variables occuring in the
      ++ power series f.
    pole?: % -> Boolean
      ++ \spad{pole?(f)} determines if the power series f has a pole.
    complete: % -> %
      ++ \spad{complete(f)} causes all terms of f to be computed.
      ++ Note: this results in an infinite loop
      ++ if f has infinitely many terms.

  add
    n:I    * ps:% == (zero? n => 0; map((r1:Coef):Coef +-> n * r1,ps))

    r:Coef * ps:% == (zero? r => 0; map((r1:Coef):Coef +-> r * r1,ps))

    ps:% * r:Coef == (zero? r => 0; map((r1:Coef):Coef +-> r1 * r,ps))

    - ps          == map((r1:Coef):Coef +-> -r1,ps)

    if Coef has Algebra Fraction Integer then
      r:RN * ps:% == (zero? r => 0; map((r1:Coef):Coef +-> r * r1,ps))

      ps:% * r:RN == (zero? r => 0; map((r1:Coef):Coef +-> r1 * r,ps))

    if Coef has Field then
      ps:% / r:Coef == map((r1:Coef):Coef +-> r1 / r,ps)
```

⟨*PSCAT.dotabb*⟩≡

```
"PSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PSCAT"];
"PSCAT" -> "AMR"
```

⟨*PSCAT.dotfull*⟩≡

```
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PSCAT"];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PSCAT"];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,IndexedExponents(b:OrderedSet),c:OrderedSet))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=PSCAT"];
"PowerSeriesCategory(a:Ring,IndexedExponents(b:OrderedSet),c:OrderedSet))"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
```

⟨*PSCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"

"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" [color=lightblue];
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" -> "RING..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "BIMODULE..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "IntegralDomain()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CharacteristicNonZero()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CommutativeRing()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "Algebra(Fraction(Integer))"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BIMODULE..."

"CharacteristicNonZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CharacteristicNonZero()" -> "RING..."

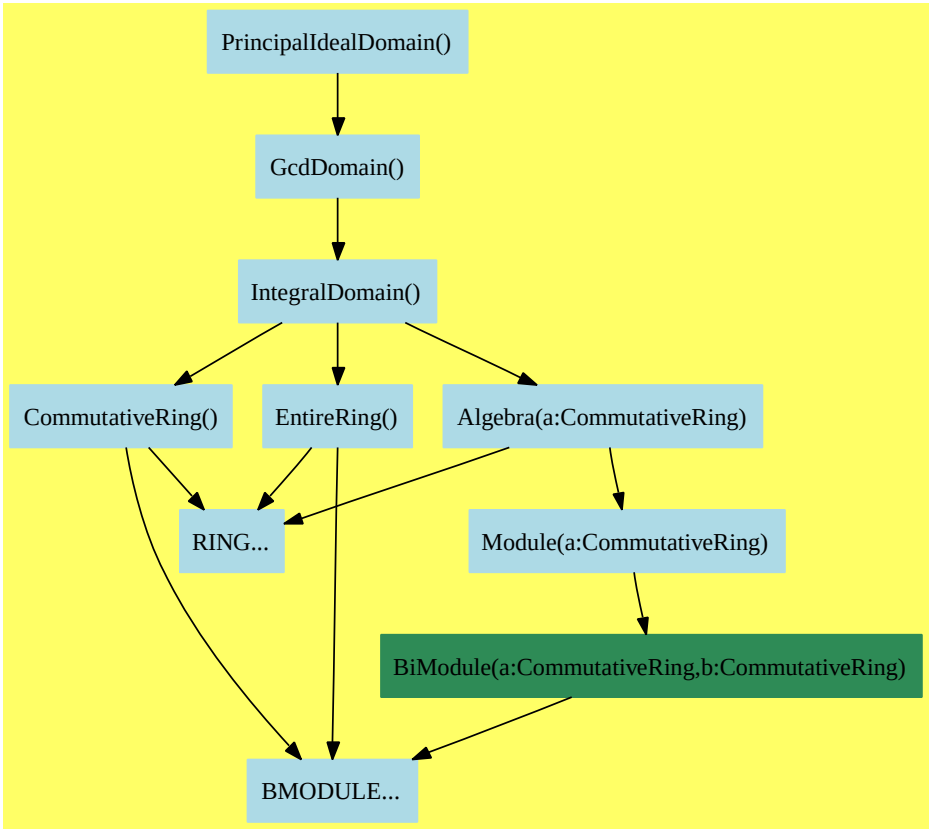
"Algebra(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."
```

```
  "CommutativeRing()" [color=lightblue];
  "CommutativeRing()" -> "RING..."
  "CommutativeRing()" -> "BIMODULE..."

  "BIMODULE..." [color=lightblue];
  "RING..." [color=lightblue];
  "MODULE..." [color=lightblue];
  }
```

14.4 PrincipalIdealDomain (PID)



Unique Factorization Domains are a subset of Principal Ideal Domains.
⇐ "UniqueFactorizationDomain" (UFD) 14.5 on page 969 Principal Ideal Domains are a subset of Euclidean Domains.
⇒ "EuclideanDomain" (EUCDOM) 15.1 on page 975

See:

⇒ "EuclideanDomain" (EUCDOM) 15.1 on page 975
⇐ "GcdDomain" (GCDDOM) 13.4 on page 932

Exports:

0	1	associates?	characteristic	coerce
expressIdealMember	exquo	gcd	gcdPolynomial	hash
latex	lcm	one?	principalIdeal	recip
sample	subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?+?	?-?	-?	?=?
?~=?	?*?	?**?	?^?	

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
expressIdealMember : (List %,%) -> Union(List %,"failed")
principalIdeal : List % -> Record(coef: List %,generator: %)
```

These exports come from (p932) GcdDomain():

```
associates? : (%,%) -> Boolean
exquo : (%,%) -> Union(%,"failed")
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
lcm : List % -> %
lcm : (%,%) -> %
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
```

```
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

⟨*category PID PrincipalIdealDomain*⟩≡

```
)abbrev category PID PrincipalIdealDomain
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of constructive principal ideal domains, i.e.
++ where a single generator can be constructively found for
++ any ideal given by a finite set of generators.
++ Note that this constructive definition only implies that
++ finitely generated ideals are principal. It is not clear
++ what we would mean by an infinitely generated ideal.

PrincipalIdealDomain(): Category == GcdDomain with
      principalIdeal: List % -> Record(coef:List %,generator:%)
         ++ principalIdeal([f1,...,fn]) returns a record whose
         ++ generator component is a generator of the ideal
         ++ generated by \spad{[f1,...,fn]} whose coef component satisfies
         ++ \spad{generator = sum (input.i * coef.i)}
      expressIdealMember: (List %,%) -> Union(List %,"failed")
         ++ expressIdealMember([f1,...,fn],h) returns a representation
         ++ of h as a linear combination of the fi or "failed" if h
         ++ is not in the ideal generated by the fi.
```

⟨*PID.dotabb*⟩≡

```
"PID"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PID"];
"PID" -> "GCDDOM"
```

⟨*PID.dotfull*⟩≡
```
  "PrincipalIdealDomain()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=PID"];
  "PrincipalIdealDomain()" -> "GcdDomain()"
```

⟨*PID.dotpic*⟩≡
```
  digraph pic {
   fontsize=10;
   bgcolor="#FFFF66";
   node [shape=box, color=white, style=filled];

  "PrincipalIdealDomain()" [color=lightblue];
  "PrincipalIdealDomain()" -> "GcdDomain()"

  "GcdDomain()" [color=lightblue];
  "GcdDomain()" -> "IntegralDomain()"

  "IntegralDomain()" [color=lightblue];
  "IntegralDomain()" -> "CommutativeRing()"
  "IntegralDomain()" -> "Algebra(a:CommutativeRing)"
  "IntegralDomain()" -> "EntireRing()"

  "CommutativeRing()" [color=lightblue];
  "CommutativeRing()" -> "RING..."
  "CommutativeRing()" -> "BMODULE..."

  "EntireRing()" [color=lightblue];
  "EntireRing()" -> "RING..."
  "EntireRing()" -> "BMODULE..."

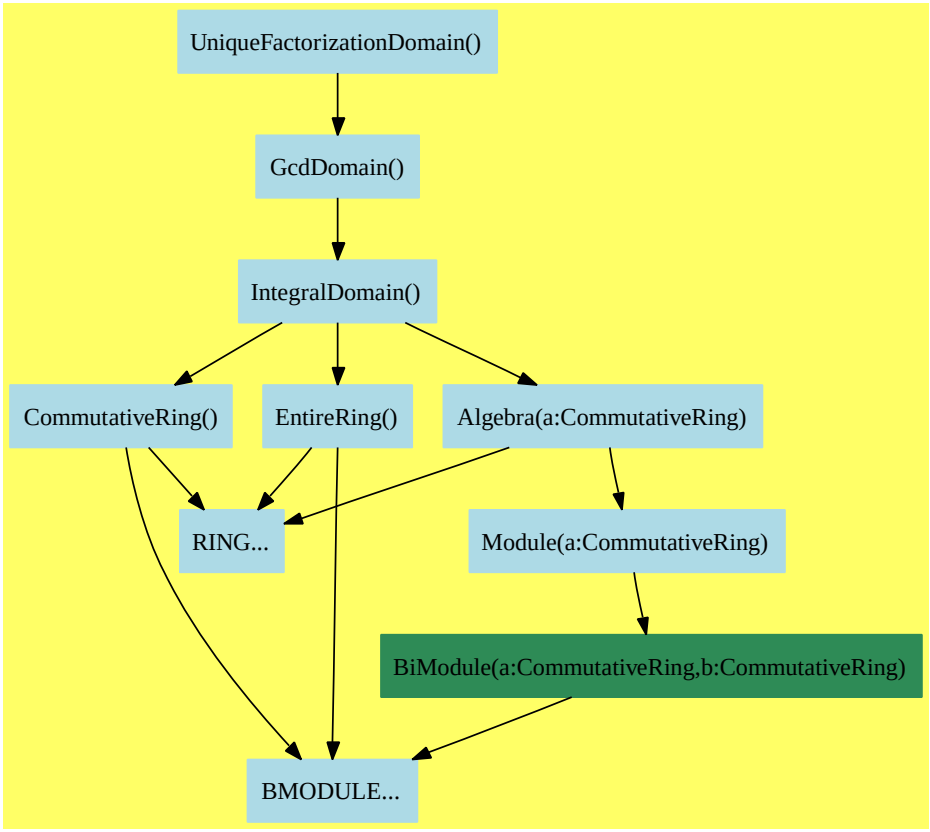
  "Algebra(a:CommutativeRing)" [color=lightblue];
  "Algebra(a:CommutativeRing)" -> "RING..."
  "Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

  "Module(a:CommutativeRing)" [color=lightblue];
  "Module(a:CommutativeRing)" ->
    "BiModule(a:CommutativeRing,b:CommutativeRing)"

  "BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
  "BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

  "BMODULE..." [color=lightblue];
  "RING..." [color=lightblue];
  }
```

14.5 UniqueFactorizationDomain (UFD)



All Integral Domains are UniqueFactorizationDomains.
⇐ "IntegralDomain" (INTDOM) 12.5 on page 857. Unique Factorization Domains are a subset of Principal Ideal Domains.
⇒ "PrincipalIdealDomain" (PID) 14.4 on page 965

See:

⇒ "Field" (FIELD) 16.1 on page 1005
⇒ "IntegerNumberSystem" (INS) 16.2 on page 1011
⇒ "PolynomialFactorizationExplicit" (PFECAT) 15.3 on page 990
⇐ "GcdDomain" (GCDDOM) 13.4 on page 932

Exports:

0	1	associates?	characteristic	coerce
exquo	factor	gcd	gcdPolynomial	hash
latex	lcm	one?	prime?	recip
sample	squareFree	squareFreePart	subtractIfCan	unit?
unitCanonical	unitNormal	zero?	?*?	?**?
?+?	?-?	-?	?=?	?~=?
?^?				

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
factor : % -> Factored %
squareFree : % -> Factored %
```

These are implemented by this category:

```
prime? : % -> Boolean
squareFreePart : % -> %
```

These exports come from (p932) GcdDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
exquo : (%,%) -> Union(%,"failed")
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
```

```
lcm : List % -> %
lcm : (%,%) -> %
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

⟨*category UFD UniqueFactorizationDomain*⟩≡

```
)abbrev category UFD UniqueFactorizationDomain
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A constructive unique factorization domain, i.e. where
++ we can constructively factor members into a product of
++ a finite number of irreducible elements.

UniqueFactorizationDomain(): Category == GcdDomain with
   prime?: % -> Boolean
     ++ prime?(x) tests if x can never be written as the product of two
     ++ non-units of the ring,
     ++ i.e., x is an irreducible element.
   squareFree    : % -> Factored(%)
     ++ squareFree(x) returns the square-free factorization of x
     ++ i.e. such that the factors are pairwise relatively prime
```

```
      ++ and each has multiple prime factors.
    squareFreePart: % -> %
      ++ squareFreePart(x) returns a product of prime factors of
      ++ x each taken with multiplicity one.
    factor: % -> Factored(%)
      ++ factor(x) returns the factorization of x into irreducibles.
  add
   squareFreePart x ==
     unit(s := squareFree x) * _*/[f.factor for f in factors s]

   prime? x == # factorList factor x = 1
```

⟨*UFD.dotabb*⟩≡

```
"UFD"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UFD"];
"UFD" -> "GCDDOM"
```

⟨*UFD.dotfull*⟩≡

```
"UniqueFactorizationDomain()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UFD"];
"UniqueFactorizationDomain()" -> "GcdDomain()"
```

⟨*UFD.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GcdDomain()"

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" ->
  "BiModule(a:CommutativeRing,b:CommutativeRing)"

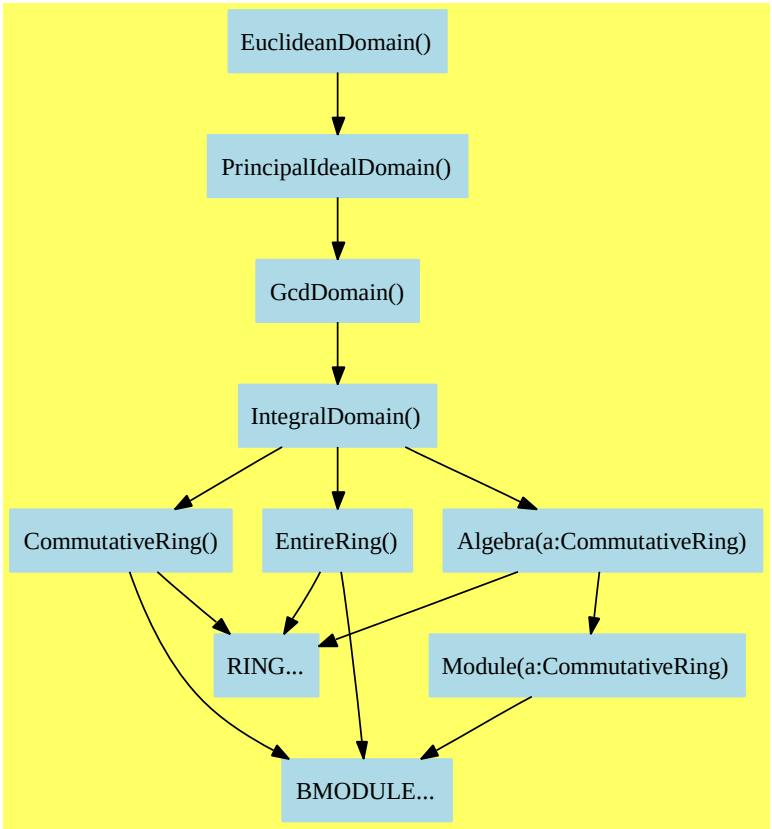
"BiModule(a:CommutativeRing,b:CommutativeRing)" [color=seagreen];
"BiModule(a:CommutativeRing,b:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

Chapter 15

Category Layer 14

15.1 EuclideanDomain (EUCDOM)



Principal Ideal Domains are a subset of Euclidean Domains.
⇐ "PrincipalIdealDomain" (PID) 14.4 on page 965. Euclidean Domains are a subset of Fields.
⇒ "Field" (FIELD) 16.1 on page 1005

See:

⇒ "Field" (FIELD) 16.1 on page 1005
⇒ "IntegerNumberSystem" (INS) 16.2 on page 1011
⇒ "PAdicIntegerCategory" (PADICCT) 16.3 on page 1021
⇒ "QuotientFieldCategory" (QFCAT) 17.6 on page 1121
⇐ "PrincipalIdealDomain" (PID) 14.4 on page 965

Exports:

1	0	associates?	characteristic	coerce
divide	euclideanSize	expressIdealMember	exquo	extendedEuclidean
gcd	gcdPolynomial	hash	latex	lcm
multiEuclidean	one?	principalIdeal	recip	sample
sizeLess?	subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?+?	?-?	-?	?=?
?quo?	?rem?	?~=?	?*?	?**?
?^?				

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
```

These are implemented by this category:

```
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
gcd : (%,%) -> %
```

```
multiEuclidean : (List %,%) -> Union(List %,"failed")
principalIdeal : List % -> Record(coef: List %,generator: %)
sizeLess? : (%,%) -> Boolean
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p965) PrincipalIdealDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

⟨*category EUCDOM EuclideanDomain*⟩≡

```
)abbrev category EUCDOM EuclideanDomain
++ Author:
++ Date Created:
++ Date Last Updated:
```

```
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A constructive euclidean domain, i.e. one can divide producing
++ a quotient and a remainder where the remainder is either zero
++ or is smaller (\spadfun{euclideanSize}) than the divisor.
++
++ Conditional attributes:
++   multiplicativeValuation\tab{25}\spad{Size(a*b)=Size(a)*Size(b)}
++   additiveValuation\tab{25}\spad{Size(a*b)=Size(a)+Size(b)}

EuclideanDomain(): Category == PrincipalIdealDomain with
   sizeLess?: (%,%) -> Boolean
     ++ sizeLess?(x,y) tests whether x is strictly
     ++ smaller than y with respect to the
     ++ \spadfunFrom{euclideanSize}{EuclideanDomain}.
   euclideanSize: % -> NonNegativeInteger
     ++ euclideanSize(x) returns the euclidean size of the element x.
     ++ Error: if x is zero.
   divide: (%,%) -> Record(quotient:%,remainder:%)
     ++ divide(x,y) divides x by y producing a record containing a
     ++ \spad{quotient} and \spad{remainder},
     ++ where the remainder is smaller (see
     ++ \spadfunFrom{sizeLess?}{EuclideanDomain}) than the divisor y.
   "quo" : (%,%) -> %
     ++ x quo y is the same as \spad{divide(x,y).quotient}.
     ++ See \spadfunFrom{divide}{EuclideanDomain}.
   "rem": (%,%) -> %
     ++ x rem y is the same as \spad{divide(x,y).remainder}.
     ++ See \spadfunFrom{divide}{EuclideanDomain}.
   extendedEuclidean: (%,%) -> Record(coef1:%,coef2:%,generator:%)
     -- formerly called princIdeal
     ++ extendedEuclidean(x,y) returns a record rec where
     ++ \spad{rec.coef1*x+rec.coef2*y = rec.generator} and
     ++ rec.generator is a gcd of x and y.
     ++ The gcd is unique only
     ++ up to associates if \spadatt{canonicalUnitNormal} is not asserted.
     ++ \spadfun{principalIdeal} provides a version of this operation
     ++ which accepts an arbitrary length list of arguments.
   extendedEuclidean: (%,%,%) -> Union(Record(coef1:%,coef2:%),"failed")
     -- formerly called expressIdealElt
     ++ extendedEuclidean(x,y,z) either returns a record rec
```

```
      ++ where \spad{rec.coef1*x+rec.coef2*y=z} or returns "failed"
      ++ if z cannot be expressed as a linear combination of x and y.
    multiEuclidean: (List %,%) -> Union(List %,"failed")
      ++ multiEuclidean([f1,...,fn],z) returns a list of coefficients
      ++ \spad{[a1, ..., an]} such that
      ++ \spad{ z / prod fi = sum aj/fj}.
      ++ If no such list of coefficients exists, "failed" is returned.
   add
       x,y,z: %
       l: List %
       sizeLess?(x,y) ==
             zero? y => false
             zero? x => true
             euclideanSize(x)<euclideanSize(y)
       x quo y == divide(x,y).quotient --divide must be user-supplied
       x rem y == divide(x,y).remainder
       x exquo y ==
          zero? x => 0
          zero? y => "failed"
          qr:=divide(x,y)
          zero?(qr.remainder) => qr.quotient
          "failed"
       gcd(x,y) ==                --Euclidean Algorithm
          x:=unitCanonical x
          y:=unitCanonical y
          while not zero? y repeat
             (x,y):= (y,x rem y)
             y:=unitCanonical y   -- this doesn't affect the
                                  -- correctness of Euclid's algorithm,
                                  -- but
                                  -- a) may improve performance
                                  -- b) ensures gcd(x,y)=gcd(y,x)
                                  --    if canonicalUnitNormal
          x
       IdealElt ==> Record(coef1:%,coef2:%,generator:%)
       unitNormalizeIdealElt(s:IdealElt):IdealElt ==
          (u,c,a):=unitNormal(s.generator)
--         one? a => s
          (a = 1) => s
          [a*s.coef1,a*s.coef2,c]$IdealElt
       extendedEuclidean(x,y) ==         --Extended Euclidean Algorithm
          s1:=unitNormalizeIdealElt([1$%,0$%,x]$IdealElt)
          s2:=unitNormalizeIdealElt([0$%,1$%,y]$IdealElt)
          zero? y => s1
          zero? x => s2
          while not zero?(s2.generator) repeat
```

```
      qr:= divide(s1.generator, s2.generator)
      s3:=[s1.coef1 - qr.quotient * s2.coef1,
           s1.coef2 - qr.quotient * s2.coef2, qr.remainder]$IdealElt
      s1:=s2
      s2:=unitNormalizeIdealElt s3
   if not(zero?(s1.coef1)) and not sizeLess?(s1.coef1,y)
     then
        qr:= divide(s1.coef1,y)
        s1.coef1:= qr.remainder
        s1.coef2:= s1.coef2 + qr.quotient * x
        s1 := unitNormalizeIdealElt s1
   s1

TwoCoefs ==> Record(coef1:%,coef2:%)
extendedEuclidean(x,y,z) ==
   zero? z => [0,0]$TwoCoefs
   s:= extendedEuclidean(x,y)
   (w:= z exquo s.generator) case "failed" => "failed"
   zero? y =>
      [s.coef1 * w, s.coef2 * w]$TwoCoefs
   qr:= divide((s.coef1 * w), y)
   [qr.remainder, s.coef2 * w + qr.quotient * x]$TwoCoefs
principalIdeal l ==
   l = [] => error "empty list passed to principalIdeal"
   rest l = [] =>
        uca:=unitNormal(first l)
        [[uca.unit],uca.canonical]
   rest rest l = [] =>
       u:= extendedEuclidean(first l,second l)
       [[u.coef1, u.coef2], u.generator]
   v:=principalIdeal rest l
   u:= extendedEuclidean(first l,v.generator)
   [[u.coef1,:[u.coef2*vv for vv in v.coef]],u.generator]
expressIdealMember(l,z) ==
   z = 0 => [0 for v in l]
   pid := principalIdeal l
   (q := z exquo (pid.generator)) case "failed" => "failed"
   [q*v for v in pid.coef]
multiEuclidean(l,z) ==
   n := #l
   zero? n => error "empty list passed to multiEuclidean"
   n = 1 => [z]
   l1 := copy l
   l2 := split!(l1, n quo 2)
   u:= extendedEuclidean(*/l1, */l2, z)
   u case "failed" => "failed"
```

```
            v1 := multiEuclidean(l1,u.coef2)
            v1 case "failed" => "failed"
            v2 := multiEuclidean(l2,u.coef1)
            v2 case "failed" => "failed"
            concat(v1,v2)
```

⟨*EUCDOM.dotabb*⟩≡

```
  "EUCDOM"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=EUCDOM"];
  "EUCDOM" -> "PID"
```

⟨*EUCDOM.dotfull*⟩≡

```
  "EuclideanDomain()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=EUCDOM"];
  "EuclideanDomain()" -> "PrincipalIdealDomain()"
```

⟨*EUCDOM.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GcdDomain()"

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

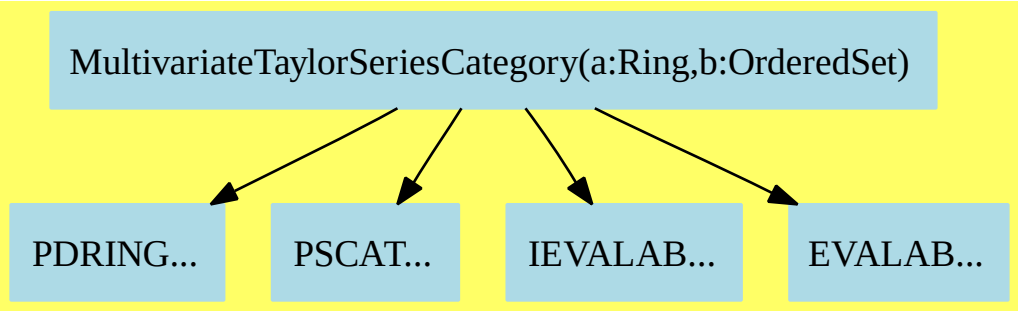
"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

15.2 MultivariateTaylorSeriesCategory (MTSCAT)



See:

⇐ "Evalable" (EVALAB) 3.4 on page 65
⇐ "InnerEvalable" (IEVALAB) 2.12 on page 29
⇐ "PartialDifferentialRing" (PDRING) 10.12 on page 720
⇐ "PowerSeriesCategory" (PSCAT) 14.3 on page 958

Exports:

0	1	acos	acosh	acot
acoth	acsc	acsch	asec	asech
asin	asinh	associates?	atan	atanh
characteristic	charthRoot	coefficient	coerce	complete
cos	cosh	cot	coth	csc
csch	D	degree	differentiate	eval
exp	exquo	extend	hash	integrate
latex	leadingCoefficient	leadingMonomial	log	map
monomial	monomial?	nthRoot	order	one?
pi	pole?	polynomial	recip	reductum
sample	sec	sech	sin	sinh
sqrt	subtractIfCan	tan	tanh	unit?
unitCanonical	unitNormal	variables	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?~=?	?/?		

Attributes Exported:

- if $ has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- if $ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
coefficient : (%,Var,NonNegativeInteger) -> %
coefficient : (%,List Var,List NonNegativeInteger) -> %
extend : (%,NonNegativeInteger) -> %
integrate : (%,Var) -> % if Coef has ALGEBRA FRAC INT
monomial : (%,Var,NonNegativeInteger) -> %
monomial : (%,List Var,List NonNegativeInteger) -> %
order : (%,Var,NonNegativeInteger) -> NonNegativeInteger
order : (%,Var) -> NonNegativeInteger
polynomial : (%,NonNegativeInteger,NonNegativeInteger) -> Polynomial Coef
polynomial : (%,NonNegativeInteger) -> Polynomial Coef
```

These exports come from (p720) PartialDifferentialRing(OrderedSet):

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
D : (%,Var) -> %
D : (%,List Var) -> %
D : (%,List Var,List NonNegativeInteger) -> %
D : (%,Var,NonNegativeInteger) -> %
differentiate : (%,Var) -> %
differentiate : (%,List Var,List NonNegativeInteger) -> %
differentiate : (%,Var,NonNegativeInteger) -> %
differentiate : (%,List Var) -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

```
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p958) PowerSeriesCategory(A,B,C)
where A:Ring, B:IndexedExponents(OrderedSet) and C:OrderedSet:

```
associates? : (%,%) -> Boolean if Coef has INTDOM
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,IndexedExponents Var) -> Coef
coerce : Coef -> % if Coef has COMRING
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : % -> % if Coef has INTDOM
complete : % -> %
degree : % -> IndexedExponents Var
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
map : ((Coef -> Coef),%) -> %
monomial : (Coef,IndexedExponents Var) -> %
monomial? : % -> Boolean
monomial : (%,List Var,List IndexedExponents Var) -> %
monomial : (%,Var,IndexedExponents Var) -> %
pole? : % -> Boolean
reductum : % -> %
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
  if Coef has INTDOM
variables : % -> List Var
?*? : (%,Coef) -> %
?*? : (Coef,%) -> %
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
```

These exports come from (p29) InnerEvalable(OrderedSet,

```
eval : (%,Var,%) -> %
eval : (%,List Var,List %) -> %
```

These exports come from (p65) Evalable(

```
eval : (%,List Equation %) -> %
eval : (%,Equation %) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
```

These exports come from (p42) RadicalCategory():

```
nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
sqrt : % -> % if Coef has ALGEBRA FRAC INT
?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
```

These exports come from (p94) TranscendentalFunctionCategory():

```
acos : % -> % if Coef has ALGEBRA FRAC INT
acosh : % -> % if Coef has ALGEBRA FRAC INT
acot : % -> % if Coef has ALGEBRA FRAC INT
acoth : % -> % if Coef has ALGEBRA FRAC INT
acsc : % -> % if Coef has ALGEBRA FRAC INT
acsch : % -> % if Coef has ALGEBRA FRAC INT
asec : % -> % if Coef has ALGEBRA FRAC INT
asech : % -> % if Coef has ALGEBRA FRAC INT
asin : % -> % if Coef has ALGEBRA FRAC INT
asinh : % -> % if Coef has ALGEBRA FRAC INT
atan : % -> % if Coef has ALGEBRA FRAC INT
atanh : % -> % if Coef has ALGEBRA FRAC INT
cos : % -> % if Coef has ALGEBRA FRAC INT
cosh : % -> % if Coef has ALGEBRA FRAC INT
cot : % -> % if Coef has ALGEBRA FRAC INT
coth : % -> % if Coef has ALGEBRA FRAC INT
csc : % -> % if Coef has ALGEBRA FRAC INT
csch : % -> % if Coef has ALGEBRA FRAC INT
exp : % -> % if Coef has ALGEBRA FRAC INT
log : % -> % if Coef has ALGEBRA FRAC INT
pi : () -> % if Coef has ALGEBRA FRAC INT
sec : % -> % if Coef has ALGEBRA FRAC INT
sech : % -> % if Coef has ALGEBRA FRAC INT
sin : % -> % if Coef has ALGEBRA FRAC INT
sinh : % -> % if Coef has ALGEBRA FRAC INT
tan : % -> % if Coef has ALGEBRA FRAC INT
tanh : % -> % if Coef has ALGEBRA FRAC INT
?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
```

⟨*category MTSCAT MultivariateTaylorSeriesCategory*⟩≡

```
)abbrev category MTSCAT MultivariateTaylorSeriesCategory
++ Author: Clifton J. Williamson
++ Date Created: 6 March 1990
++ Date Last Updated: 6 March 1990
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: multivariate, Taylor, series
++ Examples:
++ References:
++ Description:
++   \spadtype{MultivariateTaylorSeriesCategory} is the most general
```

```
++   multivariate Taylor series category.
MultivariateTaylorSeriesCategory(Coef,Var): Category == Definition where
  Coef  : Ring
  Var   : OrderedSet
  L   ==> List
  NNI ==> NonNegativeInteger

  Definition ==> Join(PartialDifferentialRing Var,_
                      PowerSeriesCategory(Coef,IndexedExponents Var,Var),_
                      InnerEvalable(Var,%),Evalable %) with
    coefficient: (%,Var,NNI) -> %
      ++ \spad{coefficient(f,x,n)} returns the coefficient of \spad{x^n} in f.
    coefficient: (%,L Var,L NNI) -> %
      ++ \spad{coefficient(f,[x1,x2,...,xk],[n1,n2,...,nk])} returns the
      ++ coefficient of \spad{x1^n1 * ... * xk^nk} in f.
    extend: (%,NNI) -> %
      ++ \spad{extend(f,n)} causes all terms of f of degree
      ++ \spad{<= n} to be computed.
    monomial: (%,Var,NNI) -> %
      ++ \spad{monomial(a,x,n)} returns  \spad{a*x^n}.
    monomial: (%,L Var,L NNI) -> %
      ++ \spad{monomial(a,[x1,x2,...,xk],[n1,n2,...,nk])} returns
      ++ \spad{a * x1^n1 * ... * xk^nk}.
    order: (%,Var) -> NNI
      ++ \spad{order(f,x)} returns the order of f viewed as a series in x
      ++ may result in an infinite loop if f has no non-zero terms.
    order: (%,Var,NNI) -> NNI
      ++ \spad{order(f,x,n)} returns \spad{min(n,order(f,x))}.
    polynomial: (%,NNI) -> Polynomial Coef
      ++ \spad{polynomial(f,k)} returns a polynomial consisting of the sum
      ++ of all terms of f of degree \spad{<= k}.
    polynomial: (%,NNI,NNI) -> Polynomial Coef
      ++ \spad{polynomial(f,k1,k2)} returns a polynomial consisting of the
      ++ sum of all terms of f of degree d with \spad{k1 <= d <= k2}.
    if Coef has Algebra Fraction Integer then
      integrate: (%,Var) -> %
        ++ \spad{integrate(f,x)} returns the anti-derivative of the power
        ++ series \spad{f(x)} with respect to the variable x with constant
        ++ coefficient 1.  We may integrate a series when we can divide
        ++ coefficients by integers.
      RadicalCategory
        --++ We provide rational powers when we can divide coefficients
        --++ by integers.
      TranscendentalFunctionCategory
        --++ We provide transcendental functions when we can divide
        --++ coefficients by integers.
```

⟨*MTSCAT.dotabb*⟩≡
```
  "MTSCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=MTSCAT"];
  "MTSCAT" -> "PDRING"
  "MTSCAT" -> "PSCAT"
  "MTSCAT" -> "IEVALAB"
  "MTSCAT" -> "EVALAB"
```

⟨*MTSCAT.dotfull*⟩≡
```
  "MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=MTSCAT"];
  "MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
      "PartialDifferentialRing(a:OrderedSet)"
  "MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
     "PowerSeriesCategory(a:Ring,IndexedExponents(b:OrderedSet),c:OrderedSet))"
  "MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
   "InnerEvalable(a:Ring,MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet))"
  "MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
   "Evalable(MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet))"
```

⟨*MTSCAT.dotpic*⟩≡

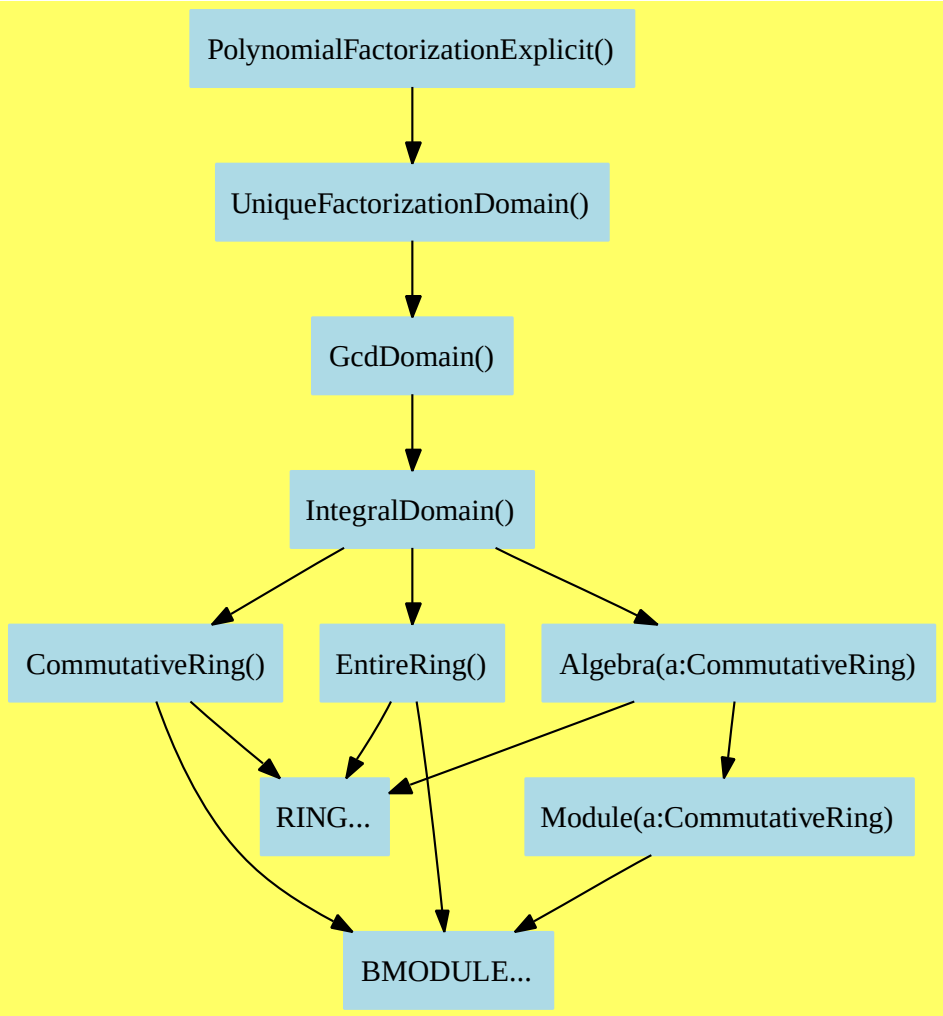
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" [color=lightblue];
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
    "PDRING..."
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
   "PSCAT..."
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
 "IEVALAB..."
"MultivariateTaylorSeriesCategory(a:Ring,b:OrderedSet)" ->
 "EVALAB..."

"PDRING..." [color=lightblue];
"PSCAT..." [color=lightblue];
"IEVALAB..." [color=lightblue];
"EVALAB..." [color=lightblue];

}
```

15.3 PolynomialFactorizationExplicit (PFECAT)



See:

⇒ "PolynomialCategory" (POLYCAT) 16.4 on page 1027
⇒ "QuotientFieldCategory" (QFCAT) 17.6 on page 1121
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.12 on page 1204
⇐ "UniqueFactorizationDomain" (UFD) 14.5 on page 969

Exports:

0	1	associates?
characteristic	charthRoot	coerce
conditionP	exquo	factor
factorPolynomial	factorSquareFreePolynomial	gcd
gcdPolynomial	hash	latex
lcm	one?	prime?
recip	sample	solveLinearPolynomialEquation
squareFree	squareFreePart	squareFreePolynomial
subtractIfCan	unit?	unitNormal
unitCanonical	zero?	?*?
?**?	?+?	?-?
-?	?=?	?^?
?~=?		

Attributes exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
conditionP : Matrix % -> Union(Vector %,"failed")
    if $ has CHARNZ
factorPolynomial :
  SparseUnivariatePolynomial % ->
     Factored SparseUnivariatePolynomial %
factorSquareFreePolynomial :
  SparseUnivariatePolynomial % ->
     Factored SparseUnivariatePolynomial %
squareFreePolynomial :
  SparseUnivariatePolynomial % ->
     Factored SparseUnivariatePolynomial %
```

These are implemented by this category:

```
charthRoot : % -> Union(%,"failed") if $ has CHARNZ
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
```

```
solveLinearPolynomialEquation :
  (List SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     Union(List SparseUnivariatePolynomial %,"failed")
```

These exports come from (p969) UniqueFactorizationDomain():

```
factor : % -> Factored %
squareFree : % -> Factored %
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
exquo : (%,%) -> Union(%,"failed")
gcd : List % -> %
gcd : (%,%) -> %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
one? : % -> Boolean
prime? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
unitCanonical : % -> %
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

⟨*category PFECAT PolynomialFactorizationExplicit*⟩≡

```
)abbrev category PFECAT PolynomialFactorizationExplicit
++ Author: James Davenport
```

```
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ This is the category of domains that know "enough" about
++ themselves in order to factor univariate polynomials over themselves.
++ This will be used in future releases for supporting factorization
++ over finitely generated coefficient fields, it is not yet available
++ in the current release of axiom.

PolynomialFactorizationExplicit(): Category == Definition where
  P ==> SparseUnivariatePolynomial %
  Definition ==>
   UniqueFactorizationDomain with
     squareFreePolynomial: P -> Factored(P)
       ++ squareFreePolynomial(p) returns the
       ++ square-free factorization of the
       ++ univariate polynomial p.
     factorPolynomial: P -> Factored(P)
       ++ factorPolynomial(p) returns the factorization
       ++ into irreducibles of the univariate polynomial p.
     factorSquareFreePolynomial: P -> Factored(P)
       ++ factorSquareFreePolynomial(p) factors the
       ++ univariate polynomial p into irreducibles
       ++ where p is known to be square free
       ++ and primitive with respect to its main variable.
     gcdPolynomial: (P, P) -> P
       ++ gcdPolynomial(p,q) returns the gcd of the univariate
       ++ polynomials p qnd q.
       -- defaults to Euclidean, but should be implemented via
       -- modular or p-adic methods.
     solveLinearPolynomialEquation: (List P, P) -> Union(List P,"failed")
       ++ solveLinearPolynomialEquation([f1, ..., fn], g)
       ++ (where the fi are relatively prime to each other)
       ++ returns a list of ai such that
       ++ \spad{g/prod fi = sum ai/fi}
       ++ or returns "failed" if no such list of ai's exists.
     if % has CharacteristicNonZero then
       conditionP: Matrix % -> Union(Vector %,"failed")
         ++ conditionP(m) returns a vector of elements, not all zero,
         ++ whose \spad{p}-th powers (p is the characteristic of the domain)
```

```
          ++ are a solution of the homogenous linear system represented
          ++ by m, or "failed" is there is no such vector.
        charthRoot: % -> Union(%,"failed")
          ++ charthRoot(r) returns the \spad{p}-th root of r, or "failed"
          ++ if none exists in the domain.
          -- this is a special case of conditionP, but often the one we want
      add
          gcdPolynomial(f,g) ==
             zero? f => g
             zero? g => f
             cf:=content f
             if not one? cf then f:=(f exquo cf)::P
             cg:=content g
             if not one? cg then g:=(g exquo cg)::P
             ans:=subResultantGcd(f,g)$P
             gcd(cf,cg)*(ans exquo content ans)::P
          if % has CharacteristicNonZero then
            charthRoot f ==
               -- to take p'th root of f, solve the system X-fY=0,
               -- so solution is [x,y]
               -- with x^p=X and y^p=Y, then (x/y)^p = f
               zero? f => 0
               m:Matrix % := matrix [[1,-f]]
               ans:= conditionP m
               ans case "failed" => "failed"
               (ans.1) exquo (ans.2)
          if % has Field then
            solveLinearPolynomialEquation(lf,g) ==
              multiEuclidean(lf,g)$P
          else solveLinearPolynomialEquation(lf,g) ==
                 LPE ==> LinearPolynomialEquationByFractions %
                 solveLinearPolynomialEquationByFractions(lf,g)$LPE
```

⟨*PFECAT.dotabb*⟩≡

```
"PFECAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PFECAT"];
"PFECAT" -> "UFD"
```

⟨*PFECAT.dotfull*⟩≡

```
"PolynomialFactorizationExplicit()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PFECAT"];
"PolynomialFactorizationExplicit()" -> "UniqueFactorizationDomain()"
```

⟨*PFECAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PolynomialFactorizationExplicit()" [color=lightblue];
"PolynomialFactorizationExplicit()" -> "UniqueFactorizationDomain()"

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GcdDomain()"

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

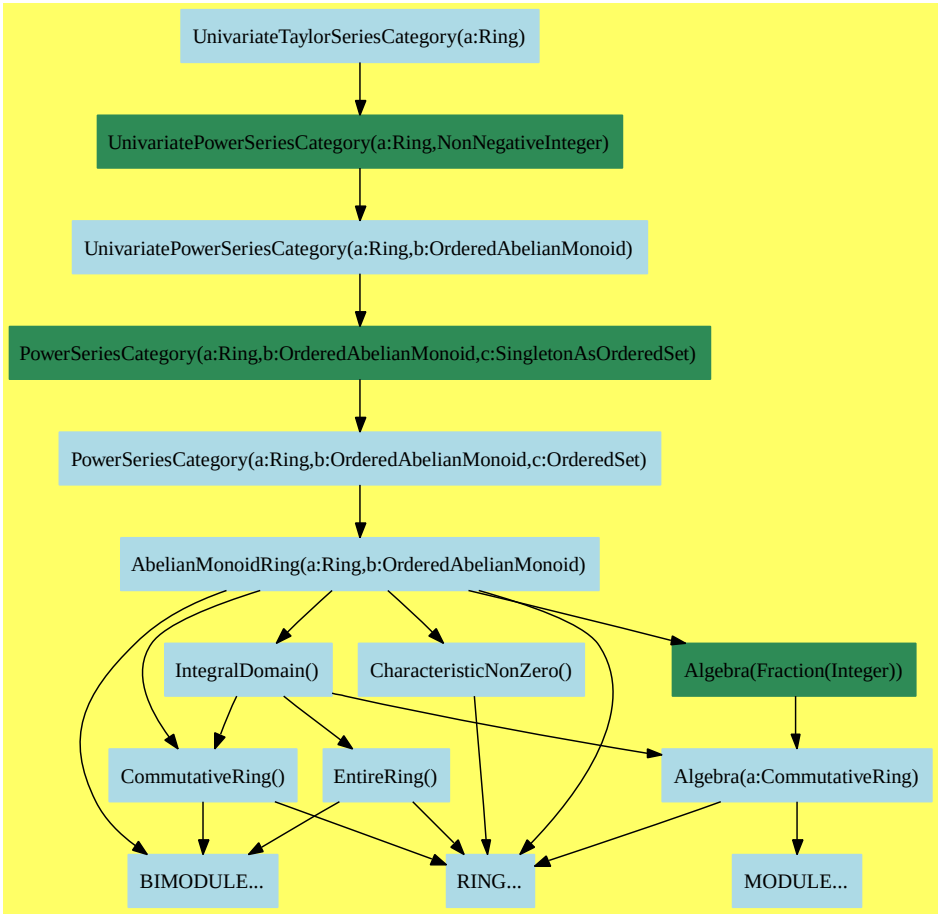
"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" -> "BMODULE..."

"BMODULE..." [color=lightblue];
"RING..." [color=lightblue];
}
```

15.4 UnivariatePowerSeriesCategory (UPSCAT)



See:

⇒ "UnivariateLaurentSeriesCategory" (ULSCAT) 17.10 on page 1186
⇒ "UnivariatePuiseuxSeriesCategory" (UPXSCAT) 17.11 on page 1195
⇒ "UnivariateTaylorSeriesCategory" (UTSCAT) 16.5 on page 1046
⇐ "PowerSeriesCategory" (PSCAT) 14.3 on page 958

Exports:

0	1	approximate	associates?	center
characteristic	charthRoot	coefficient	coerce	complete
D	degree	differentiate	eval	exquo
extend	hash	latex	leadingCoefficient	leadingMonomial
map	monomial	monomial?	multiplyExponents	one?
order	pole?	recip	reductum	sample
subtractIfCan	truncate	terms	unit?	unitCanonical
unitNormal	variable	variables	zero?	?*?
?**?	?+?	?-?	-?	?=?
?^?	?.?	?~=?	?/?	

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if #1 has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.

These are directly exported but not implemented:

```
approximate : (%,Expon) -> Coef
    if Coef has **: (Coef,Expon) -> Coef
    and Coef has coerce: Symbol -> Coef
center : % -> Coef
eval : (%,Coef) -> Stream Coef if Coef has **: (Coef,Expon) -> Coef
extend : (%,Expon) -> %
multiplyExponents : (%,PositiveInteger) -> %
order : % -> Expon
order : (%,Expon) -> Expon
terms : % -> Stream Record(k: Expon,c: Coef)
truncate : (%,Expon) -> %
truncate : (%,Expon,Expon) -> %
variable : % -> Symbol
?.? : (%,Expon) -> Coef
```

These are implemented by this category:

```
degree : % -> Expon
leadingCoefficient : % -> Coef
```

```
leadingMonomial : % -> %
monomial : (%,SingletonAsOrderedSet,Expon) -> %
reductum : % -> %
variables : % -> List SingletonAsOrderedSet
```

These exports come from (p958) PowerSeriesCategory(C,E,S)
where C:Ring, E:OrderedAbelianMonoid, S:SingletonAsOrderedSet:

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean if Coef has INTDOM
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,Expon) -> Coef
coerce : Coef -> % if Coef has COMRING
coerce : % -> % if Coef has INTDOM
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : Integer -> %
coerce : % -> OutputForm
complete : % -> %
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
hash : % -> SingleInteger
latex : % -> String
map : ((Coef -> Coef),%) -> %
monomial : (%,List Var,List Expon) -> %
monomial : (Coef,Expon) -> %
monomial? : % -> Boolean
one? : % -> Boolean
recip : % -> Union(%,"failed")
pole? : % -> Boolean
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if Coef has INTDOM
zero? : % -> Boolean
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (Integer,%) -> %
```

```
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
-? : % -> %
```

These exports come from (p25) Eltable(%,%):

```
?.? : (%,%) -> % if Expon has SGROUP
```

These exports come from (p690) DifferentialRing():

```
D : % -> %
    if Coef has *: (Expon,Coef) -> Coef
D : (%,NonNegativeInteger) -> %
    if Coef has *: (Expon,Coef) -> Coef
differentiate : (%,NonNegativeInteger) -> %
    if Coef has *: (Expon,Coef) -> Coef
differentiate : % -> %
    if Coef has *: (Expon,Coef) -> Coef
```

These exports come from (p720) PartialDifferentialRing(Symbol):

```
D : (%,Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
D : (%,List Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
D : (%,Symbol,NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
D : (%,List Symbol,List NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
differentiate : (%,List Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
differentiate : (%,Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (Expon,Coef) -> Coef
```

⟨*category UPSCAT UnivariatePowerSeriesCategory*⟩≡

```
)abbrev category UPSCAT UnivariatePowerSeriesCategory
++ Author: Clifton J. Williamson
++ Date Created: 21 December 1989
++ Date Last Updated: 20 September 1993
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ Examples:
++ References:
++ Description:
++   \spadtype{UnivariatePowerSeriesCategory} is the most general
++   univariate power series category with exponents in an ordered
++   abelian monoid.
++   Note: this category exports a substitution function if it is
++   possible to multiply exponents.
++   Note: this category exports a derivative operation if it is possible
++   to multiply coefficients by exponents.
UnivariatePowerSeriesCategory(Coef,Expon): Category == Definition where
  Coef   : Ring
  Expon  : OrderedAbelianMonoid
  Term ==> Record(k:Expon,c:Coef)

  Definition ==> PowerSeriesCategory(Coef,Expon,SingletonAsOrderedSet) with

    terms: % -> Stream Term
      ++ \spad{terms(f(x))} returns a stream of non-zero terms, where a
      ++ a term is an exponent-coefficient pair.  The terms in the stream
      ++ are ordered by increasing order of exponents.
    --series: Stream Term -> %
      --++ \spad{series(st)} creates a series from a stream of non-zero terms,
      --++ where a term is an exponent-coefficient pair.  The terms in the
      --++ stream should be ordered by increasing order of exponents.
    elt: (%,Expon) -> Coef
      ++ \spad{elt(f(x),r)} returns the coefficient of the term of degree r in
      ++ \spad{f(x)}.  This is the same as the function \spadfun{coefficient}.
    variable: % -> Symbol
      ++ \spad{variable(f)} returns the (unique) power series variable of
      ++ the power series f.
    center: % -> Coef
      ++ \spad{center(f)} returns the point about which the series f is
      ++ expanded.
    multiplyExponents: (%,PositiveInteger) -> %
      ++ \spad{multiplyExponents(f,n)} multiplies all exponents of the power
```

```
    ++ series f by the positive integer n.
  order: % -> Expon
    ++ \spad{order(f)} is the degree of the lowest order non-zero term in f.
    ++ This will result in an infinite loop if f has no non-zero terms.
  order: (%,Expon) -> Expon
    ++ \spad{order(f,n) = min(m,n)}, where m is the degree of the
    ++ lowest order non-zero term in f.
  truncate: (%,Expon) -> %
    ++ \spad{truncate(f,k)} returns a (finite) power series consisting of
    ++ the sum of all terms of f of degree \spad{<= k}.
  truncate: (%,Expon,Expon) -> %
    ++ \spad{truncate(f,k1,k2)} returns a (finite) power
    ++ series consisting of
    ++ the sum of all terms of f of degree d with \spad{k1 <= d <= k2}.
  if Coef has coerce: Symbol -> Coef then
    if Coef has "**":(Coef,Expon) -> Coef then
      approximate: (%,Expon) -> Coef
        ++ \spad{approximate(f)} returns a truncated power series with the
        ++ series variable viewed as an element of the coefficient domain.
  extend: (%,Expon) -> %
    ++ \spad{extend(f,n)} causes all terms of f of degree <= n
    ++ to be computed.
  if Expon has SemiGroup then Eltable(%,%)
  if Coef has "*": (Expon,Coef) -> Coef then
    DifferentialRing
    --!! DifferentialExtension Coef
    if Coef has PartialDifferentialRing Symbol then
      PartialDifferentialRing Symbol
  if Coef has "**": (Coef,Expon) -> Coef then
    eval: (%,Coef) -> Stream Coef
      ++ \spad{eval(f,a)} evaluates a power series at a value in the
      ++ ground ring by returning a stream of partial sums.

 add
  degree f == order f

  leadingCoefficient f == coefficient(f,order f)

  leadingMonomial f ==
    ord := order f
    monomial(coefficient(f,ord),ord)

  monomial(f:%,listVar:List SingletonAsOrderedSet,listExpon:List Expon) ==
    empty? listVar or not empty? rest listVar =>
      error "monomial: variable list must have exactly one entry"
    empty? listExpon or not empty? rest listExpon =>
```

```
          error "monomial: exponent list must have exactly one entry"
        f * monomial(1,first listExpon)

      monomial(f:%,v:SingletonAsOrderedSet,n:Expon) ==
        f * monomial(1,n)

      reductum f == f - leadingMonomial f

      variables f == list create()
```

⟨*UPSCAT.dotabb*⟩≡

```
"UPSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPSCAT"];
"UPSCAT" -> "PSCAT"
```

⟨*UPSCAT.dotfull*⟩≡

```
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPSCAT"];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=UPSCAT"];
"UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

"UnivariatePowerSeriesCategory(a:Ring,Integer)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=UPSCAT"];
"UnivariatePowerSeriesCategory(a:Ring,Integer)" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

"UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=UPSCAT"];
"UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
```

⟨*UPSCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"

"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" [color=lightblue];
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" -> "RING..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "BIMODULE..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "IntegralDomain()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CharacteristicNonZero()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CommutativeRing()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "Algebra(Fraction(Integer))"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BIMODULE..."

"CharacteristicNonZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CharacteristicNonZero()" -> "RING..."
```

```
"Algebra(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."

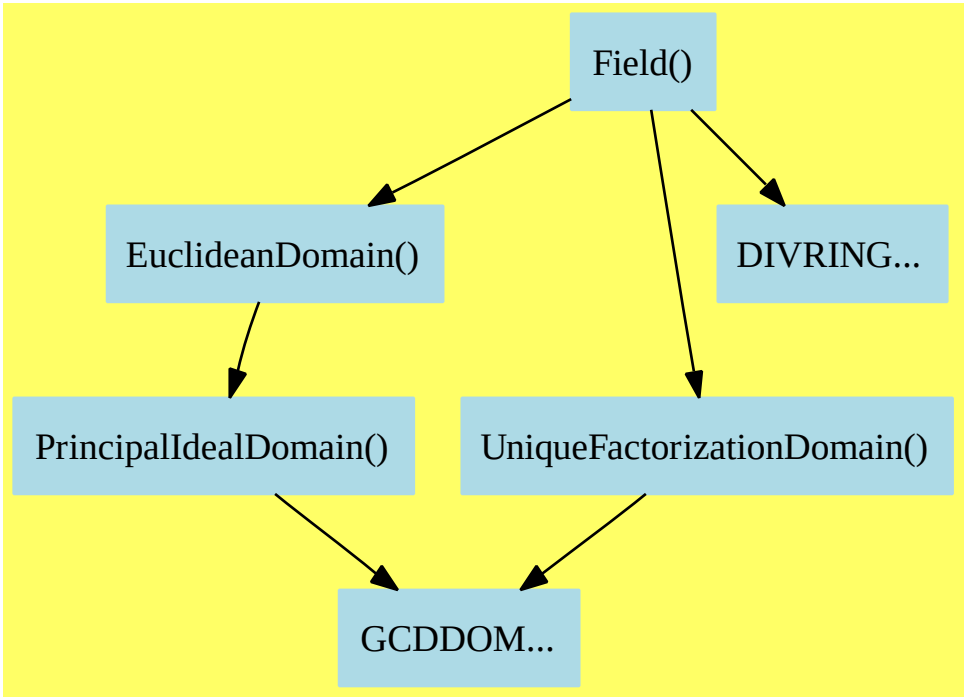
"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BIMODULE..."

"BIMODULE..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

Chapter 16

Category Layer 15

16.1 Field (FIELD)



Euclidean Domains are a subset of Fields.
⇐ “EuclideanDomain” (EUCDOM) 15.1 on page 975

See:

⇒ "AlgebraicallyClosedField" (ACF) 17.1 on page 1061
⇒ "ExtensionField" (XF) 18.2 on page 1237
⇒ "FieldOfPrimeCharacteristic" (FPC) 17.3 on page 1084
⇒ "FiniteRankAlgebra" (FINRALG) 17.4 on page 1089
⇒ "FunctionSpace" (FS) 17.5 on page 1095
⇒ "QuotientFieldCategory" (QFCAT) 17.6 on page 1121
⇒ "RealClosedField" (RCFIELD) 17.7 on page 1132
⇒ "RealNumberSystem" (RNS) 17.8 on page 1141
⇒ "UnivariateLaurentSeriesCategory" (ULSCAT) 17.10 on page 1186
⇒ "UnivariatePolynomialCategory" (UPOLYC) 17.12 on page 1204
⇒ "UnivariatePuiseuxSeriesCategory" (UPXSCAT) 17.11 on page 1195
⇐ "DivisionRing" (DIVRING) 12.2 on page 825
⇐ "EuclideanDomain" (EUCDOM) 15.1 on page 975
⇐ "UniqueFactorizationDomain" (UFD) 14.5 on page 969

Exports:

0	1	associates?	characteristic	coerce
divide	euclideanSize	expressIdealMember	exquo	extendedEuclidean
factor	gcd	gcdPolynomial	hash	inv
latex	lcm	multiEuclidean	one?	prime?
principalIdeal	recip	sample	sizeLess?	squareFree
squareFreePart	subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?*?	?**?	?+?	?-?
-?	?/?	?=?	?^?	?quo?
?rem?	?~=?			

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are implemented by this category:

```
associates? : (%,%) -> Boolean
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
exquo : (%,%) -> Union(%,"failed")
factor : % -> Factored %
gcd : (%,%) -> %
inv : % -> %
prime? : % -> Boolean
squareFree : % -> Factored %
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
?/? : (%,%) -> %
```

These exports come from (p975) EuclideanDomain():

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                     SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
```

```
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p969) UniqueFactorizationDomain():

```
squareFreePart : % -> %
```

These exports come from (p825) DivisionRing():

```
coerce : Fraction Integer -> %
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
```

⟨*category FIELD Field*⟩≡

```
)abbrev category FIELD Field
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of commutative fields, i.e. commutative rings
++ where all non-zero elements have multiplicative inverses.
++ The \spadfun{factor} operation while trivial is useful to have defined.
++
++ Axioms:
++   \spad{a*(b/a) = b}
++   \spad{inv(a) = 1/a}

Field(): Category == Join(EuclideanDomain,UniqueFactorizationDomain,
  DivisionRing) with
      "/": (%,%) -> %
        ++ x/y divides the element x by the element y.
        ++ Error: if y is 0.
      canonicalUnitNormal  ++ either 0 or 1.
      canonicalsClosed     ++ since \spad{0*0=0}, \spad{1*1=1}
    add
      x,y: %
```

```
n: Integer
UCA ==> Record(unit:%,canonical:%,associate:%)
unitNormal(x) ==
    if zero? x then [1$%,0$%,1$%]$UCA else [x,1$%,inv(x)]$UCA
unitCanonical(x) == if zero? x then x else 1
associates?(x,y) == if zero? x then zero? y else not(zero? y)
inv x ==((u:=recip x) case "failed" => error "not invertible"; u)
x exquo y == (y=0 => "failed"; x / y)
gcd(x,y) == 1
euclideanSize(x) == 0
prime? x == false
squareFree x == x::Factored(%)
factor x == x::Factored(%)
x / y == (zero? y => error "catdef: division by zero"; x * inv(y))
divide(x,y) == [x / y,0]
```

⟨*FIELD.dotabb*⟩≡

```
"FIELD"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FIELD"];
"FIELD" -> "EUCDOM"
"FIELD" -> "UFD"
"FIELD" -> "DIVRING"
```

⟨*FIELD.dotfull*⟩≡

```
"Field()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FIELD"];
"Field()" -> "EuclideanDomain()"
"Field()" -> "UniqueFactorizationDomain()"
"Field()" -> "DivisionRing()"
```

⟨*FIELD.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"Field()" [color=lightblue];
"Field()" -> "EuclideanDomain()"
"Field()" -> "UniqueFactorizationDomain()"
"Field()" -> "DIVRING..."

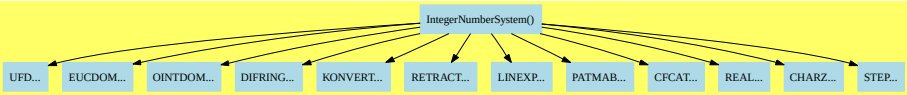
"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GCDDOM..."

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GCDDOM..."

"DIVRING..." [color=lightblue];
"GCDDOM..." [color=lightblue];
}
```

16.2 IntegerNumberSystem (INS)



See:

⇐ "CharacteristicZero" (CHARZ) 10.3 on page 681
⇐ "CombinatorialFunctionCategory" (CFCAT) 2.7 on page 17
⇐ "ConvertibleTo" (KONVERT) 2.8 on page 19
⇐ "DifferentialRing" (DIFRING) 10.5 on page 690
⇐ "EuclideanDomain" (EUCDOM) 15.1 on page 975
⇐ "LinearlyExplicitRingOver" (LINEXP) 10.9 on page 707
⇐ "OrderedIntegralDomain" (OINTDOM) 13.5 on page 937
⇐ "Patternable" (PATAB) 2.15 on page 38
⇐ "RealConstant" (REAL) 3.11 on page 85
⇐ "RetractableTo" (RETRACT) 2.18 on page 44
⇐ "StepThrough" (STEP) 4.26 on page 193
⇐ "UniqueFactorizationDomain" (UFD) 14.5 on page 969

Exports:

0	1	abs	addmod
associates?	base	binomial	bit?
characteristic	coerce	convert	copy
D	dec	differentiate	divide
euclideanSize	even?	expressIdealMember	exquo
extendedEuclidean	factor	factorial	gcd
gcdPolynomial	hash	inc	init
invmod	latex	lcm	length
mask	max	min	mulmod
multiEuclidean	negative?	nextItem	odd?
one?	patternMatch	permutation	positive?
positiveRemainder	powmod	prime?	principalIdeal
random	rational	rational?	rationalIfCan
recip	reducedSystem	retract	retractIfCan
sample	shift	sign	sizeLess?
squareFree	squareFreePart	submod	subtractIfCan
symmetricRemainder	unit?	unitCanonical	unitNormal
zero?	?*?	?**?	?+?
?-?	-?	?<?	?<=?
?=?	?>?	?>=?	?^?
?~=?	?quo?	?rem?	

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative

for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.

- **multiplicativeValuation** implies `euclideanSize(a*b)=euclideanSize(a)*euclideanSize(b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
addmod : (%,%,%) -> %
base : () -> %
dec : % -> %
hash : % -> %
inc : % -> %
length : % -> %
mulmod : (%,%,%) -> %
odd? : % -> Boolean
positiveRemainder : (%,%) -> %
random : () -> %
random : % -> %
shift : (%,%) -> %
submod : (%,%,%) -> %
```

These are implemented by this category:

```
binomial : (%,%) -> %
bit? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
convert : % -> Float
convert : % -> DoubleFloat
convert : % -> InputForm
convert : % -> Pattern Integer
copy : % -> %
differentiate : % -> %
euclideanSize : % -> NonNegativeInteger
even? : % -> Boolean
factor : % -> Factored %
factorial : % -> %
```

```
init : () -> %
invmod : (%,%) -> %
mask : % -> %
nextItem : % -> Union(%,"failed")
patternMatch :
 (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
   PatternMatchResult(Integer,%)
permutation : (%,%) -> %
positive? : % -> Boolean
powmod : (%,%,%) -> %
prime? : % -> Boolean
rational : % -> Fraction Integer
rational? : % -> Boolean
rationalIfCan : % -> Union(Fraction Integer,"failed")
retract : % -> Integer
retractIfCan : % -> Union(Integer,"failed")
squareFree : % -> Factored %
symmetricRemainder : (%,%) -> %
```

These exports come from (p969) UniqueFactorizationDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
coerce : % -> %
coerce : Integer -> %
coerce : Integer -> %
coerce : % -> OutputForm
exquo : (%,%) -> Union(%,"failed")
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial :
 (SparseUnivariatePolynomial %,
  SparseUnivariatePolynomial %) ->
    SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
```

```
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

These exports come from (p975) EuclideanDomain():

```
divide : (%,%) -> Record(quotient: %,remainder: %)
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
multiEuclidean : (List %,%) -> Union(List %,"failed")
principalIdeal : List % -> Record(coef: List %,generator: %)
sizeLess? : (%,%) -> Boolean
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p937) OrderedIntegralDomain():

```
abs : % -> %
max : (%,%) -> %
min : (%,%) -> %
negative? : % -> Boolean
sign : % -> Integer
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

These exports come from (p690) DifferentialRing():

```
D : % -> %
D : (%,NonNegativeInteger) -> %
differentiate : (%,NonNegativeInteger) -> %
```

These exports come from (p19) ConvertibleTo(Integer):

```
convert : % -> Integer
```

These exports come from (p707) LinearlyExplicitRingOver(Integer):

```
reducedSystem : Matrix % -> Matrix Integer
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Integer,vec: Vector Integer)
```

⟨*category INS IntegerNumberSystem*⟩≡

```
)abbrev category INS IntegerNumberSystem
++ Author: Stephen M. Watt
++ Date Created:
++   January 1988
++ Change History:
++ Basic Operations:
++   addmod, base, bit?, copy, dec, even?, hash, inc, invmod, length, mask,
++   positiveRemainder, symmetricRemainder, multiplicativeValuation, mulmod,
++   odd?, powmod, random, rational, rational?, rationalIfCan, shift, submod
++ Description:  An \spad{IntegerNumberSystem} is a model for the integers.
IntegerNumberSystem(): Category ==
  Join(UniqueFactorizationDomain, EuclideanDomain, OrderedIntegralDomain,
         DifferentialRing, ConvertibleTo Integer, RetractableTo Integer,
           LinearlyExplicitRingOver Integer, ConvertibleTo InputForm,
             ConvertibleTo Pattern Integer, PatternMatchable Integer,
               CombinatorialFunctionCategory, RealConstant,
                 CharacteristicZero, StepThrough) with
   odd?     : % -> Boolean
      ++ odd?(n) returns true if and only if n is odd.
   even?    : % -> Boolean
      ++ even?(n) returns true if and only if n is even.
   multiplicativeValuation
      ++ euclideanSize(a*b) returns \spad{euclideanSize(a)*euclideanSize(b)}.
   base     : () -> %
      ++ base() returns the base for the operations of
      ++ \spad{IntegerNumberSystem}.
   length   : % -> %
      ++ length(a) length of \spad{a} in digits.
   shift    : (%, %) -> %
      ++ shift(a,i) shift \spad{a} by i digits.
   bit?     : (%, %) -> Boolean
      ++ bit?(n,i) returns true if and only if i-th bit of n is a 1.
   positiveRemainder     : (%, %) -> %
      ++ positiveRemainder(a,b) (where \spad{b > 1}) yields r
      ++ where \spad{0 <= r < b} and \spad{r == a rem b}.
   symmetricRemainder     : (%, %) -> %
      ++ symmetricRemainder(a,b) (where \spad{b > 1}) yields r
      ++ where \spad{ -b/2 <= r < b/2 }.
   rational?: % -> Boolean
      ++ rational?(n) tests if n is a rational number
      ++ (see \spadtype{Fraction Integer}).
   rational : % -> Fraction Integer
      ++ rational(n) creates a rational number
      ++ (see \spadtype{Fraction Integer})..
   rationalIfCan: % -> Union(Fraction Integer, "failed")
```

```
      ++ rationalIfCan(n) creates a rational number, or returns "failed"
      ++ if this is not possible.
    random   : () -> %
      ++ random() creates a random element.
    random   : % -> %
      ++ random(a) creates a random element from 0 to \spad{n-1}.
    hash     : % -> %
      ++ hash(n) returns the hash code of n.
    copy     : % -> %
      ++ copy(n) gives a copy of n.
    inc      : % -> %
      ++ inc(x) returns \spad{x + 1}.
    dec      : % -> %
      ++ dec(x) returns \spad{x - 1}.
    mask     : % -> %
      ++ mask(n) returns \spad{2**n-1} (an n bit mask).
    addmod   : (%,%,%) -> %
      ++ addmod(a,b,p), \spad{0<=a,b<p>1}, means \spad{a+b mod p}.
    submod   : (%,%,%) -> %
      ++ submod(a,b,p), \spad{0<=a,b<p>1}, means \spad{a-b mod p}.
    mulmod   : (%,%,%) -> %
      ++ mulmod(a,b,p), \spad{0<=a,b<p>1}, means \spad{a*b mod p}.
    powmod   : (%,%,%) -> %
      ++ powmod(a,b,p), \spad{0<=a,b<p>1}, means \spad{a**b mod p}.
    invmod   : (%,%) -> %
      ++ invmod(a,b), \spad{0<=a<b>1}, \spad{(a,b)=1} means \spad{1/a mod b}.
    canonicalUnitNormal
--   commutative("*")    -- follows from the above

  add
    characteristic()         == 0

    differentiate x          == 0

    even? x                  == not odd? x

    positive? x              == x > 0

    copy x                   == x

    bit?(x, i)               == odd? shift(x, -i)

    mask n                   == dec shift(1, n)

    rational? x              == true
```

```
euclideanSize(x)         ==
     x=0 => error "euclideanSize called on zero"
     x<0 => (-convert(x)@Integer)::NonNegativeInteger
     convert(x)@Integer::NonNegativeInteger

convert(x:%):Float       == (convert(x)@Integer)::Float

convert(x:%):DoubleFloat  == (convert(x)@Integer)::DoubleFloat

convert(x:%):InputForm   == convert(convert(x)@Integer)

retract(x:%):Integer     == convert(x)@Integer

convert(x:%):Pattern(Integer)== convert(x)@Integer ::Pattern(Integer)

factor x          == factor(x)$IntegerFactorizationPackage(%)

squareFree x      == squareFree(x)$IntegerFactorizationPackage(%)

prime? x          == prime?(x)$IntegerPrimesPackage(%)

factorial x       == factorial(x)$IntegerCombinatoricFunctions(%)

binomial(n, m)    == binomial(n, m)$IntegerCombinatoricFunctions(%)

permutation(n, m) == permutation(n,m)$IntegerCombinatoricFunctions(%)

retractIfCan(x:%):Union(Integer, "failed") == convert(x)@Integer

init() == 0

-- iterates in order 0,1,-1,2,-2,3,-3,...
nextItem(n) ==
  zero? n => 1
  n>0 => -n
  1-n

patternMatch(x, p, l) ==
  patternMatch(x, p, l)$PatternMatchIntegerNumberSystem(%)

rational(x:%):Fraction(Integer) ==
  (convert(x)@Integer)::Fraction(Integer)

rationalIfCan(x:%):Union(Fraction Integer, "failed") ==
  (convert(x)@Integer)::Fraction(Integer)
```

```
    symmetricRemainder(x, n) ==
       r := x rem n
       r = 0 => r
       if n < 0 then n:=-n
       r > 0 =>
          2 * r > n => r - n
          r
       2*r + n <= 0 => r + n
       r

    invmod(a, b) ==
       if negative? a then a := positiveRemainder(a, b)
       c := a; c1:% := 1
       d := b; d1:% := 0
       while not zero? d repeat
          q := c quo d
          r := c-q*d
          r1 := c1-q*d1
          c := d; c1 := d1
          d := r; d1 := r1
--       not one? c => error "inverse does not exist"
       not (c = 1) => error "inverse does not exist"
       negative? c1 => c1 + b
       c1

    powmod(x, n, p) ==
       if negative? x then x := positiveRemainder(x, p)
       zero? x => 0
       zero? n => 1
       y:% := 1
       z := x
       repeat
          if odd? n then y := mulmod(y, z, p)
          zero?(n := shift(n, -1)) => return y
          z := mulmod(z, z, p)
```

⟨*INS.dotabb*⟩≡
```
"INS"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INS"];
"INS" -> "UFD"
"INS" -> "EUCDOM"
"INS" -> "OINTDOM"
"INS" -> "DIFRING"
"INS" -> "KONVERT"
"INS" -> "RETRACT"
"INS" -> "LINEXP"
"INS" -> "PATMAB"
"INS" -> "CFCAT"
"INS" -> "REAL"
"INS" -> "CHARZ"
"INS" -> "STEP"
```

⟨*INS.dotfull*⟩≡
```
"IntegerNumberSystem()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=INS"];
"IntegerNumberSystem()" -> "UniqueFactorizationDomain()"
"IntegerNumberSystem()" -> "EuclideanDomain()"
"IntegerNumberSystem()" -> "OrderedIntegralDomain()"
"IntegerNumberSystem()" -> "DifferentialRing()"
"IntegerNumberSystem()" -> "ConvertibleTo(Integer)"
"IntegerNumberSystem()" -> "ConvertibleTo(InputForm)"
"IntegerNumberSystem()" -> "ConvertibleTo(Pattern(Integer))"
"IntegerNumberSystem()" -> "RetractableTo(Integer)"
"IntegerNumberSystem()" -> "LinearlyExplicitRingOver(Integer)"
"IntegerNumberSystem()" -> "PatternMatchable(Integer)"
"IntegerNumberSystem()" -> "CombinatorialFunctionCategory()"
"IntegerNumberSystem()" -> "RealConstant()"
"IntegerNumberSystem()" -> "CharacteristicZero()"
"IntegerNumberSystem()" -> "StepThrough()"
```

⟨*INS.dotpic*⟩≡

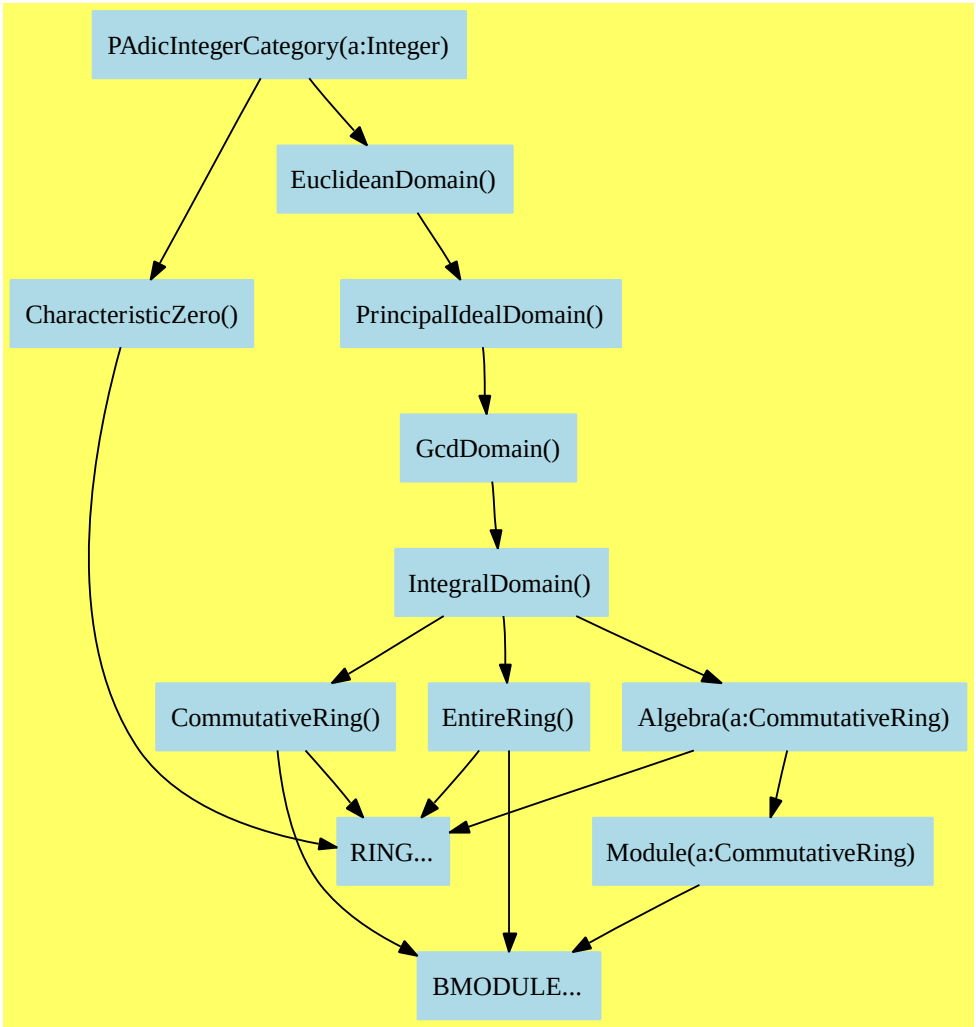
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"IntegerNumberSystem()" [color=lightblue];
"IntegerNumberSystem()" -> "UFD..."
"IntegerNumberSystem()" -> "EUCDOM..."
"IntegerNumberSystem()" -> "OINTDOM..."
"IntegerNumberSystem()" -> "DIFRING..."
"IntegerNumberSystem()" -> "KONVERT..."
"IntegerNumberSystem()" -> "RETRACT..."
"IntegerNumberSystem()" -> "LINEXP..."
"IntegerNumberSystem()" -> "PATMAB..."
"IntegerNumberSystem()" -> "CFCAT..."
"IntegerNumberSystem()" -> "REAL..."
"IntegerNumberSystem()" -> "CHARZ..."
"IntegerNumberSystem()" -> "STEP..."

"UFD..." [color=lightblue];
"EUCDOM..." [color=lightblue];
"OINTDOM..." [color=lightblue];
"DIFRING..." [color=lightblue];
"KONVERT..." [color=lightblue];
"RETRACT..." [color=lightblue];
"LINEXP..." [color=lightblue];
"PATMAB..." [color=lightblue];
"CFCAT..." [color=lightblue];
"REAL..." [color=lightblue];
"CHARZ..." [color=lightblue];
"STEP..." [color=lightblue];

}
```

16.3 PAdicIntegerCategory (PADICCT)



See:

⇐ "CharacteristicZero" (CHARZ) 10.3 on page 681
⇐ "EuclideanDomain" (EUCDOM) 15.1 on page 975

Exports:

0	1	approximate	associates?
characteristic	coerce	complete	digits
divide	euclideanSize	expressIdealMember	exquo
extend	extendedEuclidean	gcd	gcdPolynomial
hash	latex	lcm	moduloP
modulus	multiEuclidean	one?	order
principalIdeal	quotientByP	recip	root
sample	sizeLess?	sqrt	subtractIfCan
unit?	unitCanonical	unitNormal	zero?
?*?	?**?	?+?	?-?
-?	?=?	?quo?	?rem?
?~=?	?^?		

Attributes Exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
approximate : (%,Integer) -> Integer
complete : % -> %
digits : % -> Stream Integer
extend : (%,Integer) -> %
moduloP : % -> Integer
modulus : () -> Integer
order : % -> NonNegativeInteger
quotientByP : % -> %
root : (SparseUnivariatePolynomial Integer,Integer) -> %
sqrt : (%,Integer) -> %
```

These exports come from (p975) EuclideanDomain():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
```

```
coerce : % -> OutputForm
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
hash : % -> SingleInteger
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

⟨*category PADICCT PAdicIntegerCategory*⟩≡

```
)abbrev category PADICCT PAdicIntegerCategory
++ Author: Clifton J. Williamson
++ Date Created: 15 May 1990
++ Date Last Updated: 15 May 1990
++ Basic Operations:
```

```
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: p-adic, completion
++ Examples:
++ References:
++ Description: This is the category of stream-based representations of
++   the p-adic integers.
PAdicIntegerCategory(p): Category == Definition where
  p   :   Integer
  I   ==> Integer
  NNI ==> NonNegativeInteger
  ST  ==> Stream
  SUP ==> SparseUnivariatePolynomial

  Definition ==> Join(EuclideanDomain,CharacteristicZero) with
    digits: % -> ST I
      ++ \spad{digits(x)} returns a stream of p-adic digits of x.
    order: % -> NNI
      ++ \spad{order(x)} returns the exponent of the highest power of p
      ++ dividing x.
    extend: (%,I) -> %
      ++ \spad{extend(x,n)} forces the computation of digits up to order n.
    complete: % -> %
      ++ \spad{complete(x)} forces the computation of all digits.
    modulus: () -> I
      ++ \spad{modulus()} returns the value of p.
    moduloP: % -> I
      ++ \spad{modulo(x)} returns a, where \spad{x = a + b p}.
    quotientByP: % -> %
      ++ \spad{quotientByP(x)} returns b, where \spad{x = a + b p}.
    approximate: (%,I) -> I
      ++ \spad{approximate(x,n)} returns an integer y such that
      ++ \spad{y = x (mod p^n)}
      ++ when n is positive, and 0 otherwise.
    sqrt: (%,I) -> %
      ++ \spad{sqrt(b,a)} returns a square root of b.
      ++ Argument \spad{a} is a square root of b \spad{(mod p)}.
    root: (SUP I,I) -> %
      ++ \spad{root(f,a)} returns a root of the polynomial \spad{f}.
      ++ Argument \spad{a} must be a root of \spad{f} \spad{(mod p)}.
```

⟨*PADICCT.dotabb*⟩≡
```
"PADICCT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PADICCT"];
"PADICCT" -> "CHARZ"
"PADICCT" -> "EUCDOM"
```

⟨*PADICCT.dotfull*⟩≡
```
"PAdicIntegerCategory(a:Integer)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=PADICCT"];
"PAdicIntegerCategory(a:Integer)" -> "CharacteristicZero()"
"PAdicIntegerCategory(a:Integer)" -> "EuclideanDomain()"
```

⟨*PADICCT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PAdicIntegerCategory(a:Integer)" [color=lightblue];
"PAdicIntegerCategory(a:Integer)" -> "CharacteristicZero()"
"PAdicIntegerCategory(a:Integer)" -> "EuclideanDomain()"

"CharacteristicZero()" [color=lightblue];
"CharacteristicZero()" -> "RING..."

"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GcdDomain()"

"GcdDomain()" [color=lightblue];
"GcdDomain()" -> "IntegralDomain()"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
"IntegralDomain()" -> "EntireRing()"

"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BMODULE..."

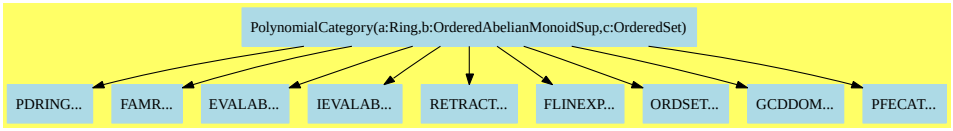
"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BMODULE..."

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "Module(a:CommutativeRing)"

"Module(a:CommutativeRing)" [color=lightblue];
"Module(a:CommutativeRing)" -> "BMODULE..."

"RING..." [color=lightblue];
"BMODULE..." [color=lightblue];
}
```

16.4 PolynomialCategory (POLYCAT)



See:

⇒ “DifferentialPolynomialCategory” (DPOLCAT) 17.2 on page 1069
⇒ “RecursivePolynomialCategory” (RPOLCAT) 17.9 on page 1148
⇒ “UnivariatePolynomialCategory” (UPOLYC) 17.12 on page 1204

⇐ “InnerEvalable” (IEVALAB) 2.12 on page 29
⇐ “Evalable” (EVALAB) 3.4 on page 65
⇐ “FiniteAbelianMonoidRing” (FAMR) 14.1 on page 941
⇐ “GcdDomain” (GCDDOM) 13.4 on page 932
⇐ “FullyLinearlyExplicitRingOver” (FLINEXP) 11.3 on page 782
⇐ “OrderedSet” (ORDSET) 4.19 on page 168
⇐ “PartialDifferentialRing” (PDRING) 10.12 on page 720
⇐ “PolynomialFactorizationExplicit” (PFECAT) 15.3 on page 990
⇐ “RetractableTo” (RETRACT) 2.18 on page 44

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
conditionP	content	convert
D	degree	differentiate
discriminant	eval	exquo
factor	factorPolynomial	factorSquareFreePolynomial
gcd	gcdPolynomial	ground
ground?	hash	isExpt
isPlus	isTimes	latex
lcm	leadingCoefficient	leadingMonomial
mainVariable	map	mapExponents
max	min	minimumDegree
monicDivide	monomial	monomial?
monomials	multivariate	numberOfMonomials
one?	patternMatch	pomopo!
prime?	primitiveMonomials	primitivePart
recip	reducedSystem	reductum
resultant	retract	retractIfCan
sample	solveLinearPolynomialEquation	squareFree
squareFreePart	squareFreePolynomial	subtractIfCan
totalDegree	unit?	unitCanonical
unitNormal	univariate	variables
zero?	?*?	?**?
?+?	?-?	-?
?=?	?^?	?~=?
?/?	?<?	?<=?
?>?	?>=?	

Attributes Exported:

-
- if R has canonicalUnitNormal then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if $ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if $ has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.

- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
degree : (%,VarSet) -> NonNegativeInteger
degree : (%,List VarSet) -> List NonNegativeInteger
mainVariable : % -> Union(VarSet,"failed")
minimumDegree : (%,List VarSet) -> List NonNegativeInteger
minimumDegree : (%,VarSet) -> NonNegativeInteger
monomial : (%,VarSet,NonNegativeInteger) -> %
multivariate : (SparseUnivariatePolynomial %,VarSet) -> %
multivariate : (SparseUnivariatePolynomial R,VarSet) -> %
univariate : (%,VarSet) -> SparseUnivariatePolynomial %
univariate : % -> SparseUnivariatePolynomial R
variables : % -> List VarSet
```

These are implemented by this category:

```
charthRoot : % -> Union(%,"failed")
 if
   and(has($,CharacteristicNonZero),
       has(R,PolynomialFactorizationExplicit))
   or R has CHARNZ
coefficient : (%,VarSet,NonNegativeInteger) -> %
coefficient : (%,List VarSet,List NonNegativeInteger) -> %
conditionP : Matrix % -> Union(Vector %,"failed")
 if
   and(has($,CharacteristicNonZero),
       has(R,PolynomialFactorizationExplicit))
content : (%,VarSet) -> % if R has GCDDOM
convert : % -> Pattern Integer
    if VarSet has KONVERT PATTERN INT
    and R has KONVERT PATTERN INT
convert : % -> Pattern Float
    if VarSet has KONVERT PATTERN FLOAT
    and R has KONVERT PATTERN FLOAT
convert : % -> InputForm
    if VarSet has KONVERT INFORM
    and R has KONVERT INFORM
discriminant : (%,VarSet) -> % if R has COMRING
eval : (%,List Equation %) -> %
factor : % -> Factored % if R has PFECAT
factorPolynomial :
   SparseUnivariatePolynomial % ->
      Factored SparseUnivariatePolynomial %
        if R has PFECAT
factorSquareFreePolynomial :
   SparseUnivariatePolynomial % ->
      Factored SparseUnivariatePolynomial %
        if R has PFECAT
```

```
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
    if R has GCDDOM
isExpt : % ->
  Union(Record(var: VarSet,exponent: NonNegativeInteger),"failed")
isPlus : % -> Union(List %,"failed")
isTimes : % -> Union(List %,"failed")
monicDivide : (%,%,VarSet) -> Record(quotient: %,remainder: %)
monomial : (%,List VarSet,List NonNegativeInteger) -> %
monomials : % -> List %
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
    PatternMatchResult(Integer,%)
      if VarSet has PATMAB INT
      and R has PATMAB INT
patternMatch :
   (%,Pattern Float,PatternMatchResult(Float,%)) ->
     PatternMatchResult(Float,%)
       if VarSet has PATMAB FLOAT
       and R has PATMAB FLOAT
primitiveMonomials : % -> List %
primitivePart : % -> % if R has GCDDOM
primitivePart : (%,VarSet) -> % if R has GCDDOM
reducedSystem : Matrix % -> Matrix R
reducedSystem : (Matrix %,Vector %) ->
   Record(mat: Matrix R,vec: Vector R)
resultant : (%,%,VarSet) -> % if R has COMRING
retract : % -> VarSet
retractIfCan : % -> Union(VarSet,"failed")
solveLinearPolynomialEquation :
   (List SparseUnivariatePolynomial %,
    SparseUnivariatePolynomial %) ->
       Union(List SparseUnivariatePolynomial %,"failed")
         if R has PFECAT
squareFree : % -> Factored % if R has GCDDOM
squareFreePart : % -> % if R has GCDDOM
totalDegree : % -> NonNegativeInteger
totalDegree : (%,List VarSet) -> NonNegativeInteger
?<? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p720) PartialDifferentialRing(VarSet)
where VarSet:OrderedSet:

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
D : (%,List VarSet) -> %
```

```
D : (%,VarSet) -> %
D : (%,List VarSet,List NonNegativeInteger) -> %
D : (%,VarSet,NonNegativeInteger) -> %
differentiate : (%,VarSet) -> %
differentiate : (%,List VarSet,List NonNegativeInteger) -> %
differentiate : (%,VarSet,NonNegativeInteger) -> %
differentiate : (%,List VarSet) -> %
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
```

These exports come from (p941) FiniteAbelianMonoidRing(R,E) where R:Ring and E:OrderedAbelianMonoidSup:

```
associates? : (%,%) -> Boolean if R has INTDOM
binomThmExpt : (%,%,NonNegativeInteger) -> %
    if R has COMRING
coefficient : (%,E) -> R
coefficients : % -> List R
coerce : R -> %
coerce : Fraction Integer -> %
    if R has RETRACT FRAC INT
    or R has ALGEBRA FRAC INT
coerce : % -> % if R has INTDOM
content : % -> R if R has GCDDOM
degree : % -> E
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
exquo : (%,%) -> Union(%,"failed") if R has INTDOM
ground : % -> R
ground? : % -> Boolean
leadingCoefficient : % -> R
leadingMonomial : % -> %
map : ((R -> R),%) -> %
```

```
mapExponents : ((E -> E),%) -> %
minimumDegree : % -> E
monomial : (R,E) -> %
monomial? : % -> Boolean
numberOfMonomials : % -> NonNegativeInteger
pomopo! : (%,R,E,%) -> %
reductum : % -> %
retract : % -> Integer if R has RETRACT INT
retract : % -> Fraction Integer
    if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
    if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if R has RETRACT FRAC INT
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if R has INTDOM
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Fraction Integer,%) -> %
    if R has ALGEBRA FRAC INT
?*? : (%,Fraction Integer) -> %
    if R has ALGEBRA FRAC INT
?/? : (%,R) -> % if R has FIELD
```

These exports come from (p65) Evalable(PolynomialCategory(...)):

```
eval : (%,Equation %) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
```

These exports come from (p29) InnerEvalable(VarSet,R)
where VarSet:OrderedSet and R:Ring

```
eval : (%,VarSet,R) -> %
eval : (%,List VarSet,List R) -> %
```

These exports come from (p29) InnerEvalable(VarSet,R)
where VarSet:OrderedSet and R:PolynomialCategory(...):

```
eval : (%,VarSet,%) -> %
eval : (%,List VarSet,List %) -> %
```

These exports come from (p44) RetractableTo(VarSet)
where VarSet:OrderedSet:

```
coerce : VarSet -> %
retract : % -> R
retractIfCan : % -> Union(R,"failed")
```

These exports come from (p782) FullyLinearlyExplicitRingOver(R)
where R:Ring:

```
reducedSystem : (Matrix %,Vector %) ->
   Record(mat: Matrix Integer,vec: Vector Integer)
      if R has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer
      if R has LINEXP INT
```

These exports come from (p168) OrderedSet():

```
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p932) GcdDomain():

```
gcd : (%,%) -> % if R has GCDDOM
gcd : List % -> % if R has GCDDOM
lcm : (%,%) -> % if R has GCDDOM
lcm : List % -> % if R has GCDDOM
```

These exports come from (p990) PolynomialFactorizationExplicit():

```
prime? : % -> Boolean if R has PFECAT
squareFreePolynomial : SparseUnivariatePolynomial % ->
   Factored SparseUnivariatePolynomial %
     if R has PFECAT
```

⟨*category POLYCAT PolynomialCategory*⟩≡

```
)abbrev category POLYCAT PolynomialCategory
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions: Ring, monomial, coefficient, differentiate, eval
++ Related Constructors: Polynomial, DistributedMultivariatePolynomial
++ Also See: UnivariatePolynomialCategory
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category for general multi-variate polynomials over a ring
++ R, in variables from VarSet, with exponents from the
++ \spadtype{OrderedAbelianMonoidSup}.

PolynomialCategory(R:Ring, E:OrderedAbelianMonoidSup, VarSet:OrderedSet):
```

```
      Category ==
  Join(PartialDifferentialRing VarSet, FiniteAbelianMonoidRing(R, E),
       Evalable %, InnerEvalable(VarSet, R),
       InnerEvalable(VarSet, %), RetractableTo VarSet,
       FullyLinearlyExplicitRingOver R) with
    degree : (%,VarSet) -> NonNegativeInteger
      ++ degree(p,v) gives the degree of polynomial p with respect
      ++ to the variable v.
    degree : (%,List(VarSet)) -> List(NonNegativeInteger)
      ++ degree(p,lv) gives the list of degrees of polynomial p
      ++ with respect to each of the variables in the list lv.
    coefficient: (%,VarSet,NonNegativeInteger) -> %
      ++ coefficient(p,v,n) views the polynomial p as a univariate
      ++ polynomial in v and returns the coefficient of the \spad{v**n} term.
    coefficient: (%,List VarSet,List NonNegativeInteger) -> %
      ++ coefficient(p, lv, ln) views the polynomial p as a polynomial
      ++ in the variables of lv and returns the coefficient of the term
      ++ \spad{lv**ln}, i.e. \spad{prod(lv_i ** ln_i)}.
    monomials: % -> List %
      ++ monomials(p) returns the list of non-zero monomials of
      ++ polynomial p, i.e.
      ++ \spad{monomials(sum(a_(i) X^(i))) = [a_(1) X^(1),...,a_(n) X^(n)]}.
    univariate   : (%,VarSet) -> SparseUnivariatePolynomial(%)
      ++ univariate(p,v) converts the multivariate polynomial p
      ++ into a univariate polynomial in v, whose coefficients are still
      ++ multivariate polynomials (in all the other variables).
    univariate   : % -> SparseUnivariatePolynomial(R)
      ++ univariate(p) converts the multivariate polynomial p,
      ++ which should actually involve only one variable,
      ++ into a univariate polynomial
      ++ in that variable, whose coefficients are in the ground ring.
      ++ Error: if polynomial is genuinely multivariate
    mainVariable  : % -> Union(VarSet,"failed")
      ++ mainVariable(p) returns the biggest variable which actually
      ++ occurs in the polynomial p, or "failed" if no variables are
      ++ present.
      ++ fails precisely if polynomial satisfies ground?
    minimumDegree : (%,VarSet) -> NonNegativeInteger
      ++ minimumDegree(p,v) gives the minimum degree of polynomial p
      ++ with respect to v, i.e. viewed a univariate polynomial in v
    minimumDegree : (%,List(VarSet)) -> List(NonNegativeInteger)
      ++ minimumDegree(p, lv) gives the list of minimum degrees of the
      ++ polynomial p with respect to each of the variables in the list lv
    monicDivide : (%,%,VarSet) -> Record(quotient:%,remainder:%)
      ++ monicDivide(a,b,v) divides the polynomial a by the polynomial b,
      ++ with each viewed as a univariate polynomial in v returning
```

```
    ++ both the quotient and remainder.
    ++ Error: if b is not monic with respect to v.
  monomial : (%,VarSet,NonNegativeInteger) -> %
    ++ monomial(a,x,n) creates the monomial \spad{a*x**n} where \spad{a} is
    ++ a polynomial, x is a variable and n is a nonnegative integer.
  monomial : (%,List VarSet,List NonNegativeInteger) -> %
    ++ monomial(a,[v1..vn],[e1..en]) returns \spad{a*prod(vi**ei)}.
  multivariate : (SparseUnivariatePolynomial(R),VarSet) -> %
    ++ multivariate(sup,v) converts an anonymous univariable
    ++ polynomial sup to a polynomial in the variable v.
  multivariate : (SparseUnivariatePolynomial(%),VarSet) -> %
    ++ multivariate(sup,v) converts an anonymous univariable
    ++ polynomial sup to a polynomial in the variable v.
  isPlus: % -> Union(List %, "failed")
    ++ isPlus(p) returns \spad{[m1,...,mn]} if polynomial
    ++ \spad{p = m1 + ... + mn} and
    ++ \spad{n >= 2} and each mi is a nonzero monomial.
  isTimes: % -> Union(List %, "failed")
    ++ isTimes(p) returns \spad{[a1,...,an]} if polynomial
    ++ \spad{p = a1 ... an} and \spad{n >= 2}, and, for each i,
    ++ ai is either a nontrivial constant in R or else of the
    ++ form \spad{x**e}, where \spad{e > 0} is an integer
    ++ and x in a member of VarSet.
  isExpt: % -> Union(Record(var:VarSet, exponent:NonNegativeInteger),_
                     "failed")
    ++ isExpt(p) returns \spad{[x, n]} if polynomial p has the
    ++ form \spad{x**n} and \spad{n > 0}.
  totalDegree : % -> NonNegativeInteger
    ++ totalDegree(p) returns the largest sum over all monomials
    ++ of all exponents of a monomial.
  totalDegree : (%,List VarSet) -> NonNegativeInteger
    ++ totalDegree(p, lv) returns the maximum sum (over all monomials
    ++ of polynomial p) of the variables in the list lv.
  variables : % -> List(VarSet)
    ++ variables(p) returns the list of those variables actually
    ++ appearing in the polynomial p.
  primitiveMonomials: % -> List %
    ++ primitiveMonomials(p) gives the list of monomials of the
    ++ polynomial p with their coefficients removed. Note:
    ++ \spad{primitiveMonomials(sum(a_(i) X^(i))) = [X^(1),...,X^(n)]}.
  if R has OrderedSet  then OrderedSet
  -- OrderedRing view removed to allow EXPR to define abs
  --if R has OrderedRing then OrderedRing
  if (R has ConvertibleTo InputForm) and
     (VarSet has ConvertibleTo InputForm) then
       ConvertibleTo InputForm
```

```
if (R has ConvertibleTo Pattern Integer) and
   (VarSet has ConvertibleTo Pattern Integer) then
     ConvertibleTo Pattern Integer
if (R has ConvertibleTo Pattern Float) and
   (VarSet has ConvertibleTo Pattern Float) then
     ConvertibleTo Pattern Float
if (R has PatternMatchable Integer) and
   (VarSet has PatternMatchable Integer) then
     PatternMatchable Integer
if (R has PatternMatchable Float) and
   (VarSet has PatternMatchable Float) then
     PatternMatchable Float
if R has CommutativeRing then
  resultant : (%,%,VarSet) -> %
     ++ resultant(p,q,v) returns the resultant of the polynomials
     ++ p and q with respect to the variable v.
  discriminant : (%,VarSet) -> %
     ++ discriminant(p,v) returns the disriminant of the polynomial p
     ++ with respect to the variable v.
if R has GcdDomain then
  GcdDomain
  content: (%,VarSet) -> %
    ++ content(p,v) is the gcd of the coefficients of the polynomial p
    ++ when p is viewed as a univariate polynomial with respect to the
    ++ variable v.
    ++ Thus, for polynomial 7*x**2*y + 14*x*y**2, the gcd of the
    ++ coefficients with respect to x is 7*y.
  primitivePart: % -> %
    ++ primitivePart(p) returns the unitCanonical associate of the
    ++ polynomial p with its content divided out.
  primitivePart: (%,VarSet) -> %
    ++ primitivePart(p,v) returns the unitCanonical associate of the
    ++ polynomial p with its content with respect to the variable v
    ++ divided out.
  squareFree: % -> Factored %
    ++ squareFree(p) returns the square free factorization of the
    ++ polynomial p.
  squareFreePart: % -> %
    ++ squareFreePart(p) returns product of all the irreducible factors
    ++ of polynomial p each taken with multiplicity one.

-- assertions
if R has canonicalUnitNormal then canonicalUnitNormal
         ++ we can choose a unique representative for each
         ++ associate class.
         ++ This normalization is chosen to be normalization of
```

```
            ++ leading coefficient (by default).
    if R has PolynomialFactorizationExplicit then
       PolynomialFactorizationExplicit
 add
    p:%
    v:VarSet
    ln:List NonNegativeInteger
    lv:List VarSet
    n:NonNegativeInteger
    pp,qq:SparseUnivariatePolynomial %

    eval(p:%, l:List Equation %) ==
      empty? l => p
      for e in l repeat
        retractIfCan(lhs e)@Union(VarSet,"failed") case "failed" =>
             error "cannot find a variable to evaluate"
      lvar:=[retract(lhs e)@VarSet for e in l]
      eval(p, lvar,[rhs e for e in l]$List(%))

    monomials p ==
--    zero? p => empty()
--    concat(leadingMonomial p, monomials reductum p)
--    replaced by sequential version for efficiency, by WMSIT, 7/30/90
      ml:= empty$List(%)
      while p ^= 0 repeat
        ml:=concat(leadingMonomial p, ml)
        p:= reductum p
      reverse ml

    isPlus p ==
      empty? rest(l := monomials p) => "failed"
      l

    isTimes p ==
      empty?(lv := variables p) or not monomial? p => "failed"
      l := [monomial(1, v, degree(p, v)) for v in lv]
--      one?(r := leadingCoefficient p) =>
      ((r := leadingCoefficient p) = 1) =>
        empty? rest lv => "failed"
        l
      concat(r::%, l)

    isExpt p ==
      (u := mainVariable p) case "failed" => "failed"
      p = monomial(1, u::VarSet, d := degree(p, u::VarSet)) =>
        [u::VarSet, d]
```

```
  "failed"

coefficient(p,v,n) == coefficient(univariate(p,v),n)

coefficient(p,lv,ln) ==
   empty? lv =>
     empty? ln => p
     error "mismatched lists in coefficient"
   empty? ln  => error "mismatched lists in coefficient"
   coefficient(coefficient(univariate(p,first lv),first ln),
               rest lv,rest ln)

monomial(p,lv,ln) ==
   empty? lv =>
     empty? ln => p
     error "mismatched lists in monomial"
   empty? ln  => error "mismatched lists in monomial"
   monomial(monomial(p,first lv, first ln),rest lv, rest ln)

retract(p:%):VarSet ==
  q := mainVariable(p)::VarSet
  q::% = p => q
  error "Polynomial is not a single variable"

retractIfCan(p:%):Union(VarSet, "failed") ==
  ((q := mainVariable p) case VarSet) and (q::VarSet::% = p) => q
  "failed"

mkPrim(p:%):% == monomial(1,degree p)

primitiveMonomials p == [mkPrim q for q in monomials p]

totalDegree p ==
    ground? p => 0
    u := univariate(p, mainVariable(p)::VarSet)
    d: NonNegativeInteger := 0
    while u ^= 0 repeat
      d := max(d, degree u + totalDegree leadingCoefficient u)
      u := reductum u
    d

totalDegree(p,lv) ==
    ground? p => 0
    u := univariate(p, v:=(mainVariable(p)::VarSet))
    d: NonNegativeInteger := 0
    w: NonNegativeInteger := 0
```

```
      if member?(v, lv) then w:=1
      while u ^= 0 repeat
        d := max(d, w*(degree u) + totalDegree(leadingCoefficient u,lv))
        u := reductum u
      d

    if R has CommutativeRing then
      resultant(p1,p2,mvar) ==
        resultant(univariate(p1,mvar),univariate(p2,mvar))

      discriminant(p,var) ==
        discriminant(univariate(p,var))

    if R has IntegralDomain then
      allMonoms(l:List %):List(%) ==
        removeDuplicates_! concat [primitiveMonomials p for p in l]

      P2R(p:%, b:List E, n:NonNegativeInteger):Vector(R) ==
        w := new(n, 0)$Vector(R)
        for i in minIndex w .. maxIndex w for bj in b repeat
          qsetelt_!(w, i, coefficient(p, bj))
        w

      eq2R(l:List %, b:List E):Matrix(R) ==
        matrix [[coefficient(p, bj) for p in l] for bj in b]

      reducedSystem(m:Matrix %):Matrix(R) ==
        l := listOfLists m
        b := removeDuplicates_!
                          concat [allMonoms r for r in l]$List(List(%))
        d := [degree bj for bj in b]
        mm := eq2R(first l, d)
        l := rest l
        while not empty? l repeat
          mm := vertConcat(mm, eq2R(first l, d))
          l := rest l
        mm

      reducedSystem(m:Matrix %, v:Vector %):
       Record(mat:Matrix R, vec:Vector R) ==
        l := listOfLists m
        r := entries v
        b : List % := removeDuplicates_! concat(allMonoms r,
                         concat [allMonoms s for s in l]$List(List(%)))
        d := [degree bj for bj in b]
        n := #d
```

```
        mm := eq2R(first l, d)
        w := P2R(first r, d, n)
        l := rest l
        r := rest r
        while not empty? l repeat
          mm := vertConcat(mm, eq2R(first l, d))
          w := concat(w, P2R(first r, d, n))
          l := rest l
          r := rest r
        [mm, w]

      if R has PolynomialFactorizationExplicit then
         -- we might be in trouble if its actually only
         -- a univariate polynomial category - have to remember to
         -- over-ride these in UnivariatePolynomialCategory

         PFBR ==>PolynomialFactorizationByRecursion(R,E,VarSet,%)

         gcdPolynomial(pp,qq) ==
            gcdPolynomial(pp,qq)$GeneralPolynomialGcdPackage(E,VarSet,R,%)

         solveLinearPolynomialEquation(lpp,pp) ==
           solveLinearPolynomialEquationByRecursion(lpp,pp)$PFBR

         factorPolynomial(pp) ==
           factorByRecursion(pp)$PFBR

         factorSquareFreePolynomial(pp) ==
           factorSquareFreeByRecursion(pp)$PFBR

         factor p ==
           v:Union(VarSet,"failed"):=mainVariable p
           v case "failed" =>
             ansR:=factor leadingCoefficient p
             makeFR(unit(ansR)::%,
                    [[w.flg,w.fctr::%,w.xpnt] for w in factorList ansR])
           up:SparseUnivariatePolynomial %:=univariate(p,v)
           ansSUP:=factorByRecursion(up)$PFBR
           makeFR(multivariate(unit(ansSUP),v),
                  [[ww.flg,multivariate(ww.fctr,v),ww.xpnt]
                   for ww in factorList ansSUP])
         if R has CharacteristicNonZero then
            mat: Matrix %

            conditionP mat ==
              ll:=listOfLists transpose mat  --hence each list corresponds to a
```

```
                                        --column, i.e. to one variable
          llR:List List R := [ empty() for z in first ll]
          monslist:List List % := empty()
          ch:=characteristic()$%
          for l in ll repeat
              mons:= "setUnion"/[primitiveMonomials u for u in l]
              redmons:List % :=[]
              for m in mons repeat
                  vars:=variables m
                  degs:=degree(m,vars)
                  deg1:List NonNegativeInteger
                  deg1:=[ ((nd:=d:Integer exquo ch:Integer)
                              case "failed" => return "failed" ;
                               nd::Integer::NonNegativeInteger)
                         for d in degs ]
                  redmons:=[monomial(1,vars,deg1),:redmons]
                  llR:=[[ground coefficient(u,vars,degs),:v]_
                          for u in l for v in llR]
              monslist:=[redmons,:monslist]
          ans:=conditionP transpose matrix llR
          ans case "failed" => "failed"
          i:NonNegativeInteger:=0
          [ +/[m*(ans.(i:=i+1))::% for m in mons ]
            for mons in monslist]

    if R has CharacteristicNonZero then
          charthRootlv:(%,List VarSet,NonNegativeInteger) ->_
                                                  Union(%,"failed")
          charthRoot p ==
            vars:= variables p
            empty? vars =>
              ans := charthRoot ground p
              ans case "failed" => "failed"
              ans::R::%
            ch:=characteristic()$%
            charthRootlv(p,vars,ch)

          charthRootlv(p,vars,ch) ==
            empty? vars =>
              ans := charthRoot ground p
              ans case "failed" => "failed"
              ans::R::%
            v:=first vars
            vars:=rest vars
            d:=degree(p,v)
            ans:% := 0
```

```
            while (d>0) repeat
               (dd:=(d::Integer exquo ch::Integer)) case "failed" =>
                      return "failed"
               cp:=coefficient(p,v,d)
               p:=p-monomial(cp,v,d)
               ansx:=charthRootlv(cp,vars,ch)
               ansx case "failed" => return "failed"
               d:=degree(p,v)
               ans:=ans+monomial(ansx,v,dd::Integer::NonNegativeInteger)
            ansx:=charthRootlv(p,vars,ch)
            ansx case "failed" => return "failed"
            return ans+ansx

        monicDivide(p1,p2,mvar) ==
           result:=monicDivide(univariate(p1,mvar),univariate(p2,mvar))
           [multivariate(result.quotient,mvar),
            multivariate(result.remainder,mvar)]

        if R has GcdDomain then
          if R has EuclideanDomain and R has CharacteristicZero then
           squareFree p == squareFree(p)$MultivariateSquareFree(E,VarSet,R,%)
          else
            squareFree p == squareFree(p)$PolynomialSquareFree(VarSet,E,R,%)

          squareFreePart p ==
            unit(s := squareFree p) * */[f.factor for f in factors s]

          content(p,v) == content univariate(p,v)

          primitivePart p ==
            zero? p => p
            unitNormal((p exquo content p) ::%).canonical

          primitivePart(p,v) ==
            zero? p => p
            unitNormal((p exquo content(p,v)) ::%).canonical

        if R has OrderedSet then
          p:% < q:% ==
            (dp:= degree p) < (dq := degree q) => (leadingCoefficient q) > 0
            dq < dp => (leadingCoefficient p) < 0
            leadingCoefficient(p - q) < 0

          if (R has PatternMatchable Integer) and
             (VarSet has PatternMatchable Integer) then
               patternMatch(p:%, pat:Pattern Integer,
```

```
            l:PatternMatchResult(Integer, %)) ==
              patternMatch(p, pat,
                l)$PatternMatchPolynomialCategory(Integer,E,VarSet,R,%)

      if (R has PatternMatchable Float) and
         (VarSet has PatternMatchable Float) then
           patternMatch(p:%, pat:Pattern Float,
            l:PatternMatchResult(Float, %)) ==
              patternMatch(p, pat,
                l)$PatternMatchPolynomialCategory(Float,E,VarSet,R,%)

    if (R has ConvertibleTo Pattern Integer) and
       (VarSet has ConvertibleTo Pattern Integer) then
         convert(x:%):Pattern(Integer) ==
           map(convert, convert,
              x)$PolynomialCategoryLifting(E,VarSet,R,%,Pattern Integer)

    if (R has ConvertibleTo Pattern Float) and
       (VarSet has ConvertibleTo Pattern Float) then
         convert(x:%):Pattern(Float) ==
           map(convert, convert,
            x)$PolynomialCategoryLifting(E, VarSet, R, %, Pattern Float)

    if (R has ConvertibleTo InputForm) and
       (VarSet has ConvertibleTo InputForm) then
         convert(p:%):InputForm ==
           map(convert, convert,
                    p)$PolynomialCategoryLifting(E,VarSet,R,%,InputForm)
```

⟨*POLYCAT.dotabb*⟩≡

```
"POLYCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=POLYCAT"];
"POLYCAT" -> "PDRING"
"POLYCAT" -> "FAMR"
"POLYCAT" -> "EVALAB"
"POLYCAT" -> "IEVALAB"
"POLYCAT" -> "RETRACT"
"POLYCAT" -> "FLINEXP"
"POLYCAT" -> "ORDSET"
"POLYCAT" -> "GCDDOM"
"POLYCAT" -> "PFECAT"
```

⟨*POLYCAT.dotfull*⟩≡

```
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=POLYCAT"];
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PartialDifferentialRing(a:OrderedSet)"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FiniteAbelianMonoidRing(a:Ring,b:OrderedAbelianMonoidSup)"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "Evalable(PolynomialCategory(...))"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "InnerEvalable(a:OrderedSet,b:Ring)"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "InnerEvalable(a:OrderedSet,b:PolynomialCategory(...))"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "RetractableTo(a:OrderedSet)"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FullyLinearlyExplicitRingOver(a:Ring)"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "OrderedSet()"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "GcdDomain()"
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PolynomialFactorizationExplicit()"

"PolynomialCategory(a:Ring,b:NonNegativeInteger,c:SingletonAsOrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=POLYCAT"];
"PolynomialCategory(a:Ring,b:NonNegativeInteger,c:SingletonAsOrderedSet)"
 -> "PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
```

⟨*POLYCAT.dotpic*⟩≡

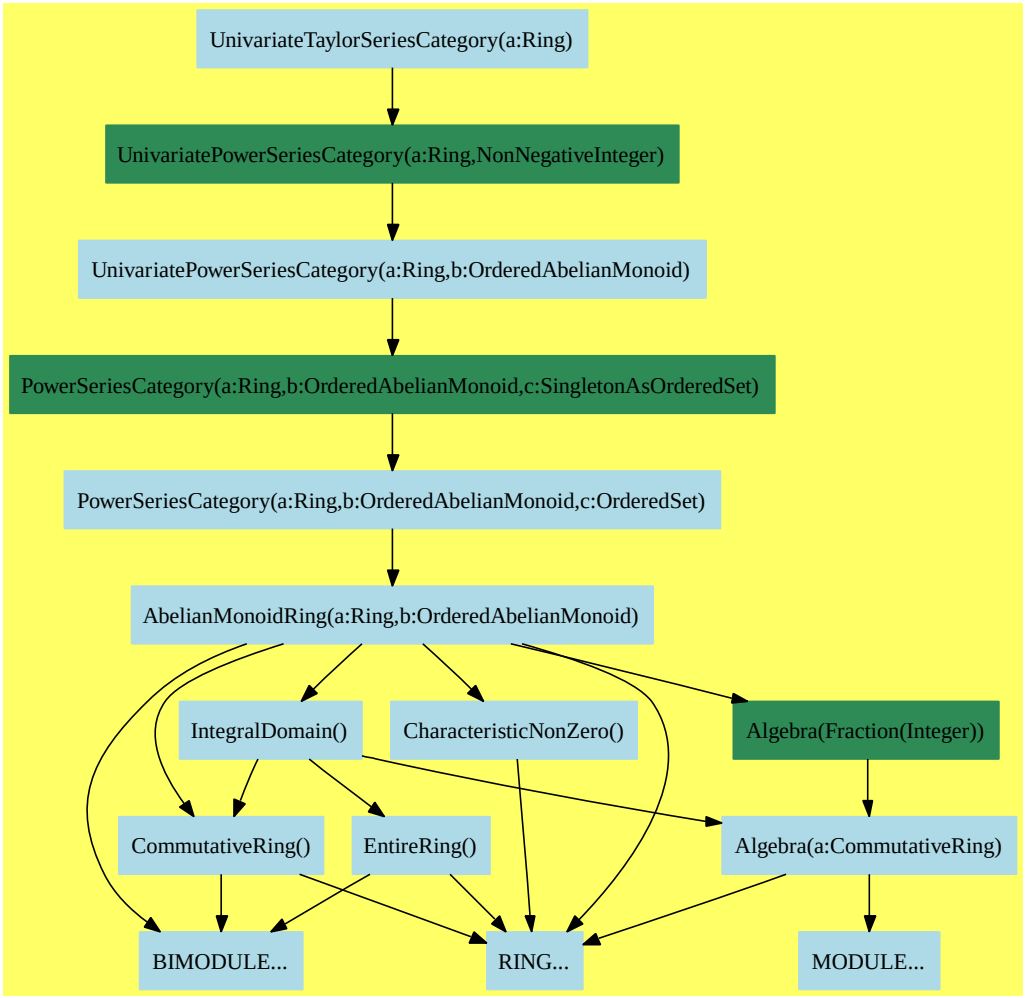
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=POLYCAT"];
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PDRING..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FAMR..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "EVALAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "IEVALAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "RETRACT..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FLINEXP..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "ORDSET..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "GCDDOM..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PFECAT..."

"PDRING..." [color=lightblue];
"FAMR..." [color=lightblue];
"EVALAB..." [color=lightblue];
"IEVALAB..." [color=lightblue];
"RETRACT..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"ORDSET..." [color=lightblue];
"GCDDOM..." [color=lightblue];
"PFECAT..." [color=lightblue];

}
```

16.5 UnivariateTaylorSeriesCategory (UTSCAT)



See:

⇐ "RadicalCategory" (RADCAT) 2.17 on page 42
⇐ "TranscendentalFunctionCategory" (TRANFUN) 3.14 on page 94
⇐ "UnivariatePowerSeriesCategory" (UPSCAT) 15.4 on page 996

Exports:

0	1	acos	acosh
acot	acoth	acsc	acsch
approximate	asec	asech	asin
asinh	associates?	atan	atanh
center	characteristic	charthRoot	coefficient
coefficients	coerce	complete	cos
cosh	cot	coth	csc
csch	D	degree	differentiate
eval	exp	exquo	extend
hash	integrate	latex	leadingCoefficient
leadingMonomial	log	map	monomial
monomial?	multiplyCoefficients	multiplyExponents	nthRoot
one?	order	pi	pole?
polynomial	quoByVar	recip	reductum
sample	sec	sech	series
sin	sinh	sqrt	subtractIfCan
tan	tanh	terms	truncate
unit?	unitCanonical	unitNormal	variable
variables	zero?	?*?	?**?
?+?	?-?	-?	?=?
?^?	?~=?	?/?	?.?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if #1 has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.

These are directly exported but not implemented:

```
coefficients : % -> Stream Coef
integrate : (%,Symbol) -> %
    if Coef has ACFS INT
    and Coef has PRIMCAT
    and Coef has TRANFUN
    and Coef has ALGEBRA FRAC INT
    or Coef has variables: Coef -> List Symbol
    and Coef has integrate: (Coef,Symbol) -> Coef
```

```
    and Coef has ALGEBRA FRAC INT
integrate : % -> % if Coef has ALGEBRA FRAC INT
multiplyCoefficients : ((Integer -> Coef),%) -> %
polynomial : (%,NonNegativeInteger,NonNegativeInteger) -> Polynomial Coef
polynomial : (%,NonNegativeInteger) -> Polynomial Coef
quoByVar : % -> %
series : Stream Coef -> %
series : Stream Record(k: NonNegativeInteger,c: Coef) -> %
```

These are implemented by this category:

```
acos : % -> % if Coef has ALGEBRA FRAC INT
acosh : % -> % if Coef has ALGEBRA FRAC INT
acot : % -> % if Coef has ALGEBRA FRAC INT
acoth : % -> % if Coef has ALGEBRA FRAC INT
acsc : % -> % if Coef has ALGEBRA FRAC INT
acsch : % -> % if Coef has ALGEBRA FRAC INT
asec : % -> % if Coef has ALGEBRA FRAC INT
asech : % -> % if Coef has ALGEBRA FRAC INT
asin : % -> % if Coef has ALGEBRA FRAC INT
asinh : % -> % if Coef has ALGEBRA FRAC INT
atan : % -> % if Coef has ALGEBRA FRAC INT
atanh : % -> % if Coef has ALGEBRA FRAC INT
coerce : % -> OutputForm
cos : % -> % if Coef has ALGEBRA FRAC INT
cosh : % -> % if Coef has ALGEBRA FRAC INT
cot : % -> % if Coef has ALGEBRA FRAC INT
coth : % -> % if Coef has ALGEBRA FRAC INT
csc : % -> % if Coef has ALGEBRA FRAC INT
csch : % -> % if Coef has ALGEBRA FRAC INT
exp : % -> % if Coef has ALGEBRA FRAC INT
log : % -> % if Coef has ALGEBRA FRAC INT
sinh : % -> % if Coef has ALGEBRA FRAC INT
sec : % -> % if Coef has ALGEBRA FRAC INT
sech : % -> % if Coef has ALGEBRA FRAC INT
sin : % -> % if Coef has ALGEBRA FRAC INT
tan : % -> % if Coef has ALGEBRA FRAC INT
tanh : % -> % if Coef has ALGEBRA FRAC INT
zero? : % -> Boolean
?**? : (%,Coef) -> % if Coef has FIELD
?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
```

These exports come from (p996) UnivariatePowerSeriesCategory(Coef,NNI) where Coef:Ring and NNI:NonNegativeInteger:

```
0 : () -> %
1 : () -> %
approximate : (%,NonNegativeInteger) -> Coef
    if Coef has **: (Coef,NonNegativeInteger) -> Coef
```

```
    and Coef has coerce: Symbol -> Coef
associates? : (%,%) -> Boolean if Coef has INTDOM
center : % -> Coef
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,NonNegativeInteger) -> Coef
coerce : Coef -> % if Coef has COMRING
coerce : % -> % if Coef has INTDOM
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : Integer -> %
complete : % -> %
D : % -> % if Coef has *: (NonNegativeInteger,Coef) -> Coef
D : (%,NonNegativeInteger) -> %
    if Coef has *: (NonNegativeInteger,Coef) -> Coef
D : (%,Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
D : (%,List Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
D : (%,Symbol,NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
D : (%,List Symbol,List NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
differentiate : (%,List Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
differentiate : (%,Symbol) -> %
    if Coef has PDRING SYMBOL
    and Coef has *: (NonNegativeInteger,Coef) -> Coef
differentiate : (%,NonNegativeInteger) -> %
    if Coef has *: (NonNegativeInteger,Coef) -> Coef
differentiate : % -> %
    if Coef has *: (NonNegativeInteger,Coef) -> Coef
degree : % -> NonNegativeInteger
extend : (%,NonNegativeInteger) -> %
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
eval : (%,Coef) -> Stream Coef
    if Coef has **: (Coef,NonNegativeInteger) -> Coef
hash : % -> SingleInteger
latex : % -> String
leadingCoefficient : % -> Coef
```

```
leadingMonomial : % -> %
map : ((Coef -> Coef),%) -> %
monomial : (Coef,NonNegativeInteger) -> %
monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
monomial : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
monomial? : % -> Boolean
multiplyExponents : (%,PositiveInteger) -> %
one? : % -> Boolean
order : % -> NonNegativeInteger
order : (%,NonNegativeInteger) -> NonNegativeInteger
pole? : % -> Boolean
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
terms : % -> Stream Record(k: NonNegativeInteger,c: Coef)
truncate : (%,NonNegativeInteger,NonNegativeInteger) -> %
truncate : (%,NonNegativeInteger) -> %
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if Coef has INTDOM
variable : % -> Symbol
variables : % -> List SingletonAsOrderedSet
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
-? : % -> %
?.? : (%,%) -> % if NonNegativeInteger has SGROUP
?.? : (%,NonNegativeInteger) -> Coef
```

These exports come from (p94) TranscendentalFunctionCategory():

```
pi : () -> % if Coef has ALGEBRA FRAC INT
```

These exports come from (p42) RadicalCategory():

```
nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
sqrt : % -> % if Coef has ALGEBRA FRAC INT
```

⟨*category UTSCAT UnivariateTaylorSeriesCategory*⟩≡

```
)abbrev category UTSCAT UnivariateTaylorSeriesCategory
++ Author: Clifton J. Williamson
++ Date Created: 21 December 1989
++ Date Last Updated: 26 May 1994
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Taylor, linebacker
++ Examples:
++ References:
++ Description:
++   \spadtype{UnivariateTaylorSeriesCategory} is the category of Taylor
++   series in one variable.
UnivariateTaylorSeriesCategory(Coef): Category == Definition where
  Coef  : Ring
  I    ==> Integer
  L    ==> List
  NNI  ==> NonNegativeInteger
  OUT  ==> OutputForm
  RN   ==> Fraction Integer
  STTA ==> StreamTaylorSeriesOperations Coef
  STTF ==> StreamTranscendentalFunctions Coef
  STNC ==> StreamTranscendentalFunctionsNonCommutative Coef
  Term ==> Record(k:NNI,c:Coef)

  Definition ==> UnivariatePowerSeriesCategory(Coef,NNI) with

    series: Stream Term -> %
      ++ \spad{series(st)} creates a series from a stream of non-zero terms,
      ++ where a term is an exponent-coefficient pair.  The terms in the
      ++ stream should be ordered by increasing order of exponents.
    coefficients: % -> Stream Coef
      ++ \spad{coefficients(a0 + a1 x + a2 x**2 + ...)} returns a stream
      ++ of coefficients: \spad{[a0,a1,a2,...]}. The entries of the stream
      ++ may be zero.
    series: Stream Coef -> %
      ++ \spad{series([a0,a1,a2,...])} is the Taylor series
      ++ \spad{a0 + a1 x + a2 x**2 + ...}.
    quoByVar: % -> %
      ++ \spad{quoByVar(a0 + a1 x + a2 x**2 + ...)}
      ++ returns \spad{a1 + a2 x + a3 x**2 + ...}
```

```
    ++ Thus, this function substracts the constant term and divides by
    ++ the series variable.  This function is used when Laurent series
    ++ are represented by a Taylor series and an order.
  multiplyCoefficients: (I -> Coef,%) -> %
    ++ \spad{multiplyCoefficients(f,sum(n = 0..infinity,a[n] * x**n))}
    ++ returns \spad{sum(n = 0..infinity,f(n) * a[n] * x**n)}.
    ++ This function is used when Laurent series are represented by
    ++ a Taylor series and an order.
  polynomial: (%,NNI) -> Polynomial Coef
    ++ \spad{polynomial(f,k)} returns a polynomial consisting of the sum
    ++ of all terms of f of degree \spad{<= k}.
  polynomial: (%,NNI,NNI) -> Polynomial Coef
    ++ \spad{polynomial(f,k1,k2)} returns a polynomial consisting of the
    ++ sum of all terms of f of degree d with \spad{k1 <= d <= k2}.

  if Coef has Field then
    "**": (%,Coef) -> %
      ++ \spad{f(x) ** a} computes a power of a power series.
      ++ When the coefficient ring is a field, we may raise a series
      ++ to an exponent from the coefficient ring provided that the
      ++ constant coefficient of the series is 1.

  if Coef has Algebra Fraction Integer then
    integrate: % -> %
      ++ \spad{integrate(f(x))} returns an anti-derivative of the power
      ++ series \spad{f(x)} with constant coefficient 0.
      ++ We may integrate a series when we can divide coefficients
      ++ by integers.
    if Coef has integrate: (Coef,Symbol) -> Coef and _
       Coef has variables: Coef -> List Symbol then
      integrate: (%,Symbol) -> %
        ++ \spad{integrate(f(x),y)} returns an anti-derivative of the
        ++ power series \spad{f(x)} with respect to the variable \spad{y}.
    if Coef has TranscendentalFunctionCategory and _
       Coef has PrimitiveFunctionCategory and _
       Coef has AlgebraicallyClosedFunctionSpace Integer then
      integrate: (%,Symbol) -> %
        ++ \spad{integrate(f(x),y)} returns an anti-derivative of
        ++ the power series \spad{f(x)} with respect to the variable
        ++ \spad{y}.
    RadicalCategory
      --++ We provide rational powers when we can divide coefficients
      --++ by integers.
    TranscendentalFunctionCategory
      --++ We provide transcendental functions when we can divide
      --++ coefficients by integers.
```

```
    add

     zero? x ==
       empty? (coefs := coefficients x) => true
       (zero? frst coefs) and (empty? rst coefs) => true
       false

  --% OutputForms

  --  We provide defaulr output functions on UTSCAT using the functions
  --  'coefficients', 'center', and 'variable'.

     factorials?: () -> Boolean
     -- check a global Lisp variable
     factorials?() == false

     termOutput: (I,Coef,OUT) -> OUT
     termOutput(k,c,vv) ==
     -- creates a term c * vv ** k
       k = 0 => c :: OUT
       mon := (k = 1 => vv; vv ** (k :: OUT))
  --       if factorials?() and k > 1 then
  --         c := factorial(k)$IntegerCombinatoricFunctions * c
  --         mon := mon / hconcat(k :: OUT,"!" :: OUT)
       c = 1 => mon
       c = -1 => -mon
       (c :: OUT) * mon

     showAll?: () -> Boolean
     -- check a global Lisp variable
     showAll?() == true

     coerce(p:%):OUT ==
       empty? (uu := coefficients p) => (0$Coef) :: OUT
       var := variable p; cen := center p
       vv :=
         zero? cen => var :: OUT
         paren(var :: OUT - cen :: OUT)
       n : NNI ; count : NNI := _$streamCount$Lisp
       l : L OUT := empty()
       for n in 0..count while not empty? uu repeat
         if frst(uu) ^= 0 then
           l := concat(termOutput(n :: I,frst uu,vv),l)
         uu := rst uu
       if showAll?() then
```

```
      for n in (count + 1).. while explicitEntries? uu and _
             not eq?(uu,rst uu) repeat
        if frst(uu) ^= 0 then
          l := concat(termOutput(n :: I,frst uu,vv),l)
        uu := rst uu
      l :=
        explicitlyEmpty? uu => l
        eq?(uu,rst uu) and frst uu = 0 => l
        concat(prefix("O" :: OUT,[vv ** (n :: OUT)]),l)
      empty? l => (0$Coef) :: OUT
      reduce("+",reverse_! l)

    if Coef has Field then
      (x:%) ** (r:Coef) == series power(r,coefficients x)$STTA

    if Coef has Algebra Fraction Integer then
      if Coef has CommutativeRing then
        (x:%) ** (y:%)    == series(coefficients x **$STTF coefficients y)

        (x:%) ** (r:RN)   == series powern(r,coefficients x)$STTA

        exp x == series exp(coefficients x)$STTF

        log x == series log(coefficients x)$STTF

        sin x == series sin(coefficients x)$STTF

        cos x == series cos(coefficients x)$STTF

        tan x == series tan(coefficients x)$STTF

        cot x == series cot(coefficients x)$STTF

        sec x == series sec(coefficients x)$STTF

        csc x == series csc(coefficients x)$STTF

        asin x == series asin(coefficients x)$STTF

        acos x == series acos(coefficients x)$STTF

        atan x == series atan(coefficients x)$STTF

        acot x == series acot(coefficients x)$STTF

        asec x == series asec(coefficients x)$STTF
```

```
        acsc x == series acsc(coefficients x)$STTF

        sinh x == series sinh(coefficients x)$STTF

        cosh x == series cosh(coefficients x)$STTF

        tanh x == series tanh(coefficients x)$STTF

        coth x == series coth(coefficients x)$STTF

        sech x == series sech(coefficients x)$STTF

        csch x == series csch(coefficients x)$STTF

        asinh x == series asinh(coefficients x)$STTF

        acosh x == series acosh(coefficients x)$STTF

        atanh x == series atanh(coefficients x)$STTF

        acoth x == series acoth(coefficients x)$STTF

        asech x == series asech(coefficients x)$STTF

        acsch x == series acsch(coefficients x)$STTF

      else
        (x:%) ** (y:%) == series(coefficients x **$STNC coefficients y)

        (x:%) ** (r:RN) ==
          coefs := coefficients x
          empty? coefs =>
            positive? r => 0
            zero? r => error "0**0 undefined"
            error "0 raised to a negative power"
--          not one? frst coefs =>
          not (frst coefs = 1) =>
            error "**: constant coefficient should be 1"
          coefs := concat(0,rst coefs)
          onePlusX := monom(1,0)$STTA + $STTA monom(1,1)$STTA
          ratPow := powern(r,onePlusX)$STTA
          series compose(ratPow,coefs)$STTA

        exp x == series exp(coefficients x)$STNC
```

```
log x == series log(coefficients x)$STNC

sin x == series sin(coefficients x)$STNC

cos x == series cos(coefficients x)$STNC

tan x == series tan(coefficients x)$STNC

cot x == series cot(coefficients x)$STNC

sec x == series sec(coefficients x)$STNC

csc x == series csc(coefficients x)$STNC

asin x == series asin(coefficients x)$STNC

acos x == series acos(coefficients x)$STNC

atan x == series atan(coefficients x)$STNC

acot x == series acot(coefficients x)$STNC

asec x == series asec(coefficients x)$STNC

acsc x == series acsc(coefficients x)$STNC

sinh x == series sinh(coefficients x)$STNC

cosh x == series cosh(coefficients x)$STNC

tanh x == series tanh(coefficients x)$STNC

coth x == series coth(coefficients x)$STNC

sech x == series sech(coefficients x)$STNC

csch x == series csch(coefficients x)$STNC

asinh x == series asinh(coefficients x)$STNC

acosh x == series acosh(coefficients x)$STNC

atanh x == series atanh(coefficients x)$STNC

acoth x == series acoth(coefficients x)$STNC
```

```
asech x == series asech(coefficients x)$STNC

acsch x == series acsch(coefficients x)$STNC
```

⟨*UTSCAT.dotabb*⟩≡

```
"UTSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UTSCAT"];
"UTSCAT" -> "UPSCAT"
```

⟨*UTSCAT.dotfull*⟩≡

```
"UnivariateTaylorSeriesCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UTSCAT"];
"UnivariateTaylorSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)"
```

⟨*UTSCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UnivariateTaylorSeriesCategory(a:Ring)" [color=lightblue];
"UnivariateTaylorSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)"

"UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=UPSCAT"];
"UnivariatePowerSeriesCategory(a:Ring,NonNegativeInteger)" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)"

"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" [color=lightblue];
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" -> "RING..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "BIMODULE..."
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "IntegralDomain()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CharacteristicNonZero()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "CommutativeRing()"
"AbelianMonoidRing(a:Ring,b:OrderedAbelianMonoid)" ->
    "Algebra(Fraction(Integer))"

"IntegralDomain()" [color=lightblue];
"IntegralDomain()" -> "CommutativeRing()"
"IntegralDomain()" -> "Algebra(a:CommutativeRing)"
```

```
"IntegralDomain()" -> "EntireRing()"

"EntireRing()" [color=lightblue];
"EntireRing()" -> "RING..."
"EntireRing()" -> "BIMODULE..."

"CharacteristicNonZero()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=CHARNZ"];
"CharacteristicNonZero()" -> "RING..."

"Algebra(Fraction(Integer))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(Fraction(Integer))" -> "Algebra(a:CommutativeRing)"

"Algebra(a:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ALGEBRA"];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."

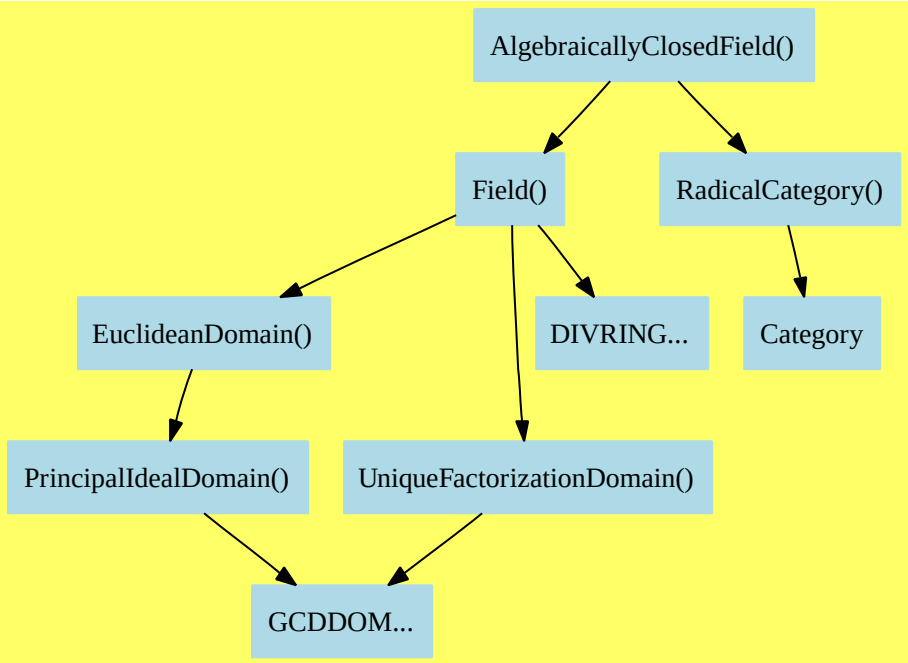
"CommutativeRing()" [color=lightblue];
"CommutativeRing()" -> "RING..."
"CommutativeRing()" -> "BIMODULE..."

"BIMODULE..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

Chapter 17

Category Layer 16

17.1 AlgebraicallyClosedField (ACF)



See:

⇒ "AlgebraicallyClosedFunctionSpace" (ACFS) 18.1 on page 1225
⇐ "Field" (FIELD) 16.1 on page 1005
⇐ "RadicalCategory" (RADCAT) 2.17 on page 42

Exports:

0	1	associates?	characteristic
coerce	divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	factor	gcd
gcdPolynomial	hash	inv	latex
lcm	multiEuclidean	nthRoot	one?
prime?	principalIdeal	recip	rootOf
rootsOf	sample	sizeLess?	sqrt
squareFree	squareFreePart	subtractIfCan	unit?
unitCanonical	unitNormal	zero?	zeroOf
zerosOf	?*?	?**?	?+?
?-?	-?	?/?	?=?
?quo?	?rem?	?~=?	?^?

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
rootOf : Polynomial % -> %
```

These are implemented by this category:

```
rootOf : SparseUnivariatePolynomial % -> %
rootOf : (SparseUnivariatePolynomial %,Symbol) -> %
rootsOf : Polynomial % -> List %
rootsOf : (SparseUnivariatePolynomial %,Symbol) -> List %
rootsOf : SparseUnivariatePolynomial % -> List %
zeroOf : (SparseUnivariatePolynomial %,Symbol) -> %
zeroOf : Polynomial % -> %
```

```
zeroOf : SparseUnivariatePolynomial % -> %
zerosOf : Polynomial % -> List %
zerosOf : (SparseUnivariatePolynomial %,Symbol) -> List %
zerosOf : SparseUnivariatePolynomial % -> List %
```

These exports come from (p1005) Field():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
      SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
```

```
?^? : (%,Integer) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?/? : (%,%) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p42) RadicalCategory():

```
nthRoot : (%,Integer) -> %
sqrt : % -> %
?**? : (%,Fraction Integer) -> %
```

⟨*category ACF AlgebraicallyClosedField*⟩≡

```
)abbrev category ACF AlgebraicallyClosedField
++ Author: Manuel Bronstein
++ Date Created: 22 Mar 1988
++ Date Last Updated: 27 November 1991
++ Description:
++   Model for algebraically closed fields.
++ Keywords: algebraic, closure, field.

AlgebraicallyClosedField(): Category == Join(Field,RadicalCategory) with
    rootOf: Polynomial $ -> $
      ++ rootOf(p) returns y such that \spad{p(y) = 0}.
      ++ Error: if p has more than one variable y.
    rootOf: SparseUnivariatePolynomial $ -> $
      ++ rootOf(p) returns y such that \spad{p(y) = 0}.
    rootOf: (SparseUnivariatePolynomial $, Symbol) -> $
      ++ rootOf(p, y) returns y such that \spad{p(y) = 0}.
      ++ The object returned displays as \spad{'y}.
    rootsOf: Polynomial $ -> List $
      ++ rootsOf(p) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
      ++ Note: the returned symbols y1,...,yn are bound in the
      ++ interpreter to respective root values.
      ++ Error: if p has more than one variable y.
    rootsOf: SparseUnivariatePolynomial $ -> List $
```

```
      ++ rootsOf(p) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
      ++ Note: the returned symbols y1,...,yn are bound in the interpreter
      ++ to respective root values.
    rootsOf: (SparseUnivariatePolynomial $, Symbol) -> List $
      ++ rootsOf(p, y) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0};
      ++ The returned roots display as \spad{'y1},...,\spad{'yn}.
      ++ Note: the returned symbols y1,...,yn are bound in the interpreter
      ++ to respective root values.
    zeroOf: Polynomial $ -> $
      ++ zeroOf(p) returns y such that \spad{p(y) = 0}.
      ++ If possible, y is expressed in terms of radicals.
      ++ Otherwise it is an implicit algebraic quantity.
      ++ Error: if p has more than one variable y.
    zeroOf: SparseUnivariatePolynomial $ -> $
      ++ zeroOf(p) returns y such that \spad{p(y) = 0};
      ++ if possible, y is expressed in terms of radicals.
      ++ Otherwise it is an implicit algebraic quantity.
    zeroOf: (SparseUnivariatePolynomial $, Symbol) -> $
      ++ zeroOf(p, y) returns y such that \spad{p(y) = 0};
      ++ if possible, y is expressed in terms of radicals.
      ++ Otherwise it is an implicit algebraic quantity which
      ++ displays as \spad{'y}.
    zerosOf: Polynomial $ -> List $
      ++ zerosOf(p) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
      ++ The yi's are expressed in radicals if possible.
      ++ Otherwise they are implicit algebraic quantities.
      ++ The returned symbols y1,...,yn are bound in the interpreter
      ++ to respective root values.
      ++ Error: if p has more than one variable y.
    zerosOf: SparseUnivariatePolynomial $ -> List $
      ++ zerosOf(p) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
      ++ The yi's are expressed in radicals if possible, and otherwise
      ++ as implicit algebraic quantities.
      ++ The returned symbols y1,...,yn are bound in the interpreter
      ++ to respective root values.
    zerosOf: (SparseUnivariatePolynomial $, Symbol) -> List $
      ++ zerosOf(p, y) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
      ++ The yi's are expressed in radicals if possible, and otherwise
      ++ as implicit algebraic quantities
      ++ which display as \spad{'yi}.
      ++ The returned symbols y1,...,yn are bound in the interpreter
      ++ to respective root values.
  add
    SUP ==> SparseUnivariatePolynomial $

    assign  : (Symbol, $) -> $
```

```
allroots: (SUP, Symbol, (SUP, Symbol) -> $) -> List $
binomialRoots: (SUP, Symbol, (SUP, Symbol) -> $) -> List $

zeroOf(p:SUP)            == assign(x := new(), zeroOf(p, x))

rootOf(p:SUP)            == assign(x := new(), rootOf(p, x))

zerosOf(p:SUP)           == zerosOf(p, new())

rootsOf(p:SUP)           == rootsOf(p, new())

rootsOf(p:SUP, y:Symbol) == allroots(p, y, rootOf)

zerosOf(p:SUP, y:Symbol) == allroots(p, y, zeroOf)

assign(x, f)             == (assignSymbol(x, f, $)$Lisp; f)

zeroOf(p:Polynomial $) ==
  empty?(l := variables p) => error "zeroOf: constant polynomial"
  zeroOf(univariate p, first l)

rootOf(p:Polynomial $) ==
  empty?(l := variables p) => error "rootOf: constant polynomial"
  rootOf(univariate p, first l)

zerosOf(p:Polynomial $) ==
  empty?(l := variables p) => error "zerosOf: constant polynomial"
  zerosOf(univariate p, first l)

rootsOf(p:Polynomial $) ==
  empty?(l := variables p) => error "rootsOf: constant polynomial"
  rootsOf(univariate p, first l)

zeroOf(p:SUP, y:Symbol) ==
  zero?(d := degree p) => error "zeroOf: constant polynomial"
  zero? coefficient(p, 0) => 0
  a := leadingCoefficient p
  d = 2 =>
    b := coefficient(p, 1)
    (sqrt(b**2 - 4 * a * coefficient(p, 0)) - b) / (2 * a)
  (r := retractIfCan(reductum p)@Union($,"failed")) case "failed" =>
    rootOf(p, y)
  nthRoot(- (r::$ / a), d)

binomialRoots(p, y, fn) ==
 -- p = a * x**n + b
```

```
        alpha := assign(x := new(y)$Symbol, fn(p, x))
  --      one?(n := degree p) =>  [ alpha ]
        ((n := degree p) = 1) =>  [ alpha ]
        cyclo := cyclotomic(n,monomial(1,1)$SUP)_
                      $NumberTheoreticPolynomialFunctions(SUP)
        beta := assign(x := new(y)$Symbol, fn(cyclo, x))
        [alpha*beta**i for i in 0..(n-1)::NonNegativeInteger]

      import PolynomialDecomposition(SUP,$)

      allroots(p, y, fn) ==
        zero? p => error "allroots: polynomial must be nonzero"
        zero? coefficient(p,0) =>
           concat(0, allroots(p quo monomial(1,1), y, fn))
        zero?(p1:=reductum p) => empty()
        zero? reductum p1 => binomialRoots(p, y, fn)
        decompList := decompose(p)
        # decompList > 1 =>
            h := last decompList
            g := leftFactor(p,h) :: SUP
            groots := allroots(g, y, fn)
            "append"/[allroots(h-r::SUP, y, fn) for r in groots]
        ans := nil()$List($)
        while not ground? p repeat
          alpha := assign(x := new(y)$Symbol, fn(p, x))
          q     := monomial(1, 1)$SUP - alpha::SUP
          if not zero?(p alpha) then
            p   := p quo q
            ans := concat(alpha, ans)
          else while zero?(p alpha) repeat
            p   := (p exquo q)::SUP
            ans := concat(alpha, ans)
        reverse_! ans
```

⟨*ACF.dotabb*⟩≡

```
"ACF"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ACF"];
"ACF" -> "FIELD"
"ACF" -> "RADCAT"
```

⟨*ACF.dotfull*⟩≡
```
"AlgebraicallyClosedField()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ACF"];
"AlgebraicallyClosedField()" -> "Field()"
"AlgebraicallyClosedField()" -> "RadicalCategory()"
```

⟨*ACF.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"AlgebraicallyClosedField()" [color=lightblue];
"AlgebraicallyClosedField()" -> "Field()"
"AlgebraicallyClosedField()" -> "RadicalCategory()"

"Field()" [color=lightblue];
"Field()" -> "EuclideanDomain()"
"Field()" -> "UniqueFactorizationDomain()"
"Field()" -> "DIVRING..."

"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GCDDOM..."

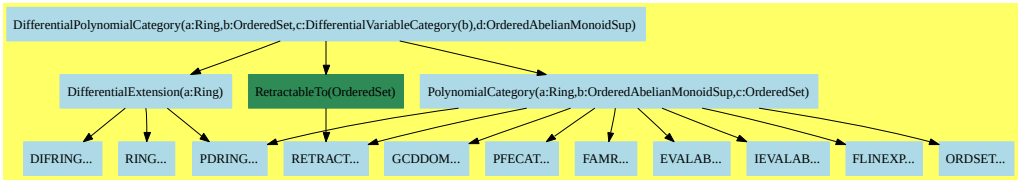
"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GCDDOM..."

"DIVRING..." [color=lightblue];
"GCDDOM..." [color=lightblue];

"RadicalCategory()" [color=lightblue];
"RadicalCategory()" -> "Category"

"Category" [color=lightblue];
}
```

17.2 DifferentialPolynomialCategory (DPOLCAT)



See:

⇐ "DifferentialExtension" (DIFEXT) 11.2 on page 777
⇐ "PolynomialCategory" (POLYCAT) 16.4 on page 1027
⇐ "RetractableTo" (RETRACT) 2.18 on page 44

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
conditionP	content	convert
D	degree	differentialVariables
differentiate	discriminant	eval
exquo	factor	factorPolynomial
factorSquareFreePolynomial	gcd	gcdPolynomial
ground	ground?	hash
initial	isExpt	isobaric?
isPlus	isTimes	latex
lcm	leader	leadingCoefficient
leadingMonomial	makeVariable	map
mapExponents	max	min
minimumDegree	monicDivide	monomial
monomial?	monomials	multivariate
numberOfMonomials	one?	order
patternMatch	pomopo!	prime?
primitiveMonomials	primitivePart	recip
reducedSystem	reductum	resultant
retract	retractIfCan	sample
separant	solveLinearPolynomialEquation	squareFree
squareFreePart	squareFreePolynomial	subtractIfCan
totalDegree	unit?	unitCanonical
unitNormal	univariate	variables
weight	weights	zero?
?*?	?**?	?+?
?-?	-?	?=?
?^?	?~=?	?/?
?<?	?<=?	?>?
?>=?		

Attributes Exported:

-
- if R has canonicalUnitNormal then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if $ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if $ has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are implemented by this category:

```
coerce : S -> %
degree : (%,S) -> NonNegativeInteger
differentialVariables : % -> List S
differentiate : (%,(R -> R)) -> %
eval : (%,List S,List R) -> % if R has DIFRING
eval : (%,List S,List %) -> % if R has DIFRING
eval : (%,List Equation %) -> %
initial : % -> %
isobaric? : % -> Boolean
leader : % -> V
makeVariable : S -> (NonNegativeInteger -> %)
makeVariable : % -> (NonNegativeInteger -> %) if R has DIFRING
order : % -> NonNegativeInteger
order : (%,S) -> NonNegativeInteger
retractIfCan : % -> Union(S,"failed")
separant : % -> %
weight : % -> NonNegativeInteger
weight : (%,S) -> NonNegativeInteger
weights : (%,S) -> List NonNegativeInteger
weights : % -> List NonNegativeInteger
```

These exports come from (p1027) PolynomialCategory(R,E,V)
where R:Ring, E:OrderedAbelianMonoidSup,
V:DifferentialVariableCategory(S:OrderedSet):

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean if R has INTDOM
binomThmExpt : (%,%,NonNegativeInteger) -> % if R has COMRING
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed")
  if and(has($,CharacteristicNonZero),
     has(R,PolynomialFactorizationExplicit))
  or R has CHARNZ
coefficient : (%,List V,List NonNegativeInteger) -> %
coefficient : (%,V,NonNegativeInteger) -> %
coefficient : (%,E) -> R
coefficients : % -> List R
coerce : R -> %
coerce : Fraction Integer -> %
  if R has RETRACT FRAC INT
  or R has ALGEBRA FRAC INT
coerce : % -> % if R has INTDOM
coerce : Integer -> %
coerce : % -> OutputForm
coerce : V -> %
conditionP : Matrix % -> Union(Vector %,"failed")
  if and(has($,CharacteristicNonZero),
         has(R,PolynomialFactorizationExplicit))
content : % -> R if R has GCDDOM
content : (%,V) -> % if R has GCDDOM
convert : % -> Pattern Integer
  if V has KONVERT PATTERN INT
  and R has KONVERT PATTERN INT
convert : % -> Pattern Float
  if V has KONVERT PATTERN FLOAT
  and R has KONVERT PATTERN FLOAT
convert : % -> InputForm
  if V has KONVERT INFORM
  and R has KONVERT INFORM
D : (%,List V) -> %
D : (%,V) -> %
D : (%,List V,List NonNegativeInteger) -> %
D : (%,V,NonNegativeInteger) -> %
degree : % -> E
degree : (%,List V) -> List NonNegativeInteger
degree : (%,V) -> NonNegativeInteger
differentiate : (%,V) -> %
differentiate : (%,List V,List NonNegativeInteger) -> %
differentiate : (%,V,NonNegativeInteger) -> %
differentiate : (%,List V) -> %
discriminant : (%,V) -> % if R has COMRING
eval : (%,Equation %) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
```

```
eval : (%,List V,List R) -> %
eval : (%,V,R) -> %
eval : (%,List V,List %) -> %
eval : (%,V,%) -> %
exquo : (%,%) -> Union(%,"failed") if R has INTDOM
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
factor : % -> Factored % if R has PFECAT
factorPolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
factorSquareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
gcd : (%,%) -> % if R has GCDDOM
gcd : List % -> % if R has GCDDOM
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
       if R has GCDDOM
ground : % -> R
ground? : % -> Boolean
hash : % -> SingleInteger
isExpt : % ->
  Union(Record(var: V,exponent: NonNegativeInteger),"failed")
isPlus : % -> Union(List %,"failed")
isTimes : % -> Union(List %,"failed")
latex : % -> String
lcm : (%,%) -> % if R has GCDDOM
lcm : List % -> % if R has GCDDOM
leadingCoefficient : % -> R
leadingMonomial : % -> %
mainVariable : % -> Union(V,"failed")
map : ((R -> R),%) -> %
mapExponents : ((E -> E),%) -> %
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
minimumDegree : % -> E
minimumDegree : (%,List V) -> List NonNegativeInteger
minimumDegree : (%,V) -> NonNegativeInteger
monicDivide : (%,%,V) -> Record(quotient: %,remainder: %)
monomial : (%,V,NonNegativeInteger) -> %
monomial : (%,List V,List NonNegativeInteger) -> %
monomial : (R,E) -> %
monomial? : % -> Boolean
monomials : % -> List %
multivariate : (SparseUnivariatePolynomial %,V) -> %
multivariate : (SparseUnivariatePolynomial R,V) -> %
```

```
numberOfMonomials : % -> NonNegativeInteger
one? : % -> Boolean
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
    PatternMatchResult(Integer,%)
      if V has PATMAB INT
      and R has PATMAB INT
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
    PatternMatchResult(Float,%)
      if V has PATMAB FLOAT
      and R has PATMAB FLOAT
pomopo! : (%,R,E,%) -> %
prime? : % -> Boolean if R has PFECAT
primitiveMonomials : % -> List %
primitivePart : (%,V) -> % if R has GCDDOM
primitivePart : % -> % if R has GCDDOM
recip : % -> Union(%,"failed")
reducedSystem : Matrix % -> Matrix R
reducedSystem :
  (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
reducedSystem :
  (Matrix %,Vector %) ->
    Record(mat: Matrix Integer,vec: Vector Integer)
      if R has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer
    if R has LINEXP INT
reductum : % -> %
resultant : (%,%,V) -> % if R has COMRING
retract : % -> R
retract : % -> Integer if R has RETRACT INT
retract : % -> Fraction Integer if R has RETRACT FRAC INT
retract : % -> V
retractIfCan : % -> Union(R,"failed")
retractIfCan : % -> Union(Integer,"failed")
    if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if R has RETRACT FRAC INT
retractIfCan : % -> Union(V,"failed")
sample : () -> %
solveLinearPolynomialEquation :
  (List SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
    Union(List SparseUnivariatePolynomial %,"failed")
      if R has PFECAT
squareFree : % -> Factored % if R has GCDDOM
squareFreePart : % -> % if R has GCDDOM
squareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
```

```
      if R has PFECAT
subtractIfCan : (%,%) -> Union(%,"failed")
totalDegree : (%,List V) -> NonNegativeInteger
totalDegree : % -> NonNegativeInteger
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if R has INTDOM
univariate : % -> SparseUnivariatePolynomial R
univariate : (%,V) -> SparseUnivariatePolynomial %
variables : % -> List V
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?/? : (%,R) -> % if R has FIELD
?-? : (%,%) -> %
-? : % -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?<? : (%,%) -> Boolean if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
```

These exports come from (p777) DifferentialExtension(R:Ring):

```
D : (%,(R -> R)) -> %
D : (%,(R -> R),NonNegativeInteger) -> %
D : % -> % if R has DIFRING
D : (%,NonNegativeInteger) -> % if R has DIFRING
D : (%,Symbol) -> % if R has PDRING SYMBOL
D : (%,List Symbol) -> % if R has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> %
    if R has PDRING SYMBOL
D : (%,List Symbol,List NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : (%,NonNegativeInteger) -> %
    if R has DIFRING
differentiate : (%,List Symbol) -> %
```

```
    if R has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : % -> % if R has DIFRING
differentiate : (%,(R -> R),NonNegativeInteger) -> %
differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
```

These exports come from (p44) RetractableTo(S:OrderedSet):

```
retract : % -> S
```

These exports come from (p29) InnerEvalable(S,R)
where S:OrderedSet, R:Ring:

```
eval : (%,S,R) -> % if R has DIFRING
```

These exports come from (p29) InnerEvalable(S,where S:OrderedSet,

```
eval : (%,S,%) -> % if R has DIFRING
```

These exports come from (p65) Evalable(

⟨*category DPOLCAT DifferentialPolynomialCategory*⟩≡

```
)abbrev category DPOLCAT DifferentialPolynomialCategory
++ Author:  William Sit
++ Date Created: 19 July 1990
++ Date Last Updated: 13 September 1991
++ Basic Operations:PolynomialCategory
++ Related Constructors:DifferentialVariableCategory
++ See Also:
++ AMS Classifications:12H05
++ Keywords: differential indeterminates, ranking, differential polynomials,
++           order, weight, leader, separant, initial, isobaric
++ References:Kolchin, E.R. "Differential Algebra and Algebraic Groups"
++   (Academic Press, 1973).
++ Description:
++   \spadtype{DifferentialPolynomialCategory} is a category constructor
++   specifying basic functions in an ordinary differential polynomial
++   ring with a given ordered set of differential indeterminates.
++   In addition, it implements defaults for the basic functions.
++   The functions \spadfun{order} and \spadfun{weight} are extended
++   from the set of derivatives of differential indeterminates
++   to the set of differential polynomials.  Other operations
++   provided on differential polynomials are
```

```
++   \spadfun{leader}, \spadfun{initial},
++   \spadfun{separant}, \spadfun{differentialVariables}, and
++   \spadfun{isobaric?}.   Furthermore, if the ground ring is
++   a differential ring, then evaluation (substitution
++   of differential indeterminates by elements of the ground ring
++   or by differential polynomials) is
++   provided by \spadfun{eval}.
++   A convenient way of referencing derivatives is provided by
++   the functions \spadfun{makeVariable}.
++
++   To construct a domain using this constructor, one needs
++   to provide a ground ring R, an ordered set S of differential
++   indeterminates, a ranking V on the set of derivatives
++   of the differential indeterminates, and a set E of
++   exponents in bijection with the set of differential monomials
++   in the given differential indeterminates.
++

DifferentialPolynomialCategory(R:Ring,S:OrderedSet,
  V:DifferentialVariableCategory S, E:OrderedAbelianMonoidSup):
             Category ==
  Join(PolynomialCategory(R,E,V),
       DifferentialExtension R, RetractableTo S) with
    -- Examples:
    -- s:=makeVariable('s)
    -- p:= 3*(s 1)**2 + s*(s 2)**3
    --  all functions below have default implementations
    --  using primitives from V

    makeVariable: S -> (NonNegativeInteger -> $)
       ++ makeVariable(s) views s as a differential
       ++ indeterminate,  in such a way that the n-th
       ++ derivative of s may be simply referenced as z.n
       ++ where z :=makeVariable(s).
       ++ Note: In the interpreter, z is
       ++ given as an internal map, which may be ignored.
       -- Example: makeVariable('s); %.5

    differentialVariables: $ ->  List S
      ++ differentialVariables(p) returns a list of differential
      ++ indeterminates occurring in a differential polynomial p.
    order : ($, S) -> NonNegativeInteger
      ++ order(p,s) returns the order of the differential
      ++ polynomial p in differential indeterminate s.
    order : $   -> NonNegativeInteger
      ++ order(p) returns the order of the differential polynomial p,
```

```
    ++ which is the maximum number of differentiations of a
    ++ differential indeterminate, among all those appearing in p.
  degree: ($, S) -> NonNegativeInteger
    ++ degree(p, s) returns the maximum degree of
    ++ the differential polynomial p viewed as a differential polynomial
    ++ in the differential indeterminate s alone.
  weights: $ -> List NonNegativeInteger
    ++ weights(p) returns a list of weights of differential monomials
    ++ appearing in differential polynomial p.
  weight: $   -> NonNegativeInteger
    ++ weight(p) returns the maximum weight of all differential monomials
    ++ appearing in the differential polynomial p.
  weights: ($, S) -> List NonNegativeInteger
    ++ weights(p, s) returns a list of
    ++ weights of differential monomials
    ++ appearing in the differential polynomial p when p is viewed
    ++ as a differential polynomial in the differential indeterminate s
    ++ alone.
  weight: ($, S) -> NonNegativeInteger
    ++ weight(p, s) returns the maximum weight of all differential
    ++ monomials appearing in the differential polynomial p
    ++ when p is viewed as a differential polynomial in
    ++ the differential indeterminate s alone.
  isobaric?: $ -> Boolean
    ++ isobaric?(p) returns true if every differential monomial appearing
    ++ in the differential polynomial p has same weight,
    ++ and returns false otherwise.
  leader: $   -> V
    ++ leader(p) returns the derivative of the highest rank
    ++ appearing in the differential polynomial p
    ++ Note: an error occurs if p is in the ground ring.
  initial:$   -> $
    ++ initial(p) returns the
    ++ leading coefficient when the differential polynomial p
    ++ is written as a univariate polynomial in its leader.
  separant:$  -> $
    ++ separant(p) returns the
    ++ partial derivative of the differential polynomial p
    ++ with respect to its leader.
  if R has DifferentialRing then
    InnerEvalable(S, R)
    InnerEvalable(S, $)
    Evalable $
    makeVariable: $ -> (NonNegativeInteger -> $)
     ++ makeVariable(p) views p as an element of a differential
     ++ ring,  in such a way that the n-th
```

```
        ++ derivative of p may be simply referenced as z.n
        ++ where z := makeVariable(p).
        ++ Note: In the interpreter, z is
        ++ given as an internal map, which may be ignored.
        -- Example: makeVariable(p); %.5; makeVariable(%**2); %.2

   add
      p:$
      s:S

      makeVariable s == n +-> makeVariable(s,n)::$

      if R has IntegralDomain then
        differentiate(p:$, d:R -> R) ==
          ans:$ := 0
          l := variables p
          while (u:=retractIfCan(p)@Union(R, "failed")) case "failed" repeat
            t := leadingMonomial p
            lc := leadingCoefficient t
            ans := ans + d(lc)::$ * (t exquo lc)::$
                + +/[differentiate(t, v) * (differentiate v)::$ for v in l]
            p := reductum p
          ans + d(u::R)::$

      order (p:$):NonNegativeInteger ==
        ground? p => 0
        "max"/[order v for v in variables p]

      order (p:$,s:S):NonNegativeInteger ==
        ground? p => 0
        empty? (vv:= [order v for v in variables p | (variable v) = s ]) =>0
        "max"/vv

      degree (p, s) ==
        d:NonNegativeInteger:=0
        for lp in monomials p repeat
          lv:= [v for v in variables lp | (variable v) = s ]
          if not empty? lv then d:= max(d, +/degree(lp, lv))
        d

      weights p ==
        ws:List NonNegativeInteger := nil
        empty? (mp:=monomials p) => ws
        for lp in mp repeat
          lv:= variables lp
          if not empty? lv then
```

```
      dv:= degree(lp, lv)
      w:=+/[(weight v) * d _
             for v in lv for d in dv]$(List NonNegativeInteger)
      ws:= concat(ws, w)
  ws

weight p ==
  empty? (ws:=weights p) => 0
  "max"/ws

weights (p, s) ==
  ws:List NonNegativeInteger := nil
  empty?(mp:=monomials p) => ws
  for lp in mp repeat
    lv:= [v for v in variables lp | (variable v) = s ]
    if not empty? lv then
      dv:= degree(lp, lv)
      w:=+/[(weight v) * d _
           for v in lv for d in dv]$(List NonNegativeInteger)
      ws:= concat(ws, w)
  ws

weight (p,s)  ==
  empty? (ws:=weights(p,s)) => 0
  "max"/ws

isobaric? p == (# removeDuplicates weights p) = 1

leader p ==             -- depends on the ranking
  vl:= variables p
  -- it's not enough just to look at leadingMonomial p
  -- the term-ordering need not respect the ranking
  empty? vl => error "leader is not defined "
  "max"/vl

initial p == leadingCoefficient univariate(p,leader p)

separant p == differentiate(p, leader p)

coerce(s:S):$   == s::V::$

retractIfCan(p:$):Union(S, "failed") ==
  (v := retractIfCan(p)@Union(V,"failed")) case "failed" => "failed"
  retractIfCan(v::V)

differentialVariables p ==
```

```
      removeDuplicates [variable v for v in variables p]

    if R has DifferentialRing then

      makeVariable p == n +-> differentiate(p, n)

      eval(p:$, sl:List S, rl:List R) ==
        ordp:= order p
        vl  := concat [[makeVariable(s,j)$V for j in  0..ordp]
                                for s in sl]$List(List V)
        rrl:=nil$List(R)
        for r in rl repeat
          t:= r
          rrl:= concat(rrl,
                concat(r, [t := differentiate t for i in 1..ordp]))
        eval(p, vl, rrl)

      eval(p:$, sl:List S, rl:List $) ==
        ordp:= order p
        vl  := concat [[makeVariable(s,j)$V for j in  0..ordp]
                                for s in sl]$List(List V)
        rrl:=nil$List($)
        for r in rl repeat
          t:=r
          rrl:=concat(rrl,
               concat(r, [t:=differentiate t for i in 1..ordp]))
        eval(p, vl, rrl)

      eval(p:$, l:List Equation $) ==
        eval(p, [retract(lhs e)@S for e in l]$List(S),
              [rhs e for e in l]$List($))
```

⟨*DPOLCAT.dotabb*⟩≡

```
"DPOLCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DPOLCAT"];
"DPOLCAT" -> "DIFEXT"
"DPOLCAT" -> "POLYCAT"
"DPOLCAT" -> "RETRACT"
```

⟨*DPOLCAT.dotfull*⟩≡

```
"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategory(b),d:Orde
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DPOLCAT"];
"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategory(b),d:Orde
 -> "DifferentialExtension(a:Ring)"
"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategory(b),d:Orde
 -> "PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategory(b),d:Orde
 -> "RetractableTo(OrderedSet)"
```

⟨*DPOLCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategor
 [color=lightblue];

"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategor
 -> "DifferentialExtension(a:Ring)"

"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategor
 -> "PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"

"DifferentialPolynomialCategory(a:Ring,b:OrderedSet,c:DifferentialVariableCategor
 -> "RetractableTo(OrderedSet)"

"DifferentialExtension(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=DIFEXT"];
"DifferentialExtension(a:Ring)" -> "RING..."
"DifferentialExtension(a:Ring)" -> "DIFRING..."
"DifferentialExtension(a:Ring)" -> "PDRING..."

"RetractableTo(OrderedSet)"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=RETRACT"];
"RetractableTo(OrderedSet)" -> "RETRACT..."

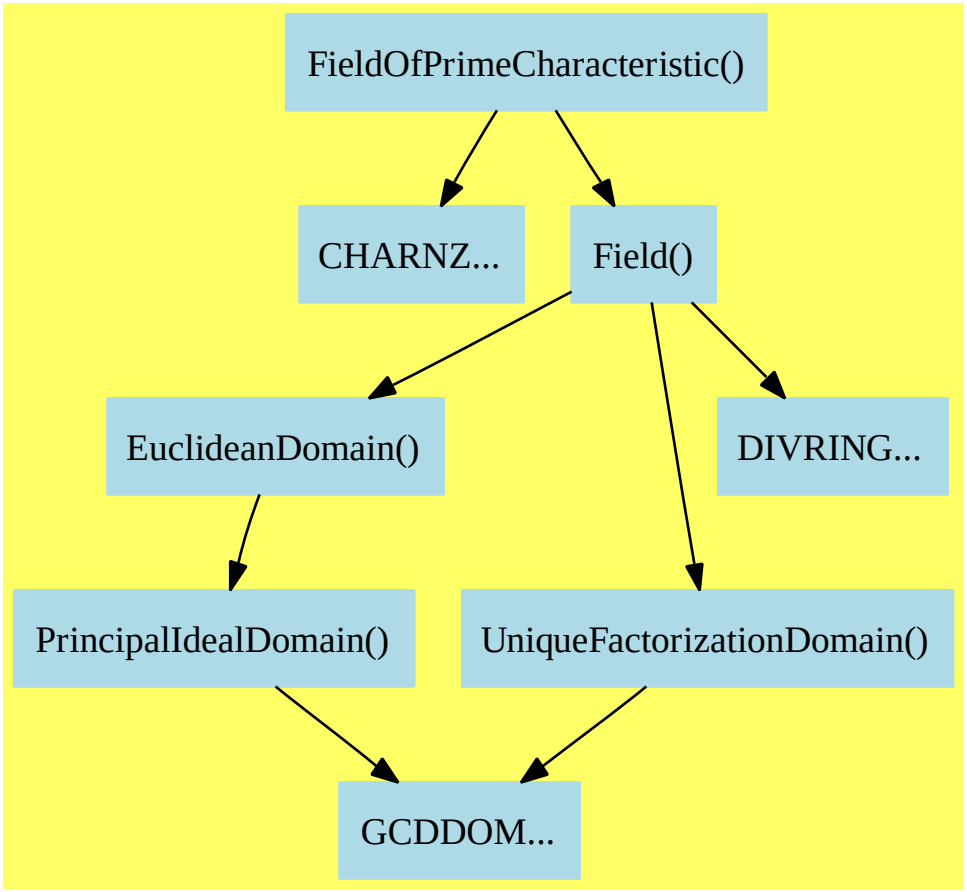
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=POLYCAT"];
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PDRING..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FAMR..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "EVALAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "IEVALAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "RETRACT..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FLINEXP..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "ORDSET..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "GCDDOM..."
```

```
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PFECAT..."

"DIFRING..." [color=lightblue];
"RING..." [color=lightblue];
"PDRING..." [color=lightblue];
"FAMR..." [color=lightblue];
"EVALAB..." [color=lightblue];
"IEVALAB..." [color=lightblue];
"RETRACT..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"ORDSET..." [color=lightblue];
"GCDDOM..." [color=lightblue];
"PFECAT..." [color=lightblue];

}
```

17.3 FieldOfPrimeCharacteristic (FPC)



See:

⇒ “FiniteFieldCategory” (FFIELDC) 18.3 on page 1244
⇐ “CharacteristicNonZero” (CHARNZ) 10.2 on page 677
⇐ “Field” (FIELD) 16.1 on page 1005

Exports:

0	1	associates?	characteristic
charthRoot	coerce	discreteLog	divide
euclideanSize	expressIdealMember	exquo	extendedEuclidean
factor	gcd	gcdPolynomial	hash
inv	latex	lcm	multiEuclidean
one?	order	prime?	primeFrobenius
principalIdeal	recip	sample	sizeLess?
squareFree	squareFreePart	subtractIfCan	unit?
unitCanonical	unitNormal	zero?	?*?
?**?	?+?	?-?	-?
?/?	?=?	?^?	?rem?
?quo?	?~=?		

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
order : % -> OnePointCompletion PositiveInteger
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
```

These are implemented by this category:

```
primeFrobenius : % -> %
primeFrobenius : (%,NonNegativeInteger) -> %
```

These exports come from (p1005) Field():

```
0 : () -> %
1 : () -> %
```

```
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
exquo : (%,%) -> Union(%,"failed")
factor : % -> Factored %
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                     SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,Integer) -> %
?**? : (%,PositiveInteger) -> %
```

```
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,Integer) -> %
?/? : (%,%) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p677) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed")
```

⟨*category FPC FieldOfPrimeCharacteristic*⟩≡

```
)abbrev category FPC FieldOfPrimeCharacteristic
++ Author: J. Grabmeier, A. Scheerhorn
++ Date Created: 10 March 1991
++ Date Last Updated: 31 March 1991
++ Basic Operations: _+, _*
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: field, finite field, prime characteristic
++ References:
++  J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++  AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++  FieldOfPrimeCharacteristic is the category of fields of prime
++  characteristic, e.g. finite fields, algebraic closures of
++  fields of prime characteristic, transcendental extensions of
++  of fields of prime characteristic.
FieldOfPrimeCharacteristic:Category == _
 Join(Field,CharacteristicNonZero) with
   order: $ -> OnePointCompletion PositiveInteger
     ++ order(a) computes the order of an element in the multiplicative
     ++ group of the field.
     ++ Error: if \spad{a} is 0.
   discreteLog: ($,$) -> Union(NonNegativeInteger,"failed")
     ++ discreteLog(b,a) computes s with \spad{b**s = a} if such an s exists.
   primeFrobenius: $ -> $
     ++ primeFrobenius(a) returns \spad{a**p} where p is the characteristic.
   primeFrobenius: ($,NonNegativeInteger) -> $
     ++ primeFrobenius(a,s) returns \spad{a**(p**s)} where p
     ++ is the characteristic.
 add
   primeFrobenius(a) == a ** characteristic()
   primeFrobenius(a,s) == a ** (characteristic()**s)
```

⟨*FPC.dotabb*⟩≡
```
"FPC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FPC"];
"FPC" -> "CHARNZ"
"FPC" -> "FIELD"
```

⟨*FPC.dotfull*⟩≡
```
"FieldOfPrimeCharacteristic()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FPC"];
"FieldOfPrimeCharacteristic()" -> "CharacteristicNonZero()"
```

⟨*FPC.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FieldOfPrimeCharacteristic()" [color=lightblue];
"FieldOfPrimeCharacteristic()" -> "CHARNZ..."
"FieldOfPrimeCharacteristic()" -> "Field()"

"Field()" [color=lightblue];
"Field()" -> "EuclideanDomain()"
"Field()" -> "UniqueFactorizationDomain()"
"Field()" -> "DIVRING..."

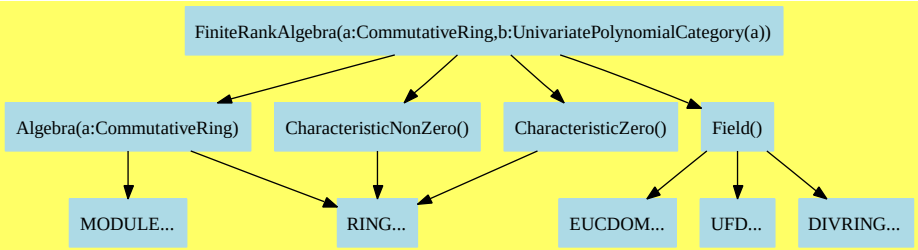
"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GCDDOM..."

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GCDDOM..."

"GCDDOM..." [color=lightblue];
"DIVRING..." [color=lightblue];
"CHARNZ..." [color=lightblue];
}
```

17.4 FiniteRankAlgebra (FINRALG)



See:

⇒ "FramedAlgebra" (FRAMALG) 18.5 on page 1261
⇐ "Algebra" (ALGEBRA) 11.1 on page 771
⇐ "CharacteristicNonZero" (CHARNZ) 10.2 on page 677
⇐ "CharacteristicZero" (CHARZ) 10.3 on page 681
⇐ "Field" (FIELD) 16.1 on page 1005

Exports:

0	1	characteristic
characteristicPolynomial	charthRoot	coerce
coordinates	discriminant	hash
latex	minimalPolynomial	norm
one?	rank	recip
regularRepresentation	represents	sample
subtractIfCan	trace	traceMatrix
zero?	?*?	?**?
?+?	?-?	-?
?=?	?^?	?~=?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
characteristicPolynomial : % -> UP
coordinates : (%,Vector %) -> Vector R
minimalPolynomial : % -> UP if R has FIELD
norm : % -> R
rank : () -> PositiveInteger
trace : % -> R
```

These are implemented by this category:

```
coordinates : (Vector %,Vector %) -> Matrix R
discriminant : Vector % -> R
regularRepresentation : (%,Vector %) -> Matrix R
represents : (Vector R,Vector %) -> %
traceMatrix : Vector % -> Matrix R
```

These exports come from (p771) Algebra(R:CommutativeRing):

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

These exports come from (p1005) Field():

```
coerce : R -> %
?*? : (R,%) -> %
?*? : (%,R) -> %
```

These exports come from (p677) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed") if R has CHARNZ
```

These exports come from (p681) CharacteristicZero():

⟨*category FINRALG FiniteRankAlgebra*⟩≡

```
)abbrev category FINRALG FiniteRankAlgebra
++ Author: Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A FiniteRankAlgebra is an algebra over a commutative ring R which
++ is a free R-module of finite rank.

FiniteRankAlgebra(R:CommutativeRing, UP:UnivariatePolynomialCategory R):
 Category == Algebra R with
    rank                    : () -> PositiveInteger
      ++ rank() returns the rank of the algebra.
    regularRepresentation   : (% , Vector %) -> Matrix R
      ++ regularRepresentation(a,basis) returns the matrix of the
      ++ linear map defined by left multiplication by \spad{a} with respect
      ++ to the basis \spad{basis}.
    trace                   : %  -> R
      ++ trace(a) returns the trace of the regular representation
      ++ of \spad{a} with respect to any basis.
    norm                    : %  -> R
      ++ norm(a) returns the determinant of the regular representation
      ++ of \spad{a} with respect to any basis.
    coordinates             : (%, Vector %) -> Vector R
      ++ coordinates(a,basis) returns the coordinates of \spad{a} with
      ++ respect to the basis \spad{basis}.
    coordinates             : (Vector %, Vector %) -> Matrix R
      ++ coordinates([v1,...,vm], basis) returns the coordinates of the
      ++ vi's with to the basis \spad{basis}.  The coordinates of vi are
      ++ contained in the ith row of the matrix returned by this
      ++ function.
    represents              : (Vector R, Vector %) -> %
      ++ represents([a1,..,an],[v1,..,vn]) returns \spad{a1*v1+...+an*vn}.
    discriminant            : Vector % -> R
      ++ discriminant([v1,..,vn]) returns
      ++ \spad{determinant(traceMatrix([v1,..,vn]))}.
    traceMatrix             : Vector % -> Matrix R
      ++ traceMatrix([v1,..,vn]) is the n-by-n matrix ( Tr(vi * vj) )
    characteristicPolynomial: % -> UP
      ++ characteristicPolynomial(a) returns the characteristic
```

```
      ++ polynomial of the regular representation of \spad{a} with respect
      ++ to any basis.
    if R has Field then minimalPolynomial : % -> UP
      ++ minimalPolynomial(a) returns the minimal polynomial of \spad{a}.
    if R has CharacteristicZero then CharacteristicZero
    if R has CharacteristicNonZero then CharacteristicNonZero

  add

    discriminant v == determinant traceMatrix v

    coordinates(v:Vector %, b:Vector %) ==
      m := new(#v, #b, 0)$Matrix(R)
      for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
        setRow_!(m, j, coordinates(qelt(v, i), b))
      m

    represents(v, b) ==
      m := minIndex v - 1
      _+/[v(i+m) * b(i+m) for i in 1..rank()]

    traceMatrix v ==
      matrix [[trace(v.i*v.j) for j in minIndex v..maxIndex v]$List(R)
               for i in minIndex v .. maxIndex v]$List(List R)

    regularRepresentation(x, b) ==
      m := minIndex b - 1
      matrix
       [parts coordinates(x*b(i+m),b) for i in 1..rank()]$List(List R)
```

⟨*FINRALG.dotabb*⟩≡

```
"FINRALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FINRALG"];
"FINRALG" -> "ALGEBRA"
"FINRALG" -> "FIELD"
"FINRALG" -> "CHARNZ"
"FINRALG" -> "CHARZ"
```

⟨*FINRALG.dotfull*⟩≡

```
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FINRALG"];
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "Algebra(a:CommutativeRing)"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "Field()"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CharacteristicNonZero()"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CharacteristicZero()"
```

⟨*FINRALG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue];
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "Algebra(a:CommutativeRing)"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "Field()"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CharacteristicNonZero()"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CharacteristicZero()"

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."

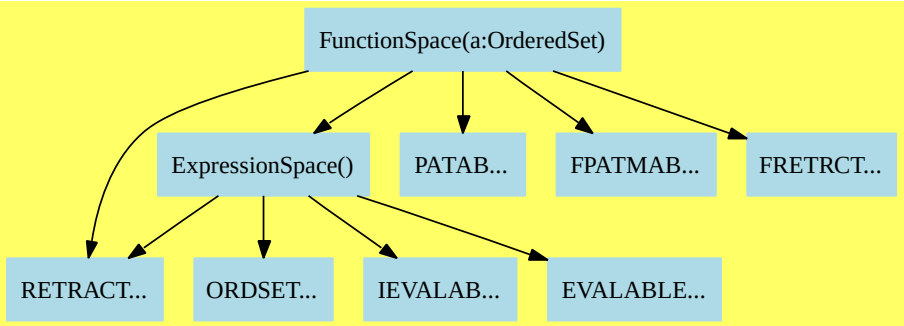
"Field()" [color=lightblue];
"Field()" -> "EUCDOM..."
"Field()" -> "UFD..."
"Field()" -> "DIVRING..."

"CharacteristicNonZero()" [color=lightblue];
"CharacteristicNonZero()" -> "RING..."

"CharacteristicZero()" [color=lightblue];
"CharacteristicZero()" -> "RING..."

"EUCDOM..." [color=lightblue];
"UFD..." [color=lightblue];
"DIVRING..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

17.5 FunctionSpace (FS)



See:

⇐ “AbelianGroup” (ABELGRP) 7.1 on page 418
⇐ “AbelianMonoid” (ABELMON) 5.1 on page 207
⇐ “AbelianSemiGroup” (ABELSG) 4.1 on page 99
⇐ “Algebra” (ALGEBRA) 11.1 on page 771
⇐ “AlgebraicallyClosedFunctionSpace” (ACFS) 18.1 on page 1225
⇐ “CharacteristicNonZero” (CHARNZ) 10.2 on page 677
⇐ “CharacteristicZero” (CHARZ) 10.3 on page 681
⇐ “CommutativeRing” (COMRING) 10.4 on page 685
⇐ “ConvertibleTo” (KONVERT) 2.8 on page 19
⇐ “ExpressionSpace” (ES) 5.6 on page 230
⇐ “Field” (FIELD) 16.1 on page 1005
⇐ “FullyLinearlyExplicitRingOver” (FLINEXP) 11.3 on page 782
⇐ “FullyPatternMatchable” (FPATMAB) 3.7 on page 74
⇐ “FullyRetractableTo” (FRETRCT) 3.6 on page 71
⇐ “Group” (GROUP) 6.5 on page 304
⇐ “IntegralDomain” (INTDOM) 12.5 on page 857
⇐ “Monoid” (MONOID) 5.10 on page 256
⇐ “PartialDifferentialRing” (PDRING) 10.12 on page 720
⇐ “Patternable” (PATAB) 2.15 on page 38
⇐ “RetractableTo” (RETRACT) 2.18 on page 44
⇐ “Ring” (RING) 9.8 on page 628
⇐ “SemiGroup” (SGROUP) 4.24 on page 186

Exports:

0	1	applyQuote	associates?
belong?	box	characteristic	charthRoot
coerce	commutator	conjugate	convert
D	definingPolynomial	denom	denominator
differentiate	distribute	divide	elt
eval	euclideanSize	even?	expressIdealMember
exquo	extendedEuclidean	factor	freeOf?
gcd	gcdPolynomial	ground	ground?
hash	height	inv	is?
isExpt	isMult	isPlus	isPower
isTimes	kernel	kernels	latex
lcm	mainKernel	map	max
min	minPoly	multiEuclidean	numer
numerator	odd?	one?	operator
operators	paren	patternMatch	prime?
principalIdeal	recip	reducedSystem	retract
retractIfCan	sample	sizeLess?	squareFree
squareFreePart	subst	subtractIfCan	tower
unit?	unitCanonical	unitNormal	univariate
variables	zero?	-?	?<?
?<=?	?=?	?>?	?>=?
?~=?	?*?	?**?	?+?
?-?	?/?	?^?	?quo?
?rem?			

Attributes Exported:

- if $ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if $ has IntegralDomain then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if $ has IntegralDomain then canonicalsClosed where **canonicalsClosed** is true if
`unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- if $ has IntegralDomain then commutative("*") where **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- if $ has Ring or Group then unitsKnown where **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- if $ has CommutativeRing then leftUnitary where **leftUnitary** is true if $1 * x = x$ for all x.

- if $ has CommutativeRing then rightUnitary where **rightUnitary** is true if $x * 1 = x$ for all x.
- **nil**

These are directly exported but not implemented:

```
coerce : SparseMultivariatePolynomial(R,Kernel %) -> %
    if R has RING
coerce : Fraction R -> % if R has INTDOM
coerce : Polynomial Fraction R -> % if R has INTDOM
denom : % -> SparseMultivariatePolynomial(R,Kernel %)
    if R has INTDOM
eval : (%,List BasicOperator,List %,Symbol) -> %
    if R has KONVERT INFORM
numer : % -> SparseMultivariatePolynomial(R,Kernel %)
    if R has RING
?/? :
  (SparseMultivariatePolynomial(R,Kernel %),
   SparseMultivariatePolynomial(R,Kernel %)) -> %
    if R has INTDOM
?**? : (%,NonNegativeInteger) -> % if R has SGROUP
```

These are implemented by this category:

```
applyQuote : (Symbol,%) -> %
applyQuote : (Symbol,%,%) -> %
applyQuote : (Symbol,%,%,%) -> %
applyQuote : (Symbol,%,%,%,%) -> %
applyQuote : (Symbol,List %) -> %
belong? : BasicOperator -> Boolean
characteristic : () -> NonNegativeInteger if R has RING
coerce : Symbol -> %
coerce : Kernel % -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
    if R has INTDOM
    or R has RETRACT INT
    and R has INTDOM
    or R has RETRACT FRAC INT
coerce : Fraction Polynomial R -> % if R has INTDOM
coerce : Fraction Polynomial Fraction R -> %
    if R has INTDOM
coerce : Polynomial R -> % if R has RING
convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT
convert : % -> Pattern Integer if R has KONVERT PATTERN INT
convert : Factored % -> % if R has INTDOM
denominator : % -> % if R has INTDOM
differentiate : (%,Symbol) -> % if R has RING
elt : (BasicOperator,List %) -> %
```

```
eval : (%,List Symbol) -> % if R has KONVERT INFORM
eval : % -> % if R has KONVERT INFORM
eval : (%,Symbol) -> % if R has KONVERT INFORM
eval : (%,BasicOperator,%,Symbol) -> %
    if R has KONVERT INFORM
eval : (%,Symbol,NonNegativeInteger,(% -> %)) -> %
    if R has RING
eval : (%,Symbol,NonNegativeInteger,(List % -> %)) -> %
    if R has RING
eval : (%,List Symbol,List NonNegativeInteger,List (List % -> %)) -> %
    if R has RING
eval : (%,List Symbol,List NonNegativeInteger,List (% -> %)) -> %
    if R has RING
eval : (%,List Kernel %,List %) -> %
ground : % -> R
ground? : % -> Boolean
isExpt : (%,BasicOperator) ->
   Union(Record(var: Kernel %,exponent: Integer),"failed")
    if R has RING
isExpt : % ->
  Union(Record(var: Kernel %,exponent: Integer),"failed")
    if R has SGROUP
isExpt : (%,Symbol) ->
  Union(Record(var: Kernel %,exponent: Integer),"failed")
    if R has RING
isMult : % ->
  Union(Record(coef: Integer,var: Kernel %),"failed")
    if R has ABELSG
isPlus : % -> Union(List %,"failed") if R has ABELSG
isPower : % ->
  Union(Record(val: %,exponent: Integer),"failed")
    if R has RING
isTimes : % -> Union(List %,"failed") if R has SGROUP
kernels : % -> List Kernel %
mainKernel : % -> Union(Kernel %,"failed")
numerator : % -> % if R has RING
operator : BasicOperator -> BasicOperator
retract : % -> Fraction Polynomial R if R has INTDOM
retract : % -> Polynomial R if R has RING
retract : % -> R
retract : % -> Symbol
retractIfCan : % -> Union(R,"failed")
retractIfCan : % -> Union(Fraction Polynomial R,"failed")
    if R has INTDOM
retractIfCan : % -> Union(Polynomial R,"failed")
    if R has RING
retractIfCan : % -> Union(Symbol,"failed")
subst : (%,List Kernel %,List %) -> %
univariate : (%,Kernel %) ->
  Fraction SparseUnivariatePolynomial %
```

```
    if R has INTDOM
variables : % -> List Symbol
?*? : (%,R) -> % if R has COMRING
?*? : (R,%) -> % if R has COMRING
```

These exports come from (p230) ExpressionSpace():

```
box : List % -> %
box : % -> %
definingPolynomial : % -> % if $ has RING
distribute : (%,%) -> %
distribute : % -> %
elt : (BasicOperator,%,%,%,%) -> %
elt : (BasicOperator,%,%,%) -> %
elt : (BasicOperator,%,%) -> %
elt : (BasicOperator,%) -> %
eval : (%,List BasicOperator,List (% -> %)) -> %
eval : (%,List Equation %) -> %
eval : (%,Symbol,(% -> %)) -> %
eval : (%,Symbol,(List % -> %)) -> %
eval : (%,BasicOperator,(% -> %)) -> %
eval : (%,BasicOperator,(List % -> %)) -> %
eval : (%,List Symbol,List (% -> %)) -> %
eval : (%,List BasicOperator,List (List % -> %)) -> %
eval : (%,List Symbol,List (List % -> %)) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
eval : (%,Equation %) -> %
eval : (%,Kernel %,%) -> %
even? : % -> Boolean if $ has RETRACT INT
freeOf? : (%,Symbol) -> Boolean
freeOf? : (%,%) -> Boolean
hash : % -> SingleInteger
height : % -> NonNegativeInteger
is? : (%,BasicOperator) -> Boolean
is? : (%,Symbol) -> Boolean
kernel : (BasicOperator,%) -> %
kernel : (BasicOperator,List %) -> %
latex : % -> String
map : ((% -> %),Kernel %) -> %
max : (%,%) -> %
min : (%,%) -> %
minPoly : Kernel % -> SparseUnivariatePolynomial %
    if $ has RING
odd? : % -> Boolean if $ has RETRACT INT
operators : % -> List BasicOperator
paren : List % -> %
paren : % -> %
retract : % -> Kernel %
retractIfCan : % -> Union(Kernel %,"failed")
```

```
subst : (%,List Equation %) -> %
subst : (%,Equation %) -> %
tower : % -> List Kernel %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

These exports come from (p74) FullyPatternMatchable(OrderedSet):

```
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
    PatternMatchResult(Integer,%)
      if R has PATMAB INT
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
    PatternMatchResult(Float,%)
      if R has PATMAB FLOAT
```

These exports come from (p71) FullyRetractableTo(OrderedSet):

```
coerce : Integer -> % if R has RING or R has RETRACT INT
coerce : R -> %
```

These exports come from (p19) ConvertibleTo(InputForm):

```
convert : % -> InputForm if R has KONVERT INFORM
```

These exports come from (p256) Monoid():

```
1 : () -> % if R has SGROUP
one? : % -> Boolean if R has SGROUP
recip : % -> Union(%,"failed") if R has SGROUP
sample : () -> % if R has SGROUP or R has ABELSG
?^? : (%,NonNegativeInteger) -> % if R has SGROUP
?*? : (%,%) -> % if R has SGROUP
?**? : (%,PositiveInteger) -> % if R has SGROUP
?^? : (%,PositiveInteger) -> % if R has SGROUP
```

These exports come from (p304) Group():

```
commutator : (%,%) -> % if R has GROUP
conjugate : (%,%) -> % if R has GROUP
inv : % -> % if R has GROUP or R has INTDOM
?/? : (%,%) -> % if R has GROUP or R has INTDOM
?^? : (%,Integer) -> % if R has GROUP or R has INTDOM
?**? : (%,Integer) -> % if R has GROUP or R has INTDOM
```

These exports come from (p207) AbelianMonoid():

```
0 : () -> % if R has ABELSG
zero? : % -> Boolean if R has ABELSG
?*? : (PositiveInteger,%) -> % if R has ABELSG
?*? : (NonNegativeInteger,%) -> % if R has ABELSG
?+? : (%,%) -> % if R has ABELSG
```

These exports come from (p418) AbelianGroup():

```
subtractIfCan : (%,%) -> Union(%,"failed") if R has ABELGRP
?*? : (Integer,%) -> % if R has ABELGRP
?-? : (%,%) -> % if R has ABELGRP
-? : % -> % if R has ABELGRP
```

These exports come from (p720) PartialDifferentialRing(Symbol):

```
D : (%,Symbol) -> % if R has RING
D : (%,List Symbol) -> % if R has RING
D : (%,Symbol,NonNegativeInteger) -> % if R has RING
D : (%,List Symbol,List NonNegativeInteger) -> % if R has RING
differentiate : (%,List Symbol) -> % if R has RING
differentiate : (%,Symbol,NonNegativeInteger) -> % if R has RING
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if R has RING
```

These exports come from (p782) FullyLinearlyExplicitRingOver(R)
where R:OrderedSet:

```
reducedSystem : Matrix % -> Matrix R if R has RING
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix R,vec: Vector R) if R has RING
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Integer,vec: Vector Integer)
    if and(has(R,LinearlyExplicitRingOver Integer),has(R,Ring))
reducedSystem : Matrix % -> Matrix Integer
    if and(has(R,LinearlyExplicitRingOver Integer),has(R,Ring))
```

These exports come from (p677) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed") if R has CHARNZ
```

These exports come from (p857) IntegralDomain():

```
associates? : (%,%) -> Boolean if R has INTDOM
exquo : (%,%) -> Union(%,"failed") if R has INTDOM
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if R has INTDOM
```

These exports come from (p1005) Field():

```
coerce : % -> % if R has INTDOM
divide : (%,%) -> Record(quotient: %,remainder: %)
    if R has INTDOM
euclideanSize : % -> NonNegativeInteger if R has INTDOM
expressIdealMember : (List %,%) -> Union(List %,"failed")
    if R has INTDOM
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
    if R has INTDOM
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
    if R has INTDOM
factor : % -> Factored % if R has INTDOM
gcd : (%,%) -> % if R has INTDOM
gcd : List % -> % if R has INTDOM
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
       if R has INTDOM
lcm : (%,%) -> % if R has INTDOM
lcm : List % -> % if R has INTDOM
multiEuclidean : (List %,%) -> Union(List %,"failed")
    if R has INTDOM
prime? : % -> Boolean if R has INTDOM
principalIdeal : List % -> Record(coef: List %,generator: %)
    if R has INTDOM
sizeLess? : (%,%) -> Boolean if R has INTDOM
squareFree : % -> Factored % if R has INTDOM
squareFreePart : % -> % if R has INTDOM
?quo? : (%,%) -> % if R has INTDOM
?rem? : (%,%) -> % if R has INTDOM
```

These exports come from (p44) RetractableTo(Integer):

```
retract : % -> Integer if R has RETRACT INT
retractIfCan : % -> Union(Integer,"failed")
    if R has RETRACT INT
```

These exports come from (p44) RetractableTo(Fraction(Integer)):

```
retract : % -> Fraction Integer
    if R has RETRACT INT
    and R has INTDOM
    or R has RETRACT FRAC INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if R has RETRACT INT
    and R has INTDOM
    or R has RETRACT FRAC INT
```

```
?*? : (%,Fraction Integer) -> % if R has INTDOM
?*? : (Fraction Integer,%) -> % if R has INTDOM
```

⟨*category FS FunctionSpace*⟩≡

```
)abbrev category FS FunctionSpace
++ Category for formal functions
++ Author: Manuel Bronstein
++ Date Created: 22 March 1988
++ Date Last Updated: 14 February 1994
++ Description:
++   A space of formal functions with arguments in an arbitrary
++   ordered set.
++ Keywords: operator, kernel, function.
FunctionSpace(R:OrderedSet): Category == Definition where
  OP ==> BasicOperator
  O  ==> OutputForm
  SY ==> Symbol
  N  ==> NonNegativeInteger
  Z  ==> Integer
  K  ==> Kernel %
  Q  ==> Fraction R
  PR ==> Polynomial R
  MP ==> SparseMultivariatePolynomial(R, K)
  QF==> PolynomialCategoryQuotientFunctions(IndexedExponents K,K,R,MP,%)

  ODD  ==> "odd"
  EVEN ==> "even"

  SPECIALDIFF  ==> "%specialDiff"
  SPECIALDISP  ==> "%specialDisp"
  SPECIALEQUAL ==> "%specialEqual"
  SPECIALINPUT ==> "%specialInput"

  Definition ==> Join(ExpressionSpace, RetractableTo SY, Patternable R,
                      FullyPatternMatchable R, FullyRetractableTo R) with
       ground?   : % -> Boolean
         ++ ground?(f) tests if f is an element of R.
       ground    : % -> R
         ++ ground(f) returns f as an element of R.
         ++ An error occurs if f is not an element of R.
       variables : %  -> List SY
         ++ variables(f) returns the list of all the variables of f.
       applyQuote: (SY, %) -> %
         ++ applyQuote(foo, x) returns \spad{'foo(x)}.
       applyQuote: (SY, %, %) -> %
         ++ applyQuote(foo, x, y) returns \spad{'foo(x,y)}.
```

```
applyQuote: (SY, %, %, %) -> %
  ++ applyQuote(foo, x, y, z) returns \spad{'foo(x,y,z)}.
applyQuote: (SY, %, %, %, %) -> %
  ++ applyQuote(foo, x, y, z, t) returns \spad{'foo(x,y,z,t)}.
applyQuote: (SY, List %) -> %
  ++ applyQuote(foo, [x1,...,xn]) returns \spad{'foo(x1,...,xn)}.
if R has ConvertibleTo InputForm then
  ConvertibleTo InputForm
  eval     : (%, SY) -> %
    ++ eval(f, foo) unquotes all the foo's in f.
  eval     : (%, List SY) -> %
    ++ eval(f, [foo1,...,foon]) unquotes all the \spad{fooi}'s in f.
  eval     : % -> %
    ++ eval(f) unquotes all the quoted operators in f.
  eval     : (%, OP, %, SY) -> %
    ++ eval(x, s, f, y) replaces every \spad{s(a)} in x by \spad{f(y)}
    ++ with \spad{y} replaced by \spad{a} for any \spad{a}.
  eval     : (%, List OP, List %, SY) -> %
    ++ eval(x, [s1,...,sm], [f1,...,fm], y) replaces every
    ++ \spad{si(a)} in x by \spad{fi(y)}
    ++ with \spad{y} replaced by \spad{a} for any \spad{a}.
if R has SemiGroup then
  Monoid
  -- the following line is necessary because of a compiler bug
  "**"   : (%, N) -> %
    ++ x**n returns x * x * x * ... * x (n times).
  isTimes: % -> Union(List %, "failed")
    ++ isTimes(p) returns \spad{[a1,...,an]}
    ++ if \spad{p = a1*...*an} and \spad{n > 1}.
  isExpt : % -> Union(Record(var:K,exponent:Z),"failed")
    ++ isExpt(p) returns \spad{[x, n]} if \spad{p = x**n}
    ++ and \spad{n <> 0}.
if R has Group then Group
if R has AbelianSemiGroup then
  AbelianMonoid
  isPlus: % -> Union(List %, "failed")
    ++ isPlus(p) returns \spad{[m1,...,mn]}
    ++ if \spad{p = m1 +...+ mn} and \spad{n > 1}.
  isMult: % -> Union(Record(coef:Z, var:K),"failed")
    ++ isMult(p) returns \spad{[n, x]} if \spad{p = n * x}
    ++ and \spad{n <> 0}.
if R has AbelianGroup then AbelianGroup
if R has Ring then
  Ring
  RetractableTo PR
  PartialDifferentialRing SY
```

```
  FullyLinearlyExplicitRingOver R
  coerce    : MP -> %
    ++ coerce(p) returns p as an element of %.
  numer     : %  -> MP
    ++ numer(f) returns the
    ++ numerator of f viewed as a polynomial in the kernels over R
    ++ if R is an integral domain. If not, then numer(f) = f viewed
    ++ as a polynomial in the kernels over R.
    -- DO NOT change this meaning of numer!  MB 1/90
  numerator : % -> %
    ++ numerator(f) returns the numerator of \spad{f} converted to %.
  isExpt:(%,OP) -> Union(Record(var:K,exponent:Z),"failed")
    ++ isExpt(p,op) returns \spad{[x, n]} if \spad{p = x**n}
    ++ and \spad{n <> 0} and \spad{x = op(a)}.
  isExpt:(%,SY) -> Union(Record(var:K,exponent:Z),"failed")
    ++ isExpt(p,f) returns \spad{[x, n]} if \spad{p = x**n}
    ++ and \spad{n <> 0} and \spad{x = f(a)}.
  isPower   : % -> Union(Record(val:%,exponent:Z),"failed")
    ++ isPower(p) returns \spad{[x, n]} if \spad{p = x**n}
    ++ and \spad{n <> 0}.
  eval: (%, List SY, List N, List(% -> %)) -> %
    ++ eval(x, [s1,...,sm], [n1,...,nm], [f1,...,fm]) replaces
    ++ every \spad{si(a)**ni} in x by \spad{fi(a)} for any \spad{a}.
  eval: (%, List SY, List N, List(List % -> %)) -> %
    ++ eval(x, [s1,...,sm], [n1,...,nm], [f1,...,fm]) replaces
    ++ every \spad{si(a1,...,an)**ni} in x by \spad{fi(a1,...,an)}
    ++ for any a1,...,am.
  eval: (%, SY, N, List % -> %) -> %
    ++ eval(x, s, n, f) replaces every \spad{s(a1,...,am)**n} in x
    ++ by \spad{f(a1,...,am)} for any a1,...,am.
  eval: (%, SY, N, % -> %) -> %
    ++ eval(x, s, n, f) replaces every \spad{s(a)**n} in x
    ++ by \spad{f(a)} for any \spad{a}.
if R has CharacteristicZero then CharacteristicZero
if R has CharacteristicNonZero then CharacteristicNonZero
if R has CommutativeRing then
  Algebra R
if R has IntegralDomain then
  Field
  RetractableTo Fraction PR
  convert   : Factored % -> %
    ++ convert(f1\^e1 ... fm\^em) returns \spad{(f1)\^e1 ... (fm)\^em}
    ++ as an element of %, using formal kernels
    ++ created using a \spadfunFrom{paren}{ExpressionSpace}.
  denom     : %  -> MP
    ++ denom(f) returns the denominator of f viewed as a
```

```
          ++ polynomial in the kernels over R.
        denominator : % -> %
          ++ denominator(f) returns the denominator of \spad{f}
          ++ converted to %.
        "/"       : (MP, MP) -> %
          ++ p1/p2 returns the quotient of p1 and p2 as an element of %.
        coerce    : Q  -> %
          ++ coerce(q) returns q as an element of %.
        coerce    : Polynomial Q -> %
          ++ coerce(p) returns p as an element of %.
        coerce    : Fraction Polynomial Q -> %
          ++ coerce(f) returns f as an element of %.
        univariate: (%, K) -> Fraction SparseUnivariatePolynomial %
          ++ univariate(f, k) returns f viewed as a univariate fraction in k.
        if R has RetractableTo Z then RetractableTo Fraction Z
      add
       import BasicOperatorFunctions1(%)

       -- these are needed in Ring only, but need to be declared here
       -- because of compiler bug: if they are declared inside the Ring
       -- case, then they are not visible inside the IntegralDomain case.
       smpIsMult : MP -> Union(Record(coef:Z, var:K),"failed")
       smpret    : MP -> Union(PR, "failed")
       smpeval   : (MP, List K, List %) -> %
       smpsubst  : (MP, List K, List %) -> %
       smpderiv  : (MP, SY) -> %
       smpunq    : (MP, List SY, Boolean) -> %
       kerderiv  : (K, SY)  -> %
       kderiv    : K -> List %
       opderiv   : (OP, N) -> List(List % -> %)
       smp2O     : MP -> O
       bestKernel: List K -> K
       worse?    : (K, K) -> Boolean
       diffArg   : (List %, OP, N) -> List %
       substArg  : (OP, List %, Z, %) -> %
       dispdiff  : List % -> Record(name:O, sub:O, arg:List O, level:N)
       ddiff     : List % -> O
       diffEval  : List % -> %
       dfeval    : (List %, K) -> %
       smprep    : (List SY, List N, List(List % -> %), MP) -> %
       diffdiff  : (List %, SY) -> %
       diffdiff0 : (List %, SY, %, K, List %) -> %
       subs      : (% -> %, K) -> %
       symsub    : (SY, Z) -> SY
       kunq      : (K, List SY, Boolean) -> %
       pushunq   : (List SY, List %) -> List %
```

```
notfound  : (K -> %, List K, K) -> %

equaldiff : (K,K)->Boolean
debugA: (List % ,List %,Boolean) -> Boolean
opdiff := operator("%diff"::SY)$CommonOperators()
opquote := operator("applyQuote"::SY)$CommonOperators

ground? x               == retractIfCan(x)@Union(R,"failed") case R

ground  x               == retract x

coerce(x:SY):%            == kernel(x)@K :: %

retract(x:%):SY           == symbolIfCan(retract(x)@K)::SY

applyQuote(s:SY, x:%)     == applyQuote(s, [x])

applyQuote(s, x, y)       == applyQuote(s, [x, y])

applyQuote(s, x, y, z)    == applyQuote(s, [x, y, z])

applyQuote(s, x, y, z, t) == applyQuote(s, [x, y, z, t])

applyQuote(s:SY, l:List %) == opquote concat(s::%, l)

belong? op                == op = opdiff or op = opquote

subs(fn, k) == kernel(operator k,[fn x for x in argument k]$List(%))

operator op ==
  is?(op, "%diff"::SY) => opdiff
  is?(op, "%quote"::SY) => opquote
  error "Unknown operator"

if R has ConvertibleTo InputForm then
  INP==>InputForm
  import MakeUnaryCompiledFunction(%, %, %)
  indiff: List % -> INP
  pint  : List INP-> INP
  differentiand: List % -> %

  differentiand l    == eval(first l, retract(second l)@K, third l)

  pint l  == convert concat(convert("D"::SY)@INP, l)

  indiff l ==
```

```
          r2:= convert([convert("::"::SY)@INP,_
                        convert(third l)@INP,_
                        convert("Symbol"::SY)@INP]@List INP)@INP
          pint [convert(differentiand l)@INP, r2]

        eval(f:%, s:SY)            == eval(f, [s])

        eval(f:%, s:OP, g:%, x:SY) == eval(f, [s], [g], x)

        eval(f:%, ls:List OP, lg:List %, x:SY) ==
          eval(f, ls, [compiledFunction(g, x) for g in lg])

        setProperty(opdiff,SPECIALINPUT,_
                    indiff@(List % -> InputForm) pretend None)

      variables x ==
        l := empty()$List(SY)
        for k in tower x repeat
          if ((s := symbolIfCan k) case SY) then l := concat(s::SY, l)
        reverse_! l

      retractIfCan(x:%):Union(SY, "failed") ==
        (k := retractIfCan(x)@Union(K,"failed")) case "failed" => "failed"
        symbolIfCan(k::K)

      if R has Ring then
        import UserDefinedPartialOrdering(SY)

    -- cannot use new()$Symbol because of possible re-instantiation
        gendiff := "%%0"::SY

        characteristic()    == characteristic()$R

        coerce(k:K):%       == k::MP::%

        symsub(sy, i)       == concat(string sy, convert(i)@String)::SY

        numerator x         == numer(x)::%

        eval(x:%, s:SY, n:N, f:% -> %) ==
          eval(x,[s],[n],[(y:List %):% +-> f(first(y))])

        eval(x:%, s:SY, n:N, f:List % -> %) == eval(x, [s], [n], [f])

        eval(x:%, l:List SY, f:List(List % -> %)) == eval(x, l, new(#l, 1), f)
```

```
elt(op:OP, args:List %) ==
  unary? op and ((od? := has?(op, ODD)) or has?(op, EVEN)) and
    leadingCoefficient(numer first args) < 0 =>
      x := op(- first args)
      od? => -x
      x
  elt(op, args)$ExpressionSpace_&(%)

eval(x:%, s:List SY, n:List N, l:List(% -> %)) ==
  eval(x, s, n, [y+-> f(first(y)) for f in l]$List(List % -> %))

-- op(arg)**m ==> func(arg)**(m quo n) * op(arg)**(m rem n)
smprep(lop, lexp, lfunc, p) ==
  (v := mainVariable p) case "failed" => p::%
  symbolIfCan(k := v::K) case SY => p::%
  g := (op := operator k)
     (arg := [eval(a,lop,lexp,lfunc) for a in argument k]$List(%))
  q := map(y+->eval(y::%, lop, lexp, lfunc),
           univariate(p, k))$SparseUnivariatePolynomialFunctions2(MP, %)
  (n := position(name op, lop)) < minIndex lop => q g
  a:%  := 0
  f    := eval((lfunc.n) arg, lop, lexp, lfunc)
  e    := lexp.n
  while q ^= 0 repeat
    m  := degree q
    qr := divide(m, e)
    t1 := f ** (qr.quotient)::N
    t2 := g ** (qr.remainder)::N
    a  := a + leadingCoefficient(q) * t1 * t2
    q  := reductum q
  a

dispdiff l ==
  s := second(l)::O
  t := third(l)::O
  a := argument(k := retract(first l)@K)
  is?(k, opdiff) =>
    rec := dispdiff a
    i   := position(s, rec.arg)
    rec.arg.i := t
    [rec.name,
       hconcat(rec.sub, hconcat(","::SY::O, (i+1-minIndex a)::O)),
                  rec.arg, (zero?(rec.level) => 0; rec.level + 1)]
  i   := position(second l, a)
  m   := [x::O for x in a]$List(O)
  m.i := t
```

```
    [name(operator k)::O, hconcat(","::SY::O, (i+1-minIndex a)::O),
                                          m, (empty? rest a => 1; 0)]

  ddiff l ==
    rec := dispdiff l
    opname :=
      zero?(rec.level) => sub(rec.name, rec.sub)
      differentiate(rec.name, rec.level)
    prefix(opname, rec.arg)

  substArg(op, l, i, g) ==
    z := copy l
    z.i := g
    kernel(op, z)


  diffdiff(l, x) ==
    f := kernel(opdiff, l)
    diffdiff0(l, x, f, retract(f)@K, empty())

  diffdiff0(l, x, expr, kd, done) ==
    op  := operator(k := retract(first l)@K)
    gg  := second l
    u   := third l
    arg := argument k
    ans:% := 0
    if (not member?(u,done)) and (ans := differentiate(u,x))^=0 then
      ans := ans * kernel(opdiff,
           [subst(expr, [kd], [kernel(opdiff, [first l, gg, gg])]),
                        gg, u])
    done := concat(gg, done)
    is?(k, opdiff) => ans + diffdiff0(arg, x, expr, k, done)
    for i in minIndex arg .. maxIndex arg for b in arg repeat
      if (not member?(b,done)) and (bp:=differentiate(b,x))^=0 then
        g   := symsub(gendiff, i)::%
        ans := ans + bp * kernel(opdiff, [subst(expr, [kd],
         [kernel(opdiff, [substArg(op, arg, i, g), gg, u])]), g, b])
    ans

  dfeval(l, g) ==
    eval(differentiate(first l, symbolIfCan(g)::SY), g, third l)

  diffEval l ==
    k:K
    g := retract(second l)@K
    ((u := retractIfCan(first l)@Union(K, "failed")) case "failed")
```

```
          or (u case K and symbolIfCan(k := u::K) case SY) => dfeval(l, g)
        op := operator k
        (ud := derivative op) case "failed" =>
             -- possible trouble
             -- make sure it is a dummy var
             dumm:%:=symsub(gendiff,1)::%
             ss:=subst(l.1,l.2=dumm)
             -- output(nl::OutputForm)$OutputPackage
             -- output("fixed"::OutputForm)$OutputPackage
             nl:=[ss,dumm,l.3]
             kernel(opdiff, nl)
        (n := position(second l,argument k)) < minIndex l =>
              dfeval(l,g)
        d := ud::List(List % -> %)
        eval((d.n)(argument k), g, third l)

      diffArg(l, op, i) ==
        n := i - 1 + minIndex l
        z := copy l
        z.n := g := symsub(gendiff, n)::%
        [kernel(op, z), g, l.n]

      opderiv(op, n) ==
--        one? n =>
        (n = 1) =>
          g := symsub(gendiff, n)::%
          [x +-> kernel(opdiff,[kernel(op, g), g, first x])]
        [y +-> kernel(opdiff, diffArg(y, op, i)) for i in 1..n]

      kderiv k ==
        zero?(n := #(args := argument k)) => empty()
        op := operator k
        grad :=
          (u := derivative op) case "failed" => opderiv(op, n)
          u::List(List % -> %)
        if #grad ^= n then grad := opderiv(op, n)
        [g args for g in grad]

    -- SPECIALDIFF contains a map (List %, Symbol) -> %
    -- it is used when the usual chain rule does not apply,
    -- for instance with implicit algebraics.
      kerderiv(k, x) ==
        (v := symbolIfCan(k)) case SY =>
          v::SY = x => 1
          0
        (fn := property(operator k, SPECIALDIFF)) case None =>
```

```
          ((fn::None) pretend ((List %, SY) -> %)) (argument k, x)
        +/[g * differentiate(y,x) for g in kderiv k for y in argument k]

      smpderiv(p, x) ==
        map((s:R):R +-> retract differentiate(s::PR, x), p)::% +
         +/[differentiate(p,k)::% * kerderiv(k, x) for k in variables p]

      coerce(p:PR):% ==
        map(s +-> s::%, r +-> r::%, p)$PolynomialCategoryLifting(
                                      IndexedExponents SY, SY, R, PR, %)

      worse?(k1, k2) ==
        (u := less?(name operator k1,name operator k2)) case "failed" =>
          k1 < k2
        u::Boolean

      bestKernel l ==
        empty? rest l => first l
        a := bestKernel rest l
        worse?(first l, a) => a
        first l

      smp20 p ==
        (r:=retractIfCan(p)@Union(R,"failed")) case R =>r::R::OutputForm
        a :=
          userOrdered?() => bestKernel variables p
          mainVariable(p)::K
        outputForm(map((x:MP):% +-> x::%, univariate(p, a))_
            $SparseUnivariatePolynomialFunctions2(MP, %), a::OutputForm)

      smpsubst(p, lk, lv) ==
        map(x +-> match(lk, lv, x,
            notfound((z:K):%+->subs(s+->subst(s, lk, lv), z), lk, x))_
             $ListToMap(K,%),y+->y::%,p)_
              $PolynomialCategoryLifting(IndexedExponents K,K,R,MP,%)

      smpeval(p, lk, lv) ==
        map(x +-> match(lk, lv, x,
            notfound((z:K):%+->map(s+->eval(s,lk,lv),z),lk,x))_
             $ListToMap(K,%),y+->y::%,p)_
              $PolynomialCategoryLifting(IndexedExponents K,K,R,MP,%)

  -- this is called on k when k is not a member of lk
      notfound(fn, lk, k) ==
        empty? setIntersection(tower(f := k::%), lk) => f
        fn k
```

```
      if R has ConvertibleTo InputForm then
        pushunq(l, arg) ==
           empty? l => [eval a for a in arg]
           [eval(a, l) for a in arg]

        kunq(k, l, givenlist?) ==
          givenlist? and empty? l => k::%
          is?(k, opquote) and
            (member?(s:=retract(first argument k)@SY, l) or empty? l) =>
              interpret(convert(concat(convert(s)@InputForm,
                [convert a for a in pushunq(l, rest argument k)
                   ]@List(InputForm)))@InputForm)$InputFormFunctions1(%)
          (operator k) pushunq(l, argument k)

        smpunq(p, l, givenlist?) ==
          givenlist? and empty? l => p::%
          map(x +-> kunq(x, l, givenlist?), y+->y::%, p)_
            $PolynomialCategoryLifting(IndexedExponents K,K,R,MP,%)

      smpret p ==
        "or"/[symbolIfCan(k) case "failed" for k in variables p] =>
          "failed"
        map(x+->symbolIfCan(x)::SY::PR, y+->y::PR,p)_
          $PolynomialCategoryLifting(IndexedExponents K, K, R, MP, PR)

      isExpt(x:%, op:OP) ==
        (u := isExpt x) case "failed" => "failed"
        is?((u::Record(var:K, exponent:Z)).var, op) => u
        "failed"

      isExpt(x:%, sy:SY) ==
        (u := isExpt x) case "failed" => "failed"
        is?((u::Record(var:K, exponent:Z)).var, sy) => u
        "failed"

      if R has RetractableTo Z then
          smpIsMult p ==
--          (u := mainVariable p) case K and one? degree(q:=univariate(p,u::K))
          (u := mainVariable p) case K and (degree(q:=univariate(p,u::K))=1)
            and zero?(leadingCoefficient reductum q)
              and ((r:=retractIfCan(leadingCoefficient q)@Union(R,"failed"))
                 case R)
                   and (n := retractIfCan(r::R)@Union(Z, "failed")) case Z =>
                     [n::Z, u::K]
          "failed"
```

```
evaluate(opdiff, diffEval)

debugA(a1,a2,t) ==
   -- uncomment for debugging
   -- output(hconcat [a1::OutputForm,_
   --                 a2::OutputForm,t::OutputForm])$OutputPackage
   t

equaldiff(k1,k2) ==
  a1:=argument k1
  a2:=argument k2
  -- check the operator
  res:=operator k1 = operator k2
  not res => debugA(a1,a2,res)
  -- check the evaluation point
  res:= (a1.3 = a2.3)
  not res => debugA(a1,a2,res)
  -- check all the arguments
  res:= (a1.1 = a2.1) and (a1.2 = a2.2)
  res => debugA(a1,a2,res)
  -- check the substituted arguments
  (subst(a1.1,[retract(a1.2)@K],[a2.2]) = a2.1) => debugA(a1,a2,true)
  debugA(a1,a2,false)

setProperty(opdiff,SPECIALEQUAL,
                    equaldiff@((K,K) -> Boolean) pretend None)

setProperty(opdiff, SPECIALDIFF,
                    diffdiff@((List %, SY) -> %) pretend None)

setProperty(opdiff, SPECIALDISP,
                        ddiff@(List % -> OutputForm) pretend None)

if not(R has IntegralDomain) then
  mainKernel x         == mainVariable numer x

  kernels x            == variables numer x

  retract(x:%):R       == retract numer x

  retract(x:%):PR      == smpret(numer x)::PR

  retractIfCan(x:%):Union(R,  "failed") == retract numer x

  retractIfCan(x:%):Union(PR, "failed") == smpret numer x
```

```
    eval(x:%, lk:List K, lv:List %)  == smpeval(numer x, lk, lv)

    subst(x:%, lk:List K, lv:List %) == smpsubst(numer x, lk, lv)

    differentiate(x:%, s:SY)         == smpderiv(numer x, s)

    coerce(x:%):OutputForm           == smp2O numer x

    if R has ConvertibleTo InputForm then
      eval(f:%, l:List SY) == smpunq(numer f, l, true)

      eval f               == smpunq(numer f, empty(), false)

    eval(x:%, s:List SY, n:List N, f:List(List % -> %)) ==
      smprep(s, n, f, numer x)

    isPlus x ==
      (u := isPlus numer x) case "failed" => "failed"
      [p::% for p in u::List(MP)]

    isTimes x ==
      (u := isTimes numer x) case "failed" => "failed"
      [p::% for p in u::List(MP)]

    isExpt x ==
      (u := isExpt numer x) case "failed" => "failed"
      r := u::Record(var:K, exponent:NonNegativeInteger)
      [r.var, r.exponent::Z]

    isPower x ==
      (u := isExpt numer x) case "failed" => "failed"
      r := u::Record(var:K, exponent:NonNegativeInteger)
      [r.var::%, r.exponent::Z]

    if R has ConvertibleTo Pattern Z then
      convert(x:%):Pattern(Z) == convert numer x

    if R has ConvertibleTo Pattern Float then
      convert(x:%):Pattern(Float) == convert numer x

    if R has RetractableTo Z then
      isMult x == smpIsMult numer x

  if R has CommutativeRing then
    r:R * x:% == r::MP::% * x
```

```
    if R has IntegralDomain then
      par   : % -> %

      mainKernel x                    == mainVariable(x)$QF

      kernels x                       == variables(x)$QF

      univariate(x:%, k:K)            == univariate(x, k)$QF

      isPlus x                        == isPlus(x)$QF

      isTimes x                       == isTimes(x)$QF

      isExpt x                        == isExpt(x)$QF

      isPower x                       == isPower(x)$QF

      denominator x                   == denom(x)::%

      coerce(q:Q):%                   == (numer q)::MP / (denom q)::MP

      coerce(q:Fraction PR):%         == (numer q)::% / (denom q)::%

      coerce(q:Fraction Polynomial Q) == (numer q)::% / (denom q)::%

      retract(x:%):PR                == retract(retract(x)@Fraction(PR))

      retract(x:%):Fraction(PR) == smpret(numer x)::PR / smpret(denom x)::PR

      retract(x:%):R == (retract(numer x)@R exquo retract(denom x)@R)::R

      coerce(x:%):OutputForm ==
--        one?(denom x) => smp2O numer x
        ((denom x) = 1) => smp2O numer x
        smp2O(numer x) / smp2O(denom x)

      retractIfCan(x:%):Union(R, "failed") ==
        (n := retractIfCan(numer x)@Union(R, "failed")) case "failed" or
          (d := retractIfCan(denom x)@Union(R, "failed")) case "failed"
            or (r := n::R exquo d::R) case "failed" => "failed"
        r::R

      eval(f:%, l:List SY) ==
        smpunq(numer f, l, true) / smpunq(denom f, l, true)
```

```
if R has ConvertibleTo InputForm then
  eval f ==
    smpunq(numer f, empty(), false) / smpunq(denom f, empty(), false)

  eval(x:%, s:List SY, n:List N, f:List(List % -> %)) ==
    smprep(s, n, f, numer x) / smprep(s, n, f, denom x)

differentiate(f:%, x:SY) ==
  (smpderiv(numer f, x) * denom(f)::% -
    numer(f)::% * smpderiv(denom f, x))
      / (denom(f)::% ** 2)

eval(x:%, lk:List K, lv:List %) ==
  smpeval(numer x, lk, lv) / smpeval(denom x, lk, lv)

subst(x:%, lk:List K, lv:List %) ==
  smpsubst(numer x, lk, lv) / smpsubst(denom x, lk, lv)

par x ==
  (r := retractIfCan(x)@Union(R, "failed")) case R => x
  paren x

convert(x:Factored %):% ==
  par(unit x) * */[par(f.factor) ** f.exponent for f in factors x]

retractIfCan(x:%):Union(PR, "failed") ==
  (u := retractIfCan(x)@Union(Fraction PR,"failed")) case "failed"
    => "failed"
  retractIfCan(u::Fraction(PR))

retractIfCan(x:%):Union(Fraction PR, "failed") ==
  (n := smpret numer x) case "failed" => "failed"
  (d := smpret denom x) case "failed" => "failed"
  n::PR / d::PR

coerce(p:Polynomial Q):% ==
  map(x+->x::%, y+->y::%,p)_
   $PolynomialCategoryLifting(IndexedExponents SY, SY,
                                              Q, Polynomial Q, %)

if R has RetractableTo Z then
  coerce(x:Fraction Z):% == numer(x)::MP / denom(x)::MP

  isMult x ==
     (u := smpIsMult numer x) case "failed"
        or (v := retractIfCan(denom x)@Union(R, "failed")) case "failed"
```

```
                or (w := retractIfCan(v::R)@Union(Z, "failed")) case "failed"
                    => "failed"
            r := u::Record(coef:Z, var:K)
            (q := r.coef exquo w::Z) case "failed" => "failed"
            [q::Z, r.var]

        if R has ConvertibleTo Pattern Z then
          convert(x:%):Pattern(Z) == convert(numer x) / convert(denom x)

        if R has ConvertibleTo Pattern Float then
          convert(x:%):Pattern(Float) ==
            convert(numer x) / convert(denom x)
```

⟨*FS.dotabb*⟩≡

```
  "FS"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FS"];
  "FS" -> "ES"
  "FS" -> "FPATMAB"
  "FS" -> "FRETRCT"
  "FS" -> "PATAB"
  "FS" -> "RETRACT"
  "FS" -> "KONVERT"
  "FS" -> "MONOID"
  "FS" -> "GROUP"
  "FS" -> "ABELMON"
  "FS" -> "ABELGRP"
  "FS" -> "PDRING"
  "FS" -> "FLINEXP"
  "FS" -> "CHARNZ"
  "FS" -> "INTDOM"
  "FS" -> "FIELD"
```

⟨*FS.dotfull*⟩≡
```
  "FunctionSpace(a:OrderedSet)"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FS"];
  "FunctionSpace(a:OrderedSet)" -> "ExpressionSpace()"
  "FunctionSpace(a:OrderedSet)" -> "RetractableTo(Symbol)"
  "FunctionSpace(a:OrderedSet)" -> "Patternable(OrderedSet)"
  "FunctionSpace(a:OrderedSet)" -> "FullyPatternMatchable(OrderedSet)"
  "FunctionSpace(a:OrderedSet)" -> "FullyRetractableTo(OrderedSet)"
  "FunctionSpace(a:OrderedSet)" -> "ConvertibleTo(InputForm)"
  "FunctionSpace(a:OrderedSet)" -> "Monoid()"
  "FunctionSpace(a:OrderedSet)" -> "Group()"
  "FunctionSpace(a:OrderedSet)" -> "AbelianMonoid()"
  "FunctionSpace(a:OrderedSet)" -> "AbelianGroup()"
  "FunctionSpace(a:OrderedSet)" -> "PartialDifferentialRing(Symbol)"
  "FunctionSpace(a:OrderedSet)" -> "FullyLinearlyExplicitRingOver(OrderedSet)"
  "FunctionSpace(a:OrderedSet)" -> "CharacteristicNonZero()"
  "FunctionSpace(a:OrderedSet)" -> "IntegralDomain()"
  "FunctionSpace(a:OrderedSet)" -> "Field()"
  "FunctionSpace(a:OrderedSet)" -> "RetractableTo(Integer)"
  "FunctionSpace(a:OrderedSet)" -> "RetractableTo(Fraction(Integer))"
```

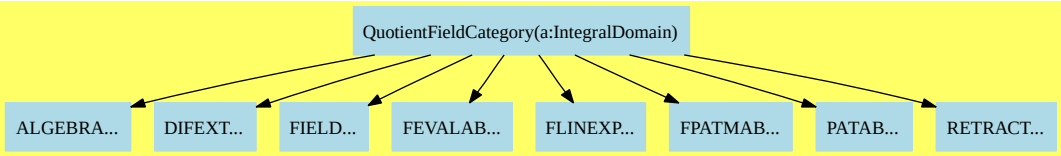
⟨*FS.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FunctionSpace(a:OrderedSet)" [color=lightblue];
"FunctionSpace(a:OrderedSet)" -> "ES..."
"FunctionSpace(a:OrderedSet)" -> "RETRACT..."
"FunctionSpace(a:OrderedSet)" -> "PATAB..."
"FunctionSpace(a:OrderedSet)" -> "FPATMAB..."
"FunctionSpace(a:OrderedSet)" -> "FRETRCT..."
"FunctionSpace(a:OrderedSet)" -> "KONVERT..."
"FunctionSpace(a:OrderedSet)" -> "MONOID..."
"FunctionSpace(a:OrderedSet)" -> "GROUP..."
"FunctionSpace(a:OrderedSet)" -> "ABELMON..."
"FunctionSpace(a:OrderedSet)" -> "ABELGRP..."
"FunctionSpace(a:OrderedSet)" -> "PDRING..."
"FunctionSpace(a:OrderedSet)" -> "FLINEXP..."
"FunctionSpace(a:OrderedSet)" -> "CHARNZ..."
"FunctionSpace(a:OrderedSet)" -> "INTDOM..."
"FunctionSpace(a:OrderedSet)" -> "FIELD..."
"FunctionSpace(a:OrderedSet)" -> "RETRACT..."

"ES..." [color=lightblue];
"EVALABLE..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"FPATMAB..." [color=lightblue];
"IEVALAB..." [color=lightblue];
"ORDSET..." [color=lightblue];
"PATAB..." [color=lightblue];
"RETRACT..." [color=lightblue];
"KONVERT..." [color=lightblue];
"MONOID..." [color=lightblue];
"GROUP..." [color=lightblue];
"ABELMON..." [color=lightblue];
"ABELGRP..." [color=lightblue];
"PDRING..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"CHARNZ..." [color=lightblue];
"INTDOM..." [color=lightblue];
"FIELD..." [color=lightblue];
"RETRACT..." [color=lightblue];
}
```

17.6 QuotientFieldCategory (QFCAT)



See:

⇐ “Algebra” (ALGEBRA) 11.1 on page 771
⇐ “CharacteristicNonZero” (CHARNZ) 10.2 on page 677
⇐ “CharacteristicZero” (CHARZ) 10.3 on page 681
⇐ “ConvertibleTo” (KONVERT) 2.8 on page 19
⇐ “DifferentialExtension” (DIFEXT) 11.2 on page 777
⇐ “EuclideanDomain” (EUCDOM) 15.1 on page 975
⇐ “Field” (FIELD) 16.1 on page 1005
⇐ “FullyEvalableOver” (FEVALAB) 4.7 on page 121
⇐ “FullyLinearlyExplicitRingOver” (FLINEXP) 11.3 on page 782
⇐ “FullyPatternMatchable” (FPATMAB) 3.7 on page 74
⇐ “OrderedIntegralDomain” (OINTDOM) 13.5 on page 937
⇐ “OrderedSet” (ORDSET) 4.19 on page 168
⇐ “Patternable” (PATAB) 2.15 on page 38
⇐ “PolynomialFactorizationExplicit” (PFECAT) 15.3 on page 990
⇐ “RealConstant” (REAL) 3.11 on page 85
⇐ “RetractableTo” (RETRACT) 2.18 on page 44
⇐ “StepThrough” (STEP) 4.26 on page 193

Exports:

0	1	abs
associates?	ceiling	characteristic
charthRoot	coerce	conditionP
convert	D	denom
denominator	differentiate	divide
euclideanSize	eval	expressIdealMember
exquo	extendedEuclidean	factor
factorPolynomial	factorSquareFreePolynomial	floor
fractionPart	gcd	gcdPolynomial
hash	init	inv
latex	lcm	map
max	min	multiEuclidean
negative?	nextItem	numer
numerator	one?	patternMatch
positive?	prime?	principalIdeal
random	recip	reducedSystem
retract	retractIfCan	sample
sign	sizeLess?	solveLinearPolynomialEquation
squareFree	squareFreePart	squareFreePolynomial
subtractIfCan	unit?	unitNormal
unitCanonical	wholePart	zero?
?.?	?*?	?**?
?+?	?-?	-?
?/?	?=?	?^?
?quo?	?rem?	?~=?
?<?	?<=?	?>?
?>=?		

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

- **nil**

These are directly exported but not implemented:

```
ceiling : % -> S if S has INS
denom : % -> S
floor : % -> S if S has INS
numer : % -> S
wholePart : % -> S if S has EUCDOM
?/? : (S,S) -> %
```

These are implemented by this category:

```
characteristic : () -> NonNegativeInteger
coerce : Symbol -> % if S has RETRACT SYMBOL
coerce : Fraction Integer -> %
convert : % -> InputForm if S has KONVERT INFORM
convert : % -> DoubleFloat if S has REAL
convert : % -> Float if S has REAL
convert : % -> Pattern Integer if S has KONVERT PATTERN INT
convert : % -> Pattern Float if S has KONVERT PATTERN FLOAT
denominator : % -> %
differentiate : (%,(S -> S)) -> %
fractionPart : % -> % if S has EUCDOM
init : () -> % if S has STEP
map : ((S -> S),%) -> %
nextItem : % -> Union(%,"failed") if S has STEP
numerator : % -> %
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
    PatternMatchResult(Float,%)
     if S has PATMAB FLOAT
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
    PatternMatchResult(Integer,%)
      if S has PATMAB INT
random : () -> % if S has INS
reducedSystem : Matrix % -> Matrix S
reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix S,vec: Vector S)
retract : % -> Symbol if S has RETRACT SYMBOL
retract : % -> Integer if S has RETRACT INT
retractIfCan : % -> Union(Integer,"failed") if S has RETRACT INT
retractIfCan : % -> Union(Symbol,"failed") if S has RETRACT SYMBOL
?<? : (%,%) -> Boolean if S has ORDSET
```

These exports come from (p1005) Field():

```
0 : () -> %
1 : () -> %
```

```
associates? : (%,%) -> Boolean
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
exquo : (%,%) -> Union(%,"failed")
factor : % -> Factored %
gcd : (%,%) -> %
gcd : List % -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
```

```
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?/? : (%,%) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p771) Algebra(S:IntegralDomain):

```
coerce : S -> %
?*? : (%,S) -> %
?*? : (S,%) -> %
```

These exports come from (p44) RetractableTo(S:IntegralDomain):

```
retract : % -> S
retractIfCan : % -> Union(S,"failed")
```

These exports come from (p121) FullyEvalableOver(S:IntegralDomain):

```
?.? : (%,S) -> % if S has ELTAB(S,S)
eval : (%,Equation S) -> % if S has EVALAB S
eval : (%,List Symbol,List S) -> % if S has IEVALAB(SYMBOL,S)
eval : (%,List Equation S) -> % if S has EVALAB S
eval : (%,S,S) -> % if S has EVALAB S
eval : (%,List S,List S) -> % if S has EVALAB S
eval : (%,Symbol,S) -> % if S has IEVALAB(SYMBOL,S)
```

These exports come from (p777) DifferentialExtension(S:IntegralDomain):

```
D : (%,(S -> S)) -> %
D : (%,(S -> S),NonNegativeInteger) -> %
D : % -> % if S has DIFRING
D : (%,NonNegativeInteger) -> % if S has DIFRING
D : (%,List Symbol,List NonNegativeInteger) -> %
    if S has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> %
    if S has PDRING SYMBOL
D : (%,List Symbol) -> % if S has PDRING SYMBOL
D : (%,Symbol) -> % if S has PDRING SYMBOL
differentiate : (%,List Symbol) -> %
    if S has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if S has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if S has PDRING SYMBOL
differentiate : (%,NonNegativeInteger) -> % if S has DIFRING
differentiate : % -> % if S has DIFRING
differentiate : (%,Symbol) -> % if S has PDRING SYMBOL
differentiate : (%,(S -> S),NonNegativeInteger) -> %
```

These exports come from (p782) FullyLinearlyExplicitRingOver(S:IntegralDomain):

```
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Integer,vec: Vector Integer)
    if S has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer if S has LINEXP INT
```

These exports come from (p44) RetractableTo(Fraction(Integer)):

```
retract : % -> Fraction Integer if S has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
  if S has RETRACT INT
```

These exports come from (p168) OrderedSet():

```
max : (%,%) -> % if S has ORDSET
min : (%,%) -> % if S has ORDSET
?<=? : (%,%) -> Boolean if S has ORDSET
?>? : (%,%) -> Boolean if S has ORDSET
?>=? : (%,%) -> Boolean if S has ORDSET
```

These exports come from (p937) OrderedIntegralDomain():

```
abs : % -> % if S has OINTDOM
negative? : % -> Boolean if S has OINTDOM
positive? : % -> Boolean if S has OINTDOM
sign : % -> Integer if S has OINTDOM
```

These exports come from (p677) CharacteristicNonZero():

```
charthRoot : % -> Union(%,"failed")
  if S has CHARNZ
  or and(has($,CharacteristicNonZero),
         has(S,PolynomialFactorizationExplicit))
```

These exports come from (p990) PolynomialFactorizationExplicit():

```
conditionP : Matrix % -> Union(Vector %,"failed")
  if and(has($,CharacteristicNonZero),
         has(S,PolynomialFactorizationExplicit))
factorPolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if S has PFECAT
factorSquareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if S has PFECAT
solveLinearPolynomialEquation :
```

```
   (List SparseUnivariatePolynomial %,
    SparseUnivariatePolynomial %) ->
      Union(List SparseUnivariatePolynomial %,"failed")
       if S has PFECAT
 squareFreePolynomial :
   SparseUnivariatePolynomial % ->
     Factored SparseUnivariatePolynomial %
       if S has PFECAT
```

⟨*category QFCAT QuotientFieldCategory*⟩≡

```
  )abbrev category QFCAT QuotientFieldCategory
  ++ Author:
  ++ Date Created:
  ++ Date Last Updated: 5th March 1996
  ++ Basic Functions: + - * / numer denom
  ++ Related Constructors:
  ++ Also See:
  ++ AMS Classifications:
  ++ Keywords:
  ++ References:
  ++ Description: QuotientField(S) is the
  ++ category of fractions of an Integral Domain S.
  QuotientFieldCategory(S: IntegralDomain): Category ==
    Join(Field, Algebra S, RetractableTo S, FullyEvalableOver S,
           DifferentialExtension S, FullyLinearlyExplicitRingOver S,
             Patternable S, FullyPatternMatchable S) with
      _/     : (S, S) -> %
         ++ d1 / d2 returns the fraction d1 divided by d2.
      numer  : % -> S
         ++ numer(x) returns the numerator of the fraction x.
      denom  : % -> S
         ++ denom(x) returns the denominator of the fraction x.
      numerator : % -> %
         ++ numerator(x) is the numerator of the fraction x converted to %.
      denominator : % -> %
         ++ denominator(x) is the denominator of the fraction x converted to %.
      if S has StepThrough then StepThrough
      if S has RetractableTo Integer then
               RetractableTo Integer
               RetractableTo Fraction Integer
      if S has OrderedSet then OrderedSet
      if S has OrderedIntegralDomain then OrderedIntegralDomain
      if S has RealConstant then RealConstant
      if S has ConvertibleTo InputForm then ConvertibleTo InputForm
      if S has CharacteristicZero then CharacteristicZero
      if S has CharacteristicNonZero then CharacteristicNonZero
      if S has RetractableTo Symbol then RetractableTo Symbol
```

```
    if S has EuclideanDomain then
      wholePart: % -> S
        ++ wholePart(x) returns the whole part of the fraction x
        ++ i.e. the truncated quotient of the numerator by the denominator.
      fractionPart: % -> %
        ++ fractionPart(x) returns the fractional part of x.
        ++ x = wholePart(x) + fractionPart(x)
    if S has IntegerNumberSystem then
      random: () -> %
        ++ random() returns a random fraction.
      ceiling : % -> S
        ++ ceiling(x) returns the smallest integral element above x.
      floor: % -> S
        ++ floor(x) returns the largest integral element below x.
    if S has PolynomialFactorizationExplicit then
      PolynomialFactorizationExplicit

  add
    import MatrixCommonDenominator(S, %)

    numerator(x) == numer(x)::%

    denominator(x) == denom(x) ::%

    if S has StepThrough then
       init() == init()$S / 1$S

       nextItem(n) ==
         m:= nextItem(numer(n))
         m case "failed" =>
           error "We seem to have a Fraction of a finite object"
         m / 1

    map(fn, x)                          == (fn numer x) / (fn denom x)

    reducedSystem(m:Matrix %):Matrix S == clearDenominator m

    characteristic()                    == characteristic()$S

    differentiate(x:%, deriv:S -> S) ==
        n := numer x
        d := denom x
        (deriv n * d - n * deriv d) / (d**2)

    if S has ConvertibleTo InputForm then
      convert(x:%):InputForm == (convert numer x) / (convert denom x)
```

```
if S has RealConstant then
  convert(x:%):Float == (convert numer x) / (convert denom x)

  convert(x:%):DoubleFloat == (convert numer x) / (convert denom x)

-- Note that being a Join(OrderedSet,IntegralDomain) is not the same
-- as being an OrderedIntegralDomain.
if S has OrderedIntegralDomain then
   if S has canonicalUnitNormal then
       x:% < y:% ==
         (numer x  * denom y) < (numer y * denom x)
     else
       x:% < y:% ==
         if denom(x) < 0 then (x,y):=(y,x)
         if denom(y) < 0 then (x,y):=(y,x)
         (numer x  * denom y) < (numer y * denom x)
else if S has OrderedSet then
   x:% < y:% ==
     (numer x  * denom y) < (numer y * denom x)

if (S has EuclideanDomain) then
  fractionPart x == x - (wholePart(x)::%)

if S has RetractableTo Symbol then
  coerce(s:Symbol):%  == s::S::%

  retract(x:%):Symbol == retract(retract(x)@S)

  retractIfCan(x:%):Union(Symbol, "failed") ==
    (r := retractIfCan(x)@Union(S,"failed")) case "failed" =>"failed"
    retractIfCan(r::S)

if (S has ConvertibleTo Pattern Integer) then
  convert(x:%):Pattern(Integer)==(convert numer x)/(convert denom x)

  if (S has PatternMatchable Integer) then
    patternMatch(x:%, p:Pattern Integer,
     l:PatternMatchResult(Integer, %)) ==
       patternMatch(x, p,
                 l)$PatternMatchQuotientFieldCategory(Integer, S, %)

if (S has ConvertibleTo Pattern Float) then
  convert(x:%):Pattern(Float) == (convert numer x)/(convert denom x)

  if (S has PatternMatchable Float) then
```

```
      patternMatch(x:%, p:Pattern Float,
       l:PatternMatchResult(Float, %)) ==
         patternMatch(x, p,
                      l)$PatternMatchQuotientFieldCategory(Float, S, %)

    if S has RetractableTo Integer then
      coerce(x:Fraction Integer):% == numer(x)::% / denom(x)::%

      if not(S is Integer) then
        retract(x:%):Integer == retract(retract(x)@S)

        retractIfCan(x:%):Union(Integer, "failed") ==
          (u := retractIfCan(x)@Union(S, "failed")) case "failed" =>
            "failed"
          retractIfCan(u::S)

    if S has IntegerNumberSystem then
      random():% ==
        while zero?(d:=random()$S) repeat d
        random()$S / d

    reducedSystem(m:Matrix %, v:Vector %):
      Record(mat:Matrix S, vec:Vector S) ==
        n := reducedSystem(horizConcat(v::Matrix(%), m))@Matrix(S)
        [subMatrix(n, minRowIndex n, maxRowIndex n, 1 + minColIndex n,
                                  maxColIndex n), column(n, minColIndex n)]
```

⟨*QFCAT.dotabb*⟩≡

```
"QFCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=QFCAT"];
"QFCAT" -> "ALGEBRA"
"QFCAT" -> "DIFEXT"
"QFCAT" -> "FIELD"
"QFCAT" -> "FEVALAB"
"QFCAT" -> "FLINEXP"
"QFCAT" -> "FPATMAB"
"QFCAT" -> "PATAB"
"QFCAT" -> "RETRACT"
```

⟨*QFCAT.dotfull*⟩≡
```
"QuotientFieldCategory(a:IntegralDomain)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=QFCAT"];
"QuotientFieldCategory(a:IntegralDomain)" -> "Field()"
"QuotientFieldCategory(a:IntegralDomain)" -> "Algebra(IntegralDomain)"
"QuotientFieldCategory(a:IntegralDomain)" -> "RetractableTo(IntegralDomain)"
"QuotientFieldCategory(a:IntegralDomain)" ->
  "FullyEvalableOver(IntegralDomain)"
"QuotientFieldCategory(a:IntegralDomain)" ->
  "DifferentialExtension(IntegralDomain)"
"QuotientFieldCategory(a:IntegralDomain)" ->
  "FullyLinearlyExplicitRingOver(IntegralDomain)"
"QuotientFieldCategory(a:IntegralDomain)" ->
  "Patternable(IntegralDomain)"
"QuotientFieldCategory(a:IntegralDomain)" ->
  "FullyPatternMatchable(IntegralDomain)"
```

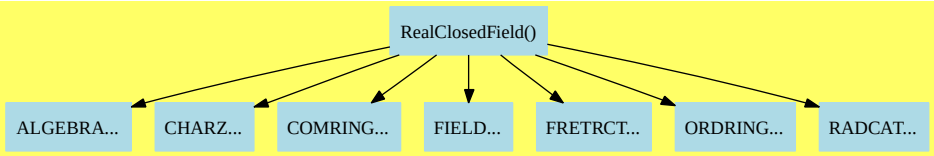
⟨*QFCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"QuotientFieldCategory(a:IntegralDomain)" [color=lightblue];
"QuotientFieldCategory(a:IntegralDomain)" -> "ALGEBRA..."
"QuotientFieldCategory(a:IntegralDomain)" -> "DIFEXT..."
"QuotientFieldCategory(a:IntegralDomain)" -> "FIELD..."
"QuotientFieldCategory(a:IntegralDomain)" -> "FEVALAB..."
"QuotientFieldCategory(a:IntegralDomain)" -> "FLINEXP..."
"QuotientFieldCategory(a:IntegralDomain)" -> "FPATMAB..."
"QuotientFieldCategory(a:IntegralDomain)" -> "PATAB..."
"QuotientFieldCategory(a:IntegralDomain)" -> "RETRACT..."

"ALGEBRA..." [color=lightblue];
"DIFEXT..." [color=lightblue];
"FIELD..." [color=lightblue];
"FEVALAB..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"FPATMAB..." [color=lightblue];
"PATAB..." [color=lightblue];
"RETRACT..." [color=lightblue];

}
```

17.7 RealClosedField (RCFIELD)



See:

⇐ "Algebra" (ALGEBRA) 11.1 on page 771
⇐ "CharacteristicZero" (CHARZ) 10.3 on page 681
⇐ "CommutativeRing" (COMRING) 10.4 on page 685
⇐ "Field" (FIELD) 16.1 on page 1005
⇐ "FullyRetractableTo" (FRETRCT) 3.6 on page 71
⇐ "OrderedRing" (ORDRING) 10.11 on page 715
⇐ "RadicalCategory" (RADCAT) 2.17 on page 42

Exports:

0	1	abs	allRootsOf
approximate	associates?	characteristic	coerce
divide	euclideanSize	expressIdealMember	exquo
extendedEuclidean	factor	gcd	gcdPolynomial
hash	inv	latex	lcm
mainDefiningPolynomial	mainForm	mainValue	max
min	multiEuclidean	negative?	nthRoot
one?	positive?	prime?	principalIdeal
recip	rename	rename!	retract
retractIfCan	rootOf	sample	sign
sizeLess?	sqrt	squareFree	squareFreePart
subtractIfCan	unit?	unitCanonical	unitNormal
zero?	?*?	?**?	?+?
?-?	-?	?/?	?<?
?<=?	?=?	?>?	?>=?
?^?	?~=?	?quo?	?rem?

Attributes Exported:

- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1)

has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.

- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.

These are directly exported but not implemented:

```
allRootsOf : SparseUnivariatePolynomial % -> List %
approximate : (%,%) -> Fraction Integer
mainDefiningPolynomial :
   % -> Union(SparseUnivariatePolynomial %,"failed")
mainForm : % -> Union(OutputForm,"failed")
mainValue : % -> Union(SparseUnivariatePolynomial %,"failed")
rename : (%,OutputForm) -> %
rename! : (%,OutputForm) -> %
```

These are implemented by this category:

```
allRootsOf : Polynomial Integer -> List %
allRootsOf : Polynomial Fraction Integer -> List %
allRootsOf : Polynomial % -> List %
allRootsOf : SparseUnivariatePolynomial Integer -> List %
allRootsOf : SparseUnivariatePolynomial Fraction Integer -> List %
characteristic : () -> NonNegativeInteger
nthRoot : (%,Integer) -> %
rootOf :
  (SparseUnivariatePolynomial %,PositiveInteger) ->
     Union(%,"failed")
rootOf :
  (SparseUnivariatePolynomial %,PositiveInteger,OutputForm) ->
     Union(%,"failed")
sqrt : (%,NonNegativeInteger) -> %
sqrt : Integer -> %
sqrt : Fraction Integer -> %
sqrt : % -> %
?**? : (%,Fraction Integer) -> %
```

These exports come from (p681) CharacteristicZero():

```
0 : () -> %
1 : () -> %
coerce : Integer -> %
coerce : % -> OutputForm
hash : % -> SingleInteger
latex : % -> String
```

```
one? : % -> Boolean
recip : % -> Union(%,"failed")
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
zero? : % -> Boolean
?~=? : (%,%) -> Boolean
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (%,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
```

These exports come from (p715) OrderedRing():

```
abs : % -> %
coerce : Integer -> %
max : (%,%) -> %
min : (%,%) -> %
negative? : % -> Boolean
positive? : % -> Boolean
sign : % -> Integer
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?*? : (Integer,%) -> %
```

These exports come from (p1005) Field():

```
associates? : (%,%) -> Boolean
coerce : % -> %
coerce : Fraction Integer -> %
coerce : Fraction Integer -> %
coerce : Fraction Integer -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
factor : % -> Factored %
gcd : (%,%) -> %
gcd : List % -> %
```

```
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
      SparseUnivariatePolynomial %
inv : % -> %
lcm : (%,%) -> %
lcm : List % -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
?/? : (%,%) -> %
?*? : (Fraction Integer,%) -> %
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p71) FullyRetractableTo(Fraction(Integer)):

```
retract : % -> Fraction Integer
retract : % -> Fraction Integer
  if Fraction Integer has RETRACT FRAC INT
retract : % -> Integer if Fraction Integer has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
retractIfCan : % -> Union(Fraction Integer,"failed")
  if Fraction Integer has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
  if Fraction Integer has RETRACT INT
```

These exports come from (p771) Algebra(Integer):

```
?*? : (%,Integer) -> %
```

⟨*category RCFIELD RealClosedField*⟩≡

```
)abbrev category RCFIELD RealClosedField
++ Author: Renaud Rioboo
++ Date Created: may 1993
++ Date Last Updated: January 2004
++ Basic Functions: provides computations with generic real roots of
++                  polynomials
```

```
++ Related Constructors: SimpleOrderedAlgebraicExtension, RealClosure
++ Also See:
++ AMS Classifications:
++ Keywords: Real Algebraic Numbers
++ References:
++ Description:
++ \axiomType{RealClosedField} provides common acces
++ functions for all real closed fields.
RealClosedField : Category == PUB where

    E ==> OutputForm
    SUP ==> SparseUnivariatePolynomial
    OFIELD ==> Join(OrderedRing,Field)
    PME ==> SUP($)
    N ==> NonNegativeInteger
    PI ==> PositiveInteger
    RN ==> Fraction(Integer)
    Z  ==> Integer
    POLY ==> Polynomial
    PACK ==> SparseUnivariatePolynomialFunctions2

    PUB == Join(CharacteristicZero,
                OrderedRing,
                CommutativeRing,
                Field,
                FullyRetractableTo(Fraction(Integer)),
                Algebra Integer,
                Algebra(Fraction(Integer)),
                RadicalCategory) with

        mainForm :   $ -> Union(E,"failed")
             ++ \axiom{mainForm(x)} is the main algebraic quantity name of
             ++ \axiom{x}

        mainDefiningPolynomial :   $ -> Union(PME,"failed")
             ++ \axiom{mainDefiningPolynomial(x)} is the defining
             ++ polynomial for the main algebraic quantity of \axiom{x}

        mainValue :   $ -> Union(PME,"failed")
             ++ \axiom{mainValue(x)} is the expression of \axiom{x} in terms
             ++ of \axiom{SparseUnivariatePolynomial($)}

        rootOf:          (PME,PI,E)           -> Union($,"failed")
             ++ \axiom{rootOf(pol,n,name)} creates the nth root for the order
             ++ of \axiom{pol} and names it \axiom{name}
```

```
rootOf:          (PME,PI)             -> Union($,"failed")
     ++ \axiom{rootOf(pol,n)} creates the nth root for the order
     ++ of \axiom{pol} and gives it unique name

allRootsOf:      PME                  ->  List $
     ++ \axiom{allRootsOf(pol)} creates all the roots
     ++ of \axiom{pol} naming each uniquely

allRootsOf:      (SUP(RN))            ->  List $
     ++ \axiom{allRootsOf(pol)} creates all the roots
     ++ of \axiom{pol} naming each uniquely

allRootsOf:      (SUP(Z))             ->  List $
     ++ \axiom{allRootsOf(pol)} creates all the roots
     ++ of \axiom{pol} naming each uniquely

allRootsOf:      (POLY($))            ->  List $
     ++ \axiom{allRootsOf(pol)} creates all the roots
     ++ of \axiom{pol} naming each uniquely

allRootsOf:      (POLY(RN))           ->  List $
     ++ \axiom{allRootsOf(pol)} creates all the roots
     ++ of \axiom{pol} naming each uniquely

allRootsOf:      (POLY(Z))            ->  List $
     ++ \axiom{allRootsOf(pol)} creates all the roots
     ++ of \axiom{pol} naming each uniquely

sqrt:            ($,N)                ->     $
     ++ \axiom{sqrt(x,n)} is \axiom{x ** (1/n)}

sqrt:             $                   ->     $
     ++ \axiom{sqrt(x)} is \axiom{x ** (1/2)}

sqrt:            RN                   ->     $
     ++ \axiom{sqrt(x)} is \axiom{x ** (1/2)}

sqrt:             Z                   ->     $
     ++ \axiom{sqrt(x)} is \axiom{x ** (1/2)}

rename! :        ($,E)                ->     $
     ++ \axiom{rename!(x,name)} changes the way \axiom{x} is printed

rename :         ($,E)                ->     $
     ++ \axiom{rename(x,name)} gives a new number that prints as name
```

```
    approximate:       ($,$) -> RN
          ++ \axiom{approximate(n,p)} gives an approximation of \axiom{n}
          ++ that has precision \axiom{p}

  add

    sqrt(a:$):$ == sqrt(a,2)

    sqrt(a:RN):$ == sqrt(a::$,2)

    sqrt(a:Z):$ == sqrt(a::$,2)

    characteristic() == 0

    rootOf(pol,n,o) ==
      r := rootOf(pol,n)
      r case "failed" => "failed"
      rename!(r,o)

    rootOf(pol,n) ==
      liste:List($):= allRootsOf(pol)
      # liste > n => "failed"
      liste.n


    sqrt(x,n) ==
      n = 0 => 1
      n = 1 => x
      zero?(x) => 0
      one?(x) => 1
      if odd?(n)
      then
        r := rootOf(monomial(1,n) - (x :: PME), 1)
      else
        r := rootOf(monomial(1,n) - (x :: PME), 2)
      r case "failed" => error "no roots"
      n = 2 => rename(r,root(x::E)$E)
      rename(r,root(x :: E, n :: E)$E)

    (x : $) ** (rn : RN) == sqrt(x**numer(rn),denom(rn)::N)

    nthRoot(x, n) ==
      zero?(n) => x
      negative?(n) => inv(sqrt(x,(-n) :: N))
      sqrt(x,n :: N)
```

```
allRootsOf(p:SUP(RN)) == allRootsOf(map(z +-> z::$ ,p)$PACK(RN,$))

allRootsOf(p:SUP(Z)) == allRootsOf(map(z +-> z::$ ,p)$PACK(Z,$))

allRootsOf(p:POLY($)) == allRootsOf(univariate(p))

allRootsOf(p:POLY(RN)) == allRootsOf(univariate(p))

allRootsOf(p:POLY(Z)) == allRootsOf(univariate(p))
```

⟨*RCFIELD.dotabb*⟩≡

```
"RCFIELD"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RCFIELD"];
"RCFIELD" -> "ALGEBRA"
"RCFIELD" -> "CHARZ"
"RCFIELD" -> "COMRING"
"RCFIELD" -> "FIELD"
"RCFIELD" -> "FRETRCT"
"RCFIELD" -> "ORDRING"
"RCFIELD" -> "RADCAT"
```

⟨*RCFIELD.dotfull*⟩≡

```
"RealClosedField()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RCFIELD"];
"RealClosedField()" -> "Algebra(Integer)"
"RealClosedField()" -> "Algebra(Fraction(Integer))"
"RealClosedField()" -> "CharacteristicZero()"
"RealClosedField()" -> "CommutativeRing()"
"RealClosedField()" -> "Field()"
"RealClosedField()" -> "FullyRetractableTo(Fraction(Integer))"
"RealClosedField()" -> "OrderedRing()"
"RealClosedField()" -> "RadicalCategory()"
```

⟨*RCFIELD.dotpic*⟩≡

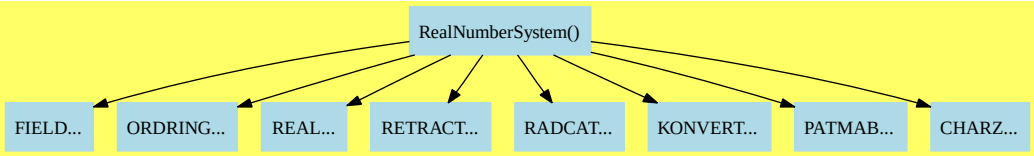
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"RealClosedField()" [color=lightblue];
"RealClosedField()" -> "ALGEBRA..."
"RealClosedField()" -> "CHARZ..."
"RealClosedField()" -> "COMRING..."
"RealClosedField()" -> "FIELD..."
"RealClosedField()" -> "FRETRCT..."
"RealClosedField()" -> "ORDRING..."
"RealClosedField()" -> "RADCAT..."

"ALGEBRA..." [color=lightblue];
"CHARZ..." [color=lightblue];
"COMRING..." [color=lightblue];
"FIELD..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"ORDRING..." [color=lightblue];
"RADCAT..." [color=lightblue];

}
```

17.8 RealNumberSystem (RNS)



See:

⇒ “FloatingPointSystem” (FPS) 18.4 on page 1254
⇐ “CharacteristicZero” (CHARZ) 10.3 on page 681
⇐ “ConvertibleTo” (KONVERT) 2.8 on page 19
⇐ “Field” (FIELD) 16.1 on page 1005
⇐ “OrderedRing” (ORDRING) 10.11 on page 715
⇐ “PatternMatchable” (PATMAB) 4.21 on page 176
⇐ “RadicalCategory” (RADCAT) 2.17 on page 42
⇐ “RealConstant” (REAL) 3.11 on page 85
⇐ “RetractableTo” (RETRACT) 2.18 on page 44

Exports:

0	1	abs	associates?
ceiling	characteristic	coerce	convert
divide	euclideanSize	expressIdealMember	exquo
extendedEuclidean	factor	floor	fractionPart
gcd	gcdPolynomial	hash	inv
latex	lcm	max	min
multiEuclidean	negative?	norm	nthRoot
one?	patternMatch	positive?	prime?
principalIdeal	recip	retract	retractIfCan
round	sample	sign	sizeLess?
sqrt	squareFree	squareFreePart	subtractIfCan
truncate	unit?	unitCanonical	unitNormal
wholePart	zero?	?*?	?**?
?+?	?-?	-?	?/?
?<?	?<=?	?=?	?>?
?>=?	?^?	?quo?	?rem?
?~=?			

These are directly exported but not implemented:

```
abs : % -> %
wholePart : % -> Integer
```

These are implemented by this category:

```
characteristic : () -> NonNegativeInteger
ceiling : % -> %
```

```
coerce : Fraction Integer -> %
convert : % -> Pattern Float
floor : % -> %
fractionPart : % -> %
norm : % -> %
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
    PatternMatchResult(Float,%)
round : % -> %
truncate : % -> %
```

These exports come from (p1005) Field():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
coerce : % -> %
coerce : Integer -> %
coerce : Integer -> %
coerce : Fraction Integer -> %
coerce : % -> OutputForm
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
exquo : (%,%) -> Union(%,"failed")
factor : % -> Factored %
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
      SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
```

```
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?**? : (%,Fraction Integer) -> %
?^? : (%,Integer) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?/? : (%,%) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p715) OrderedRing():

```
negative? : % -> Boolean
positive? : % -> Boolean
sign : % -> Integer
max : (%,%) -> %
min : (%,%) -> %
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
```

These exports come from (p85) RealConstant():

```
convert : % -> DoubleFloat
convert : % -> Float
```

These exports come from (p44) RetractableTo(Integer):

```
retract : % -> Integer
retractIfCan : % -> Union(Integer,"failed")
```

These exports come from (p44) RetractableTo(Fraction(Integer)):

```
retract : % -> Fraction Integer
retractIfCan : % -> Union(Fraction Integer,"failed")
```

These exports come from (p42) RadicalCategory():

```
nthRoot : (%,Integer) -> %
sqrt : % -> %
```

These exports come from (p19) ConvertibleTo(Pattern(Float)):

These exports come from (p176) PatternMatchable(Float):

These exports come from (p681) CharacteristicZero():

⟨*category RNS RealNumberSystem*⟩≡

```
)abbrev category RNS RealNumberSystem
++ Author: Michael Monagan and Stephen M. Watt
++ Date Created:
++   January 1988
++ Change History:
++ Basic Operations: abs, ceiling, wholePart, floor, fractionPart, norm, round, t
++ Related Constructors:
++ Keywords: real numbers
++ Description:
++ The real number system category is intended as a model for the real
++ numbers.  The real numbers form an ordered normed field.  Note that
++ we have purposely not included \spadtype{DifferentialRing} or
++ the elementary functions (see \spadtype{TranscendentalFunctionCategory})
++ in the definition.
RealNumberSystem(): Category ==
  Join(Field, OrderedRing, RealConstant, RetractableTo Integer,
       RetractableTo Fraction Integer, RadicalCategory,
        ConvertibleTo Pattern Float, PatternMatchable Float,
          CharacteristicZero) with
    norm : % -> %
      ++ norm x returns the same as absolute value.
    ceiling : % -> %
      ++ ceiling x returns the small integer \spad{>= x}.
    floor: % -> %
      ++ floor x returns the largest integer \spad{<= x}.
    wholePart  : % -> Integer
      ++ wholePart x returns the integer part of x.
    fractionPart : % -> %
      ++ fractionPart x returns the fractional part of x.
```

```
    truncate: % -> %
      ++ truncate x returns the integer between x and 0 closest to x.
    round: % -> %
      ++ round x computes the integer closest to x.
    abs  : % -> %
      ++ abs x returns the absolute value of x.
  add
    characteristic() == 0

    fractionPart x == x - truncate x

    truncate x == (negative? x => -floor(-x); floor x)

    round x == (negative? x => truncate(x-1/2::%); truncate(x+1/2::%))

    norm x == abs x

    coerce(x:Fraction Integer):% == numer(x)::% / denom(x)::%

    convert(x:%):Pattern(Float)  == convert(x)@Float :: Pattern(Float)

    floor x ==
       x1 := (wholePart x) :: %
       x = x1 => x
       x < 0 => (x1 - 1)
       x1

    ceiling x ==
       x1 := (wholePart x)::%
       x = x1 => x
       x >= 0 => (x1 + 1)
       x1

    patternMatch(x, p, l) ==
      generic? p => addMatch(p, x, l)
      constant? p =>
        (r := retractIfCan(p)@Union(Float, "failed")) case Float =>
          convert(x)@Float = r::Float => l
          failed()
        failed()
      failed()
```

⟨*RNS.dotabb*⟩≡
```
  "RNS"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=RNS"];
  "RNS" -> "FIELD"
  "RNS" -> "ORDRING"
  "RNS" -> "REAL"
  "RNS" -> "RETRACT"
  "RNS" -> "RADCAT"
  "RNS" -> "KONVERT"
  "RNS" -> "PATMAB"
  "RNS" -> "CHARZ"
```

⟨*RNS.dotfull*⟩≡
```
  "RealNumberSystem()"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=RNS"];
  "RealNumberSystem()" -> "Field()"
  "RealNumberSystem()" -> "OrderedRing()"
  "RealNumberSystem()" -> "RealConstant()"
  "RealNumberSystem()" -> "RetractableTo(Integer)"
  "RealNumberSystem()" -> "RetractableTo(Fraction(Integer))"
  "RealNumberSystem()" -> "RadicalCategory()"
  "RealNumberSystem()" -> "ConvertibleTo(Pattern(Float))"
  "RealNumberSystem()" -> "PatternMatchable(Float)"
  "RealNumberSystem()" -> "CharacteristicZero()"
```

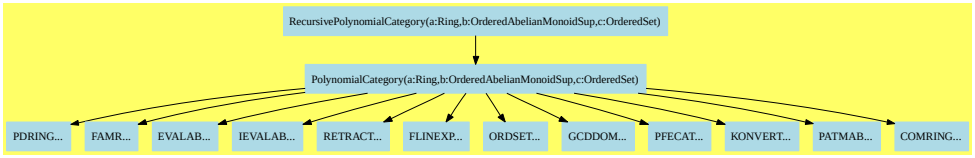
⟨*RNS.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"RealNumberSystem()" [color=lightblue];
"RealNumberSystem()" -> "FIELD..."
"RealNumberSystem()" -> "ORDRING..."
"RealNumberSystem()" -> "REAL..."
"RealNumberSystem()" -> "RETRACT..."
"RealNumberSystem()" -> "RADCAT..."
"RealNumberSystem()" -> "KONVERT..."
"RealNumberSystem()" -> "PATMAB..."
"RealNumberSystem()" -> "CHARZ..."

"FIELD..." [color=lightblue];
"ORDRING..." [color=lightblue];
"REAL..." [color=lightblue];
"RETRACT..." [color=lightblue];
"RADCAT..." [color=lightblue];
"KONVERT..." [color=lightblue];
"PATMAB..." [color=lightblue];
"CHARZ..." [color=lightblue];
}
```

17.9 RecursivePolynomialCategory (RPOLCAT)



See:

⇐ “PolynomialCategory” (POLYCAT) 16.4 on page 1027

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
conditionP	convert	D
deepestInitial	deepestTail	degree
differentiate	discriminant	eval
exactQuotient	exactQuotient!	exquo
extendedSubResultantGcd	factor	factorPolynomial
factorSquareFreePolynomial	gcd	gcdPolynomial
ground	ground?	halfExtendedSubResultantGcd1
halfExtendedSubResultantGcd2	hash	head
headReduce	headReduced?	infRittWu?
init	initiallyReduce	initiallyReduced?
isExpt	isPlus	isTimes
iteratedInitials	lastSubResultant	latex
LazardQuotient	LazardQuotient2	lazyPquo
lazyPrem	lazyPremWithDefault	lazyPseudoDivide
lazyResidueClass	lcm	leadingCoefficient
leadingMonomial	leastMonomial	mainCoefficients
mainContent	mainMonomial	mainPrimitivePart
mainSquareFreePart	mainVariable	map
mapExponents	max	mdeg
min	minimumDegree	monic?
monicDivide	monicModulo	monomial
monomial?	monomials	multivariate
mvar	nextsubResultant2	normalized?
numberOfMonomials	one?	patternMatch
pomopo!	pquo	prem
primPartElseUnitCanonical	primPartElseUnitCanonical!	prime?
primitiveMonomials	primitivePart	primitivePart!
pseudoDivide	quasiMonic?	recip
reduced?	reducedSystem	reductum
resultant	retract	retractIfCan
RittWuCompare	sample	solveLinearPolynomialEquation
squareFree	squareFreePart	squareFreePolynomial
subResultantChain	subResultantGcd	subtractIfCan
supRittWu?	tail	totalDegree
unit?	unitCanonical	unitNormal
univariate	variables	zero?
?*?	?**?	?+?
?-?	-?	?=?
?^?	?~=?	?/?
?<?	?<=?	?>?
?>=?		

Attributes Exported:

- if #1 has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if #1 has canonicalUnitNormal then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
exactQuotient! : (%,R) -> % if R has INTDOM
extendedSubResultantGcd : (%,%) -> Record(gcd: %,coef1: %,coef2: %)
  if R has INTDOM
halfExtendedSubResultantGcd1 : (%,%) -> Record(gcd: %,coef1: %)
  if R has INTDOM
halfExtendedSubResultantGcd2 : (%,%) -> Record(gcd: %,coef2: %)
  if R has INTDOM
lastSubResultant : (%,%) -> % if R has INTDOM
LazardQuotient : (%,%,NonNegativeInteger) -> %
  if R has INTDOM
LazardQuotient2 : (%,%,%,NonNegativeInteger) -> %
  if R has INTDOM
nextsubResultant2 : (%,%,%,%) -> % if R has INTDOM
resultant : (%,%) -> % if R has INTDOM
subResultantChain : (%,%) -> List % if R has INTDOM
subResultantGcd : (%,%) -> % if R has INTDOM
```

These are implemented by this category:

```
coerce : % -> OutputForm
coerce : % -> Polynomial R if V has KONVERT SYMBOL
convert : % -> String
  if R has RETRACT INT
  and V has KONVERT SYMBOL
convert : % -> Polynomial R if V has KONVERT SYMBOL
convert : Polynomial R -> % if V has KONVERT SYMBOL
convert : Polynomial Integer -> %
  if not has(R,Algebra Fraction Integer)
```

```
  and R has ALGEBRA INT
  and V has KONVERT SYMBOL
  or R has ALGEBRA FRAC INT
  and V has KONVERT SYMBOL
convert : Polynomial Fraction Integer -> %
  if R has ALGEBRA FRAC INT
  and V has KONVERT SYMBOL
deepestInitial : % -> %
deepestTail : % -> %
exactQuotient : (%,R) -> % if R has INTDOM
exactQuotient : (%,%) -> % if R has INTDOM
exactQuotient! : (%,%) -> % if R has INTDOM
gcd : (R,%) -> R if R has GCDDOM
head : % -> %
headReduce : (%,%) -> %
headReduced? : (%,List %) -> Boolean
headReduced? : (%,%) -> Boolean
infRittWu? : (%,%) -> Boolean
init : % -> %
initiallyReduce : (%,%) -> %
initiallyReduced? : (%,%) -> Boolean
initiallyReduced? : (%,List %) -> Boolean
iteratedInitials : % -> List %
lazyPremWithDefault : (%,%) ->
  Record(coef: %,gap: NonNegativeInteger,remainder: %)
lazyPremWithDefault : (%,%,V) ->
  Record(coef: %,gap: NonNegativeInteger,remainder: %)
lazyPquo : (%,%) -> %
lazyPquo : (%,%,V) -> %
lazyPrem : (%,%,V) -> %
lazyPrem : (%,%) -> %
lazyPseudoDivide : (%,%) ->
  Record(coef: %,gap: NonNegativeInteger,quotient: %,remainder: %)
lazyPseudoDivide : (%,%,V) ->
  Record(coef: %,gap: NonNegativeInteger,quotient: %,remainder: %)
lazyResidueClass : (%,%) ->
  Record(polnum: %,polden: %,power: NonNegativeInteger)
leadingCoefficient : (%,V) -> %
leastMonomial : % -> %
mainCoefficients : % -> List %
mainContent : % -> % if R has GCDDOM
mainMonomial : % -> %
mainMonomials : % -> List %
mainPrimitivePart : % -> % if R has GCDDOM
mainSquareFreePart : % -> % if R has GCDDOM
mdeg : % -> NonNegativeInteger
monic? : % -> Boolean
monicModulo : (%,%) -> %
mvar : % -> V
normalized? : (%,%) -> Boolean
```

```
normalized? : (%,List %) -> Boolean
pquo : (%,%) -> %
pquo : (%,%,V) -> %
prem : (%,%,V) -> %
prem : (%,%) -> %
primitivePart! : % -> % if R has GCDDOM
primPartElseUnitCanonical : % -> % if R has INTDOM
primPartElseUnitCanonical! : % -> % if R has INTDOM
pseudoDivide : (%,%) -> Record(quotient: %,remainder: %)
quasiMonic? : % -> Boolean
reduced? : (%,%) -> Boolean
reduced? : (%,List %) -> Boolean
reductum : (%,V) -> %
retract : Polynomial R -> %
  if not has(R,Algebra Fraction Integer)
  and not has(R,Algebra Integer)
  and V has KONVERT SYMBOL
  or not has(R,IntegerNumberSystem)
  and not has(R,Algebra Fraction Integer)
  and R has ALGEBRA INT
  and V has KONVERT SYMBOL
  or not has(R,QuotientFieldCategory Integer)
  and R has ALGEBRA FRAC INT
  and V has KONVERT SYMBOL
retract : Polynomial Integer -> %
  if not has(R,Algebra Fraction Integer)
  and R has ALGEBRA INT
  and V has KONVERT SYMBOL
  or R has ALGEBRA FRAC INT
  and V has KONVERT SYMBOL
retract : Polynomial Fraction Integer -> %
  if R has ALGEBRA FRAC INT and V has KONVERT SYMBOL
retractIfCan : Polynomial R -> Union(%,"failed")
  if not has(R,Algebra Fraction Integer)
  and not has(R,Algebra Integer)
  and V has KONVERT SYMBOL
  or not has(R,IntegerNumberSystem)
  and not has(R,Algebra Fraction Integer)
  and R has ALGEBRA INT
  and V has KONVERT SYMBOL
  or not has(R,QuotientFieldCategory Integer)
  and R has ALGEBRA FRAC INT
  and V has KONVERT SYMBOL
retractIfCan : Polynomial Fraction Integer -> Union(%,"failed")
  if R has ALGEBRA FRAC INT
  and V has KONVERT SYMBOL
retractIfCan : Polynomial Integer -> Union(%,"failed")
  if not has(R,Algebra Fraction Integer)
  and R has ALGEBRA INT
  and V has KONVERT SYMBOL
```

```
  or R has ALGEBRA FRAC INT
  and V has KONVERT SYMBOL
RittWuCompare : (%,%) -> Union(Boolean,"failed")
supRittWu? : (%,%) -> Boolean
tail : % -> %
```

These exports come from (p1027) PolynomialCategory(R,E,V)
where R:Ring, E:OrderedAbelianMonoidSup, V:OrderedSet:

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
  if R has INTDOM
binomThmExpt : (%,%,NonNegativeInteger) -> %
  if R has COMRING
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed")
  if and(has($,CharacteristicNonZero),
     has(R,PolynomialFactorizationExplicit))
  or R has CHARNZ
coefficient : (%,List V,List NonNegativeInteger) -> %
coefficient : (%,V,NonNegativeInteger) -> %
coefficient : (%,E) -> R
coefficients : % -> List R
coerce : R -> %
coerce : Fraction Integer -> %
  if R has RETRACT FRAC INT or R has ALGEBRA FRAC INT
coerce : V -> %
coerce : % -> % if R has INTDOM
coerce : Integer -> %
conditionP : Matrix % -> Union(Vector %,"failed")
  if and(has($,CharacteristicNonZero),
         has(R,PolynomialFactorizationExplicit))
content : % -> R if R has GCDDOM
content : (%,V) -> % if R has GCDDOM
convert : % -> Pattern Integer
  if V has KONVERT PATTERN INT and R has KONVERT PATTERN INT
convert : % -> Pattern Float
  if V has KONVERT PATTERN FLOAT and R has KONVERT PATTERN FLOAT
convert : % -> InputForm
  if V has KONVERT INFORM and R has KONVERT INFORM
D : (%,List V) -> %
D : (%,V) -> %
D : (%,List V,List NonNegativeInteger) -> %
D : (%,V,NonNegativeInteger) -> %
degree : % -> E
degree : (%,List V) -> List NonNegativeInteger
degree : (%,V) -> NonNegativeInteger
differentiate : (%,List V,List NonNegativeInteger) -> %
differentiate : (%,V,NonNegativeInteger) -> %
```

```
differentiate : (%,List V) -> %
differentiate : (%,V) -> %
discriminant : (%,V) -> % if R has COMRING
eval : (%,List Equation %) -> %
eval : (%,Equation %) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
eval : (%,V,R) -> %
eval : (%,List V,List R) -> %
eval : (%,V,%) -> %
eval : (%,List V,List %) -> %
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
exquo : (%,%) -> Union(%,"failed") if R has INTDOM
factor : % -> Factored % if R has PFECAT
factorPolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
     if R has PFECAT
factorSquareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
gcd : (%,%) -> % if R has GCDDOM
gcd : List % -> % if R has GCDDOM
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
       if R has GCDDOM
ground : % -> R
ground? : % -> Boolean
hash : % -> SingleInteger
isExpt : % -> Union(Record(var: V,exponent: NonNegativeInteger),"failed")
isPlus : % -> Union(List %,"failed")
isTimes : % -> Union(List %,"failed")
latex : % -> String
lcm : (%,%) -> % if R has GCDDOM
lcm : List % -> % if R has GCDDOM
leadingCoefficient : % -> R
leadingMonomial : % -> %
mainVariable : % -> Union(V,"failed")
map : ((R -> R),%) -> %
mapExponents : ((E -> E),%) -> %
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
minimumDegree : % -> E
minimumDegree : (%,List V) -> List NonNegativeInteger
minimumDegree : (%,V) -> NonNegativeInteger
monicDivide : (%,%,V) -> Record(quotient: %,remainder: %)
monomial : (%,V,NonNegativeInteger) -> %
```

```
monomial : (%,List V,List NonNegativeInteger) -> %
monomial : (R,E) -> %
monomial? : % -> Boolean
monomials : % -> List %
multivariate : (SparseUnivariatePolynomial %,V) -> %
multivariate : (SparseUnivariatePolynomial R,V) -> %
numberOfMonomials : % -> NonNegativeInteger
one? : % -> Boolean
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
    PatternMatchResult(Integer,%)
      if V has PATMAB INT and R has PATMAB INT
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
    PatternMatchResult(Float,%)
     if V has PATMAB FLOAT and R has PATMAB FLOAT
pomopo! : (%,R,E,%) -> %
prime? : % -> Boolean if R has PFECAT
primitiveMonomials : % -> List %
primitivePart : (%,V) -> % if R has GCDDOM
primitivePart : % -> % if R has GCDDOM
recip : % -> Union(%,"failed")
reducedSystem : Matrix % -> Matrix R
reducedSystem : (Matrix %,Vector %) ->
   Record(mat: Matrix R,vec: Vector R)
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Integer,vec: Vector Integer)
    if R has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer
  if R has LINEXP INT
reductum : % -> %
resultant : (%,%,V) -> % if R has COMRING
retract : % -> R
retract : % -> Integer if R has RETRACT INT
retract : % -> Fraction Integer
  if R has RETRACT FRAC INT
retract : % -> V
retractIfCan : % -> Union(R,"failed")
retractIfCan : % -> Union(Integer,"failed")
  if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
  if R has RETRACT FRAC INT
retractIfCan : % -> Union(V,"failed")
sample : () -> %
solveLinearPolynomialEquation :
  (List SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
    Union(List SparseUnivariatePolynomial %,"failed")
      if R has PFECAT
squareFree : % -> Factored % if R has GCDDOM
```

```
squareFreePart : % -> % if R has GCDDOM
squareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
subtractIfCan : (%,%) -> Union(%,"failed")
totalDegree : (%,List V) -> NonNegativeInteger
totalDegree : % -> NonNegativeInteger
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
  if R has INTDOM
univariate : % -> SparseUnivariatePolynomial R
univariate : (%,V) -> SparseUnivariatePolynomial %
variables : % -> List V
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Fraction Integer,%) -> % if R has ALGEBRA FRAC INT
?*? : (%,Fraction Integer) -> % if R has ALGEBRA FRAC INT
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?/? : (%,R) -> % if R has FIELD
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?<? : (%,%) -> Boolean if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
```

⟨*category RPOLCAT RecursivePolynomialCategory*⟩≡

```
)abbrev category RPOLCAT RecursivePolynomialCategory
++ Author: Marc Moreno Maza
++ Date Created: 04/22/1994
++ Date Last Updated: 14/12/1998
++ Basic Functions: mvar, mdeg, init, head, tail, prem, lazyPrem
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: polynomial, multivariate, ordered variables set
```

```
++ References:
++ Description:
++ A category for general multi-variate polynomials with coefficients
++ in a ring, variables in an ordered set, and exponents from an
++ ordered abelian monoid, with a \axiomOp{sup} operation.
++ When not constant, such a polynomial is viewed as a univariate polynomial
++ in its main variable w. r. t. to the total ordering on the elements in
++ the ordered set, so that some operations usually defined for univariate
++ polynomials make sense here.

RecursivePolynomialCategory(R:Ring, E:OrderedAbelianMonoidSup, V:OrderedSet):_
 Category ==
  PolynomialCategory(R, E, V) with
     mvar : $ -> V
         ++ \axiom{mvar(p)} returns an error if \axiom{p} belongs to
         ++ \axiom{R}, otherwise returns its main variable w. r. t. to the
         ++ total ordering on the elements in \axiom{V}.
     mdeg  : $ -> NonNegativeInteger
         ++ \axiom{mdeg(p)} returns an error if \axiom{p} is \axiom{0},
         ++ otherwise, if \axiom{p} belongs to \axiom{R} returns \axiom{0},
         ++ otherwise, returns the degree of \axiom{p} in its main variable.
     init : $ -> $
         ++ \axiom{init(p)} returns an error if \axiom{p} belongs to
         ++ \axiom{R}, otherwise returns its leading coefficient, where
         ++ \axiom{p} is viewed as a univariate polynomial in its main
         ++ variable.
     head  : $ -> $
         ++ \axiom{head(p)} returns \axiom{p} if \axiom{p} belongs to
         ++ \axiom{R}, otherwise returns its leading term (monomial in the
         ++ AXIOM sense), where \axiom{p} is viewed as a univariate polynomial
         ++  in its main variable.
     tail  : $ -> $
         ++ \axiom{tail(p)} returns its reductum, where \axiom{p} is viewed
         ++ as a univariate polynomial in its main variable.
     deepestTail : $ -> $
         ++ \axiom{deepestTail(p)} returns \axiom{0} if \axiom{p} belongs to
         ++ \axiom{R}, otherwise returns tail(p), if \axiom{tail(p)} belongs
         ++ to  \axiom{R} or \axiom{mvar(tail(p)) < mvar(p)}, otherwise
         ++ returns \axiom{deepestTail(tail(p))}.
     iteratedInitials : $ -> List $
         ++ \axiom{iteratedInitials(p)} returns \axiom{[]} if \axiom{p}
         ++ belongs to \axiom{R},
         ++ otherwise returns the list of the iterated initials of \axiom{p}.
     deepestInitial : $ -> $
         ++ \axiom{deepestInitial(p)} returns an error if \axiom{p} belongs
         ++ to \axiom{R},
```

```
    ++ otherwise returns the last term of \axiom{iteratedInitials(p)}.
leadingCoefficient : ($,V) -> $
    ++ \axiom{leadingCoefficient(p,v)} returns the leading coefficient
    ++ of \axiom{p}, where \axiom{p} is viewed as A univariate
    ++ polynomial in \axiom{v}.
reductum  : ($,V) -> $
    ++ \axiom{reductum(p,v)} returns the reductum of \axiom{p}, where
    ++ \axiom{p} is viewed as a univariate polynomial in \axiom{v}.
monic? : $ -> Boolean
    ++ \axiom{monic?(p)} returns false if \axiom{p} belongs to \axiom{R},
    ++ otherwise returns true iff \axiom{p} is monic as a univariate
    ++ polynomial in its main variable.
quasiMonic? : $ -> Boolean
    ++ \axiom{quasiMonic?(p)} returns false if \axiom{p} belongs to
    ++ \axiom{R}, otherwise returns true iff the initial of \axiom{p}
    ++ lies in the base ring \axiom{R}.
mainMonomial : $ -> $
    ++ \axiom{mainMonomial(p)} returns an error if \axiom{p} is
    ++ \axiom{0}, otherwise, if \axiom{p} belongs to \axiom{R} returns
    ++ \axiom{1}, otherwise, \axiom{mvar(p)} raised to the power
    ++ \axiom{mdeg(p)}.
leastMonomial : $ -> $
    ++ \axiom{leastMonomial(p)} returns an error if \axiom{p} is
    ++ \axiom{0}, otherwise, if \axiom{p} belongs to \axiom{R} returns
    ++ \axiom{1}, otherwise, the monomial of \axiom{p} with lowest
    ++ degree, where \axiom{p} is viewed as a univariate polynomial in
    ++ its main variable.
mainCoefficients : $ -> List $
    ++ \axiom{mainCoefficients(p)} returns an error if \axiom{p} is
    ++ \axiom{0}, otherwise, if \axiom{p} belongs to \axiom{R} returns
    ++ [p], otherwise returns the list of the coefficients of \axiom{p},
    ++ where \axiom{p} is viewed as a univariate polynomial in its main
    ++ variable.
mainMonomials : $ -> List $
    ++ \axiom{mainMonomials(p)} returns an error if \axiom{p} is
    ++ \axiom{0}, otherwise, if \axiom{p} belongs to \axiom{R} returns
    ++ [1], otherwise returns the list of the monomials of \axiom{p},
    ++ where \axiom{p} is viewed as a univariate polynomial in its main
    ++ variable.
RittWuCompare : ($, $) -> Union(Boolean,"failed")
    ++ \axiom{RittWuCompare(a,b)} returns \axiom{"failed"} if \axiom{a}
    ++ and \axiom{b} have same rank w.r.t.
    ++ Ritt and Wu Wen Tsun ordering using the refinement of Lazard,
    ++ otherwise returns \axiom{infRittWu?(a,b)}.
infRittWu?  : ($, $) -> Boolean
    ++ \axiom{infRittWu?(a,b)} returns true if \axiom{a} is less than
```

```
    ++ \axiom{b} w.r.t. the Ritt and Wu Wen Tsun ordering using the
    ++ refinement of Lazard.
supRittWu? : ($, $) -> Boolean
    ++ \axiom{supRittWu?(a,b)} returns true if \axiom{a} is greater
    ++ than \axiom{b} w.r.t. the Ritt and Wu Wen Tsun ordering using the
    ++ refinement of Lazard.
reduced? : ($,$) -> Boolean
    ++ \axiom{reduced?(a,b)} returns true iff
    ++ \axiom{degree(a,mvar(b)) < mdeg(b)}.
reduced? : ($,List($)) -> Boolean
    ++ \axiom{reduced?(q,lp)} returns true iff \axiom{reduced?(q,p)}
    ++ holds for every \axiom{p} in \axiom{lp}.
headReduced? : ($,$) -> Boolean
    ++ \axiom{headReduced?(a,b)} returns true iff
    ++ \axiom{degree(head(a),mvar(b)) < mdeg(b)}.
headReduced? : ($,List($)) -> Boolean
    ++ \axiom{headReduced?(q,lp)} returns true iff
    ++ \axiom{headReduced?(q,p)} holds for every \axiom{p} in \axiom{lp}.
initiallyReduced? : ($,$) -> Boolean
    ++ \axiom{initiallyReduced?(a,b)} returns false iff there exists an
    ++ iterated initial of \axiom{a} which is not reduced w.r.t \axiom{b}.
initiallyReduced? :  ($,List($)) -> Boolean
    ++ \axiom{initiallyReduced?(q,lp)} returns true iff
    ++ \axiom{initiallyReduced?(q,p)} holds for every \axiom{p} in
    ++ \axiom{lp}.
normalized? : ($,$) -> Boolean
    ++ \axiom{normalized?(a,b)} returns true iff \axiom{a} and its
    ++ iterated initials have degree zero w.r.t. the main variable of
    ++ \axiom{b}
normalized? : ($,List($)) -> Boolean
    ++ \axiom{normalized?(q,lp)} returns true iff
    ++ \axiom{normalized?(q,p)} holds
    ++ for every \axiom{p} in \axiom{lp}.
prem : ($, $) -> $
    ++ \axiom{prem(a,b)} computes the pseudo-remainder of \axiom{a} by
    ++ \axiom{b}, both viewed as univariate polynomials in the main
    ++ variable of \axiom{b}.
pquo : ($, $) -> $
    ++ \axiom{pquo(a,b)} computes the pseudo-quotient of \axiom{a} by
    ++ \axiom{b}, both viewed as univariate polynomials in the main
    ++ variable of \axiom{b}.
prem : ($, $, V) -> $
    ++ \axiom{prem(a,b,v)} computes the pseudo-remainder of \axiom{a}
    ++ by \axiom{b}, both viewed as univariate polynomials in \axiom{v}.
pquo : ($, $, V) -> $
    ++ \axiom{pquo(a,b,v)} computes the pseudo-quotient of \axiom{a} by
```

```
    ++ \axiom{b}, both viewed as univariate polynomials in \axiom{v}.
lazyPrem : ($, $) ->  $
    ++ \axiom{lazyPrem(a,b)} returns the polynomial \axiom{r} reduced
    ++ w.r.t. \axiom{b} and such that \axiom{b} divides
    ++ \axiom{init(b)^e a - r} where \axiom{e}
    ++ is the number of steps of this pseudo-division.
lazyPquo : ($, $) ->  $
    ++ \axiom{lazyPquo(a,b)} returns the polynomial \axiom{q} such that
    ++ \axiom{lazyPseudoDivide(a,b)} returns \axiom{[c,g,q,r]}.
lazyPrem : ($, $, V) -> $
    ++ \axiom{lazyPrem(a,b,v)} returns the polynomial \axiom{r}
    ++ reduced w.r.t. \axiom{b} viewed as univariate polynomials in the
    ++ variable \axiom{v} such that \axiom{b} divides
    ++ \axiom{init(b)^e a - r} where \axiom{e} is the number of steps of
    ++ this pseudo-division.
lazyPquo : ($, $, V) ->  $
    ++ \axiom{lazyPquo(a,b,v)} returns the polynomial \axiom{q} such that
    ++ \axiom{lazyPseudoDivide(a,b,v)} returns \axiom{[c,g,q,r]}.
lazyPremWithDefault : ($, $) -> _
  Record (coef : $, gap : NonNegativeInteger, remainder : $)
    ++ \axiom{lazyPremWithDefault(a,b)} returns \axiom{[c,g,r]}
    ++ such that \axiom{r = lazyPrem(a,b)} and
    ++ \axiom{(c**g)*r = prem(a,b)}.
lazyPremWithDefault : ($, $, V) -> _
  Record (coef : $, gap : NonNegativeInteger, remainder : $)
    ++ \axiom{lazyPremWithDefault(a,b,v)} returns \axiom{[c,g,r]}
    ++ such that \axiom{r = lazyPrem(a,b,v)} and
    ++ \axiom{(c**g)*r = prem(a,b,v)}.
lazyPseudoDivide : ($,$) -> _
  Record(coef:$, gap: NonNegativeInteger,quotient:$, remainder:$)
    ++ \axiom{lazyPseudoDivide(a,b)} returns \axiom{[c,g,q,r]}
    ++ such that \axiom{[c,g,r] = lazyPremWithDefault(a,b)} and
    ++ \axiom{q} is the pseudo-quotient computed in this lazy
    ++ pseudo-division.
lazyPseudoDivide : ($,$,V) -> _
  Record(coef:$, gap:NonNegativeInteger, quotient:$, remainder: $)
    ++ \axiom{lazyPseudoDivide(a,b,v)} returns \axiom{[c,g,q,r]} such
    ++ that  \axiom{r = lazyPrem(a,b,v)}, \axiom{(c**g)*r = prem(a,b,v)}
    ++ and \axiom{q} is the pseudo-quotient computed in this lazy
    ++ pseudo-division.
pseudoDivide : ($, $) -> Record (quotient : $, remainder : $)
    ++ \axiom{pseudoDivide(a,b)} computes \axiom{[pquo(a,b),prem(a,b)]},
    ++ both polynomials viewed as univariate polynomials in the main
    ++ variable of \axiom{b}, if \axiom{b} is not a constant polynomial.
monicModulo : ($, $) -> $
    ++ \axiom{monicModulo(a,b)} computes \axiom{a mod b}, if \axiom{b} is
```

```
    ++ monic as univariate polynomial in its main variable.
lazyResidueClass : ($,$) -> _
 Record(polnum:$, polden:$, power:NonNegativeInteger)
    ++ \axiom{lazyResidueClass(a,b)} returns \axiom{[p,q,n]} where
    ++ \axiom{p / q**n} represents the residue class of \axiom{a}
    ++ modulo \axiom{b} and \axiom{p} is reduced w.r.t. \axiom{b} and
    ++ \axiom{q} is \axiom{init(b)}.
headReduce: ($, $) ->  $
    ++ \axiom{headReduce(a,b)} returns a polynomial \axiom{r} such that
    ++ \axiom{headReduced?(r,b)} holds and there exists an integer
    ++ \axiom{e} such that \axiom{init(b)^e a - r} is zero modulo
    ++ \axiom{b}.
initiallyReduce: ($, $) ->  $
    ++ \axiom{initiallyReduce(a,b)} returns a polynomial \axiom{r} such
    ++ that \axiom{initiallyReduced?(r,b)} holds and there exists an
    ++ integer \axiom{e} such that \axiom{init(b)^e a - r} is zero
    ++ modulo \axiom{b}.

if (V has ConvertibleTo(Symbol))
then
  CoercibleTo(Polynomial R)
  ConvertibleTo(Polynomial R)
  if R has Algebra Fraction Integer
    then
      retractIfCan : Polynomial Fraction Integer -> Union($,"failed")
          ++ \axiom{retractIfCan(p)} returns \axiom{p} as an element of
          ++ the current domain if all its variables belong to \axiom{V}.
      retract : Polynomial Fraction Integer -> $
          ++ \axiom{retract(p)} returns \axiom{p} as an element of the
          ++ current domain if \axiom{retractIfCan(p)} does not return
          ++ "failed", otherwise an error is produced.
      convert : Polynomial Fraction Integer -> $
          ++ \axiom{convert(p)} returns the same as \axiom{retract(p)}.
      retractIfCan : Polynomial Integer -> Union($,"failed")
          ++ \axiom{retractIfCan(p)} returns \axiom{p} as an element of
          ++ the current domain if all its variables belong to \axiom{V}.
      retract : Polynomial Integer -> $
          ++ \axiom{retract(p)} returns \axiom{p} as an element of the
          ++ current domain if \axiom{retractIfCan(p)} does not return
          ++ "failed", otherwise an error is produced.
      convert : Polynomial Integer -> $
          ++ \axiom{convert(p)} returns the same as \axiom{retract(p)}
      if not (R has QuotientFieldCategory(Integer))
        then
          retractIfCan : Polynomial R -> Union($,"failed")
            ++ \axiom{retractIfCan(p)} returns \axiom{p} as an element
```

```
            ++ of the current domain if all its variables belong to
            ++ \axiom{V}.
          retract : Polynomial R -> $
            ++ \axiom{retract(p)} returns \axiom{p} as an element of the
            ++ current domain if \axiom{retractIfCan(p)} does not
            ++ return "failed", otherwise an error is produced.
      if (R has Algebra Integer) and not(R has Algebra Fraction Integer)
        then
          retractIfCan : Polynomial Integer -> Union($,"failed")
              ++ \axiom{retractIfCan(p)} returns \axiom{p} as an element of
              ++ the current domain if all its variables belong to \axiom{V}.
          retract : Polynomial Integer -> $
              ++ \axiom{retract(p)} returns \axiom{p} as an element of the
              ++ current domain if \axiom{retractIfCan(p)} does not return
              ++ "failed", otherwise an error is produced.
          convert : Polynomial Integer -> $
              ++ \axiom{convert(p)} returns the same as \axiom{retract(p)}.
          if not (R has IntegerNumberSystem)
            then
              retractIfCan : Polynomial R -> Union($,"failed")
                ++ \axiom{retractIfCan(p)} returns \axiom{p} as an element
                ++ of the current domain if all its variables belong to
                ++ \axiom{V}.
              retract : Polynomial R -> $
                ++ \axiom{retract(p)} returns \axiom{p} as an element of the
                ++ current domain if \axiom{retractIfCan(p)} does not
                ++ return "failed", otherwise an error is produced.
      if not(R has Algebra Integer) and not(R has Algebra Fraction Integer)
        then
          retractIfCan : Polynomial R -> Union($,"failed")
            ++ \axiom{retractIfCan(p)} returns \axiom{p} as an element of
            ++ the current domain if all its variables belong to \axiom{V}.
          retract : Polynomial R -> $
            ++ \axiom{retract(p)} returns \axiom{p} as an element of the
            ++ current domain if \axiom{retractIfCan(p)} does not return
            ++ "failed", otherwise an error is produced.
      convert : Polynomial R -> $
        ++ \axiom{convert(p)} returns \axiom{p} as an element of the current
        ++ domain if all its variables belong to \axiom{V}, otherwise an
        ++ error is produced.

      if R has RetractableTo(Integer)
      then
        ConvertibleTo(String)

    if R has IntegralDomain
```

```
then
  primPartElseUnitCanonical : $ -> $
      ++ \axiom{primPartElseUnitCanonical(p)} returns
      ++ \axiom{primitivePart(p)} if \axiom{R} is a gcd-domain,
      ++ otherwise \axiom{unitCanonical(p)}.
  primPartElseUnitCanonical! : $ -> $
      ++ \axiom{primPartElseUnitCanonical!(p)} replaces  \axiom{p}
      ++ by \axiom{primPartElseUnitCanonical(p)}.
  exactQuotient : ($,R) -> $
      ++ \axiom{exactQuotient(p,r)} computes the exact quotient of
      ++ \axiom{p} by \axiom{r}, which is assumed to be a divisor of
      ++ \axiom{p}. No error is returned if this exact quotient fails!
  exactQuotient! : ($,R) -> $
      ++ \axiom{exactQuotient!(p,r)} replaces \axiom{p} by
      ++ \axiom{exactQuotient(p,r)}.
  exactQuotient : ($,$) -> $
      ++ \axiom{exactQuotient(a,b)} computes the exact quotient of
      ++ \axiom{a} by \axiom{b}, which is assumed to be a divisor of
      ++ \axiom{a}. No error is returned if this exact quotient fails!
  exactQuotient! : ($,$) -> $
      ++ \axiom{exactQuotient!(a,b)} replaces \axiom{a} by
      ++ \axiom{exactQuotient(a,b)}
  subResultantGcd : ($, $) -> $
      ++ \axiom{subResultantGcd(a,b)} computes a gcd of \axiom{a} and
      ++ \axiom{b} where \axiom{a} and \axiom{b} are assumed to have the
      ++ same main variable \axiom{v} and are viewed as univariate
      ++ polynomials in \axiom{v} with coefficients in the fraction
      ++ field of the polynomial ring generated by their other variables
      ++ over \axiom{R}.
  extendedSubResultantGcd : ($, $) -> _
   Record (gcd : $, coef1 : $, coef2 : $)
      ++ \axiom{extendedSubResultantGcd(a,b)} returns \axiom{[ca,cb,r]}
      ++ such that \axiom{r} is \axiom{subResultantGcd(a,b)} and we have
      ++ \axiom{ca * a + cb * cb = r} .
  halfExtendedSubResultantGcd1: ($, $) -> Record (gcd : $, coef1 : $)
      ++ \axiom{halfExtendedSubResultantGcd1(a,b)} returns \axiom{[g,ca]}
      ++ if \axiom{extendedSubResultantGcd(a,b)} returns \axiom{[g,ca,cb]}
      ++ otherwise produces an error.
  halfExtendedSubResultantGcd2: ($, $) -> Record (gcd : $, coef2 : $)
      ++ \axiom{halfExtendedSubResultantGcd2(a,b)} returns \axiom{[g,cb]}
      ++ if \axiom{extendedSubResultantGcd(a,b)} returns \axiom{[g,ca,cb]}
      ++ otherwise produces an error.
  resultant  : ($, $) -> $
      ++ \axiom{resultant(a,b)} computes the resultant of \axiom{a} and
      ++ \axiom{b} where \axiom{a} and \axiom{b} are assumed to have the
      ++ same main variable \axiom{v} and are viewed as univariate
```

```
        ++ polynomials in \axiom{v}.
    subResultantChain : ($, $) -> List $
        ++ \axiom{subResultantChain(a,b)}, where \axiom{a} and \axiom{b}
        ++ are not contant polynomials with the same main variable, returns
        ++ the subresultant chain of \axiom{a} and \axiom{b}.
    lastSubResultant: ($, $) -> $
        ++ \axiom{lastSubResultant(a,b)} returns the last non-zero
        ++ subresultant of \axiom{a} and \axiom{b} where \axiom{a} and
        ++ \axiom{b} are assumed to have the same main variable \axiom{v}
        ++ and are viewed as univariate polynomials in \axiom{v}.
    LazardQuotient: ($, $, NonNegativeInteger) -> $
        ++ \axiom{LazardQuotient(a,b,n)} returns \axiom{a**n exquo b**(n-1)}
        ++ assuming that this quotient does not fail.
    LazardQuotient2: ($, $, $, NonNegativeInteger) -> $
        ++ \axiom{LazardQuotient2(p,a,b,n)} returns
        ++ \axiom{(a**(n-1) * p) exquo b**(n-1)}
        ++ assuming that this quotient does not fail.
    next_subResultant2: ($, $, $, $) -> $
        ++ \axiom{nextsubResultant2(p,q,z,s)} is the multivariate version
        ++ of the operation
        ++ \axiomOpFrom{next_sousResultant2}{PseudoRemainderSequence} from
        ++ the \axiomType{PseudoRemainderSequence} constructor.

  if R has GcdDomain
  then
    gcd : (R,$) -> R
        ++ \axiom{gcd(r,p)} returns the gcd of \axiom{r} and the content
        ++ of \axiom{p}.
    primitivePart! : $ -> $
        ++ \axiom{primitivePart!(p)} replaces \axiom{p}  by its primitive
        ++ part.
    mainContent : $ -> $
        ++ \axiom{mainContent(p)} returns the content of \axiom{p} viewed
        ++ as a univariate polynomial in its main variable and with
        ++ coefficients in the polynomial ring generated by its other
        ++ variables over \axiom{R}.
    mainPrimitivePart : $ -> $
        ++ \axiom{mainPrimitivePart(p)} returns the primitive part of
        ++ \axiom{p} viewed as a univariate polynomial in its main
        ++ variable and with coefficients in the polynomial ring generated
        ++ by its other variables over \axiom{R}.
    mainSquareFreePart : $ -> $
        ++ \axiom{mainSquareFreePart(p)} returns the square free part of
        ++ \axiom{p} viewed as a univariate polynomial in its main
        ++ variable and with coefficients in the polynomial ring
        ++ generated by its other variables over \axiom{R}.
```

```
 add
     O ==> OutputForm
     NNI ==> NonNegativeInteger
     INT ==> Integer

     exactQuo : (R,R) -> R

     coerce(p:$):O ==
       ground? (p) => (ground(p))::O
--       if one?((ip := init(p)))
       if (((ip := init(p))) = 1)
         then
           if zero?((tp := tail(p)))
             then
--               if one?((dp := mdeg(p)))
               if (((dp := mdeg(p))) = 1)
                 then
                   return((mvar(p))::O)
                 else
                   return(((mvar(p))::O **$O (dp::O)))
             else
--               if one?((dp := mdeg(p)))
               if (((dp := mdeg(p))) = 1)
                 then
                   return((mvar(p))::O +$O (tp::O))
                 else
                   return(((mvar(p))::O **$O (dp::O)) +$O (tp::O))
          else
           if zero?((tp := tail(p)))
             then
--               if one?((dp := mdeg(p)))
               if (((dp := mdeg(p))) = 1)
                 then
                   return((ip::O) *$O  (mvar(p))::O)
                 else
                   return((ip::O) *$O ((mvar(p))::O **$O (dp::O)))
             else
--               if one?(mdeg(p))
               if ((mdeg(p)) = 1)
                 then
                   return(((ip::O) *$O  (mvar(p))::O) +$O (tp::O))
       ((ip)::O *$O ((mvar(p))::O **$O ((mdeg(p)::O))) +$O (tail(p)::O))

     mvar p ==
       ground?(p) => error"Error in mvar from RPOLCAT : #1 is constant."
```

```
  mainVariable(p)::V

mdeg p ==
  ground?(p) => 0$NNI
  degree(p,mainVariable(p)::V)

init p ==
  ground?(p) => error"Error in mvar from RPOLCAT : #1 is constant."
  v := mainVariable(p)::V
  coefficient(p,v,degree(p,v))

leadingCoefficient (p,v) ==
  zero? (d := degree(p,v)) => p
  coefficient(p,v,d)

head p ==
  ground? p => p
  v := mainVariable(p)::V
  d := degree(p,v)
  monomial(coefficient(p,v,d),v,d)

reductum(p,v) ==
  zero? (d := degree(p,v)) => 0$$
  p - monomial(coefficient(p,v,d),v,d)

tail p ==
  ground? p => 0$$
  p - head(p)

deepestTail p ==
  ground? p => 0$$
  ground? tail(p) => tail(p)
  mvar(p) > mvar(tail(p)) => tail(p)
  deepestTail(tail(p))

iteratedInitials p ==
  ground? p => []
  p := init(p)
  cons(p,iteratedInitials(p))

localDeepestInitial (p : $) : $ ==
  ground? p => p
  localDeepestInitial init p

deepestInitial p ==
  ground? p => _
```

```
    error"Error in deepestInitial from RPOLCAT : #1 is constant."
  localDeepestInitial init p

monic? p ==
  ground? p => false
  (recip(init(p))$$ case $)@Boolean

quasiMonic?  p ==
  ground? p => false
  ground?(init(p))

mainMonomial p ==
  zero? p => error"Error in mainMonomial from RPOLCAT : #1 is zero"
  ground? p => 1$$
  v := mainVariable(p)::V
  monomial(1$$,v,degree(p,v))

leastMonomial p ==
  zero? p => error"Error in leastMonomial from RPOLCAT : #1 is zero"
  ground? p => 1$$
  v := mainVariable(p)::V
  monomial(1$$,v,minimumDegree(p,v))

mainCoefficients p ==
  zero? p => error"Error in mainCoefficients from RPOLCAT : #1 is zero"
  ground? p => [p]
  v := mainVariable(p)::V
  coefficients(univariate(p,v)@SparseUnivariatePolynomial($))

mainMonomials p ==
  zero? p => error"Error in mainMonomials from RPOLCAT : #1 is zero"
  ground? p => [1$$]
  v := mainVariable(p)::V
  lm := monomials(univariate(p,v)@SparseUnivariatePolynomial($))
  [monomial(1$$,v,degree(m)) for m in lm]

RittWuCompare (a,b) ==
  (ground? b and  ground? a) => "failed"::Union(Boolean,"failed")
  ground? b => false::Union(Boolean,"failed")
  ground? a => true::Union(Boolean,"failed")
  mvar(a) < mvar(b) => true::Union(Boolean,"failed")
  mvar(a) > mvar(b) => false::Union(Boolean,"failed")
  mdeg(a) < mdeg(b) => true::Union(Boolean,"failed")
  mdeg(a) > mdeg(b) => false::Union(Boolean,"failed")
  lc := RittWuCompare(init(a),init(b))
  lc case Boolean => lc
```

```
  RittWuCompare(tail(a),tail(b))

infRittWu? (a,b) ==
  lc : Union(Boolean,"failed") := RittWuCompare(a,b)
  lc case Boolean => lc::Boolean
  false

supRittWu? (a,b) ==
  infRittWu? (b,a)

prem (a:$, b:$)  : $ ==
  cP := lazyPremWithDefault (a,b)
  ((cP.coef) ** (cP.gap)) * cP.remainder

pquo (a:$, b:$)  : $ ==
  cPS := lazyPseudoDivide (a,b)
  c := (cPS.coef) ** (cPS.gap)
  c * cPS.quotient

prem (a:$, b:$, v:V) : $ ==
  cP := lazyPremWithDefault (a,b,v)
  ((cP.coef) ** (cP.gap)) * cP.remainder

pquo (a:$, b:$, v:V)  : $ ==
  cPS := lazyPseudoDivide (a,b,v)
  c := (cPS.coef) ** (cPS.gap)
  c * cPS.quotient

lazyPrem (a:$, b:$) : $ ==
  (not ground?(b)) and (monic?(b)) => monicModulo(a,b)
  (lazyPremWithDefault (a,b)).remainder

lazyPquo (a:$, b:$) : $ ==
  (lazyPseudoDivide (a,b)).quotient

lazyPrem (a:$, b:$, v:V) : $ ==
  zero? b => _
    error"Error in lazyPrem : ($,$,V) -> $ from RPOLCAT : #2 is zero"
  ground?(b) => 0$$
  (v = mvar(b)) => lazyPrem(a,b)
  dbv : NNI := degree(b,v)
  zero? dbv => 0$$
  dav : NNI  := degree(a,v)
  zero? dav => a
  test : INT := dav::INT - dbv
  lcbv : $ := leadingCoefficient(b,v)
```

```
      while not zero?(a) and not negative?(test) repeat
        lcav := leadingCoefficient(a,v)
        term := monomial(lcav,v,test::NNI)
        a := lcbv * a - term * b
        test := degree(a,v)::INT - dbv
      a

    lazyPquo (a:$, b:$, v:V) : $ ==
      (lazyPseudoDivide (a,b,v)).quotient

    headReduce (a:$,b:$) ==
      ground? b => error _
       "Error in headReduce : ($,$) -> Boolean from TSETCAT : #2 is constant"
      ground? a => a
      mvar(a) = mvar(b) => lazyPrem(a,b)
      while not reduced?((ha := head a),b) repeat
        lrc := lazyResidueClass(ha,b)
        if zero? tail(a)
          then
            a := lrc.polnum
          else
            a := lrc.polnum +  (lrc.polden)**(lrc.power) * tail(a)
      a

    initiallyReduce(a:$,b:$) ==
      ground? b => error _
  "Error in initiallyReduce : ($,$) -> Boolean from TSETCAT : #2 is constant"
      ground? a => a
      v := mvar(b)
      mvar(a) = v => lazyPrem(a,b)
      ia := a
      ma := 1$$
      ta := 0$$
      while (not ground?(ia)) and (mvar(ia) >= mvar(b)) repeat
        if (mvar(ia) = mvar(b)) and (mdeg(ia) >= mdeg(b))
          then
            iamodb := lazyResidueClass(ia,b)
            ia := iamodb.polnum
            if not zero? ta
              then
                ta :=  (iamodb.polden)**(iamodb.power) * ta
        if zero? ia
          then
            ia := ta
            ma := 1$$
            ta := 0$$
```

```
      else
        if not ground?(ia)
          then
            ta := tail(ia) * ma + ta
            ma := mainMonomial(ia) * ma
            ia := init(ia)
  ia * ma + ta

lazyPremWithDefault (a,b) ==
  ground?(b) => error _
    "Error in lazyPremWithDefault from RPOLCAT : #2 is constant"
  ground?(a) => [1$$,0$NNI,a]
  xa := mvar a
  xb := mvar b
  xa < xb => [1$$,0$NNI,a]
  lcb : $ := init b
  db : NNI := mdeg b
  test : INT := degree(a,xb)::INT - db
  delta : INT := max(test + 1$INT, 0$INT)
  if xa = xb
    then
      b := tail b
      while not zero?(a) and not negative?(test) repeat
        term := monomial(init(a),xb,test::NNI)
        a := lcb * tail(a) - term * b
        delta := delta - 1$INT
        test := degree(a,xb)::INT - db
    else
      while not zero?(a) and not negative?(test) repeat
        term := monomial(leadingCoefficient(a,xb),xb,test::NNI)
        a := lcb * a - term * b
        delta := delta - 1$INT
        test := degree(a,xb)::INT - db
  [lcb, (delta::NNI), a]

lazyPremWithDefault (a,b,v) ==
  zero? b =>  error _
   "Error in lazyPremWithDefault : ($,$,V) -> $ from RPOLCAT : #2 is zero"
  ground?(b) => [b,1$NNI,0$$]
  (v = mvar(b)) => lazyPremWithDefault(a,b)
  dbv : NNI := degree(b,v)
  zero? dbv => [b,1$NNI,0$$]
  dav : NNI  := degree(a,v)
  zero? dav => [1$$,0$NNI,a]
  test : INT := dav::INT - dbv
  delta : INT := max(test + 1$INT, 0$INT)
```

```
    lcbv : $ := leadingCoefficient(b,v)
    while not zero?(a) and not negative?(test) repeat
      lcav := leadingCoefficient(a,v)
      term := monomial(lcav,v,test::NNI)
      a := lcbv * a - term * b
      delta := delta - 1$INT
      test := degree(a,v)::INT - dbv
    [lcbv, (delta::NNI), a]

  pseudoDivide (a,b) ==
    cPS := lazyPseudoDivide (a,b)
    c := (cPS.coef) ** (cPS.gap)
    [c * cPS.quotient, c * cPS.remainder]

  lazyPseudoDivide (a,b) ==
    ground?(b) => error _
       "Error in lazyPseudoDivide from RPOLCAT : #2 is constant"
    ground?(a) => [1$$,0$NNI,0$$,a]
    xa := mvar a
    xb := mvar b
    xa < xb => [1$$,0$NNI,0$$, a]
    lcb : $ := init b
    db : NNI := mdeg b
    q := 0$$
    test : INT := degree(a,xb)::INT - db
    delta : INT := max(test + 1$INT, 0$INT)
    if xa = xb
      then
        b := tail b
        while not zero?(a) and not negative?(test) repeat
          term := monomial(init(a),xb,test::NNI)
          a := lcb * tail(a) - term * b
          q := lcb * q + term
          delta := delta - 1$INT
          test := degree(a,xb)::INT - db
      else
        while not zero?(a) and not negative?(test) repeat
          term := monomial(leadingCoefficient(a,xb),xb,test::NNI)
          a := lcb * a - term * b
          q := lcb * q + term
          delta := delta - 1$INT
          test := degree(a,xb)::INT - db
    [lcb, (delta::NNI), q, a]

  lazyPseudoDivide (a,b,v) ==
    zero? b =>  error _
```

```
    "Error in lazyPseudoDivide : ($,$,V) -> $ from RPOLCAT : #2 is zero"
  ground?(b) => [b,1$NNI,a,0$$]
  (v = mvar(b)) => lazyPseudoDivide(a,b)
  dbv : NNI := degree(b,v)
  zero? dbv => [b,1$NNI,a,0$$]
  dav : NNI  := degree(a,v)
  zero? dav => [1$$,0$NNI,0$$, a]
  test : INT := dav::INT - dbv
  delta : INT := max(test + 1$INT, 0$INT)
  lcbv : $ := leadingCoefficient(b,v)
  q := 0$$
  while not zero?(a) and not negative?(test) repeat
    lcav := leadingCoefficient(a,v)
    term := monomial(lcav,v,test::NNI)
    a := lcbv * a - term * b
    q := lcbv * q + term
    delta := delta - 1$INT
    test := degree(a,v)::INT - dbv
  [lcbv, (delta::NNI), q, a]

monicModulo (a,b) ==
  ground?(b) => error"Error in monicModulo from RPOLCAT : #2 is constant"
  rec : Union($,"failed")
  rec := recip((ib := init(b)))$$
  (rec case "failed")@Boolean => error _
    "Error in monicModulo from RPOLCAT : #2 is not monic"
  ground? a => a
  ib * ((lazyPremWithDefault ((rec::$) * a,(rec::$) * b)).remainder)

lazyResidueClass(a,b) ==
  zero? b => [a,1$$,0$NNI]
  ground? b => [0$$,1$$,0$NNI]
  ground? a => [a,1$$,0$NNI]
  xa := mvar a
  xb := mvar b
  xa < xb => [a,1$$,0$NNI]
  monic?(b) => [monicModulo(a,b),1$$,0$NNI]
  lcb : $ := init b
  db : NNI := mdeg b
  test : INT := degree(a,xb)::INT - db
  pow : NNI := 0
  if xa = xb
    then
      b := tail b
      while not zero?(a) and not negative?(test) repeat
        term := monomial(init(a),xb,test::NNI)
```

```
            a := lcb * tail(a) - term * b
            pow := pow + 1$NNI
            test := degree(a,xb)::INT - db
        else
          while not zero?(a) and not negative?(test) repeat
            term := monomial(leadingCoefficient(a,xb),xb,test::NNI)
            a := lcb * a - term * b
            pow := pow + 1$NNI
            test := degree(a,xb)::INT - db
      [a,lcb,pow]

    reduced? (a:$,b:$) : Boolean ==
      degree(a,mvar(b)) < mdeg(b)

    reduced? (p:$, lq : List($)) : Boolean ==
      ground? p => true
      while (not empty? lq) and (reduced?(p, first lq)) repeat
        lq := rest lq
      empty? lq

    headReduced? (a:$,b:$) : Boolean ==
      reduced?(head(a),b)

    headReduced? (p:$, lq : List($)) : Boolean ==
      reduced?(head(p),lq)

    initiallyReduced? (a:$,b:$) : Boolean ==
      ground? b => error _
  "Error in initiallyReduced? : ($,$) -> Bool. from RPOLCAT : #2 is constant"
      ground?(a) => true
      mvar(a) < mvar(b) => true
      (mvar(a) = mvar(b)) => reduced?(a,b)
      initiallyReduced?(init(a),b)

    initiallyReduced? (p:$, lq : List($)) : Boolean ==
      ground? p => true
      while (not empty? lq) and (initiallyReduced?(p, first lq)) repeat
        lq := rest lq
      empty? lq

    normalized?(a:$,b:$) : Boolean ==
      ground? b => error _
     "Error in  normalized? : ($,$) -> Boolean from TSETCAT : #2 is constant"
      ground? a => true
      mvar(a) < mvar(b) => true
      (mvar(a) = mvar(b)) => false
```

```
        normalized?(init(a),b)

      normalized? (p:$, lq : List($)) : Boolean ==
        while (not empty? lq) and (normalized?(p, first lq)) repeat
          lq := rest lq
        empty? lq

      if R has IntegralDomain
      then

        if R has EuclideanDomain
          then
            exactQuo(r:R,s:R):R ==
              r quo$R s
          else
            exactQuo(r:R,s:R):R ==
              (r exquo$R s)::R

        exactQuotient (p:$,r:R) ==
          (p exquo$$ r)::$

        exactQuotient (a:$,b:$) ==
          ground? b => exactQuotient(a,ground(b))
          (a exquo$$ b)::$

        exactQuotient! (a:$,b:$) ==
          ground? b => exactQuotient!(a,ground(b))
          a := (a exquo$$ b)::$

        if (R has GcdDomain) and not(R has Field)
        then

          primPartElseUnitCanonical p ==
            primitivePart p

          primitivePart! p ==
            zero? p => p
--            if one?(cp := content(p))
            if ((cp := content(p)) = 1)
              then
                p := unitCanonical p
              else
                p := unitCanonical exactQuotient!(p,cp)
            p

          primPartElseUnitCanonical! p ==
```

```
            primitivePart! p

        else
          primPartElseUnitCanonical p ==
            unitCanonical p

          primPartElseUnitCanonical! p ==
            p := unitCanonical p


      if R has GcdDomain
      then

        gcd(r:R,p:$):R ==
--          one? r => r
          (r = 1) => r
          zero? p => r
          ground? p => gcd(r,ground(p))$R
          gcd(gcd(r,init(p)),tail(p))

        mainContent p ==
          zero? p => p
          "gcd"/mainCoefficients(p)

        mainPrimitivePart p ==
          zero? p => p
          (unitNormal((p exquo$$ mainContent(p))::$)).canonical

        mainSquareFreePart p ==
          ground? p => p
          v := mainVariable(p)::V
          sfp : SparseUnivariatePolynomial($)
          sfp := squareFreePart(univariate(p,v)@SparseUnivariatePolynomial($))
          multivariate(sfp,v)

      if (V has ConvertibleTo(Symbol))
        then

          PR ==> Polynomial R
          PQ ==> Polynomial Fraction Integer
          PZ ==> Polynomial Integer
          IES ==> IndexedExponents(Symbol)
          Q ==> Fraction Integer
          Z ==> Integer

          convert(p:$) : PR ==
```

```
    ground? p => (ground(p)$$)::PR
    v : V := mvar(p)
    d : NNI := mdeg(p)
    convert(init(p))@PR *$PR _
                ((convert(v)@Symbol)::PR)**d +$PR convert(tail(p))@PR

  coerce(p:$) : PR ==
    convert(p)@PR

  localRetract : PR -> $
  localRetractPQ : PQ -> $
  localRetractPZ : PZ -> $
  localRetractIfCan : PR -> Union($,"failed")
  localRetractIfCanPQ : PQ -> Union($,"failed")
  localRetractIfCanPZ : PZ -> Union($,"failed")

  if V has Finite
    then

      sizeV : NNI := size()$V
      lv : List Symbol
      lv := _
        [convert(index(i::PositiveInteger)$V)@Symbol for i in 1..sizeV]

      localRetract(p : PR) : $ ==
        ground? p => (ground(p)$PR)::$
        mvp : Symbol := (mainVariable(p)$PR)::Symbol
        d : NNI
        imvp : PositiveInteger := _
                    (position(mvp,lv)$(List Symbol))::PositiveInteger
        vimvp : V := index(imvp)$V
        xvimvp,c : $
        newp := 0$$
        while (not zero? (d := degree(p,mvp))) repeat
          c := localRetract(coefficient(p,mvp,d)$PR)
          xvimvp := monomial(c,vimvp,d)$$
          newp := newp +$$ xvimvp
          p := p -$PR monomial(coefficient(p,mvp,d)$PR,mvp,d)$PR
        newp +$$ localRetract(p)

      if R has Algebra Fraction Integer
        then
          localRetractPQ(pq:PQ):$ ==
            ground? pq => ((ground(pq)$PQ)::R)::$
            mvp : Symbol := (mainVariable(pq)$PQ)::Symbol
            d : NNI
```

```
      imvp : PositiveInteger := _
                (position(mvp,lv)$(List Symbol))::PositiveInteger
      vimvp : V := index(imvp)$V
      xvimvp,c : $
      newp := 0$$
      while (not zero? (d := degree(pq,mvp))) repeat
        c := localRetractPQ(coefficient(pq,mvp,d)$PQ)
        xvimvp := monomial(c,vimvp,d)$$
        newp := newp +$$ xvimvp
        pq := pq -$PQ monomial(coefficient(pq,mvp,d)$PQ,mvp,d)$PQ
      newp +$$ localRetractPQ(pq)

  if R has Algebra Integer
    then
      localRetractPZ(pz:PZ):$ ==
        ground? pz => ((ground(pz)$PZ)::R)::$
        mvp : Symbol := (mainVariable(pz)$PZ)::Symbol
        d : NNI
        imvp : PositiveInteger := _
                  (position(mvp,lv)$(List Symbol))::PositiveInteger
        vimvp : V := index(imvp)$V
        xvimvp,c : $
        newp := 0$$
        while (not zero? (d := degree(pz,mvp))) repeat
          c := localRetractPZ(coefficient(pz,mvp,d)$PZ)
          xvimvp := monomial(c,vimvp,d)$$
          newp := newp +$$ xvimvp
          pz := pz -$PZ monomial(coefficient(pz,mvp,d)$PZ,mvp,d)$PZ
        newp +$$ localRetractPZ(pz)

  retractable?(p:PR):Boolean ==
    lvp := variables(p)$PR
    while not empty? lvp and member?(first lvp,lv) repeat
      lvp := rest lvp
    empty? lvp

  retractablePQ?(p:PQ):Boolean ==
    lvp := variables(p)$PQ
    while not empty? lvp and member?(first lvp,lv) repeat
      lvp := rest lvp
    empty? lvp

  retractablePZ?(p:PZ):Boolean ==
    lvp := variables(p)$PZ
    while not empty? lvp and member?(first lvp,lv) repeat
      lvp := rest lvp
```

```
      empty? lvp

    localRetractIfCan(p : PR): Union($,"failed") ==
      not retractable?(p) => "failed"::Union($,"failed")
      localRetract(p)::Union($,"failed")

    localRetractIfCanPQ(p : PQ): Union($,"failed") ==
      not retractablePQ?(p) => "failed"::Union($,"failed")
      localRetractPQ(p)::Union($,"failed")

    localRetractIfCanPZ(p : PZ): Union($,"failed") ==
      not retractablePZ?(p) => "failed"::Union($,"failed")
      localRetractPZ(p)::Union($,"failed")

  if R has Algebra Fraction Integer
    then

    mpc2Z := MPolyCatFunctions2(Symbol,IES,IES,Z,R,PZ,PR)
    mpc2Q := MPolyCatFunctions2(Symbol,IES,IES,Q,R,PQ,PR)
    ZToR (z:Z):R == coerce(z)@R
    QToR (q:Q):R == coerce(q)@R
    PZToPR (pz:PZ):PR == map(ZToR,pz)$mpc2Z
    PQToPR (pq:PQ):PR == map(QToR,pq)$mpc2Q

    retract(pz:PZ) ==
      rif : Union($,"failed") := retractIfCan(pz)@Union($,"failed")
      (rif case "failed") => error _
                          "failed in retract: POLY Z -> $ from RPOLCAT"
      rif::$

    convert(pz:PZ) ==
      retract(pz)@$

    retract(pq:PQ) ==
      rif : Union($,"failed") := retractIfCan(pq)@Union($,"failed")
      (rif case "failed") => error _
                          "failed in retract: POLY Z -> $ from RPOLCAT"
      rif::$

    convert(pq:PQ) ==
      retract(pq)@$

    if not (R has QuotientFieldCategory(Integer))
      then
        -- the only operation to implement is
        -- retractIfCan : PR -> Union($,"failed")
```

```
    -- when V does not have Finite

    if V has Finite
      then
        retractIfCan(pr:PR) ==
          localRetractIfCan(pr)@Union($,"failed")

        retractIfCan(pq:PQ) ==
          localRetractIfCanPQ(pq)@Union($,"failed")
      else
        retractIfCan(pq:PQ) ==
          pr : PR := PQToPR(pq)
          retractIfCan(pr)@Union($,"failed")

    retractIfCan(pz:PZ) ==
      pr : PR := PZToPR(pz)
      retractIfCan(pr)@Union($,"failed")

    retract(pr:PR) ==
      rif : Union($,"failed") := _
                              retractIfCan(pr)@Union($,"failed")
      (rif case "failed") => error _
                   "failed in retract: POLY Z -> $ from RPOLCAT"
      rif::$

    convert(pr:PR) ==
      retract(pr)@$

  else
    -- the only operation to implement is
    -- retractIfCan : PQ -> Union($,"failed")
    -- when V does not have Finite
    mpc2ZQ := MPolyCatFunctions2(Symbol,IES,IES,Z,Q,PZ,PQ)
    mpc2RQ := MPolyCatFunctions2(Symbol,IES,IES,R,Q,PR,PQ)
    ZToQ(z:Z):Q == coerce(z)@Q
    RToQ(r:R):Q == retract(r)@Q

    PZToPQ (pz:PZ):PQ == map(ZToQ,pz)$mpc2ZQ
    PRToPQ (pr:PR):PQ == map(RToQ,pr)$mpc2RQ

    retractIfCan(pz:PZ) ==
      pq : PQ := PZToPQ(pz)
      retractIfCan(pq)@Union($,"failed")

    if V has Finite
      then
```

```
                    retractIfCan(pq:PQ) ==
                      localRetractIfCanPQ(pq)@Union($,"failed")

                    convert(pr:PR) ==
                      lrif : Union($,"failed") := _
                                    localRetractIfCan(pr)@Union($,"failed")
                      (lrif case "failed") => error _
                                    "failed in convert: PR->$ from RPOLCAT"
                      lrif::$
                  else
                    convert(pr:PR) ==
                      pq : PQ := PRToPQ(pr)
                      retract(pq)@$

                if (R has Algebra Integer) and not(R has Algebra Fraction Integer)
                  then

                    mpc2Z := MPolyCatFunctions2(Symbol,IES,IES,Z,R,PZ,PR)
                    ZToR (z:Z):R == coerce(z)@R
                    PZToPR (pz:PZ):PR == map(ZToR,pz)$mpc2Z

                    retract(pz:PZ) ==
                      rif : Union($,"failed") := retractIfCan(pz)@Union($,"failed")
                      (rif case "failed") => error _
                                       "failed in retract: POLY Z -> $ from RPOLCAT"
                      rif::$

                    convert(pz:PZ) ==
                      retract(pz)@$

                    if not (R has IntegerNumberSystem)
                      then
                        -- the only operation to implement is
                        -- retractIfCan : PR -> Union($,"failed")
                        -- when V does not have Finite

                        if V has Finite
                          then
                            retractIfCan(pr:PR) ==
                              localRetractIfCan(pr)@Union($,"failed")

                            retractIfCan(pz:PZ) ==
                              localRetractIfCanPZ(pz)@Union($,"failed")
                          else
                            retractIfCan(pz:PZ) ==
                              pr : PR := PZToPR(pz)
```

```
                retractIfCan(pr)@Union($,"failed")

            retract(pr:PR) ==
              rif : Union($,"failed"):=retractIfCan(pr)@Union($,"failed")
              (rif case "failed") => error _
                            "failed in retract: POLY Z -> $ from RPOLCAT"
              rif::$

            convert(pr:PR) ==
              retract(pr)@$

          else
            -- the only operation to implement is
            -- retractIfCan : PZ -> Union($,"failed")
            -- when V does not have Finite

            mpc2RZ := MPolyCatFunctions2(Symbol,IES,IES,R,Z,PR,PZ)
            RToZ(r:R):Z == retract(r)@Z
            PRToPZ (pr:PR):PZ == map(RToZ,pr)$mpc2RZ

            if V has Finite
              then
                convert(pr:PR) ==
                  lrif : Union($,"failed") := _
                                localRetractIfCan(pr)@Union($,"failed")
                  (lrif case "failed") => error _
                                "failed in convert: PR->$ from RPOLCAT"
                  lrif::$
                retractIfCan(pz:PZ) ==
                  localRetractIfCanPZ(pz)@Union($,"failed")
              else
                convert(pr:PR) ==
                  pz : PZ := PRToPZ(pr)
                  retract(pz)@$


      if not(R has Algebra Integer) and not(R has Algebra Fraction Integer)
        then
          -- the only operation to implement is
          -- retractIfCan : PR -> Union($,"failed")

          if V has Finite
            then
              retractIfCan(pr:PR) ==
                localRetractIfCan(pr)@Union($,"failed")
```

```
            retract(pr:PR) ==
              rif : Union($,"failed") := retractIfCan(pr)@Union($,"failed")
              (rif case "failed") => error _
                              "failed in retract: POLY Z -> $ from RPOLCAT"
              rif::$

            convert(pr:PR) ==
              retract(pr)@$

          if (R has RetractableTo(INT))
            then

            convert(pol:$):String ==
              ground?(pol) => convert(retract(ground(pol))@INT)@String
              ipol : $ := init(pol)
              vpol : V := mvar(pol)
              dpol : NNI := mdeg(pol)
              tpol: $  := tail(pol)
              sipol,svpol,sdpol,stpol : String
--              if one? ipol
              if (ipol = 1)
                then
                  sipol := empty()$String
                else
--                  if one?(-ipol)
                  if ((-ipol) = 1)
                    then
                      sipol := "-"
                    else
                      sipol := convert(ipol)@String
                      if not monomial?(ipol)
                        then
                          sipol := concat(["(",sipol,")*"])$String
                        else
                          sipol := concat(sipol,"*")$String
              svpol := string(convert(vpol)@Symbol)
--              if one? dpol
              if (dpol = 1)
                then
                  sdpol :=  empty()$String
                else
                  sdpol := _
                       concat("**",convert(convert(dpol)@INT)@String )$String
              if zero? tpol
                then
                  stpol :=  empty()$String
```

```
                else
                  if ground?(tpol)
                    then
                      n := retract(ground(tpol))@INT
                      if n > 0
                        then
                          stpol :=  concat(" +",convert(n)@String)$String
                        else
                          stpol := convert(n)@String
                    else
                      stpol := convert(tpol)@String
                      if _
                        not member?((stpol.1)::String,["+","-"])$(List String)
                        then
                          stpol :=  concat(" + ",stpol)$String
              concat([sipol,svpol,sdpol,stpol])$String
```

⟨*RPOLCAT.dotabb*⟩≡

```
"RPOLCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RPOLCAT"];
"RPOLCAT" -> "POLYCAT"
```

⟨*RPOLCAT.dotfull*⟩≡

```
"RecursivePolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=RPOLCAT"];
"RecursivePolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 -> "PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
```

⟨*RPOLCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"RecursivePolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=lightblue];
"RecursivePolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 -> "PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"

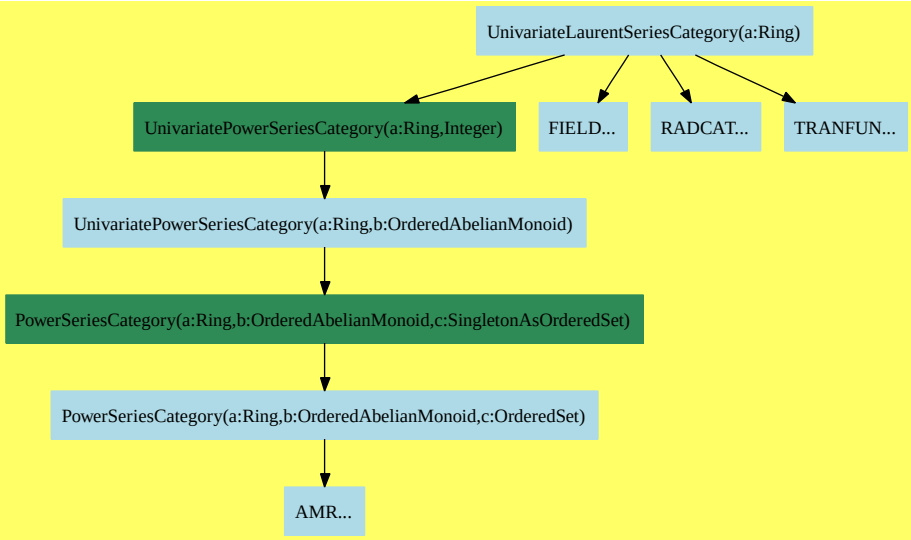
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
 [color=lightblue];
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PDRING..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FAMR..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "EVALAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "IEVALAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "RETRACT..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "FLINEXP..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "ORDSET..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "GCDDOM..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PFECAT..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "KONVERT..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "PATMAB..."
"PolynomialCategory(a:Ring,b:OrderedAbelianMonoidSup,c:OrderedSet)"
  -> "COMRING..."

"PDRING..." [color=lightblue];
"FAMR..." [color=lightblue];
"EVALAB..." [color=lightblue];
"IEVALAB..." [color=lightblue];
"RETRACT..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"ORDSET..." [color=lightblue];
"GCDDOM..." [color=lightblue];
```

```
"PFECAT..." [color=lightblue];
"KONVERT..." [color=lightblue];
"PATMAB..." [color=lightblue];
"COMRING..." [color=lightblue];

}
```

17.10 UnivariateLaurentSeriesCategory (ULSCAT)



See:

⇒ "UnivariateLaurentSeriesConstructorCategory" (ULSCCAT) 18.6 on page 1267

⇐ "Field" (FIELD) 16.1 on page 1005
⇐ "RadicalCategory" (RADCAT) 2.17 on page 42
⇐ "TranscendentalFunctionCategory" (TRANFUN) 3.14 on page 94
⇐ "UnivariatePowerSeriesCategory" (UPSCAT) 15.4 on page 996

Exports:

0	1	acos	acosh
acot	acoth	acsc	acsch
approximate	asec	asech	asin
asinh	associates?	atan	atanh
center	characteristic	charthRoot	coefficient
coerce	complete	cos	cosh
cot	coth	csc	csch
D	degree	differentiate	divide
euclideanSize	eval	exp	expressIdealMember
exquo	extend	extendedEuclidean	factor
gcd	gcdPolynomial	hash	integrate
inv	latex	lcm	leadingCoefficient
leadingMonomial	log	map	monomial
monomial?	multiEuclidean	multiplyCoefficients	multiplyExponents
nthRoot	one?	order	pi
pole?	prime?	principalIdeal	rationalFunction
recip	reductum	sample	sec
sech	series	sin	sinh
sizeLess?	sqrt	squareFree	squareFreePart
subtractIfCan	tan	tanh	terms
truncate	unit?	unitCanonical	unitNormal
variable	variables	zero?	?*?
?**?	?+?	?-? * -?	
?=?	?^?	?~=?	?/?
?.?	?quo?	?rem?	

Attributes Exported:

- if #1 has Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if #1 has Field then canonicalsClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if #1 has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.

These are directly exported but not implemented:

```
integrate : % -> % if Coef has ALGEBRA FRAC INT
integrate : (%,Symbol) -> %
  if Coef has ACFS INT
  and Coef has PRIMCAT
  and Coef has TRANFUN
  and Coef has ALGEBRA FRAC INT
  or Coef has variables: Coef -> List Symbol
  and Coef has integrate: (Coef,Symbol) -> Coef
  and Coef has ALGEBRA FRAC INT
multiplyCoefficients : ((Integer -> Coef),%) -> %
rationalFunction : (%,Integer) -> Fraction Polynomial Coef
  if Coef has INTDOM
rationalFunction : (%,Integer,Integer) -> Fraction Polynomial Coef
  if Coef has INTDOM
series : Stream Record(k: Integer,c: Coef) -> %
```

These exports come from (p996) UnivariatePowerSeriesCategory(Coef,Integer) where Coef:Ring:

```
0 : () -> %
1 : () -> %
approximate : (%,Integer) -> Coef
  if Coef has **: (Coef,Integer) -> Coef
  and Coef has coerce: Symbol -> Coef
associates? : (%,%) -> Boolean if Coef has INTDOM
center : % -> Coef
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,Integer) -> Coef
coerce : % -> % if Coef has INTDOM
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : Coef -> % if Coef has COMRING
coerce : Integer -> %
coerce : % -> OutputForm
complete : % -> %
D : % -> % if Coef has *: (Integer,Coef) -> Coef
D : (%,NonNegativeInteger) -> %
  if Coef has *: (Integer,Coef) -> Coef
D : (%,Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
D : (%,List Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
D : (%,Symbol,NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
D : (%,List Symbol,List NonNegativeInteger) -> %
```

```
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
degree : % -> Integer
differentiate : (%,Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,List Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,Symbol,NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : % -> %
  if Coef has *: (Integer,Coef) -> Coef
differentiate : (%,NonNegativeInteger) -> %
  if Coef has *: (Integer,Coef) -> Coef
eval : (%,Coef) -> Stream Coef
  if Coef has **: (Coef,Integer) -> Coef
exquo : (%,%) -> Union(%,"failed")
  if Coef has INTDOM
extend : (%,Integer) -> %
hash : % -> SingleInteger
latex : % -> String
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
map : ((Coef -> Coef),%) -> %
monomial : (%,SingletonAsOrderedSet,Integer) -> %
monomial : (%,List SingletonAsOrderedSet,List Integer) -> %
monomial : (Coef,Integer) -> %
monomial? : % -> Boolean
multiplyExponents : (%,PositiveInteger) -> %
one? : % -> Boolean
order : (%,Integer) -> Integer
order : % -> Integer
pole? : % -> Boolean
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
terms : % -> Stream Record(k: Integer,c: Coef)
truncate : (%,Integer,Integer) -> %
truncate : (%,Integer) -> %
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
  if Coef has INTDOM
variable : % -> Symbol
```

```
variables : % -> List SingletonAsOrderedSet
zero? : % -> Boolean
?.? : (%,Integer) -> Coef
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
-? : % -> %
?.? : (%,%) -> % if Integer has SGROUP
```

These exports come from (p94) TranscendentalFunctionCategory():

```
acos : % -> % if Coef has ALGEBRA FRAC INT
acosh : % -> % if Coef has ALGEBRA FRAC INT
acot : % -> % if Coef has ALGEBRA FRAC INT
acoth : % -> % if Coef has ALGEBRA FRAC INT
acsc : % -> % if Coef has ALGEBRA FRAC INT
acsch : % -> % if Coef has ALGEBRA FRAC INT
asec : % -> % if Coef has ALGEBRA FRAC INT
asech : % -> % if Coef has ALGEBRA FRAC INT
asin : % -> % if Coef has ALGEBRA FRAC INT
asinh : % -> % if Coef has ALGEBRA FRAC INT
atanh : % -> % if Coef has ALGEBRA FRAC INT
atan : % -> % if Coef has ALGEBRA FRAC INT
cos : % -> % if Coef has ALGEBRA FRAC INT
cosh : % -> % if Coef has ALGEBRA FRAC INT
cot : % -> % if Coef has ALGEBRA FRAC INT
coth : % -> % if Coef has ALGEBRA FRAC INT
csc : % -> % if Coef has ALGEBRA FRAC INT
csch : % -> % if Coef has ALGEBRA FRAC INT
exp : % -> % if Coef has ALGEBRA FRAC INT
log : % -> % if Coef has ALGEBRA FRAC INT
pi : () -> % if Coef has ALGEBRA FRAC INT
sec : % -> % if Coef has ALGEBRA FRAC INT
sech : % -> % if Coef has ALGEBRA FRAC INT
sin : % -> % if Coef has ALGEBRA FRAC INT
sinh : % -> % if Coef has ALGEBRA FRAC INT
```

```
tan : % -> % if Coef has ALGEBRA FRAC INT
tanh : % -> % if Coef has ALGEBRA FRAC INT
?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
```

These exports come from (p42) RadicalCategory():

```
nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
sqrt : % -> % if Coef has ALGEBRA FRAC INT
?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
```

These exports come from (p1005) Field():

```
divide : (%,%) -> Record(quotient: %,remainder: %)
  if Coef has FIELD
euclideanSize : % -> NonNegativeInteger
  if Coef has FIELD
expressIdealMember : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
  if Coef has FIELD
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
  if Coef has FIELD
factor : % -> Factored % if Coef has FIELD
gcd : (%,%) -> % if Coef has FIELD
gcd : List % -> % if Coef has FIELD
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
       if Coef has FIELD
inv : % -> % if Coef has FIELD
lcm : (%,%) -> % if Coef has FIELD
lcm : List % -> % if Coef has FIELD
multiEuclidean : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
prime? : % -> Boolean if Coef has FIELD
principalIdeal : List % -> Record(coef: List %,generator: %)
  if Coef has FIELD
sizeLess? : (%,%) -> Boolean if Coef has FIELD
squareFree : % -> Factored % if Coef has FIELD
squareFreePart : % -> % if Coef has FIELD
?**? : (%,Integer) -> % if Coef has FIELD
?^? : (%,Integer) -> % if Coef has FIELD
?/? : (%,%) -> % if Coef has FIELD
?quo? : (%,%) -> % if Coef has FIELD
?rem? : (%,%) -> % if Coef has FIELD
```

⟨*category ULSCAT UnivariateLaurentSeriesCategory*⟩≡

```
)abbrev category ULSCAT UnivariateLaurentSeriesCategory
++ Author: Clifton J. Williamson
```

```
++ Date Created: 21 December 1989
++ Date Last Updated: 20 September 1993
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Laurent
++ Examples:
++ References:
++ Description:
++   \spadtype{UnivariateLaurentSeriesCategory} is the category of
++   Laurent series in one variable.
UnivariateLaurentSeriesCategory(Coef): Category == Definition where
  Coef  : Ring
  I    ==> Integer
  NNI  ==> NonNegativeInteger
  Term ==> Record(k:I,c:Coef)

  Definition ==> UnivariatePowerSeriesCategory(Coef,Integer) with

    series: Stream Term -> %
      ++ \spad{series(st)} creates a series from a stream of non-zero terms,
      ++ where a term is an exponent-coefficient pair.  The terms in the
      ++ stream should be ordered by increasing order of exponents.
    multiplyCoefficients: (I -> Coef,%) -> %
      ++ \spad{multiplyCoefficients(f,sum(n = n0..infinity,a[n] * x**n)) =
      ++ sum(n = 0..infinity,f(n) * a[n] * x**n)}.
      ++ This function is used when Puiseux series are represented by
      ++ a Laurent series and an exponent.
    if Coef has IntegralDomain then
      rationalFunction: (%,I) -> Fraction Polynomial Coef
        ++ \spad{rationalFunction(f,k)} returns a rational function
        ++ consisting of the sum of all terms of f of degree <= k.
      rationalFunction: (%,I,I) -> Fraction Polynomial Coef
        ++ \spad{rationalFunction(f,k1,k2)} returns a rational function
        ++ consisting of the sum of all terms of f of degree d with
        ++ \spad{k1 <= d <= k2}.

    if Coef has Algebra Fraction Integer then
      integrate: % -> %
        ++ \spad{integrate(f(x))} returns an anti-derivative of the power
        ++ series \spad{f(x)} with constant coefficient 1.
        ++ We may integrate a series when we can divide coefficients
        ++ by integers.
      if Coef has integrate: (Coef,Symbol) -> Coef and _
         Coef has variables: Coef -> List Symbol then
```

```
      integrate: (%,Symbol) -> %
        ++ \spad{integrate(f(x),y)} returns an anti-derivative of the power
        ++ series \spad{f(x)} with respect to the variable \spad{y}.
    if Coef has TranscendentalFunctionCategory and _
       Coef has PrimitiveFunctionCategory and _
       Coef has AlgebraicallyClosedFunctionSpace Integer then
      integrate: (%,Symbol) -> %
        ++ \spad{integrate(f(x),y)} returns an anti-derivative of
        ++ the power series \spad{f(x)} with respect to the variable
        ++ \spad{y}.
    RadicalCategory
      --++ We provide rational powers when we can divide coefficients
      --++ by integers.
    TranscendentalFunctionCategory
      --++ We provide transcendental functions when we can divide
      --++ coefficients by integers.
  if Coef has Field then Field
      --++ Univariate Laurent series over a field form a field.
      --++ In fact, K((x)) is the quotient field of K[[x]].
```

⟨*ULSCAT.dotabb*⟩≡

```
"ULSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ULSCAT"];
"ULSCAT" -> "UPSCAT"
"ULSCAT" -> "FIELD"
"ULSCAT" -> "TRANFUN"
"ULSCAT" -> "RADCAT"
```

⟨*ULSCAT.dotfull*⟩≡

```
"UnivariateLaurentSeriesCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ULSCAT"];
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "Field()"
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "RadicalCategory()"
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "TranscendentalFunctionCategory()"
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,Integer)"
```

⟨*ULSCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UnivariateLaurentSeriesCategory(a:Ring)" [color=lightblue];
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,Integer)"
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "FIELD..."
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "RADCAT..."
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "TRANFUN..."

"UnivariatePowerSeriesCategory(a:Ring,Integer)" [color=seagreen];
"UnivariatePowerSeriesCategory(a:Ring,Integer)" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

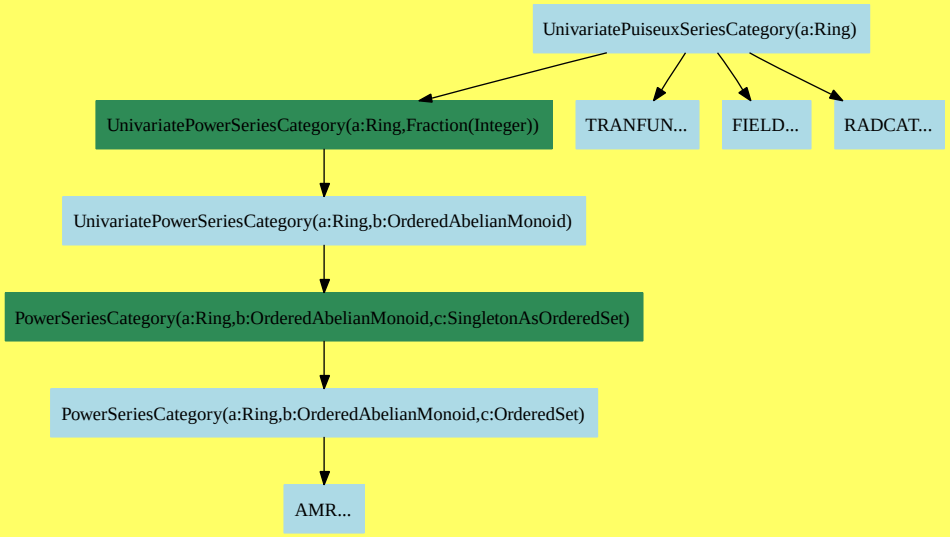
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AMR..."

"AMR..." [color=lightblue];
"FIELD..." [color=lightblue];
"TRANFUN..." [color=lightblue];
"RADCAT..." [color=lightblue];
}
```

17.11 UnivariatePuiseuxSeriesCategory (UPXSCAT)



See:

⇒ "UnivariatePuiseuxSeriesConstructorCategory" (UPXSCCA) 18.7 on page 1280

⇐ "TranscendentalFunctionCategory" (TRANFUN) 3.14 on page 94
⇐ "Field" (FIELD) 16.1 on page 1005
⇐ "RadicalCategory" (RADCAT) 2.17 on page 42
⇐ "UnivariatePowerSeriesCategory" (UPSCAT) 15.4 on page 996

Exports:

0	1	acos	acosh
acot	acoth	acsc	acsch
approximate	asec	asech	asin
asinh	associates?	atan	atanh
center	characteristic	charthRoot	coefficient
coerce	complete	cos	cosh
cot	coth	csc	csch
D	degree	differentiate	divide
euclideanSize	eval	exp	expressIdealMember
exquo	extend	extendedEuclidean	factor
gcd	gcdPolynomial	hash	integrate
inv	latex	lcm	leadingCoefficient
leadingMonomial	log	map	monomial
monomial?	multiEuclidean	multiplyExponents	nthRoot
one?	order	pi	pole?
prime?	principalIdeal	recip	reductum
sample	sec	sech	series
sin	sinh	sizeLess?	sqrt
squareFree	squareFreePart	subtractIfCan	tan
tanh	terms	truncate	unit?
unitCanonical	unitNormal	variable	variables
zero?	?*?	?**?	?+?
?-?	-?	?=?	?~=?
?/?	?^?	?.?	?quo?
?rem?			

Attributes Exported:

- if #1 has Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if #1 has Field then canonicalsClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.

- if #1 has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.

These are directly exported but not implemented:

```
integrate : % -> % if Coef has ALGEBRA FRAC INT
integrate : (%,Symbol) -> %
  if Coef has ACFS INT
  and Coef has PRIMCAT
  and Coef has TRANFUN
  and Coef has ALGEBRA FRAC INT
  or Coef has variables: Coef -> List Symbol
  and Coef has integrate: (Coef,Symbol) -> Coef
  and Coef has ALGEBRA FRAC INT
multiplyExponents : (%,Fraction Integer) -> %
series : (NonNegativeInteger,Stream Record(k: Fraction Integer,c: Coef)) -> %
```

These exports come from (p996) UnivariatePowerSeriesCategory(Coef,RN) where Coef:Ring and RN:Fraction(Integer):

```
0 : () -> %
1 : () -> %
approximate : (%,Fraction Integer) -> Coef
  if Coef has **: (Coef,Fraction Integer) -> Coef
  and Coef has coerce: Symbol -> Coef
associates? : (%,%) -> Boolean if Coef has INTDOM
center : % -> Coef
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,Fraction Integer) -> Coef
coerce : % -> % if Coef has INTDOM
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : Coef -> % if Coef has COMRING
coerce : Integer -> %
coerce : % -> OutputForm
complete : % -> %
D : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,NonNegativeInteger) -> %
  if Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,List Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,Symbol,NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
```

```
D : (%,List Symbol,List NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
degree : % -> Fraction Integer
differentiate : (%,Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,List Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,Symbol,NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : % -> %
  if Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,NonNegativeInteger) -> %
  if Coef has *: (Fraction Integer,Coef) -> Coef
eval : (%,Coef) -> Stream Coef
  if Coef has **: (Coef,Fraction Integer) -> Coef
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
extend : (%,Fraction Integer) -> %
hash : % -> SingleInteger
latex : % -> String
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
map : ((Coef -> Coef),%) -> %
monomial : (%,List SingletonAsOrderedSet,List Fraction Integer) -> %
monomial : (Coef,Fraction Integer) -> %
monomial : (%,SingletonAsOrderedSet,Fraction Integer) -> %
monomial? : % -> Boolean
multiplyExponents : (%,PositiveInteger) -> %
one? : % -> Boolean
order : (%,Fraction Integer) -> Fraction Integer
order : % -> Fraction Integer
pole? : % -> Boolean
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
terms : % -> Stream Record(k: Fraction Integer,c: Coef)
truncate : (%,Fraction Integer,Fraction Integer) -> %
truncate : (%,Fraction Integer) -> %
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
  if Coef has INTDOM
variable : % -> Symbol
```

```
variables : % -> List SingletonAsOrderedSet
zero? : % -> Boolean
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
-? : % -> %
?.? : (%,%) -> % if Fraction Integer has SGROUP
?.? : (%,Fraction Integer) -> Coef
```

These exports come from (p94) TranscendentalFunctionCategory():

```
acos : % -> % if Coef has ALGEBRA FRAC INT
acosh : % -> % if Coef has ALGEBRA FRAC INT
acot : % -> % if Coef has ALGEBRA FRAC INT
acoth : % -> % if Coef has ALGEBRA FRAC INT
acsc : % -> % if Coef has ALGEBRA FRAC INT
acsch : % -> % if Coef has ALGEBRA FRAC INT
asec : % -> % if Coef has ALGEBRA FRAC INT
asech : % -> % if Coef has ALGEBRA FRAC INT
asin : % -> % if Coef has ALGEBRA FRAC INT
asinh : % -> % if Coef has ALGEBRA FRAC INT
atan : % -> % if Coef has ALGEBRA FRAC INT
atanh : % -> % if Coef has ALGEBRA FRAC INT
cos : % -> % if Coef has ALGEBRA FRAC INT
cosh : % -> % if Coef has ALGEBRA FRAC INT
cot : % -> % if Coef has ALGEBRA FRAC INT
coth : % -> % if Coef has ALGEBRA FRAC INT
csc : % -> % if Coef has ALGEBRA FRAC INT
csch : % -> % if Coef has ALGEBRA FRAC INT
exp : % -> % if Coef has ALGEBRA FRAC INT
log : % -> % if Coef has ALGEBRA FRAC INT
pi : () -> % if Coef has ALGEBRA FRAC INT
sec : % -> % if Coef has ALGEBRA FRAC INT
sech : % -> % if Coef has ALGEBRA FRAC INT
sin : % -> % if Coef has ALGEBRA FRAC INT
sinh : % -> % if Coef has ALGEBRA FRAC INT
```

```
tan : % -> % if Coef has ALGEBRA FRAC INT
tanh : % -> % if Coef has ALGEBRA FRAC INT
?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
```

These exports come from (p1005) Field():

```
divide : (%,%) -> Record(quotient: %,remainder: %)
  if Coef has FIELD
euclideanSize : % -> NonNegativeInteger if Coef has FIELD
expressIdealMember : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
  if Coef has FIELD
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
  if Coef has FIELD
factor : % -> Factored % if Coef has FIELD
gcd : (%,%) -> % if Coef has FIELD
gcd : List % -> % if Coef has FIELD
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
       if Coef has FIELD
inv : % -> % if Coef has FIELD
lcm : (%,%) -> % if Coef has FIELD
lcm : List % -> % if Coef has FIELD
multiEuclidean : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
prime? : % -> Boolean if Coef has FIELD
principalIdeal : List % -> Record(coef: List %,generator: %)
  if Coef has FIELD
sizeLess? : (%,%) -> Boolean if Coef has FIELD
squareFree : % -> Factored % if Coef has FIELD
squareFreePart : % -> % if Coef has FIELD
?**? : (%,Integer) -> % if Coef has FIELD
?^? : (%,Integer) -> % if Coef has FIELD
?/? : (%,%) -> % if Coef has FIELD
?quo? : (%,%) -> % if Coef has FIELD
?rem? : (%,%) -> % if Coef has FIELD
```

These exports come from (p42) RadicalCategory():

```
nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
sqrt : % -> % if Coef has ALGEBRA FRAC INT
?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
```

⟨*category UPXSCAT UnivariatePuiseuxSeriesCategory*⟩≡

```
)abbrev category UPXSCAT UnivariatePuiseuxSeriesCategory
++ Author: Clifton J. Williamson
++ Date Created: 21 December 1989
```

```
++ Date Last Updated: 20 September 1993
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Puiseux
++ Examples:
++ References:
++ Description:
++   \spadtype{UnivariatePuiseuxSeriesCategory} is the category of Puiseux
++   series in one variable.
UnivariatePuiseuxSeriesCategory(Coef): Category == Definition where
  Coef : Ring
  NNI  ==> NonNegativeInteger
  RN   ==> Fraction Integer
  Term ==> Record(k:RN,c:Coef)

  Definition ==> UnivariatePowerSeriesCategory(Coef,RN) with

    series: (NNI,Stream Term) -> %
      ++ \spad{series(n,st)} creates a series from a common denomiator and
      ++ a stream of non-zero terms, where a term is an exponent-coefficient
      ++ pair.  The terms in the stream should be ordered by increasing order
      ++ of exponents and \spad{n} should be a common denominator for the
      ++ exponents in the stream of terms.
    multiplyExponents: (%,Fraction Integer) -> %
      ++ \spad{multiplyExponents(f,r)} multiplies all exponents of the power
      ++ series f by the positive rational number r.

    if Coef has Algebra Fraction Integer then
      integrate: % -> %
        ++ \spad{integrate(f(x))} returns an anti-derivative of the power
        ++ series \spad{f(x)} with constant coefficient 1.
        ++ We may integrate a series when we can divide coefficients
        ++ by rational numbers.
      if Coef has integrate: (Coef,Symbol) -> Coef and _
         Coef has variables: Coef -> List Symbol then
        integrate: (%,Symbol) -> %
          ++ \spad{integrate(f(x),var)} returns an anti-derivative of the power
          ++ series \spad{f(x)} with respect to the variable \spad{var}.
      if Coef has TranscendentalFunctionCategory and _
         Coef has PrimitiveFunctionCategory and _
         Coef has AlgebraicallyClosedFunctionSpace Integer then
        integrate: (%,Symbol) -> %
          ++ \spad{integrate(f(x),y)} returns an anti-derivative of
          ++ the power series \spad{f(x)} with respect to the variable
```

```
          ++ \spad{y}.
      RadicalCategory
        --++ We provide rational powers when we can divide coefficients
        --++ by integers.
      TranscendentalFunctionCategory
        --++ We provide transcendental functions when we can divide
        --++ coefficients by integers.
    if Coef has Field then Field
        --++ Univariate Puiseux series over a field form a field.
```

⟨*UPXSCAT.dotabb*⟩≡

```
"UPXSCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPXSCAT"];
"UPXSCAT" -> "TRANFUN"
"UPXSCAT" -> "FIELD"
"UPXSCAT" -> "RADCAT"
"UPXSCAT" -> "UPSCAT"
```

⟨*UPXSCAT.dotfull*⟩≡

```
"UnivariatePuiseuxSeriesCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPXSCAT"];
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))"
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "TranscendentalFunctionCategory()"
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "Field()"
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "RadicalCategory()"
```

⟨*UPXSCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UnivariatePuiseuxSeriesCategory(a:Ring)" [color=lightblue];
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))"
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "TRANFUN..."
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "FIELD..."
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "RADCAT..."

"UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))" [color=seagreen];
"UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

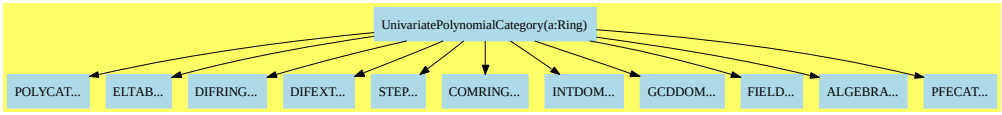
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AMR..."

"TRANFUN..." [color=lightblue];
"FIELD..." [color=lightblue];
"RADCAT..." [color=lightblue];
"AMR..." [color=lightblue];
}
```

17.12 UnivariatePolynomialCategory (UPOLYC)



See:

⇐ "Algebra" (ALGEBRA) 11.1 on page 771
⇐ "CommutativeRing" (COMRING) 10.4 on page 685
⇐ "DifferentialExtension" (DIFEXT) 11.2 on page 777
⇐ "DifferentialRing" (DIFRING) 10.5 on page 690
⇐ "Eltable" (ELTAB) 2.10 on page 25
⇐ "Field" (FIELD) 16.1 on page 1005
⇐ "GcdDomain" (GCDDOM) 13.4 on page 932
⇐ "IntegralDomain" (INTDOM) 12.5 on page 857
⇐ "PolynomialCategory" (POLYCAT) 16.4 on page 1027
⇐ "PolynomialFactorizationExplicit" (PFECAT) 15.3 on page 990
⇐ "StepThrough" (STEP) 4.26 on page 193

Exports:

0	1	associates?
binomThmExpt	characteristic	charthRoot
coefficient	coefficients	coerce
composite	conditionP	content
convert	D	degree
differentiate	discriminant	divide
divideExponents	elt	euclideanSize
eval	expressIdealMember	exquo
extendedEuclidean	factor	factorPolynomial
factorSquareFreePolynomial	gcd	gcdPolynomial
ground	ground?	hash
init	integrate	isExpt
isPlus	isTimes	karatsubaDivide
latex	lcm	leadingCoefficient
leadingMonomial	mainVariable	makeSUP
map	mapExponents	max
min	minimumDegree	monicDivide
monomial	monomial?	monomials
multiEuclidean	multiplyExponents	multivariate
nextItem	numberOfMonomials	one?
order	patternMatch	pomopo!
prime?	primitiveMonomials	primitivePart
principalIdeal	pseudoDivide	pseudoQuotient
pseudoRemainder	recip	reducedSystem
reductum	resultant	retract
retractIfCan	sample	separate
shiftLeft	shiftRight	sizeLess?
solveLinearPolynomialEquation	squareFree	squareFreePart
squareFreePolynomial	subResultantGcd	subtractIfCan
totalDegree	unit?	unitCanonical
unitNormal	univariate	unmakeSUP
variables	vectorise	zero?
?*?	?**?	?+?
?-?	-?	?=?
?^?	?.?	?~=?
?/?	?<?	?<=?
?>?	?>=?	?quo?
?rem?		

Attributes exported:

- if $ has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- if $ has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if R has Field then additiveValuation where **additiveValuation** implies

`euclideanSize(a*b)=euclideanSize(a)+euclideanSize(b)`.

- if $ has canonicalUnitNormal then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
discriminant : % -> R if R has COMRING
divideExponents : (%,NonNegativeInteger) -> Union(%,"failed")
monicDivide : (%,%) -> Record(quotient: %,remainder: %)
multiplyExponents : (%,NonNegativeInteger) -> %
pseudoRemainder : (%,%) -> %
resultant : (%,%) -> R if R has COMRING
subResultantGcd : (%,%) -> % if R has INTDOM
```

These are implemented by this category:

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean if R has INTDOM
binomThmExpt : (%,%,NonNegativeInteger) -> %
    if R has COMRING
characteristic : () -> NonNegativeInteger
coefficient : (%,NonNegativeInteger) -> R
coefficients : % -> List R
coerce : R -> %
coerce : Fraction Integer -> %
    if R has RETRACT FRAC INT
    or R has ALGEBRA FRAC INT
coerce : % -> % if R has INTDOM
content : % -> R if R has GCDDOM
coerce : Integer -> %
coerce : % -> OutputForm
coerce : SingletonAsOrderedSet -> %
composite : (Fraction %,%) -> Union(Fraction %,"failed")
    if R has INTDOM
composite : (%,%) -> Union(%,"failed")
    if R has INTDOM
content : (%,SingletonAsOrderedSet) -> %
    if R has GCDDOM
```

```
D : (%,List SingletonAsOrderedSet) -> %
D : (%,SingletonAsOrderedSet) -> %
D : (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
D : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
degree : % -> NonNegativeInteger
degree :
  (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
differentiate :
  (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
differentiate :
  (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
differentiate : (%,List SingletonAsOrderedSet) -> %
differentiate : (%,(R -> R),%) -> %
differentiate : (%,(R -> R)) -> %
differentiate : % -> %
differentiate : (%,SingletonAsOrderedSet) -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
    if R has FIELD
elt : (Fraction %,Fraction %) -> Fraction %
    if R has INTDOM
elt : (Fraction %,R) -> R if R has FIELD
euclideanSize : % -> NonNegativeInteger
    if R has FIELD
eval : (%,List SingletonAsOrderedSet,List %) -> %
eval : (%,SingletonAsOrderedSet,%) -> %
eval : (%,List SingletonAsOrderedSet,List R) -> %
eval : (%,SingletonAsOrderedSet,R) -> %
eval : (%,List Equation %) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
eval : (%,Equation %) -> %
exquo : (%,R) -> Union(%,"failed")
    if R has INTDOM
exquo : (%,%) -> Union(%,"failed")
    if R has INTDOM
factor : % -> Factored % if R has PFECAT
factorPolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
factorSquareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
      if R has PFECAT
gcd : (%,%) -> % if R has GCDDOM
gcd : List % -> % if R has GCDDOM
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
```

```
       if R has GCDDOM
ground : % -> R
ground? : % -> Boolean
hash : % -> SingleInteger
init : () -> % if R has STEP
integrate : % -> % if R has ALGEBRA FRAC INT
karatsubaDivide :
  (%,NonNegativeInteger) -> Record(quotient: %,remainder: %)
latex : % -> String
lcm : (%,%) -> % if R has GCDDOM
lcm : List % -> % if R has GCDDOM
leadingCoefficient : % -> R
leadingMonomial : % -> %
mainVariable : % -> Union(SingletonAsOrderedSet,"failed")
makeSUP : % -> SparseUnivariatePolynomial R
map : ((R -> R),%) -> %
mapExponents :
  ((NonNegativeInteger -> NonNegativeInteger),%) -> %
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
minimumDegree :
  (%,SingletonAsOrderedSet) -> NonNegativeInteger
minimumDegree :
  (%,List SingletonAsOrderedSet) -> List NonNegativeInteger
monomial : (R,NonNegativeInteger) -> %
monomial : (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
monomial? : % -> Boolean
nextItem : % -> Union(%,"failed") if R has STEP
numberOfMonomials : % -> NonNegativeInteger
one? : % -> Boolean
order : (%,%) -> NonNegativeInteger
    if R has INTDOM
pomopo! : (%,R,NonNegativeInteger,%) -> %
prime? : % -> Boolean if R has PFECAT
pseudoDivide :
  (%,%) -> Record(coef: R,quotient: %,remainder: %)
    if R has INTDOM
pseudoQuotient : (%,%) -> % if R has INTDOM
recip : % -> Union(%,"failed")
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Integer,vec: Vector Integer)
    if R has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer
    if R has LINEXP INT
reductum : % -> %
retract : % -> R
retract : % -> Integer if R has RETRACT INT
retract : % -> Fraction Integer
    if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
```

```
    if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if R has RETRACT FRAC INT
retractIfCan : % -> Union(R,"failed")
sample : () -> %
separate : (%,%) -> Record(primePart: %,commonPart: %)
    if R has GCDDOM
shiftLeft : (%,NonNegativeInteger) -> %
shiftRight : (%,NonNegativeInteger) -> %
solveLinearPolynomialEquation :
  (List SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     Union(List SparseUnivariatePolynomial %,"failed")
       if R has PFECAT
squareFree : % -> Factored % if R has GCDDOM
squareFreePart : % -> % if R has GCDDOM
squareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
     if R has PFECAT
subtractIfCan : (%,%) -> Union(%,"failed")
totalDegree :
  (%,List SingletonAsOrderedSet) -> NonNegativeInteger
unit? : % -> Boolean if R has INTDOM
unitCanonical : % -> % if R has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if R has INTDOM
unmakeSUP : SparseUnivariatePolynomial R -> %
variables : % -> List SingletonAsOrderedSet
vectorise : (%,NonNegativeInteger) -> Vector R
zero? : % -> Boolean
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,R) -> %
?*? : (R,%) -> %
?*? : (Fraction Integer,%) -> %
    if R has ALGEBRA FRAC INT
?*? : (%,Fraction Integer) -> %
    if R has ALGEBRA FRAC INT
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?/? : (%,R) -> % if R has FIELD
?-? : (%,%) -> %
-? : % -> %
```

```
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?.? : (%,Fraction %) -> Fraction %
    if R has INTDOM
```

These exports come from (p1027) PolynomialCategory(R,N,S)
where R:Ring, N:NonNegativeInteger, S:SingletonAsOrderedSet:

```
charthRoot : % -> Union(%,"failed")
  if
   and(has($,CharacteristicNonZero),
       has(R,PolynomialFactorizationExplicit))
   or R has CHARNZ
coefficient :
  (%,SingletonAsOrderedSet,NonNegativeInteger) -> %
coefficient :
  (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
conditionP : Matrix % -> Union(Vector %,"failed")
 if and(has($,CharacteristicNonZero),
        has(R,PolynomialFactorizationExplicit))
convert : % -> Pattern Integer
    if SingletonAsOrderedSet has KONVERT PATTERN INT
    and R has KONVERT PATTERN INT
convert : % -> Pattern Float
    if SingletonAsOrderedSet has KONVERT PATTERN FLOAT
    and R has KONVERT PATTERN FLOAT
convert : % -> InputForm
    if SingletonAsOrderedSet has KONVERT INFORM
    and R has KONVERT INFORM
degree : (%,SingletonAsOrderedSet) -> NonNegativeInteger
discriminant : (%,SingletonAsOrderedSet) -> %
    if R has COMRING
isExpt : % ->
  Union(
   Record(var: SingletonAsOrderedSet,exponent: NonNegativeInteger),
  "failed")
isPlus : % -> Union(List %,"failed")
isTimes : % -> Union(List %,"failed")
minimumDegree : % -> NonNegativeInteger
monicDivide :
  (%,%,SingletonAsOrderedSet) -> Record(quotient: %,remainder: %)
monomial :
  (%,List SingletonAsOrderedSet,List NonNegativeInteger) -> %
monomials : % -> List %
multivariate :
  (SparseUnivariatePolynomial %,SingletonAsOrderedSet) -> %
multivariate :
  (SparseUnivariatePolynomial R,SingletonAsOrderedSet) -> %
```

```
patternMatch :
 (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
   PatternMatchResult(Integer,%)
    if SingletonAsOrderedSet has PATMAB INT
    and R has PATMAB INT
patternMatch :
 (%,Pattern Float,PatternMatchResult(Float,%)) ->
   PatternMatchResult(Float,%)
    if SingletonAsOrderedSet has PATMAB FLOAT
    and R has PATMAB FLOAT
primitiveMonomials : % -> List %
primitivePart : (%,SingletonAsOrderedSet) -> %
    if R has GCDDOM
primitivePart : % -> % if R has GCDDOM
reducedSystem : Matrix % -> Matrix R
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix R,vec: Vector R)
resultant : (%,%,SingletonAsOrderedSet) -> %
    if R has COMRING
retract : % -> SingletonAsOrderedSet
retractIfCan : % -> Union(SingletonAsOrderedSet,"failed")
totalDegree : % -> NonNegativeInteger
univariate : % -> SparseUnivariatePolynomial R
univariate :
  (%,SingletonAsOrderedSet) -> SparseUnivariatePolynomial %
?<? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p25) Eltable(R:Ring,R:Ring):

```
?.? : (%,R) -> R
```

These exports come from (p25) Eltable(R:UPOLYC,R:UPOLYC):

```
?.? : (%,%) -> %
```

These exports come from (p690) DifferentialRing():

```
D : % -> %
D : (%,NonNegativeInteger) -> %
differentiate : (%,NonNegativeInteger) -> %
```

These exports come from (p777) DifferentialExtension(R:Ring):

```
D : (%,(R -> R)) -> %
D : (%,(R -> R),NonNegativeInteger) -> %
D : (%,Symbol) -> % if R has PDRING SYMBOL
D : (%,List Symbol) -> % if R has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> %
    if R has PDRING SYMBOL
```

```
D : (%,List Symbol,List NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : (%,(R -> R),NonNegativeInteger) -> %
differentiate : (%,Symbol) -> %
    if R has PDRING SYMBOL
differentiate : (%,List Symbol) -> %
    if R has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if R has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if R has PDRING SYMBOL
```

These exports come from (p193) StepThrough()

These exports come from (p685) CommutativeRing()

These exports come from (p857) IntegralDomain()

These exports come from (p932) GcdDomain()

These exports come from (p1005) Field()

```
expressIdealMember : (List %,%) -> Union(List %,"failed")
    if R has FIELD
extendedEuclidean : (%,%) ->
  Record(coef1: %,coef2: %,generator: %)
    if R has FIELD
extendedEuclidean :
  (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
    if R has FIELD
multiEuclidean : (List %,%) -> Union(List %,"failed")
    if R has FIELD
principalIdeal : List % -> Record(coef: List %,generator: %)
    if R has FIELD
sizeLess? : (%,%) -> Boolean if R has FIELD
?quo? : (%,%) -> % if R has FIELD
?rem? : (%,%) -> % if R has FIELD
```

These exports come from (p771) Algebra(Fraction(Integer))

These exports come from (p990) PolynomialFactorizationExplicit()

⟨*category UPOLYC UnivariatePolynomialCategory*⟩≡

```
)abbrev category UPOLYC UnivariatePolynomialCategory
++ Author:
++ Date Created:
++ Date Last Updated:
++ Basic Functions: Ring, monomial, coefficient, reductum, differentiate,
++ elt, map, resultant, discriminant
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ The category of univariate polynomials over a ring R.
++ No particular model is assumed - implementations can be either
++ sparse or dense.

UnivariatePolynomialCategory(R:Ring): Category ==
 Join(PolynomialCategory(R, NonNegativeInteger, SingletonAsOrderedSet),
      Eltable(R, R), Eltable(%, %), DifferentialRing,
      DifferentialExtension R) with
    vectorise        : (%,NonNegativeInteger) -> Vector R
      ++ vectorise(p, n) returns \spad{[a0,...,a(n-1)]} where
      ++ \spad{p = a0 + a1*x + ... + a(n-1)*x**(n-1)} + higher order terms.
      ++ The degree of polynomial p can be different from \spad{n-1}.
    makeSUP: % -> SparseUnivariatePolynomial R
      ++ makeSUP(p) converts the polynomial p to be of type
      ++ SparseUnivariatePolynomial over the same coefficients.
    unmakeSUP: SparseUnivariatePolynomial R -> %
      ++ unmakeSUP(sup) converts sup of type
      ++ \spadtype{SparseUnivariatePolynomial(R)}
      ++ to be a member of the given type.
      ++ Note: converse of makeSUP.
    multiplyExponents: (%,NonNegativeInteger) -> %
      ++ multiplyExponents(p,n) returns a new polynomial resulting from
      ++ multiplying all exponents of the polynomial p by the non negative
      ++ integer n.
    divideExponents: (%,NonNegativeInteger) -> Union(%,"failed")
      ++ divideExponents(p,n) returns a new polynomial resulting from
      ++ dividing all exponents of the polynomial p by the non negative
      ++ integer n, or "failed" if some exponent is not exactly divisible
      ++ by n.
    monicDivide: (%,%) -> Record(quotient:%,remainder:%)
```

```
      ++ monicDivide(p,q) divide the polynomial p by the monic polynomial q,
      ++ returning the pair \spad{[quotient, remainder]}.
      ++ Error: if q isn't monic.
  -- These three are for Karatsuba
    karatsubaDivide: (%,NonNegativeInteger) -> Record(quotient:%,remainder:%)
      ++ \spad{karatsubaDivide(p,n)} returns the same as
      ++ \spad{monicDivide(p,monomial(1,n))}
    shiftRight: (%,NonNegativeInteger) -> %
      ++ \spad{shiftRight(p,n)} returns
      ++ \spad{monicDivide(p,monomial(1,n)).quotient}
    shiftLeft: (%,NonNegativeInteger) -> %
      ++ \spad{shiftLeft(p,n)} returns \spad{p * monomial(1,n)}
    pseudoRemainder: (%,%) -> %
       ++ pseudoRemainder(p,q) = r, for polynomials p and q, returns the
       ++ remainder when
       ++ \spad{p' := p*lc(q)**(deg p - deg q + 1)}
       ++ is pseudo right-divided by q, i.e. \spad{p' = s q + r}.
    differentiate: (%, R -> R, %) -> %
       ++ differentiate(p, d, x') extends the R-derivation d to an
       ++ extension D in \spad{R[x]} where Dx is given by x', and
       ++ returns \spad{Dp}.
    if R has StepThrough then StepThrough
    if R has CommutativeRing then
      discriminant: % -> R
        ++ discriminant(p) returns the discriminant of the polynomial p.
      resultant: (%,%) -> R
        ++ resultant(p,q) returns the resultant of the polynomials p and q.
    if R has IntegralDomain then
        Eltable(Fraction %, Fraction %)
        elt  : (Fraction %, Fraction %) -> Fraction %
             ++ elt(a,b) evaluates the fraction of univariate polynomials
             ++ \spad{a} with the distinguished variable replaced by b.
        order: (%, %) -> NonNegativeInteger
             ++ order(p, q) returns the largest n such that \spad{q**n}
             ++ divides polynomial p
             ++ i.e. the order of \spad{p(x)} at \spad{q(x)=0}.
        subResultantGcd: (%,%) -> %
           ++ subResultantGcd(p,q) computes the gcd of the polynomials p
           ++ and q using the SubResultant GCD algorithm.
        composite: (%, %) -> Union(%, "failed")
           ++ composite(p, q) returns h if \spad{p = h(q)}, and "failed"
           ++ no such h exists.
        composite: (Fraction %, %) -> Union(Fraction %, "failed")
           ++ composite(f, q) returns h if f = h(q), and "failed" is
           ++ no such h exists.
        pseudoQuotient: (%,%) -> %
```

```
          ++ pseudoQuotient(p,q) returns r, the quotient when
          ++ \spad{p' := p*lc(q)**(deg p - deg q + 1)}
          ++ is pseudo right-divided by q, i.e. \spad{p' = s q + r}.
       pseudoDivide: (%, %) -> Record(coef:R, quotient: %, remainder:%)
          ++ pseudoDivide(p,q) returns \spad{[c, q, r]}, when
          ++ \spad{p' := p*lc(q)**(deg p - deg q + 1) = c * p}
          ++ is pseudo right-divided by q, i.e. \spad{p' = s q + r}.
    if R has GcdDomain then
       separate: (%, %) -> Record(primePart:%, commonPart: %)
          ++ separate(p, q) returns \spad{[a, b]} such that polynomial
          ++ \spad{p = a b} and \spad{a} is relatively prime to q.
    if R has Field then
       EuclideanDomain
       additiveValuation
         ++ euclideanSize(a*b) = euclideanSize(a) + euclideanSize(b)
       elt      : (Fraction %, R) -> R
         ++ elt(a,r) evaluates the fraction of univariate polynomials
         ++ \spad{a} with the distinguished variable replaced by the
         ++ constant r.
    if R has Algebra Fraction Integer then
      integrate: % -> %
        ++ integrate(p) integrates the univariate polynomial p with respect
        ++ to its distinguished variable.
  add
    pp,qq: SparseUnivariatePolynomial %

    variables(p) ==
      zero? p or zero?(degree p) => []
      [create()]

    degree(p:%,v:SingletonAsOrderedSet) == degree p

    totalDegree(p:%,lv:List SingletonAsOrderedSet) ==
       empty? lv => 0
       totalDegree p

    degree(p:%,lv:List SingletonAsOrderedSet) ==
       empty? lv => []
       [degree p]

    eval(p:%,lv: List SingletonAsOrderedSet,lq: List %):% ==
      empty? lv => p
      not empty? rest lv => _
        error "can only eval a univariate polynomial once"
      eval(p,first lv,first lq)$%
```

```
eval(p:%,v:SingletonAsOrderedSet,q:%):% == p(q)

eval(p:%,lv: List SingletonAsOrderedSet,lr: List R):% ==
  empty? lv => p
  not empty? rest lv => _
     error "can only eval a univariate polynomial once"
  eval(p,first lv,first lr)$%

eval(p:%,v:SingletonAsOrderedSet,r:R):% == p(r)::%

eval(p:%,le:List Equation %):% ==
  empty? le  => p
  not empty? rest le => _
     error "can only eval a univariate polynomial once"
  mainVariable(lhs first le) case "failed" => p
  p(rhs first le)

mainVariable(p:%) ==
  zero? degree p =>  "failed"
  create()$SingletonAsOrderedSet

minimumDegree(p:%,v:SingletonAsOrderedSet) == minimumDegree p

minimumDegree(p:%,lv:List SingletonAsOrderedSet) ==
   empty? lv => []
   [minimumDegree p]

monomial(p:%,v:SingletonAsOrderedSet,n:NonNegativeInteger) ==
   mapExponents(x1+->x1+n,p)

coerce(v:SingletonAsOrderedSet):% == monomial(1,1)

makeSUP p ==
  zero? p => 0
  monomial(leadingCoefficient p,degree p) + makeSUP reductum p

unmakeSUP sp ==
  zero? sp => 0
  monomial(leadingCoefficient sp,degree sp) + unmakeSUP reductum sp

karatsubaDivide(p:%,n:NonNegativeInteger) == monicDivide(p,monomial(1,n))

shiftRight(p:%,n:NonNegativeInteger) ==
   monicDivide(p,monomial(1,n)).quotient

shiftLeft(p:%,n:NonNegativeInteger) == p * monomial(1,n)
```

```
if R has PolynomialFactorizationExplicit then
   PFBRU ==> PolynomialFactorizationByRecursionUnivariate(R,%)
   pp,qq:SparseUnivariatePolynomial %
   lpp:List SparseUnivariatePolynomial %
   SupR ==> SparseUnivariatePolynomial R
   sp:SupR

   solveLinearPolynomialEquation(lpp,pp) ==
     solveLinearPolynomialEquationByRecursion(lpp,pp)$PFBRU

   factorPolynomial(pp) ==
     factorByRecursion(pp)$PFBRU

   factorSquareFreePolynomial(pp) ==
     factorSquareFreeByRecursion(pp)$PFBRU

   import FactoredFunctions2(SupR,S)

   factor p ==
     zero? degree p  =>
       ansR:=factor leadingCoefficient p
       makeFR(unit(ansR)::%,
              [[w.flg,w.fctr::%,w.xpnt] for w in factorList ansR])
     map(unmakeSUP,factorPolynomial(makeSUP p)$R)

vectorise(p, n) ==
  m := minIndex(v := new(n, 0)$Vector(R))
  for i in minIndex v .. maxIndex v repeat
    qsetelt_!(v, i, coefficient(p, (i - m)::NonNegativeInteger))
  v

retract(p:%):R ==
  zero? p => 0
  zero? degree p => leadingCoefficient p
  error "Polynomial is not of degree 0"

retractIfCan(p:%):Union(R, "failed") ==
  zero? p => 0
  zero? degree p => leadingCoefficient p
  "failed"

if R has StepThrough then
   init() == init()$R::%

   nextItemInner: % -> Union(%,"failed")
```

```
    nextItemInner(n) ==
      zero? n => nextItem(0$R)::R::% -- assumed not to fail
      zero? degree n =>
        nn:=nextItem leadingCoefficient n
        nn case "failed" => "failed"
        nn::R::%
      n1:=reductum n
      n2:=nextItemInner n1 -- try stepping the reductum
      n2 case % => monomial(leadingCoefficient n,degree n) + n2
      1+degree n1 < degree n => -- there was a hole between lt n and n1
        monomial(leadingCoefficient n,degree n)+
          monomial(nextItem(init()$R)::R,1+degree n1)
      n3:=nextItem leadingCoefficient n
      n3 case "failed" => "failed"
      monomial(n3,degree n)

    nextItem(n) ==
      n1:=nextItemInner n
      n1 case "failed" => monomial(nextItem(init()$R)::R,1+degree(n))
      n1

  if R has GcdDomain then

    content(p:%,v:SingletonAsOrderedSet) == content(p)::%

    primeFactor: (%, %) -> %

    primeFactor(p, q) ==
      (p1 := (p exquo gcd(p, q))::%) = p => p
      primeFactor(p1, q)

    separate(p, q) ==
      a := primeFactor(p, q)
      [a, (p exquo a)::%]

  if R has CommutativeRing then
    differentiate(x:%, deriv:R -> R, x':%) ==
      d:% := 0
      while (dg := degree x) > 0 repeat
        lc := leadingCoefficient x
        d := d + x' * monomial(dg * lc, (dg - 1)::NonNegativeInteger)
               + monomial(deriv lc, dg)
        x := reductum x
      d + deriv(leadingCoefficient x)::%
  else
```

```
    ncdiff: (NonNegativeInteger, %) -> %
    -- computes d(x**n) given dx = x', non-commutative case
    ncdiff(n, x') ==
      zero? n => 0
      zero?(n1 := (n - 1)::NonNegativeInteger) => x'
      x' * monomial(1, n1) + monomial(1, 1) * ncdiff(n1, x')

    differentiate(x:%, deriv:R -> R, x':%) ==
      d:% := 0
      while (dg := degree x) > 0 repeat
        lc := leadingCoefficient x
        d := d + monomial(deriv lc, dg) + lc * ncdiff(dg, x')
        x := reductum x
      d + deriv(leadingCoefficient x)::%

  differentiate(x:%, deriv:R -> R) == differentiate(x, deriv, 1$%)$%

  differentiate(x:%) ==
      d:% := 0
      while (dg := degree x) > 0 repeat
        d:=d+monomial(dg*leadingCoefficient x,(dg-1)::NonNegativeInteger)
        x := reductum x
      d

  differentiate(x:%,v:SingletonAsOrderedSet) == differentiate x

  if R has IntegralDomain then

    elt(g:Fraction %, f:Fraction %) == ((numer g) f) / ((denom g) f)

    pseudoQuotient(p, q) ==
      (n := degree(p)::Integer - degree q + 1) < 1 => 0
      ((leadingCoefficient(q)**(n::NonNegativeInteger) * p
        - pseudoRemainder(p, q)) exquo q)::%

    pseudoDivide(p, q) ==
      (n := degree(p)::Integer - degree q + 1) < 1 => [1, 0, p]
      prem := pseudoRemainder(p, q)
      lc   := leadingCoefficient(q)**(n::NonNegativeInteger)
      [lc,((lc*p - prem) exquo q)::%, prem]

    composite(f:Fraction %, q:%) ==
      (n := composite(numer f, q)) case "failed" => "failed"
      (d := composite(denom f, q)) case "failed" => "failed"
      n::% / d::%
```

```
    composite(p:%, q:%) ==
      ground? p => p
      cqr := pseudoDivide(p, q)
      ground?(cqr.remainder) and
        ((v := cqr.remainder exquo cqr.coef) case %) and
          ((u := composite(cqr.quotient, q)) case %) and
            ((w := (u::%) exquo cqr.coef) case %) =>
              v::% + monomial(1, 1) * w::%
      "failed"

    elt(p:%, f:Fraction %) ==
      zero? p => 0
      ans:Fraction(%) := (leadingCoefficient p)::%::Fraction(%)
      n := degree p
      while not zero?(p:=reductum p) repeat
        ans := ans * f ** (n - (n := degree p))::NonNegativeInteger +
                  (leadingCoefficient p)::%::Fraction(%)
      zero? n => ans
      ans * f ** n

    order(p, q) ==
      zero? p => error "order: arguments must be nonzero"
      degree(q) < 1 => error "order: place must be non-trivial"
      ans:NonNegativeInteger := 0
      repeat
        (u  := p exquo q) case "failed" => return ans
        p   := u::%
        ans := ans + 1

  if R has GcdDomain then
    squareFree(p:%) ==
      squareFree(p)$UnivariatePolynomialSquareFree(R, %)

    squareFreePart(p:%) ==
      squareFreePart(p)$UnivariatePolynomialSquareFree(R, %)

  if R has PolynomialFactorizationExplicit then

    gcdPolynomial(pp,qq) ==
          zero? pp => unitCanonical qq  -- subResultantGcd can't handle 0
          zero? qq => unitCanonical pp
          unitCanonical(gcd(content (pp),content(qq))*
                 primitivePart
                    subResultantGcd(primitivePart pp,primitivePart qq))

    squareFreePolynomial pp ==
```

```
      squareFree(pp)$UnivariatePolynomialSquareFree(%,
                                SparseUnivariatePolynomial %)

    if R has Field then
      elt(f:Fraction %, r:R) == ((numer f) r) / ((denom f) r)

      euclideanSize x ==
            zero? x =>
              error "euclideanSize called on 0 in Univariate Polynomial"
            degree x

      divide(x,y) ==
            zero? y => error "division by 0 in Univariate Polynomials"
            quot:=0
            lc := inv leadingCoefficient y
            while not zero?(x) and (degree x >= degree y) repeat
               f:=lc*leadingCoefficient x
               n:=(degree x - degree y)::NonNegativeInteger
               quot:=quot+monomial(f,n)
               x:=x-monomial(f,n)*y
            [quot,x]

    if R has Algebra Fraction Integer then

      integrate p ==
        ans:% := 0
        while p ^= 0 repeat
          l := leadingCoefficient p
          d := 1 + degree p
          ans := ans + inv(d::Fraction(Integer)) * monomial(l, d)
          p := reductum p
        ans
```

⟨*UPOLYC.dotabb*⟩≡
```
"UPOLYC"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPOLYC"];
"UPOLYC" -> "POLYCAT"
"UPOLYC" -> "ELTAB"
"UPOLYC" -> "DIFRING"
"UPOLYC" -> "DIFEXT"
"UPOLYC" -> "STEP"
"UPOLYC" -> "COMRING"
"UPOLYC" -> "INTDOM"
"UPOLYC" -> "GCDDOM"
"UPOLYC" -> "FIELD"
"UPOLYC" -> "ALGEBRA"
"UPOLYC" -> "PFECAT"
```

⟨*UPOLYC.dotfull*⟩≡
```
"UnivariatePolynomialCategory(a:Ring)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPOLYC"];
"UnivariatePolynomialCategory(a:Ring)" ->
   "PolynomialCategory(a:Ring,b:NonNegativeInteger,c:SingletonAsOrderedSet)"
"UnivariatePolynomialCategory(a:Ring)" ->
   "Eltable(a:Ring,b:Ring)"
"UnivariatePolynomialCategory(a:Ring)" ->
   "Eltable(a:UnivariatePolynomialCategory(a:Ring),b:UnivariatePolynomialCategory
"UnivariatePolynomialCategory(a:Ring)" ->
   "DifferentialRing()"
"UnivariatePolynomialCategory(a:Ring)" ->
   "DifferentialExtension(a:Ring)"
"UnivariatePolynomialCategory(a:Ring)" ->
   "StepThrough()"
"UnivariatePolynomialCategory(a:Ring)" ->
   "CommutativeRing()"
"UnivariatePolynomialCategory(a:Ring)" ->
   "IntegralDomain()"
"UnivariatePolynomialCategory(a:Ring)" ->
   "GcdDomain()"
"UnivariatePolynomialCategory(a:Ring)" ->
   "Field()"
"UnivariatePolynomialCategory(a:Ring)" ->
   "Algebra(Fraction(Integer))"
"UnivariatePolynomialCategory(a:Ring)" ->
   "PolynomialFactorizationExplicit()"
```

⟨*UPOLYC.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UnivariatePolynomialCategory(a:Ring)" [color=lightblue];
"UnivariatePolynomialCategory(a:Ring)" -> "POLYCAT..."
"UnivariatePolynomialCategory(a:Ring)" -> "ELTAB..."
"UnivariatePolynomialCategory(a:Ring)" -> "DIFRING..."
"UnivariatePolynomialCategory(a:Ring)" -> "DIFEXT..."
"UnivariatePolynomialCategory(a:Ring)" -> "STEP..."
"UnivariatePolynomialCategory(a:Ring)" -> "COMRING..."
"UnivariatePolynomialCategory(a:Ring)" -> "INTDOM..."
"UnivariatePolynomialCategory(a:Ring)" -> "GCDDOM..."
"UnivariatePolynomialCategory(a:Ring)" -> "FIELD..."
"UnivariatePolynomialCategory(a:Ring)" -> "ALGEBRA..."
"UnivariatePolynomialCategory(a:Ring)" -> "PFECAT..."

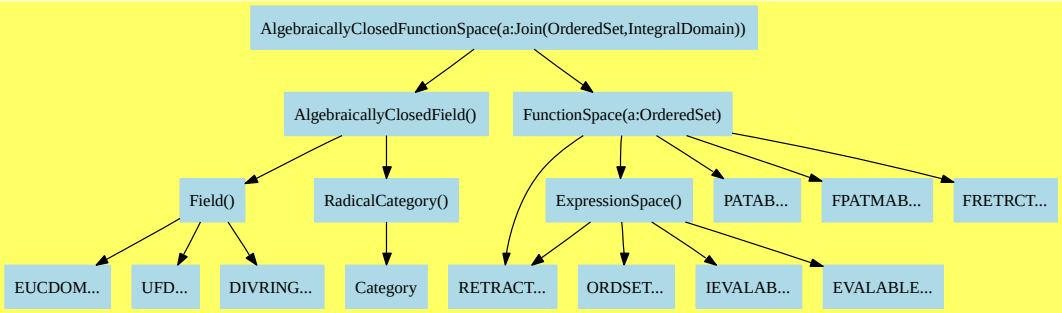
"POLYCAT..." [color=lightblue];
"ELTAB..." [color=lightblue];
"DIFRING..." [color=lightblue];
"DIFEXT..." [color=lightblue];
"STEP..." [color=lightblue];
"COMRING..." [color=lightblue];
"INTDOM..." [color=lightblue];
"GCDDOM..." [color=lightblue];
"FIELD..." [color=lightblue];
"ALGEBRA..." [color=lightblue];
"PFECAT..." [color=lightblue];

}
```

Chapter 18

Category Layer 17

18.1 AlgebraicallyClosedFunctionSpace (ACFS)



See:

⇐ "AlgebraicallyClosedField" (ACF) 17.1 on page 1061
⇐ "FunctionSpace" (FS) 17.5 on page 1095

Exports:

0	1	applyQuote	associates?
belong?	box	characteristic	charthRoot
coerce	commutator	conjugate	convert
D	definingPolynomial	denom	denominator
differentiate	distribute	divide	elt
euclideanSize	eval	even?	expressIdealMember
exquo	extendedEuclidean	factor	freeOf?
gcd	gcdPolynomial	ground	ground?
hash	height	inv	is?
isExpt	isMult	isPlus	isPower
isTimes	kernel	kernels	latex
lcm	mainKernel	map	max
min	minPoly	multiEuclidean	nthRoot
numer	numerator	odd?	one?
operator	operators	paren	patternMatch
prime?	principalIdeal	recip	reducedSystem
retract	retractIfCan	rootOf	rootsOf
sample	sizeLess?	sqrt	squareFree
squareFreePart	subst	subtractIfCan	tower
unit?	unitCanonical	unitNormal	univariate
variables	zero?	zeroOf	zerosOf
?*?	?**?	?+?	?-?
-?	?/?	?<?	?<=?
?=?	?>?	?>=?	?^?
?~=?	?quo?	?rem?	

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **nil**

These are implemented by this category:

```
rootOf : % -> %
rootOf : (%,Symbol) -> %
rootsOf : % -> List %
rootsOf : (%,Symbol) -> List %
rootsOf : (SparseUnivariatePolynomial %,Symbol) -> List %
zeroOf : % -> %
zeroOf : (%,Symbol) -> %
zeroOf : (SparseUnivariatePolynomial %,Symbol) -> %
zerosOf : % -> List %
zerosOf : (%,Symbol) -> List %
zerosOf : (SparseUnivariatePolynomial %,Symbol) -> List %
```

These exports come from (p1061) AlgebraicallyClosedField():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
nthRoot : (%,Integer) -> %
one? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
rootOf : SparseUnivariatePolynomial % -> %
rootOf : Polynomial % -> %
```

```
rootOf : (SparseUnivariatePolynomial %,Symbol) -> %
rootsOf : SparseUnivariatePolynomial % -> List %
rootsOf : Polynomial % -> List %
sample : () -> %
sizeLess? : (%,%) -> Boolean
sqrt : % -> %
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
zeroOf : Polynomial % -> %
zeroOf : SparseUnivariatePolynomial % -> %
zerosOf : Polynomial % -> List %
zerosOf : SparseUnivariatePolynomial % -> List %
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?**? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?/? : (%,%) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p1095) FunctionSpace(R)
where R:Join(OrderedSet, IntegralDomain)):

```
applyQuote : (Symbol,%) -> %
applyQuote : (Symbol,%,%) -> %
applyQuote : (Symbol,%,%,%) -> %
applyQuote : (Symbol,%,%,%,%) -> %
applyQuote : (Symbol,List %) -> %
belong? : BasicOperator -> Boolean
box : % -> %
box : List % -> %
```

```
charthRoot : % -> Union(%,"failed") if R has CHARNZ
coerce : R -> %
coerce : Symbol -> %
coerce : Kernel % -> %
coerce : Fraction Polynomial R -> % if R has INTDOM
coerce : Fraction Polynomial Fraction R -> % if R has INTDOM
coerce : SparseMultivariatePolynomial(R,Kernel %) -> %
  if R has RING
coerce : Fraction R -> % if R has INTDOM
coerce : Polynomial Fraction R -> % if R has INTDOM
coerce : Polynomial R -> % if R has RING
commutator : (%,%) -> % if R has GROUP
conjugate : (%,%) -> % if R has GROUP
convert : % -> InputForm if R has KONVERT INFORM
convert : % -> Pattern Integer if R has KONVERT PATTERN INT
convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT
convert : Factored % -> % if R has INTDOM
D : (%,List Symbol,List NonNegativeInteger) -> % if R has RING
D : (%,Symbol,NonNegativeInteger) -> % if R has RING
D : (%,List Symbol) -> % if R has RING
D : (%,Symbol) -> % if R has RING
definingPolynomial : % -> % if $ has RING
denom : % -> SparseMultivariatePolynomial(R,Kernel %) if R has INTDOM
denominator : % -> % if R has INTDOM
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if R has RING
differentiate : (%,Symbol,NonNegativeInteger) -> % if R has RING
differentiate : (%,List Symbol) -> % if R has RING
distribute : % -> %
distribute : (%,%) -> %
differentiate : (%,Symbol) -> % if R has RING
elt : (BasicOperator,%,%,%) -> %
elt : (BasicOperator,%,%,%,%) -> %
elt : (BasicOperator,%) -> %
elt : (BasicOperator,%,%) -> %
elt : (BasicOperator,List %) -> %
eval : (%,List BasicOperator,List (% -> %)) -> %
eval : (%,List Equation %) -> %
eval : (%,Symbol,(% -> %)) -> %
eval : (%,Symbol,(List % -> %)) -> %
eval : (%,BasicOperator,%,Symbol) -> % if R has KONVERT INFORM
eval : (%,BasicOperator,(List % -> %)) -> %
eval : (%,BasicOperator,(% -> %)) -> %
eval : (%,List Symbol,List (% -> %)) -> %
eval : (%,List BasicOperator,List (List % -> %)) -> %
eval : (%,List Symbol,List (List % -> %)) -> %
eval : (%,List %,List %) -> %
eval : (%,%,%) -> %
eval : (%,Equation %) -> %
eval : (%,Kernel %,%) -> %
```

```
eval : (%,List Symbol) -> % if R has KONVERT INFORM
eval : % -> % if R has KONVERT INFORM
eval : (%,Symbol) -> % if R has KONVERT INFORM
eval : (%,Symbol,NonNegativeInteger,(% -> %)) -> % if R has RING
eval : (%,Symbol,NonNegativeInteger,(List % -> %)) -> % if R has RING
eval :
  (%,List Symbol,List NonNegativeInteger,List (List % -> %)) -> %
    if R has RING
eval :
  (%,List Symbol,List NonNegativeInteger,List (% -> %)) -> %
    if R has RING
eval : (%,List Kernel %,List %) -> %
eval : (%,List BasicOperator,List %,Symbol) -> %
  if R has KONVERT INFORM
even? : % -> Boolean if $ has RETRACT INT
freeOf? : (%,%) -> Boolean
freeOf? : (%,Symbol) -> Boolean
ground : % -> R
ground? : % -> Boolean
height : % -> NonNegativeInteger
is? : (%,BasicOperator) -> Boolean
is? : (%,Symbol) -> Boolean
isExpt : % ->
  Union(Record(var: Kernel %,exponent: Integer),"failed")
    if R has SGROUP
isExpt :
  (%,BasicOperator) ->
    Union(Record(var: Kernel %,exponent: Integer),"failed")
      if R has RING
isExpt :
  (%,Symbol) ->
    Union(Record(var: Kernel %,exponent: Integer),"failed")
      if R has RING
isMult : % ->
  Union(Record(coef: Integer,var: Kernel %),"failed")
    if R has ABELSG
isPlus : % -> Union(List %,"failed") if R has ABELSG
isPower : % -> Union(Record(val: %,exponent: Integer),"failed")
    if R has RING
isTimes : % -> Union(List %,"failed") if R has SGROUP
kernel : (BasicOperator,List %) -> %
kernel : (BasicOperator,%) -> %
kernels : % -> List Kernel %
mainKernel : % -> Union(Kernel %,"failed")
map : ((% -> %),Kernel %) -> %
max : (%,%) -> %
min : (%,%) -> %
minPoly : Kernel % -> SparseUnivariatePolynomial % if $ has RING
numer : % -> SparseMultivariatePolynomial(R,Kernel %) if R has RING
numerator : % -> % if R has RING
```

```
odd? : % -> Boolean if $ has RETRACT INT
operator : BasicOperator -> BasicOperator
operators : % -> List BasicOperator
paren : % -> %
paren : List % -> %
patternMatch :
  (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
     PatternMatchResult(Integer,%)
       if R has PATMAB INT
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
     PatternMatchResult(Float,%)
       if R has PATMAB FLOAT
reducedSystem : Matrix % -> Matrix Integer
  if and(has(R,Ring),has(R,LinearlyExplicitRingOver Integer))
  or and(has(R,LinearlyExplicitRingOver Integer),has(R,Ring))
reducedSystem :
 (Matrix %,Vector %) ->
   Record(mat: Matrix Integer,vec: Vector Integer)
     if and(has(R,Ring),has(R,LinearlyExplicitRingOver Integer))
     or and(has(R,LinearlyExplicitRingOver Integer),has(R,Ring))
reducedSystem :
 (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
   if R has RING
reducedSystem : Matrix % -> Matrix R if R has RING
retract : % -> Kernel %
retract : % -> Fraction Polynomial R if R has INTDOM
retract : % -> Polynomial R if R has RING
retract : % -> R
retract : % -> Symbol
retract : % -> Integer if R has RETRACT INT
retract : % -> Fraction Integer
  if R has RETRACT INT
  and R has INTDOM
  or R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed")
  if R has RETRACT INT
  and R has INTDOM
  or R has RETRACT FRAC INT
retractIfCan : % -> Union(Kernel %,"failed")
retractIfCan : % -> Union(R,"failed")
retractIfCan : % -> Union(Fraction Polynomial R,"failed")
  if R has INTDOM
retractIfCan : % -> Union(Polynomial R,"failed") if R has RING
retractIfCan : % -> Union(Symbol,"failed")
subst : (%,List Kernel %,List %) -> %
subst : (%,List Equation %) -> %
subst : (%,Equation %) -> %
tower : % -> List Kernel %
```

```
univariate : (%,Kernel %) -> Fraction SparseUnivariatePolynomial %
  if R has INTDOM
variables : % -> List Symbol
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?*? : (R,%) -> % if R has COMRING
?*? : (%,R) -> % if R has COMRING
?/? :
 (SparseMultivariatePolynomial(R,Kernel %),
  SparseMultivariatePolynomial(R,Kernel %)) -> %
    if R has INTDOM
```

⟨*category ACFS AlgebraicallyClosedFunctionSpace*⟩≡

```
)abbrev category ACFS AlgebraicallyClosedFunctionSpace
++ Author: Manuel Bronstein
++ Date Created: 31 October 1988
++ Date Last Updated: 7 October 1991
++ Description:
++   Model for algebraically closed function spaces.
++ Keywords: algebraic, closure, field.
AlgebraicallyClosedFunctionSpace(R:Join(OrderedSet, IntegralDomain)):
 Category == Join(AlgebraicallyClosedField, FunctionSpace R) with
   rootOf : $ -> $
     ++ rootOf(p) returns y such that \spad{p(y) = 0}.
     ++ Error: if p has more than one variable y.
   rootsOf: $ -> List $
     ++ rootsOf(p, y) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0};
     ++ Note: the returned symbols y1,...,yn are bound in the interpreter
     ++ to respective root values.
     ++ Error: if p has more than one variable y.
   rootOf : ($, Symbol) -> $
     ++ rootOf(p,y) returns y such that \spad{p(y) = 0}.
     ++ The object returned displays as \spad{'y}.
   rootsOf: ($, Symbol) -> List $
     ++ rootsOf(p, y) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0};
     ++ The returned roots display as \spad{'y1},...,\spad{'yn}.
     ++ Note: the returned symbols y1,...,yn are bound in the interpreter
     ++ to respective root values.
   zeroOf : $ -> $
     ++ zeroOf(p) returns y such that \spad{p(y) = 0}.
     ++ The value y is expressed in terms of radicals if possible,and otherwise
     ++ as an implicit algebraic quantity.
     ++ Error: if p has more than one variable.
   zerosOf: $ -> List $
     ++ zerosOf(p) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
```

```
    ++ The yi's are expressed in radicals if possible.
    ++ The returned symbols y1,...,yn are bound in the interpreter
    ++ to respective root values.
    ++ Error: if p has more than one variable.
  zeroOf : ($, Symbol) -> $
    ++ zeroOf(p, y) returns y such that \spad{p(y) = 0}.
    ++ The value y is expressed in terms of radicals if possible,and otherwise
    ++ as an implicit algebraic quantity
    ++ which displays as \spad{'y}.
  zerosOf: ($, Symbol) -> List $
    ++ zerosOf(p, y) returns \spad{[y1,...,yn]} such that \spad{p(yi) = 0}.
    ++ The yi's are expressed in radicals if possible, and otherwise
    ++ as implicit algebraic quantities
    ++ which display as \spad{'yi}.
    ++ The returned symbols y1,...,yn are bound in the interpreter
    ++ to respective root values.
  add
    rootOf(p:$) ==
      empty?(l := variables p) => error "rootOf: constant expression"
      rootOf(p, first l)

    rootsOf(p:$) ==
      empty?(l := variables p) => error "rootsOf: constant expression"
      rootsOf(p, first l)

    zeroOf(p:$) ==
      empty?(l := variables p) => error "zeroOf: constant expression"
      zeroOf(p, first l)

    zerosOf(p:$) ==
      empty?(l := variables p) => error "zerosOf: constant expression"
      zerosOf(p, first l)

    zeroOf(p:$, x:Symbol) ==
      n := numer(f := univariate(p, kernel(x)$Kernel($)))
      degree denom f > 0 => error "zeroOf: variable appears in denom"
      degree n = 0 => error "zeroOf: constant expression"
      zeroOf(n, x)

    rootOf(p:$, x:Symbol) ==
      n := numer(f := univariate(p, kernel(x)$Kernel($)))
      degree denom f > 0 => error "roofOf: variable appears in denom"
      degree n = 0 => error "rootOf: constant expression"
      rootOf(n, x)

    zerosOf(p:$, x:Symbol) ==
```

```
    n := numer(f := univariate(p, kernel(x)$Kernel($)))
    degree denom f > 0 => error "zerosOf: variable appears in denom"
    degree n = 0 => empty()
    zerosOf(n, x)

  rootsOf(p:$, x:Symbol) ==
    n := numer(f := univariate(p, kernel(x)$Kernel($)))
    degree denom f > 0 => error "roofsOf: variable appears in denom"
    degree n = 0 => empty()
    rootsOf(n, x)

  rootsOf(p:SparseUnivariatePolynomial $, y:Symbol) ==
    (r := retractIfCan(p)@Union($,"failed")) case $ => rootsOf(r::$,y)
    rootsOf(p, y)$AlgebraicallyClosedField_&($)

  zerosOf(p:SparseUnivariatePolynomial $, y:Symbol) ==
    (r := retractIfCan(p)@Union($,"failed")) case $ => zerosOf(r::$,y)
    zerosOf(p, y)$AlgebraicallyClosedField_&($)

  zeroOf(p:SparseUnivariatePolynomial $, y:Symbol) ==
    (r := retractIfCan(p)@Union($,"failed")) case $ => zeroOf(r::$, y)
    zeroOf(p, y)$AlgebraicallyClosedField_&($)
```

⟨*ACFS.dotabb*⟩≡

```
"ACFS"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ACFS"];
"ACFS" -> "ACF"
"ACFS" -> "FS"
```

⟨*ACFS.dotfull*⟩≡

```
"AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralDomain))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ACFS"];
"AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralDomain))"
  -> "AlgebraicallyClosedField()"
"AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralDomain))"
  -> "FunctionSpace(a:OrderedSet)"
```

⟨*ACFS.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralDomain))"
 [color=lightblue];
"AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralDomain))"
  -> "AlgebraicallyClosedField()"
"AlgebraicallyClosedFunctionSpace(a:Join(OrderedSet,IntegralDomain))"
  -> "FunctionSpace(a:OrderedSet)"

"AlgebraicallyClosedField()" [color=lightblue];
"AlgebraicallyClosedField()" -> "Field()"
"AlgebraicallyClosedField()" -> "RadicalCategory()"

"Field()" [color=lightblue];
"Field()" -> "EUCDOM..."
"Field()" -> "UFD..."
"Field()" -> "DIVRING..."

"RadicalCategory()" [color=lightblue];
"RadicalCategory()" -> "Category"

"Category" [color=lightblue];

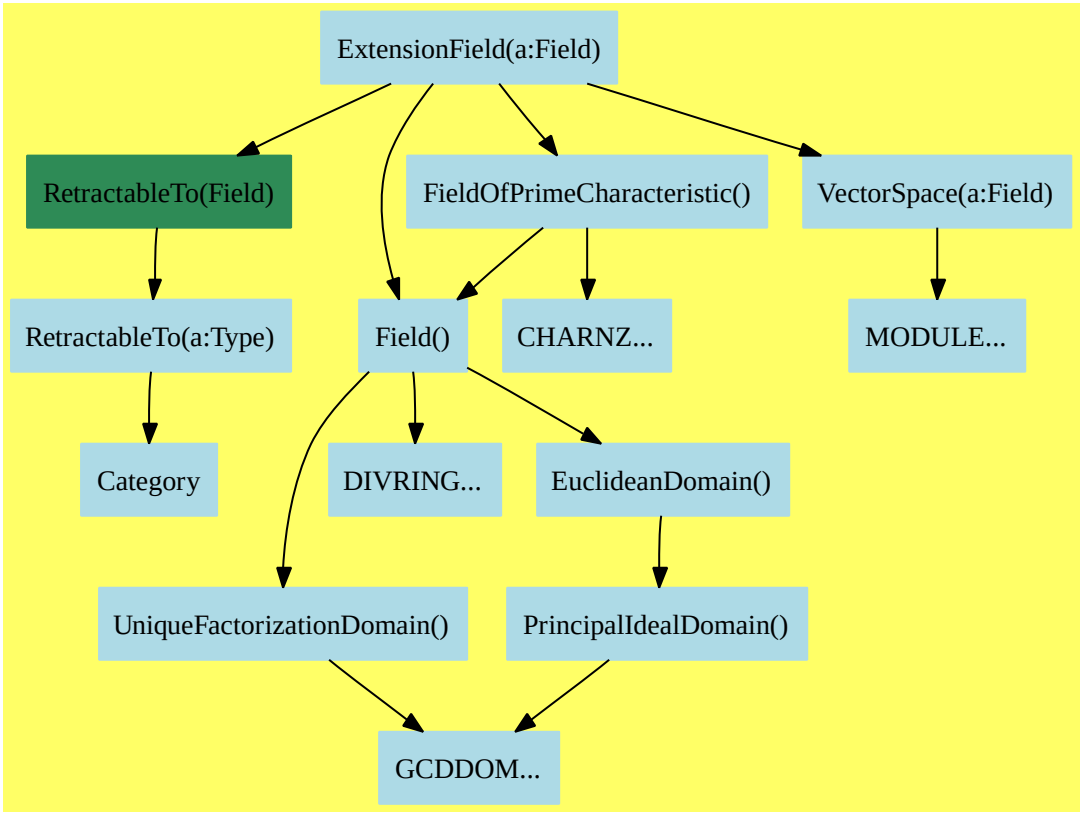
"FunctionSpace(a:OrderedSet)" [color=lightblue];
"FunctionSpace(a:OrderedSet)" -> "ExpressionSpace()"
"FunctionSpace(a:OrderedSet)" -> "RETRACT..."
"FunctionSpace(a:OrderedSet)" -> "PATAB..."
"FunctionSpace(a:OrderedSet)" -> "FPATMAB..."
"FunctionSpace(a:OrderedSet)" -> "FRETRCT..."

"ExpressionSpace()" [color=lightblue];
"ExpressionSpace()" -> "ORDSET..."
"ExpressionSpace()" -> "RETRACT..."
"ExpressionSpace()" -> "IEVALAB..."
"ExpressionSpace()" -> "EVALABLE..."

"UFD..." [color=lightblue];
"EUCDOM..." [color=lightblue];
"DIVRING..." [color=lightblue];
"EVALABLE..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"FPATMAB..." [color=lightblue];
```

```
  "IEVALAB..." [color=lightblue];
  "ORDSET..." [color=lightblue];
  "PATAB..." [color=lightblue];
  "RETRACT..." [color=lightblue];
  }
```

18.2 ExtensionField (XF)



See:

⇒ "FiniteAlgebraicExtensionField" (FAXF) 19.1 on page 1291
⇐ "Field" (FIELD) 16.1 on page 1005
⇐ "RetractableTo" (RETRACT) 2.18 on page 44
⇐ "VectorSpace" (VSPACE) 11.7 on page 803

Exports:

0	1	algebraic?	associates?
characteristic	charthRoot	coerce	degree
discreteLog	divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	extensionDegree	dimension
factor	Frobenius	gcd	gcdPolynomial
hash	inGroundField?	inv	latex
lcm	multiEuclidean	one?	order
prime?	primeFrbenius	principalIdeal	recip
retract	retractIfCan	sample	sizeLess?
squareFree	squareFreePart	subtractIfCan	transcendenceDegree
transcendent?	unit?	unitCanonical	unitNormal
zero?	?*?	?**?	?+?
?-?	-?	?/?	?=?
?^?	?quo?	?rem?	?~=?

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
inGroundField? : % -> Boolean
degree : % -> OnePointCompletion PositiveInteger
extensionDegree : () -> OnePointCompletion PositiveInteger
transcendenceDegree : () -> NonNegativeInteger
```

These are implemented by this category:

```
algebraic? : % -> Boolean
Frobenius : % -> % if F has FINITE
Frobenius : (%,NonNegativeInteger) -> % if F has FINITE
transcendent? : % -> Boolean
```

These exports come from (p1005) Field():

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%,%) ->
  Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) ->
  Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                     SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
squareFree : % -> Factored %
squareFreePart : % -> %
sizeLess? : (%,%) -> Boolean
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?/? : (%,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
```

```
?*? : (NonNegativeInteger,%) -> %
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (F,%) -> %
?*? : (%,F) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p44) RetractableTo(F:Field):

```
coerce : F -> %
retract : % -> F
retractIfCan : % -> Union(F,"failed")
```

These exports come from (p803) VectorSpace(F:Field):

```
dimension : () -> CardinalNumber
?/? : (%,F) -> %
```

These exports come from (p1084) FieldOfPrimeCharacteristic():

```
charthRoot : % -> Union(%,"failed")
    if F has CHARNZ or F has FINITE
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
    if F has CHARNZ or F has FINITE
order : % -> OnePointCompletion PositiveInteger
    if F has CHARNZ or F has FINITE
primeFrobenius : % -> %
    if F has CHARNZ or F has FINITE
primeFrobenius : (%,NonNegativeInteger) -> %
    if F has CHARNZ or F has FINITE
```

⟨*category XF ExtensionField*⟩≡

```
)abbrev category XF ExtensionField
++ Author: J. Grabmeier, A. Scheerhorn
++ Date Created: 10 March 1991
++ Date Last Updated: 31 March 1991
++ Basic Operations: _+, _*, extensionDegree, algebraic?, transcendent?
++ Related Constructors:
++ Also See:
++ AMS Classifications:
```

```
++ Keywords: field, extension field
++ References:
++  J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++  AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++  ExtensionField {\em F} is the category of fields which extend
++  the field F
ExtensionField(F:Field) : Category  == _
  Join(Field,RetractableTo F,VectorSpace F) with
    if F has CharacteristicZero then CharacteristicZero
    if F has CharacteristicNonZero then FieldOfPrimeCharacteristic
    algebraic? : $ -> Boolean
      ++ algebraic?(a) tests whether an element \spad{a} is algebraic with
      ++ respect to the ground field F.
    transcendent? : $ -> Boolean
      ++ transcendent?(a) tests whether an element \spad{a} is transcendent
      ++ with respect to the ground field F.
    inGroundField?: $ -> Boolean
      ++ inGroundField?(a) tests whether an element \spad{a}
      ++ is already in the ground field F.
    degree : $ -> OnePointCompletion PositiveInteger
      ++ degree(a) returns the degree of minimal polynomial of an element
      ++ \spad{a} if \spad{a} is algebraic
      ++ with respect to the ground field F, and \spad{infinity} otherwise.
    extensionDegree : () -> OnePointCompletion PositiveInteger
      ++ extensionDegree() returns the degree of the field extension if the
      ++ extension is algebraic, and \spad{infinity} if it is not.
    transcendenceDegree : () -> NonNegativeInteger
      ++ transcendenceDegree() returns the transcendence degree of the
      ++ field extension, 0 if the extension is algebraic.
    -- perhaps more absolute degree functions
    if F has Finite then
      FieldOfPrimeCharacteristic
      Frobenius: $ -> $
        ++ Frobenius(a) returns \spad{a ** q} where q is the \spad{size()$F}.
      Frobenius:   ($,NonNegativeInteger) -> $
        ++ Frobenius(a,s) returns \spad{a**(q**s)} where q is the size()$F.
  add
    algebraic?(a) == not infinite? (degree(a)@OnePointCompletion_
      (PositiveInteger))$OnePointCompletion(PositiveInteger)
    transcendent? a == infinite?(degree(a)@OnePointCompletion _
      (PositiveInteger))$OnePointCompletion(PositiveInteger)
    if F has Finite then
      Frobenius(a) == a ** size()$F
      Frobenius(a,s) == a ** (size()$F ** s)
```

⟨*XF.dotabb*⟩≡
```
"XF"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XF"];
"XF" -> "FIELD"
"XF" -> "RETRACT"
"XF" -> "VSPACE"
"XF" -> "FPC"
```

⟨*XF.dotfull*⟩≡
```
"ExtensionField(a:Field)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=XF"];
"ExtensionField(a:Field)" -> "Field()"
"ExtensionField(a:Field)" -> "RetractableTo(Field)"
"ExtensionField(a:Field)" -> "VectorSpace(a:Field)"
"ExtensionField(a:Field)" -> "FieldOfPrimeCharacteristic()"
```

⟨*XF.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ExtensionField(a:Field)" [color=lightblue];
"ExtensionField(a:Field)" -> "Field()"
"ExtensionField(a:Field)" -> "RetractableTo(Field)"
"ExtensionField(a:Field)" -> "VectorSpace(a:Field)"

"FieldOfPrimeCharacteristic()" [color=lightblue];
"FieldOfPrimeCharacteristic()" -> "CHARNZ..."
"FieldOfPrimeCharacteristic()" -> "Field()"

"Field()" [color=lightblue];
"Field()" -> "EuclideanDomain()"
"Field()" -> "UniqueFactorizationDomain()"
"Field()" -> "DIVRING..."

"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GCDDOM..."

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GCDDOM..."

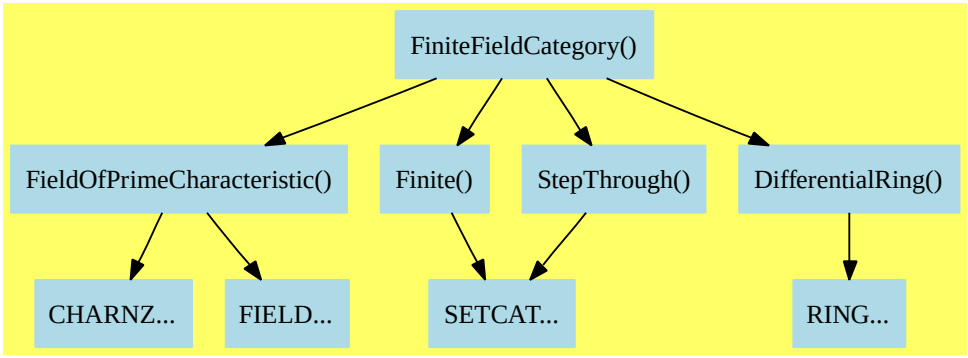
"RetractableTo(Field)" [color=seagreen];
"RetractableTo(Field)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"VectorSpace(a:Field)" [color=lightblue];
"VectorSpace(a:Field)" -> "MODULE..."

"MODULE..." [color=lightblue];
"DIVRING..." [color=lightblue];
"GCDDOM..." [color=lightblue];
"CHARNZ..." [color=lightblue];
"Category" [color=lightblue];
}
```

18.3 FiniteFieldCategory (FFIELDC)



See:

⇐ "DifferentialRing" (DIFRING) 10.5 on page 690
⇐ "FieldOfPrimeCharacteristic" (FPC) 17.3 on page 1084
⇐ "Finite" (FINITE) 4.9 on page 129
⇐ "StepThrough" (STEP) 4.26 on page 193

Exports:

0	1	associates?
characteristic	charthRoot	coerce
conditionP	createPrimitiveElement	D
differentiate	discreteLog	divide
euclideanSize	expressIdealMember	exquo
extendedEuclidean	factor	factorsOfCyclicGroupSize
gcd	gcdPolynomial	hash
index	init	inv
latex	lcm	lookup
multiEuclidean	nextItem	one?
order	prime?	primeFrobenius
primitive?	primitiveElement	principalIdeal
random	recip	representationType
sample	size	sizeLess?
squareFree	squareFreePart	subtractIfCan
tableForDiscreteLogarithm	unit?	unitCanonical
unitNormal	zero?	?*?
?**?	?+?	?-?
-?	?/?	?=?
?^?	?quo?	?rem?
?~=?		

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative

for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.

- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
factorsOfCyclicGroupSize : () ->
   List Record(factor: Integer,exponent: Integer)
primitiveElement : () -> %
representationType : () -> Union("prime",polynomial,normal,cyclic)
tableForDiscreteLogarithm : Integer ->
   Table(PositiveInteger,NonNegativeInteger)
```

These are implemented by this category:

```
charthRoot : % -> %
charthRoot : % -> Union(%,"failed")
conditionP : Matrix % -> Union(Vector %,"failed")
createPrimitiveElement : () -> %
differentiate : % -> %
discreteLog : % -> NonNegativeInteger
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                     SparseUnivariatePolynomial %
init : () -> %
nextItem : % -> Union(%,"failed")
order : % -> OnePointCompletion PositiveInteger
order : % -> PositiveInteger
primitive? : % -> Boolean
```

These exports come from (p1084) FieldOfPrimeCharacteristic():

```
0 : () -> %
1 : () -> %
```

```
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
exquo : (%,%) -> Union(%,"failed")
factor : % -> Factored %
gcd : List % -> %
gcd : (%,%) -> %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
prime? : % -> Boolean
primeFrobenius : % -> %
primeFrobenius : (%,NonNegativeInteger) -> %
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
sample : () -> %
sizeLess? : (%,%) -> Boolean
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,Integer) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
```

```
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,Integer) -> %
?/? : (%,%) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

These exports come from (p129) Finite():

```
index : PositiveInteger -> %
lookup : % -> PositiveInteger
random : () -> %
size : () -> NonNegativeInteger
```

These exports come from (p193) StepThrough():

These exports come from (p690) DifferentialRing():

```
D : % -> %
D : (%,NonNegativeInteger) -> %
differentiate : (%,NonNegativeInteger) -> %
```

⟨*category FFIELDC FiniteFieldCategory*⟩≡

```
)abbrev category FFIELDC FiniteFieldCategory
++ Author: J. Grabmeier, A. Scheerhorn
++ Date Created: 11 March 1991
++ Date Last Updated: 31 March 1991
++ Basic Operations: _+, _*, extensionDegree, order, primitiveElement
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: field, extension field, algebraic extension, finite field
++  Galois field
++ References:
++  D.Lipson, Elements of Algebra and Algebraic Computing, The
++  Benjamin/Cummings Publishing Company, Inc.-Menlo Park, California, 1981.
++  J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
++  AXIOM Technical Report Series, ATR/5 NP2522.
++ Description:
++  FiniteFieldCategory is the category of finite fields

FiniteFieldCategory() : Category ==_
  Join(FieldOfPrimeCharacteristic,Finite,StepThrough,DifferentialRing) with
--                 ,PolynomialFactorizationExplicit) with
    charthRoot: $ -> $
      ++ charthRoot(a) takes the characteristic'th root of {\em a}.
```

```
  ++ Note: such a root is alway defined in finite fields.
conditionP: Matrix $ -> Union(Vector $,"failed")
  ++ conditionP(mat), given a matrix representing a homogeneous system
  ++ of equations, returns a vector whose characteristic'th powers
  ++ is a non-trivial solution, or "failed" if no such vector exists.
-- the reason for implementing the following function is that we
-- can implement the functions order, getGenerator and primitive? on
-- category level without computing the, may be time intensive,
-- factorization of size()-1 at every function call again.
factorsOfCyclicGroupSize:_
  () -> List Record(factor:Integer,exponent:Integer)
  ++ factorsOfCyclicGroupSize() returns the factorization of size()-1
-- the reason for implementing the function tableForDiscreteLogarithm
-- is that we can implement the functions discreteLog and
-- shanksDiscLogAlgorithm on category level
-- computing the necessary exponentiation tables in the respective
-- domains once and for all
-- absoluteDegree : $ -> PositiveInteger
--  ++ degree of minimal polynomial, if algebraic with respect
--  ++ to the prime subfield
tableForDiscreteLogarithm: Integer -> _
         Table(PositiveInteger,NonNegativeInteger)
  ++ tableForDiscreteLogarithm(a,n) returns a table of the discrete
  ++ logarithms of \spad{a**0} up to \spad{a**(n-1)} which, called with
  ++ key \spad{lookup(a**i)} returns i for i in \spad{0..n-1}.
  ++ Error: if not called for prime divisors of order of
  ++        multiplicative group.
createPrimitiveElement: () -> $
  ++ createPrimitiveElement() computes a generator of the (cyclic)
  ++ multiplicative group of the field.
  -- RDJ: Are these next lines to be included?
  -- we run through the field and test, algorithms which construct
  -- elements of larger order were found to be too slow
primitiveElement: () -> $
  ++ primitiveElement() returns a primitive element stored in a global
  ++ variable in the domain.
  ++ At first call, the primitive element is computed
  ++ by calling \spadfun{createPrimitiveElement}.
primitive?: $ -> Boolean
  ++ primitive?(b) tests whether the element b is a generator of the
  ++ (cyclic) multiplicative group of the field, i.e. is a primitive
  ++ element.
  ++ Implementation Note: see ch.IX.1.3, th.2 in D. Lipson.
discreteLog: $ -> NonNegativeInteger
  ++ discreteLog(a) computes the discrete logarithm of \spad{a}
  ++ with respect to \spad{primitiveElement()} of the field.
```

```
    order: $ -> PositiveInteger
      ++ order(b) computes the order of an element b in the multiplicative
      ++ group of the field.
      ++ Error: if b equals 0.
    representationType: () -> Union("prime","polynomial","normal","cyclic")
      ++ representationType() returns the type of the representation, one of:
      ++ \spad{prime}, \spad{polynomial}, \spad{normal}, or \spad{cyclic}.
  add
    I   ==> Integer
    PI  ==> PositiveInteger
    NNI ==> NonNegativeInteger
    SUP ==> SparseUnivariatePolynomial
    DLP ==> DiscreteLogarithmPackage

    -- exported functions

    differentiate x          == 0

    init() == 0

    nextItem(a) ==
      zero?(a:=index(lookup(a)+1)) => "failed"
      a

    order(e):OnePointCompletion(PositiveInteger) ==
      (order(e)@PI)::OnePointCompletion(PositiveInteger)

    conditionP(mat:Matrix $) ==
      l:=nullSpace mat
      empty? l or every?(zero?, first l) => "failed"
      map(charthRoot,first l)

    charthRoot(x:$):$ == x**(size() quo characteristic())

    charthRoot(x:%):Union($,"failed") ==
        (charthRoot(x)@$)::Union($,"failed")

    createPrimitiveElement() ==
      sm1  : PositiveInteger := (size()$$-1) pretend PositiveInteger
      start : Integer :=
        -- in the polynomial case, index from 1 to characteristic-1
        -- gives prime field elements
        representationType = "polynomial" => characteristic()::Integer
        1
      found : Boolean := false
      for i in start..  while not found repeat
```

```
        e : $ := index(i::PositiveInteger)
        found := (order(e) = sm1)
      e

    primitive? a ==
      -- add special implementation for prime field case
      zero?(a) => false
      explist := factorsOfCyclicGroupSize()
      q:=(size()-1)@Integer
      equalone : Boolean := false
      for exp in explist while not equalone repeat
--        equalone := one?(a**(q quo exp.factor))
        equalone := ((a**(q quo exp.factor)) = 1)
      not equalone

    order e ==
      e = 0 => error "order(0) is not defined "
      ord:Integer:= size()-1 -- order e divides ord
      a:Integer:= 0
      lof:=factorsOfCyclicGroupSize()
      for rec in lof repeat -- run through prime divisors
        a := ord quo (primeDivisor := rec.factor)
--        goon := one?(e**a)
        goon := ((e**a) = 1)
        -- run through exponents of the prime divisors
        for j in 0..(rec.exponent)-2 while goon repeat
          -- as long as we get (e**ord = 1) we
          -- continue dividing by primeDivisor
          ord := a
          a := ord quo primeDivisor
--          goon := one?(e**a)
          goon := ((e**a) = 1)
        if goon then ord := a
        -- as we do a top down search we have found the
        -- correct exponent of primeDivisor in order e
        -- and continue with next prime divisor
      ord pretend PositiveInteger

    discreteLog(b) ==
      zero?(b) => error "discreteLog: logarithm of zero"
      faclist:=factorsOfCyclicGroupSize()
      a:=b
      gen:=primitiveElement()
      -- in GF(2) its necessary to have discreteLog(1) = 1
      b = gen => 1
      disclog:Integer:=0
```

```
      mult:Integer:=1
      groupord := (size() - 1)@Integer
      exp:Integer:=groupord
      for f in faclist repeat
        fac:=f.factor
        for t in 0..f.exponent-1 repeat
          exp:=exp quo fac
          -- shanks discrete logarithm algorithm
          exptable:=tableForDiscreteLogarithm(fac)
          n:=#exptable
          c:=a**exp
          end:=(fac - 1) quo n
          found:=false
          disc1:Integer:=0
          for i in 0..end while not found repeat
            rho:= search(lookup(c),exptable)_
                  $Table(PositiveInteger,NNI)
            rho case NNI =>
              found := true
              disc1:=((n * i + rho)@Integer) * mult
            c:=c* gen**((groupord quo fac) * (-n))
          not found => error "discreteLog: ?? discrete logarithm"
          -- end of shanks discrete logarithm algorithm
          mult := mult * fac
          disclog:=disclog+disc1
          a:=a * (gen ** (-disc1))
      disclog pretend NonNegativeInteger

    discreteLog(logbase,b) ==
      zero?(b) =>
        messagePrint("discreteLog: logarithm of zero")$OutputForm
        "failed"
      zero?(logbase) =>
        messagePrint("discreteLog: logarithm to base zero")$OutputForm
        "failed"
      b = logbase => 1
      not zero?((groupord:=order(logbase)@PI) rem order(b)@PI) =>
         messagePrint("discreteLog: second argument not in cyclic group_
generated by first argument")$OutputForm
         "failed"
      faclist:=factors factor groupord
      a:=b
      disclog:Integer:=0
      mult:Integer:=1
      exp:Integer:= groupord
      for f in faclist repeat
```

```
        fac:=f.factor
        primroot:= logbase ** (groupord quo fac)
        for t in 0..f.exponent-1 repeat
          exp:=exp quo fac
          rhoHelp:= shanksDiscLogAlgorithm(primroot,_
                a**exp,fac pretend NonNegativeInteger)$DLP($)
          rhoHelp case "failed" => return "failed"
          rho := (rhoHelp :: NNI) * mult
          disclog := disclog + rho
          mult := mult * fac
          a:=a * (logbase ** (-rho))
      disclog pretend NonNegativeInteger

    FP ==> SparseUnivariatePolynomial($)
    FRP ==> Factored FP
    f,g:FP

    squareFreePolynomial(f:FP):FRP ==
          squareFree(f)$UnivariatePolynomialSquareFree($,FP)

    factorPolynomial(f:FP):FRP == factor(f)$DistinctDegreeFactorize($,FP)

    factorSquareFreePolynomial(f:FP):FRP ==
        f = 0 => 0
        flist := distdfact(f,true)$DistinctDegreeFactorize($,FP)
        (flist.cont :: FP) *
            (*/[primeFactor(u.irr,u.pow) for u in flist.factors])

    gcdPolynomial(f:FP,g:FP):FP ==
         gcd(f,g)$EuclideanDomain_&(FP)
```

⟨*FFIELDC.dotabb*⟩≡

```
  "FFIELDC"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FFIELDC"];
  "FFIELDC" -> "FPC"
  "FFIELDC" -> "FINITE"
  "FFIELDC" -> "STEP"
  "FFIELDC" -> "DIFRING"
```

⟨*FFIELDC.dotfull*⟩≡
```
"FiniteFieldCategory()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FFIELDC"];
"FiniteFieldCategory()" -> "FieldOfPrimeCharacteristic()"
"FiniteFieldCategory()" -> "Finite()"
"FiniteFieldCategory()" -> "StepThrough()"
"FiniteFieldCategory()" -> "DifferentialRing()"
```

⟨*FFIELDC.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FiniteFieldCategory()" [color=lightblue];
"FiniteFieldCategory()" -> "FieldOfPrimeCharacteristic()"
"FiniteFieldCategory()" -> "Finite()"
"FiniteFieldCategory()" -> "StepThrough()"
"FiniteFieldCategory()" -> "DifferentialRing()"

"FieldOfPrimeCharacteristic()" [color=lightblue];
"FieldOfPrimeCharacteristic()" -> "CHARNZ..."
"FieldOfPrimeCharacteristic()" -> "FIELD..."

"Finite()" [color=lightblue];
"Finite()" -> "SETCAT..."

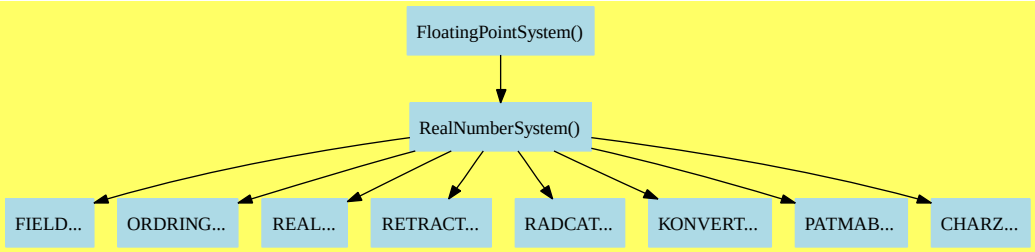
"StepThrough()" [color=lightblue];
"StepThrough()" -> "SETCAT..."

"DifferentialRing()" [color=lightblue];
"DifferentialRing()" -> "RING..."

"RING..." [color=lightblue];
"FIELD..." [color=lightblue];
"CHARNZ..." [color=lightblue];
"SETCAT..." [color=lightblue];

}
```

18.4 FloatingPointSystem (FPS)



See:

⇐ "RealNumberSystem" (RNS) 17.8 on page 1141

Exports:

0	1	abs	associates?
base	bits	characteristic	ceiling
coerce	convert	decreasePrecision	digits
divide	euclideanSize	exponent	expressIdealMember
exquo	extendedEuclidean	factor	float
floor	fractionPart	gcd	gcdPolynomial
hash	increasePrecision	inv	latex
lcm	mantissa	max	min
multiEuclidean	negative?	norm	nthRoot
one?	order	patternMatch	positive?
precision	prime?	principalIdeal	recip
retract	retractIfCan	round	sample
sign	sizeLess?	sqrt	squareFree
squareFreePart	subtractIfCan	truncate	unit?
unitCanonical	unitNormal	wholePart	zero?
?*?	?**?	?+?	?-?
-?	?/?	?<?	?<=?
?=?	?>?	?>=?	?^?
?~=?	?quo?	?rem?	

Attributes Exported:

- **approximate** means "is an approximation to the real numbers".
- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.

- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
base : () -> PositiveInteger
bits : () -> PositiveInteger
bits : PositiveInteger -> PositiveInteger
   if $ has arbitraryPrecision
decreasePrecision : Integer -> PositiveInteger
   if $ has arbitraryPrecision
digits : PositiveInteger -> PositiveInteger
   if $ has arbitraryPrecision
exponent : % -> Integer
float : (Integer,Integer,PositiveInteger) -> %
increasePrecision : Integer -> PositiveInteger
   if $ has arbitraryPrecision
mantissa : % -> Integer
max : () -> %
   if not has($,arbitraryPrecision)
   and not has($,arbitraryExponent)
min : () -> %
   if not has($,arbitraryPrecision)
   and not has($,arbitraryExponent)
order : % -> Integer
precision : () -> PositiveInteger
precision : PositiveInteger -> PositiveInteger
   if $ has arbitraryPrecision
```

These are implemented by this category:

```
digits : () -> PositiveInteger
float : (Integer,Integer) -> %
```

These exports come from (p1141) RealNumberSystem():

```
0 : () -> %
1 : () -> %
abs : % -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
ceiling : % -> %
```

```
coerce : Fraction Integer -> %
coerce : Integer -> %
coerce : Fraction Integer -> %
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
convert : % -> Pattern Float
convert : % -> DoubleFloat
convert : % -> Float
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
floor : % -> %
fractionPart : % -> %
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
      SparseUnivariatePolynomial %
hash : % -> SingleInteger
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
max : (%,%) -> %
min : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
negative? : % -> Boolean
norm : % -> %
nthRoot : (%,Integer) -> %
one? : % -> Boolean
patternMatch :
  (%,Pattern Float,PatternMatchResult(Float,%)) ->
     PatternMatchResult(Float,%)
positive? : % -> Boolean
prime? : % -> Boolean
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
retract : % -> Fraction Integer
retract : % -> Integer
retractIfCan : % -> Union(Fraction Integer,"failed")
retractIfCan : % -> Union(Integer,"failed")
round : % -> %
sample : () -> %
sign : % -> Integer
```

```
sizeLess? : (%,%) -> Boolean
sqrt : % -> %
squareFree : % -> Factored %
squareFreePart : % -> %
subtractIfCan : (%,%) -> Union(%,"failed")
truncate : % -> %
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
wholePart : % -> Integer
zero? : % -> Boolean
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,Fraction Integer) -> %
?**? : (%,Integer) -> %
?**? : (%,PositiveInteger) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?/? : (%,%) -> %
?<? : (%,%) -> Boolean
?<=? : (%,%) -> Boolean
?=? : (%,%) -> Boolean
?>? : (%,%) -> Boolean
?>=? : (%,%) -> Boolean
?^? : (%,Integer) -> %
?^? : (%,PositiveInteger) -> %
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
```

⟨*category FPS FloatingPointSystem*⟩≡

```
)abbrev category FPS FloatingPointSystem
++ Author:
++ Date Created:
++ Change History:
++ Basic Operations: approximate, base, bits, digits, exponent, float,
++    mantissa, order, precision, round?
++ Related Constructors:
++ Keywords: float, floating point
++ Description:
++ This category is intended as a model for floating point systems.
++ A floating point system is a model for the real numbers.  In fact,
```

```
++ it is an approximation in the sense that not all real numbers are
++ exactly representable by floating point numbers.
++ A floating point system is characterized by the following:
++
++   1: \spadfunFrom{base}{FloatingPointSystem} of the
++      \spadfunFrom{exponent}{FloatingPointSystem}.
++      (actual implemenations are usually binary or decimal)
++   2: \spadfunFrom{precision}{FloatingPointSystem} of the
++       \spadfunFrom{mantissa}{FloatingPointSystem} (arbitrary or fixed)
++   3: rounding error for operations
--++   4:  when, and what happens if exponent overflow/underflow occurs
++
++ Because a Float is an approximation to the real numbers, even though
++ it is defined to be a join of a Field and OrderedRing, some of
++ the attributes do not hold.  In particular associative("+")
++ does not hold.  Algorithms defined over a field need special
++ considerations when the field is a floating point system.
FloatingPointSystem(): Category == RealNumberSystem() with
   approximate
      ++ \spad{approximate} means "is an approximation to the real numbers".
   float: (Integer,Integer) -> %
      ++ float(a,e) returns \spad{a * base() ** e}.
   float: (Integer,Integer,PositiveInteger) -> %
      ++ float(a,e,b) returns \spad{a * b ** e}.
   order: % -> Integer
      ++ order x is the order of magnitude of x.
      ++ Note: \spad{base ** order x <= |x| < base ** (1 + order x)}.
   base: () -> PositiveInteger
      ++ base() returns the base of the
      ++\spadfunFrom{exponent}{FloatingPointSystem}.

   exponent: % -> Integer
      ++ exponent(x) returns the
      ++ \spadfunFrom{exponent}{FloatingPointSystem} part of x.

   mantissa: % -> Integer
      ++ mantissa(x) returns the mantissa part of x.
  -- round?: () -> B
  --    ++ round?() returns the rounding or chopping.

   bits: () -> PositiveInteger
      ++ bits() returns ceiling's precision in bits.
   digits: () -> PositiveInteger
      ++ digits() returns ceiling's precision in decimal digits.
   precision: () -> PositiveInteger
      ++ precision() returns the precision in digits base.
```

```
    if % has arbitraryPrecision then
       bits: PositiveInteger -> PositiveInteger
         ++ bits(n) set the \spadfunFrom{precision}{FloatingPointSystem}
         ++ to n bits.

       digits: PositiveInteger -> PositiveInteger
         ++ digits(d) set the \spadfunFrom{precision}{FloatingPointSystem}
         ++ to d digits.

       precision: PositiveInteger -> PositiveInteger
         ++ precision(n) set the precision in the base to n decimal digits.

       increasePrecision: Integer -> PositiveInteger
         ++ increasePrecision(n) increases the current
         ++ \spadfunFrom{precision}{FloatingPointSystem} by n decimal digits.

       decreasePrecision: Integer -> PositiveInteger
         ++ decreasePrecision(n) decreases the current
         ++ \spadfunFrom{precision}{FloatingPointSystem} precision
         ++ by n decimal digits.

    if not (% has arbitraryExponent) then
     --  overflow: (()->Exit) -> Void
     --    ++ overflow() returns the Exponent overflow of float
     --  underflow: (()->Exit) -> Void
     --    ++ underflow() returns the Exponent underflow of float
     --  maxExponent: () -> Integer
     --    ++ maxExponent() returns the max Exponent of float
       if not (% has arbitraryPrecision) then
          min: () -> %
           ++ min() returns the minimum floating point number.
          max: () -> %
           ++ max() returns the maximum floating point number.
  add
    float(ma, ex) == float(ma, ex, base())
    digits() == max(1,4004 * (bits()-1) quo 13301)::PositiveInteger
```

⟨*FPS.dotabb*⟩≡

```
  "FPS"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FPS"];
  "FPS" -> "RNS"
```

⟨*FPS.dotfull*⟩≡
```
"FloatingPointSystem()"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FPS"];
"FloatingPointSystem()" -> "RealNumberSystem()"
```

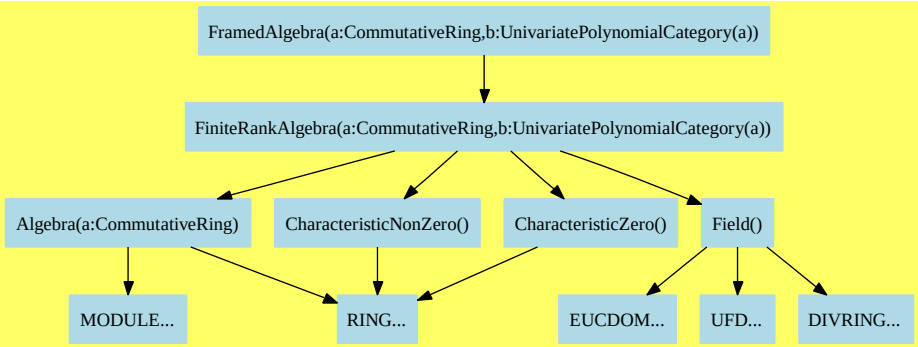
⟨*FPS.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FloatingPointSystem()" [color=lightblue];
"FloatingPointSystem()" -> "RealNumberSystem()"

"RealNumberSystem()" [color=lightblue];
"RealNumberSystem()" -> "FIELD..."
"RealNumberSystem()" -> "ORDRING..."
"RealNumberSystem()" -> "REAL..."
"RealNumberSystem()" -> "RETRACT..."
"RealNumberSystem()" -> "RADCAT..."
"RealNumberSystem()" -> "KONVERT..."
"RealNumberSystem()" -> "PATMAB..."
"RealNumberSystem()" -> "CHARZ..."

"FIELD..." [color=lightblue];
"ORDRING..." [color=lightblue];
"REAL..." [color=lightblue];
"RETRACT..." [color=lightblue];
"RADCAT..." [color=lightblue];
"KONVERT..." [color=lightblue];
"PATMAB..." [color=lightblue];
"CHARZ..." [color=lightblue];
}
```

18.5 FramedAlgebra (FRAMALG)



See:

⇒ "MonogenicAlgebra" (MONOGEN) 19.2 on page 1305
⇐ "FiniteRankAlgebra" (FINRALG) 17.4 on page 1089

Exports:

0	1	basis
characteristic	characteristicPolynomial	charthRoot
coerce	convert	coordinates
discriminant	hash	latex
minimalPolynomial	norm	one?
rank	recip	regularRepresentation
represents	sample	subtractIfCan
trace	traceMatrix	zero?
?*?	?**?	?+?
?-?	-?	?=?
?^?	?~=?	

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
basis : () -> Vector %
represents : Vector R -> %
```

These are implemented by this category:

```
convert : Vector R -> %
convert : % -> Vector R
coordinates : Vector % -> Matrix R
coordinates : % -> Vector R
discriminant : () -> R
regularRepresentation : % -> Matrix R
traceMatrix : () -> Matrix R
```

These exports come from (p1089) FiniteRankAlgebra(R, UP)
where R:CommutativeRing and UP:UnivariatePolynomialCategory R):

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
characteristicPolynomial : % -> UP
charthRoot : % -> Union(%,"failed") if R has CHARNZ
coerce : R -> %
coerce : Integer -> %
coerce : % -> OutputForm
coordinates : (%,Vector %) -> Vector R
coordinates : (Vector %,Vector %) -> Matrix R
discriminant : Vector % -> R
hash : % -> SingleInteger
latex : % -> String
minimalPolynomial : % -> UP if R has FIELD
norm : % -> R
one? : % -> Boolean
rank : () -> PositiveInteger
recip : % -> Union(%,"failed")
regularRepresentation : (%,Vector %) -> Matrix R
represents : (Vector R,Vector %) -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
trace : % -> R
traceMatrix : Vector % -> Matrix R
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (R,%) -> %
?*? : (%,R) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

```
?^? : (%,NonNegativeInteger) -> %
```

⟨*category FRAMALG FramedAlgebra*⟩≡

```
)abbrev category FRAMALG FramedAlgebra
++ Author: Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A \spadtype{FramedAlgebra} is a \spadtype{FiniteRankAlgebra} together
++ with a fixed R-module basis.

FramedAlgebra(R:CommutativeRing, UP:UnivariatePolynomialCategory R):
 Category == FiniteRankAlgebra(R, UP) with
      basis                 : () -> Vector %
        ++ basis() returns the fixed R-module basis.
      coordinates           : % -> Vector R
        ++ coordinates(a) returns the coordinates of \spad{a} with
        ++ respect to the fixed R-module basis.
      coordinates           : Vector % -> Matrix R
        ++ coordinates([v1,...,vm]) returns the coordinates of the
        ++ vi's with to the fixed basis.  The coordinates of vi are
        ++ contained in the ith row of the matrix returned by this
        ++ function.
      represents            : Vector R -> %
        ++ represents([a1,..,an]) returns \spad{a1*v1 + ... + an*vn}, where
        ++ v1, ..., vn are the elements of the fixed basis.
      convert               : % -> Vector R
        ++ convert(a) returns the coordinates of \spad{a} with respect to the
        ++ fixed R-module basis.
      convert               : Vector R -> %
        ++ convert([a1,..,an]) returns \spad{a1*v1 + ... + an*vn}, where
        ++ v1, ..., vn are the elements of the fixed basis.
      traceMatrix           : () -> Matrix R
        ++ traceMatrix() is the n-by-n matrix ( \spad{Tr(vi * vj)} ), where
        ++ v1, ..., vn are the elements of the fixed basis.
      discriminant          : () -> R
        ++ discriminant() = determinant(traceMatrix()).
      regularRepresentation : % -> Matrix R
        ++ regularRepresentation(a) returns the matrix of the linear
        ++ map defined by left multiplication by \spad{a} with respect
```

```
      ++ to the fixed basis.
   --attributes
     --separable <=> discriminant() ^= 0
 add
  convert(x:%):Vector(R)  == coordinates(x)
  convert(v:Vector R):%   == represents(v)
  traceMatrix()           == traceMatrix basis()
  discriminant()          == discriminant basis()
  regularRepresentation x == regularRepresentation(x, basis())
  coordinates x           == coordinates(x, basis())
  represents x            == represents(x, basis())

  coordinates(v:Vector %) ==
    m := new(#v, rank(), 0)$Matrix(R)
    for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
      setRow_!(m, j, coordinates qelt(v, i))
    m

  regularRepresentation x ==
    m := new(n := rank(), n, 0)$Matrix(R)
    b := basis()
    for i in minIndex b .. maxIndex b for j in minRowIndex m .. repeat
      setRow_!(m, j, coordinates(x * qelt(b, i)))
    m

  characteristicPolynomial x ==
     mat00 := (regularRepresentation x)
     mat0 := map(y+->y::UP,mat00)$MatrixCategoryFunctions2(R, Vector R,
                 Vector R, Matrix R, UP, Vector UP,Vector UP, Matrix UP)
     mat1 : Matrix UP := scalarMatrix(rank(),monomial(1,1)$UP)
     determinant(mat1 - mat0)

  if R has Field then
   -- depends on the ordering of results from nullSpace, also see FFP
     minimalPolynomial(x:%):UP ==
       y:%:=1
       n:=rank()
       m:Matrix R:=zero(n,n+1)
       for i in 1..n+1 repeat
         setColumn_!(m,i,coordinates(y))
         y:=y*x
       v:=first nullSpace(m)
       +/[monomial(v.(i+1),i) for i in 0..#v-1]
```

⟨*FRAMALG.dotabb*⟩≡
```
"FRAMALG"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRAMALG"];
"FRAMALG" -> "FINRALG"
```

⟨*FRAMALG.dotfull*⟩≡
```
"FramedAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FRAMALG"];
"FramedAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
   "FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
```

⟨*FRAMALG.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FramedAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue];
"FramedAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
   "FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"

"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue];
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "Algebra(a:CommutativeRing)"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "Field()"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CharacteristicNonZero()"
"FiniteRankAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CharacteristicZero()"

"Algebra(a:CommutativeRing)" [color=lightblue];
"Algebra(a:CommutativeRing)" -> "RING..."
"Algebra(a:CommutativeRing)" -> "MODULE..."

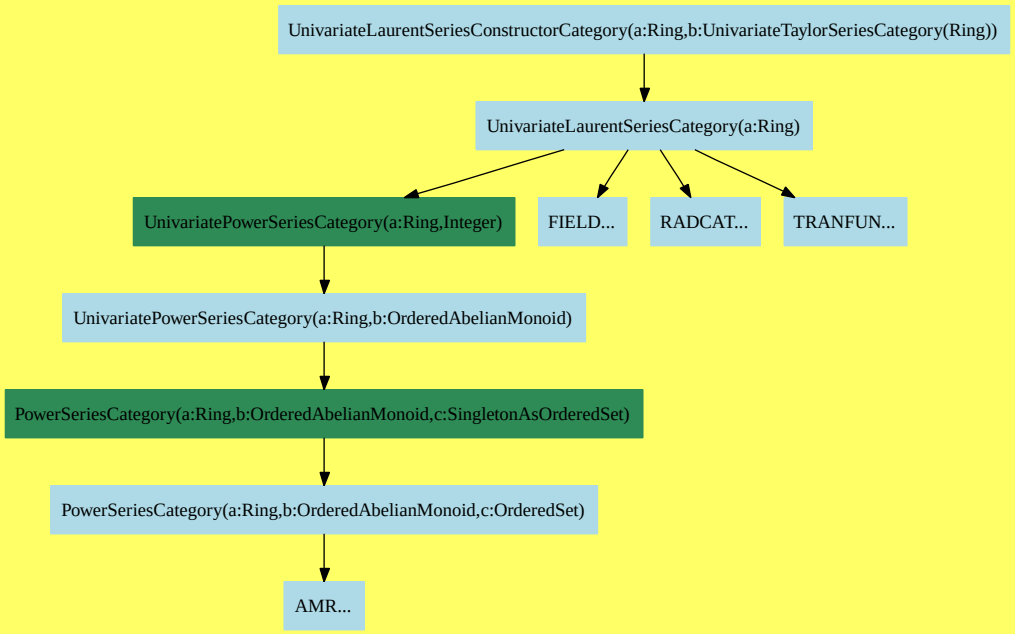
"Field()" [color=lightblue];
"Field()" -> "EUCDOM..."
"Field()" -> "UFD..."
"Field()" -> "DIVRING..."

"CharacteristicNonZero()" [color=lightblue];
"CharacteristicNonZero()" -> "RING..."

"CharacteristicZero()" [color=lightblue];
"CharacteristicZero()" -> "RING..."

"EUCDOM..." [color=lightblue];
"UFD..." [color=lightblue];
"DIVRING..." [color=lightblue];
"RING..." [color=lightblue];
"MODULE..." [color=lightblue];
}
```

18.6 UnivariateLaurentSeriesConstructorCategory (ULSCCAT)



See:

⇐ "RetractableTo" (RETRACT) 2.18 on page 44
⇐ "UnivariateLaurentSeriesCategory" (ULSCAT) 17.10 on page 1186

Exports:

0	1	abs
acos	acosh	acot
acoth	acsc	acsch
approximate	asec	asech
asin	asinh	associates?
atan	atanh	ceiling
center	characteristic	charthRoot
coefficient	coerce	complete
conditionP	convert	cos
cosh	cot	coth
csc	csch	D
degree	denom	denominator
differentiate	divide	euclideanSize
eval	exp	expressIdealMember
exquo	extend	extendedEuclidean
factor	factorPolynomial	factorSquareFreePolynomial
floor	fractionPart	gcd
gcdPolynomial	hash	init
integrate	inv	latex
laurent	lcm	leadingCoefficient
leadingMonomial	log	map
max	min	monomial
monomial?	multiEuclidean	multiplyCoefficients
multiplyExponents	negative?	nextItem
nthRoot	numer	numerator
one?	order	patternMatch
pi	pole?	positive?
prime?	principalIdeal	random
rationalFunction	recip	reducedSystem
reductum	removeZeroes	retract
retractIfCan	sample	sec
sech	series	sign
sin	sinh	sizeLess?
solveLinearPolynomialEquation	sqrt	squareFree
squareFreePart	squareFreePolynomial	subtractIfCan
tan	tanh	taylor
taylorIfCan	taylorRep	terms
truncate	unit?	unitCanonical
unitNormal	variable	variables
wholePart	zero?	?*?
?**?	?+?	?-?
-?	?=?	?^?
?~=?	?/?	?<?
?<=?	?>?	?>=?
?.?	?quo?	?rem?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has Field then canonicalClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- if #1 has Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if #1 has CommutativeRing then commutative("*") where **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if #1 has Field then nil

These are directly exported but not implemented:

```
coerce : UTS -> %
degree : % -> Integer
laurent : (Integer,UTS) -> %
removeZeroes : % -> %
removeZeroes : (Integer,%) -> %
taylor : % -> UTS
taylorIfCan : % -> Union(UTS,"failed")
taylorRep : % -> UTS
```

These are implemented by this category:

```
retract : % -> UTS
retractIfCan : % -> Union(UTS,"failed")
zero? : % -> Boolean
```

These exports come from (p1186) UnivariateLaurentSeriesCategory(Coef:Ring)

```
0 : () -> %
1 : () -> %
acos : % -> % if Coef has ALGEBRA FRAC INT
acosh : % -> % if Coef has ALGEBRA FRAC INT
```

```
acot : % -> % if Coef has ALGEBRA FRAC INT
acoth : % -> % if Coef has ALGEBRA FRAC INT
acsc : % -> % if Coef has ALGEBRA FRAC INT
acsch : % -> % if Coef has ALGEBRA FRAC INT
approximate : (%,Integer) -> Coef
  if Coef has **: (Coef,Integer) -> Coef
  and Coef has coerce: Symbol -> Coef
asec : % -> % if Coef has ALGEBRA FRAC INT
asech : % -> % if Coef has ALGEBRA FRAC INT
asin : % -> % if Coef has ALGEBRA FRAC INT
asinh : % -> % if Coef has ALGEBRA FRAC INT
associates? : (%,%) -> Boolean if Coef has INTDOM
atan : % -> % if Coef has ALGEBRA FRAC INT
atanh : % -> % if Coef has ALGEBRA FRAC INT
center : % -> Coef
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed")
  if and(OR(has(UTS,CharacteristicNonZero),
            and(has($,CharacteristicNonZero),
                has(UTS,PolynomialFactorizationExplicit))),
         has(Coef,Field))
  or Coef has CHARNZ
coefficient : (%,Integer) -> Coef
coerce : % -> % if Coef has INTDOM
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : Coef -> % if Coef has COMRING
coerce : Integer -> %
coerce : % -> OutputForm
complete : % -> %
cos : % -> % if Coef has ALGEBRA FRAC INT
cosh : % -> % if Coef has ALGEBRA FRAC INT
cot : % -> % if Coef has ALGEBRA FRAC INT
coth : % -> % if Coef has ALGEBRA FRAC INT
csc : % -> % if Coef has ALGEBRA FRAC INT
csch : % -> % if Coef has ALGEBRA FRAC INT
D : % -> %
  if and(has(UTS,DifferentialRing),has(Coef,Field))
  or Coef has *: (Integer,Coef) -> Coef
D : (%,NonNegativeInteger) -> %
  if and(has(UTS,DifferentialRing),has(Coef,Field))
  or Coef has *: (Integer,Coef) -> Coef
D : (%,Symbol) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
D : (%,List Symbol) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
D : (%,Symbol,NonNegativeInteger) -> %
```

```
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
D : (%,List Symbol,List NonNegativeInteger) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,Symbol) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,List Symbol) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,Symbol,NonNegativeInteger) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if and(has(UTS,PartialDifferentialRing Symbol),has(Coef,Field))
  or Coef has PDRING SYMBOL
  and Coef has *: (Integer,Coef) -> Coef
differentiate : % -> %
  if and(has(UTS,DifferentialRing),has(Coef,Field))
  or Coef has *: (Integer,Coef) -> Coef
differentiate : (%,NonNegativeInteger) -> %
  if and(has(UTS,DifferentialRing),has(Coef,Field))
  or Coef has *: (Integer,Coef) -> Coef
divide : (%,%) -> Record(quotient: %,remainder: %)
  if Coef has FIELD
euclideanSize : % -> NonNegativeInteger if Coef has FIELD
eval : (%,Coef) -> Stream Coef
  if Coef has **: (Coef,Integer) -> Coef
exp : % -> % if Coef has ALGEBRA FRAC INT
expressIdealMember : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
extend : (%,Integer) -> %
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
  if Coef has FIELD
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
  if Coef has FIELD
factor : % -> Factored % if Coef has FIELD
gcd : (%,%) -> % if Coef has FIELD
gcd : List % -> % if Coef has FIELD
gcdPolynomial :
 (SparseUnivariatePolynomial %,
  SparseUnivariatePolynomial %) ->
    SparseUnivariatePolynomial %
```

```
      if Coef has FIELD
hash : % -> SingleInteger
integrate : (%,Symbol) -> %
  if Coef has ACFS INT
  and Coef has PRIMCAT
  and Coef has TRANFUN
  and Coef has ALGEBRA FRAC INT
  or Coef has variables: Coef -> List Symbol
  and Coef has integrate: (Coef,Symbol) -> Coef
  and Coef has ALGEBRA FRAC INT
integrate : % -> % if Coef has ALGEBRA FRAC INT
inv : % -> % if Coef has FIELD
latex : % -> String
lcm : (%,%) -> % if Coef has FIELD
lcm : List % -> % if Coef has FIELD
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
log : % -> % if Coef has ALGEBRA FRAC INT
map : ((Coef -> Coef),%) -> %
monomial : (%,List SingletonAsOrderedSet,List Integer) -> %
monomial : (%,SingletonAsOrderedSet,Integer) -> %
monomial : (Coef,Integer) -> %
monomial? : % -> Boolean
multiEuclidean : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
multiplyCoefficients : ((Integer -> Coef),%) -> %
multiplyExponents : (%,PositiveInteger) -> %
nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
one? : % -> Boolean
order : (%,Integer) -> Integer
order : % -> Integer
pi : () -> % if Coef has ALGEBRA FRAC INT
pole? : % -> Boolean
prime? : % -> Boolean if Coef has FIELD
principalIdeal : List % -> Record(coef: List %,generator: %)
  if Coef has FIELD
rationalFunction : (%,Integer) -> Fraction Polynomial Coef
  if Coef has INTDOM
rationalFunction : (%,Integer,Integer) -> Fraction Polynomial Coef
  if Coef has INTDOM
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
sec : % -> % if Coef has ALGEBRA FRAC INT
sech : % -> % if Coef has ALGEBRA FRAC INT
series : Stream Record(k: Integer,c: Coef) -> %
sin : % -> % if Coef has ALGEBRA FRAC INT
sinh : % -> % if Coef has ALGEBRA FRAC INT
sizeLess? : (%,%) -> Boolean if Coef has FIELD
squareFree : % -> Factored % if Coef has FIELD
```

```
squareFreePart : % -> % if Coef has FIELD
sqrt : % -> % if Coef has ALGEBRA FRAC INT
subtractIfCan : (%,%) -> Union(%,"failed")
tan : % -> % if Coef has ALGEBRA FRAC INT
tanh : % -> % if Coef has ALGEBRA FRAC INT
terms : % -> Stream Record(k: Integer,c: Coef)
truncate : (%,Integer,Integer) -> %
truncate : (%,Integer) -> %
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
  if Coef has INTDOM
variable : % -> Symbol
variables : % -> List SingletonAsOrderedSet
?.? : (%,Integer) -> Coef
?**? : (%,Integer) -> % if Coef has FIELD
?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?**? : (%,NonNegativeInteger) -> %
?^? : (%,Integer) -> % if Coef has FIELD
?^? : (%,NonNegativeInteger) -> %
?/? : (%,%) -> % if Coef has FIELD
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
?^? : (%,PositiveInteger) -> %
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
?.? : (%,%) -> % if Integer has SGROUP
?quo? : (%,%) -> % if Coef has FIELD
?rem? : (%,%) -> % if Coef has FIELD
```

These exports come from (p1121) QuotientFieldCategory(UTS)
where UTS:UnivariateLaurentSeriesCategory(Coef:Ring)

```
abs : % -> %
  if and(has(UTS,OrderedIntegralDomain),has(Coef,Field))
ceiling : % -> UTS
  if and(has(UTS,IntegerNumberSystem),has(Coef,Field))
conditionP : Matrix % -> Union(Vector %,"failed")
```

```
  if and(and(has($,CharacteristicNonZero),
             has(UTS,PolynomialFactorizationExplicit)),
         has(Coef,Field))
coerce : Symbol -> %
  if and(has(UTS,RetractableTo Symbol),has(Coef,Field))
convert : % -> Pattern Integer
  if and(has(UTS,ConvertibleTo Pattern Integer),has(Coef,Field))
convert : % -> Pattern Float
  if and(has(UTS,ConvertibleTo Pattern Float),has(Coef,Field)\\
convert : % -> InputForm
  if and(has(UTS,ConvertibleTo InputForm),has(Coef,Field))
convert : % -> Float
  if and(has(UTS,RealConstant),has(Coef,Field))
convert : % -> DoubleFloat
  if and(has(UTS,RealConstant),has(Coef,Field))
D : (%,(UTS -> UTS),NonNegativeInteger) -> %
  if Coef has FIELD
D : (%,(UTS -> UTS)) -> % if Coef has FIELD
denom : % -> UTS if Coef has FIELD
denominator : % -> % if Coef has FIELD
differentiate : (%,(UTS -> UTS)) -> % if Coef has FIELD
differentiate : (%,(UTS -> UTS),NonNegativeInteger) -> %
  if Coef has FIELD
eval : (%,Equation UTS) -> %
  if and(has(UTS,Evalable UTS),has(Coef,Field))
eval : (%,List Symbol,List UTS) -> %
  if and(has(UTS,InnerEvalable(Symbol,UTS)),has(Coef,Field))
eval : (%,List Equation UTS) -> %
  if and(has(UTS,Evalable UTS),has(Coef,Field))
eval : (%,UTS,UTS) -> %
  if and(has(UTS,Evalable UTS),has(Coef,Field))
eval : (%,List UTS,List UTS) -> %
  if and(has(UTS,Evalable UTS),has(Coef,Field))
eval : (%,Symbol,UTS) -> %
  if and(has(UTS,InnerEvalable(Symbol,UTS)),has(Coef,Field))
factorPolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
     if and(has(UTS,PolynomialFactorizationExplicit),has(Coef,Field))
factorSquareFreePolynomial :
  SparseUnivariatePolynomial % ->
    Factored SparseUnivariatePolynomial %
     if and(has(UTS,PolynomialFactorizationExplicit),has(Coef,Field))
floor : % -> UTS
  if and(has(UTS,IntegerNumberSystem),has(Coef,Field))
fractionPart : % -> %
  if and(has(UTS,EuclideanDomain),has(Coef,Field))
init : () -> % if and(has(UTS,StepThrough),has(Coef,Field))
map : ((UTS -> UTS),%) -> % if Coef has FIELD
max : (%,%) -> % if and(has(UTS,OrderedSet),has(Coef,Field))
```

```
min : (%,%) -> % if and(has(UTS,OrderedSet),has(Coef,Field))
negative? : % -> Boolean
  if and(has(UTS,OrderedIntegralDomain),has(Coef,Field))
nextItem : % -> Union(%,"failed")
  if and(has(UTS,StepThrough),has(Coef,Field))
numer : % -> UTS if Coef has FIELD
numerator : % -> % if Coef has FIELD
patternMatch :
 (%,Pattern Integer,PatternMatchResult(Integer,%)) ->
   PatternMatchResult(Integer,%)
     if and(has(UTS,PatternMatchable Integer),has(Coef,Field))
patternMatch :
 (%,Pattern Float,PatternMatchResult(Float,%)) ->
   PatternMatchResult(Float,%)
     if and(has(UTS,PatternMatchable Float),has(Coef,Field))
positive? : % -> Boolean
  if and(has(UTS,OrderedIntegralDomain),has(Coef,Field))
random : () -> %
  if and(has(UTS,IntegerNumberSystem),has(Coef,Field))
reducedSystem :
 (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer)
  if and(has(UTS,LinearlyExplicitRingOver Integer),has(Coef,Field))
reducedSystem : Matrix % -> Matrix Integer
  if and(has(UTS,LinearlyExplicitRingOver Integer),has(Coef,Field))
reducedSystem :
 (Matrix %,Vector %) -> Record(mat: Matrix UTS,vec: Vector UTS)
   if Coef has FIELD
reducedSystem : Matrix % -> Matrix UTS if Coef has FIELD
retract : % -> Symbol
  if and(has(UTS,RetractableTo Symbol),has(Coef,Field))
retract : % -> Integer
  if and(has(UTS,RetractableTo Integer),has(Coef,Field))
retract : % -> Fraction Integer
  if and(has(UTS,RetractableTo Integer),has(Coef,Field))
retractIfCan : % -> Union(Fraction Integer,"failed")
  if and(has(UTS,RetractableTo Integer),has(Coef,Field))
retractIfCan : % -> Union(Symbol,"failed")
  if and(has(UTS,RetractableTo Symbol),has(Coef,Field))
retractIfCan : % -> Union(Integer,"failed")
  if and(has(UTS,RetractableTo Integer),has(Coef,Field))
sign : % -> Integer
  if and(has(UTS,OrderedIntegralDomain),has(Coef,Field))
solveLinearPolynomialEquation :
  (List SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     Union(List SparseUnivariatePolynomial %,"failed")
      if and(has(UTS,PolynomialFactorizationExplicit),has(Coef,Field))
squareFreePolynomial :
  SparseUnivariatePolynomial % ->
     Factored SparseUnivariatePolynomial %
```

```
      if and(has(UTS,PolynomialFactorizationExplicit),has(Coef,Field))
 wholePart : % -> UTS
   if and(has(UTS,EuclideanDomain),has(Coef,Field))
 ?*? : (UTS,%) -> % if Coef has FIELD
 ?*? : (%,UTS) -> % if Coef has FIELD
 ?<? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
 ?/? : (UTS,UTS) -> % if Coef has FIELD
 ?.? : (%,UTS) -> % if and(has(UTS,Eltable(UTS,UTS)),has(Coef,Field))
 ?<=? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
 ?>? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
 ?>=? : (%,%) -> Boolean if and(has(UTS,OrderedSet),has(Coef,Field))
```

⟨*category ULSCCAT UnivariateLaurentSeriesConstructorCategory*⟩≡

```
 )abbrev category ULSCCAT UnivariateLaurentSeriesConstructorCategory
 ++ Author: Clifton J. Williamson
 ++ Date Created: 6 February 1990
 ++ Date Last Updated: 10 May 1990
 ++ Basic Operations:
 ++ Related Domains:
 ++ Also See:
 ++ AMS Classifications:
 ++ Keywords: series, Laurent, Taylor
 ++ Examples:
 ++ References:
 ++ Description:
 ++   This is a category of univariate Laurent series constructed from
 ++   univariate Taylor series.  A Laurent series is represented by a pair
 ++   \spad{[n,f(x)]}, where n is an arbitrary integer and \spad{f(x)}
 ++   is a Taylor series.  This pair represents the Laurent series
 ++   \spad{x**n * f(x)}.
 UnivariateLaurentSeriesConstructorCategory(Coef,UTS):_
  Category == Definition where
   Coef: Ring
   UTS : UnivariateTaylorSeriesCategory Coef
   I ==> Integer

   Definition ==> Join(UnivariateLaurentSeriesCategory(Coef),_
                       RetractableTo UTS) with
     laurent: (I,UTS) -> %
       ++ \spad{laurent(n,f(x))} returns \spad{x**n * f(x)}.
     degree: % -> I
       ++ \spad{degree(f(x))} returns the degree of the lowest order term of
       ++ \spad{f(x)}, which may have zero as a coefficient.
     taylorRep: % -> UTS
       ++ \spad{taylorRep(f(x))} returns \spad{g(x)}, where
       ++ \spad{f = x**n * g(x)} is represented by \spad{[n,g(x)]}.
     removeZeroes: % -> %
```

```
      ++ \spad{removeZeroes(f(x))} removes leading zeroes from the
      ++ representation of the Laurent series \spad{f(x)}.
      ++ A Laurent series is represented by (1) an exponent and
      ++ (2) a Taylor series which may have leading zero coefficients.
      ++ When the Taylor series has a leading zero coefficient, the
      ++ 'leading zero' is removed from the Laurent series as follows:
      ++ the series is rewritten by increasing the exponent by 1 and
      ++ dividing the Taylor series by its variable.
      ++ Note: \spad{removeZeroes(f)} removes all leading zeroes from f
    removeZeroes: (I,%) -> %
      ++ \spad{removeZeroes(n,f(x))} removes up to n leading zeroes from
      ++ the Laurent series \spad{f(x)}.
      ++ A Laurent series is represented by (1) an exponent and
      ++ (2) a Taylor series which may have leading zero coefficients.
      ++ When the Taylor series has a leading zero coefficient, the
      ++ 'leading zero' is removed from the Laurent series as follows:
      ++ the series is rewritten by increasing the exponent by 1 and
      ++ dividing the Taylor series by its variable.
    coerce: UTS -> %
      ++ \spad{coerce(f(x))} converts the Taylor series \spad{f(x)} to a
      ++ Laurent series.
    taylor: % -> UTS
      ++ taylor(f(x)) converts the Laurent series f(x) to a Taylor series,
      ++ if possible.  Error: if this is not possible.
    taylorIfCan: % -> Union(UTS,"failed")
      ++ \spad{taylorIfCan(f(x))} converts the Laurent series \spad{f(x)}
      ++ to a Taylor series, if possible. If this is not possible,
      ++ "failed" is returned.
    if Coef has Field then QuotientFieldCategory(UTS)
      --++ the quotient field of univariate Taylor series over a field is
      --++ the field of Laurent series

   add

    zero? x == zero? taylorRep x

    retract(x:%):UTS == taylor x

    retractIfCan(x:%):Union(UTS,"failed") == taylorIfCan x
```

⟨*ULSCCAT.dotabb*⟩≡

```
"ULSCCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ULSCCAT"];
"ULSCCAT" -> "ULSCAT"
```

⟨*ULSCCAT.dotfull*⟩≡

```
"UnivariateLaurentSeriesConstructorCategory(a:Ring,b:UnivariateTaylorSeriesCatego
 [color=lightblue,href="bookvol10.2.pdf#nameddest=ULSCCAT"];
"UnivariateLaurentSeriesConstructorCategory(a:Ring,b:UnivariateTaylorSeriesCatego
  -> "UnivariateLaurentSeriesCategory(a:Ring)"
```

⟨*ULSCCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UnivariateLaurentSeriesConstructorCategory(a:Ring,b:UnivariateTaylorSeriesCategory(Ring))"
 [color=lightblue];
"UnivariateLaurentSeriesConstructorCategory(a:Ring,b:UnivariateTaylorSeriesCategory(Ring))"
  -> "UnivariateLaurentSeriesCategory(a:Ring)"

"UnivariateLaurentSeriesCategory(a:Ring)" [color=lightblue];
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,Integer)"
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "FIELD..."
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "RADCAT..."
"UnivariateLaurentSeriesCategory(a:Ring)" ->
    "TRANFUN..."

"UnivariatePowerSeriesCategory(a:Ring,Integer)" [color=seagreen];
"UnivariatePowerSeriesCategory(a:Ring,Integer)" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

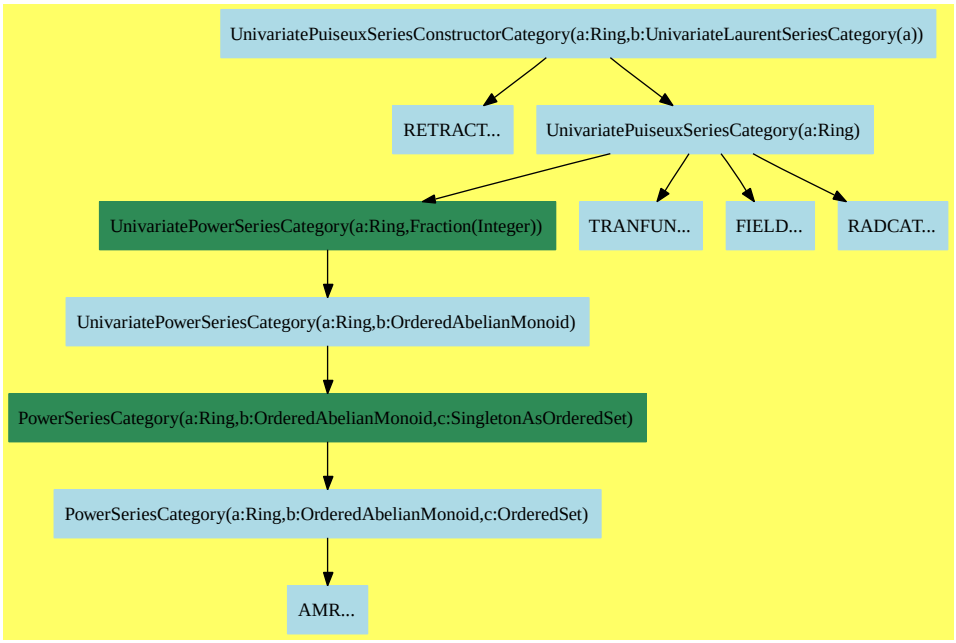
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AMR..."

"AMR..." [color=lightblue];
"FIELD..." [color=lightblue];
"TRANFUN..." [color=lightblue];
"RADCAT..." [color=lightblue];
}
```

18.7 UnivariatePuiseuxSeriesConstructorCategory (UPXSCCA)



See:

⇐ "RetractableTo" (RETRACT) 2.18 on page 44
⇐ "UnivariatePuiseuxSeriesCategory" (UPXSCAT) 17.11 on page 1195

Exports:

0	1	acos	acosh
acot	acoth	acsc	acsch
approximate	asec	asech	asin
asinh	associates?	atan	atanh
center	characteristic	charthRoot	coefficient
coerce	complete	cos	cosh
cot	coth	csc	csch
D	degree	differentiate	divide
euclideanSize	eval	exp	expressIdealMember
exquo	extend	extendedEuclidean	factor
gcd	gcdPolynomial	hash	integrate
inv	latex	laurent	laurentIfCan
laurentRep	lcm	leadingCoefficient	leadingMonomial
log	map	monomial	monomial?
multiEuclidean	multiplyExponents	nthRoot	one?
order	pi	pole?	prime?
principalIdeal	puiseux	rationalPower	recip
reductum	retract	retractIfCan	sample
sec	sech	series	sin
sinh	sizeLess?	sqrt	squareFree
squareFreePart	subtractIfCan	tan	tanh
terms	truncate	unit?	unitCanonical
unitNormal	variable	variables	zero?
?.?	?*?	?**?	?+?
?-?	-?	?=?	?^?
?~=?	?/?	?quo?	?rem?

Attributes Exported:

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- if #1 has Field then canonicalsClosed where
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- if #1 has Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if #1 has IntegralDomain then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.

- if #1 has CommutativeRing then commututive("*") where **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.

These are directly exported but not implemented:

```
coerce : ULS -> %
degree : % -> Fraction Integer
laurent : % -> ULS
laurentIfCan : % -> Union(ULS,"failed")
laurentRep : % -> ULS
puiseux : (Fraction Integer,ULS) -> %
rationalPower : % -> Fraction Integer
```

These are implemented by this category:

```
retract : % -> ULS
retractIfCan : % -> Union(ULS,"failed")
zero? : % -> Boolean
```

These exports come from (p1195) UnivariatePuiseuxSeriesCategory(Coef:Ring):

```
0 : () -> %
1 : () -> %
approximate : (%,Fraction Integer) -> Coef
  if Coef has **: (Coef,Fraction Integer) -> Coef
  and Coef has coerce: Symbol -> Coef
associates? : (%,%) -> Boolean if Coef has INTDOM
acos : % -> % if Coef has ALGEBRA FRAC INT
acosh : % -> % if Coef has ALGEBRA FRAC INT
acot : % -> % if Coef has ALGEBRA FRAC INT
acoth : % -> % if Coef has ALGEBRA FRAC INT
acsc : % -> % if Coef has ALGEBRA FRAC INT
acsch : % -> % if Coef has ALGEBRA FRAC INT
asec : % -> % if Coef has ALGEBRA FRAC INT
asech : % -> % if Coef has ALGEBRA FRAC INT
asin : % -> % if Coef has ALGEBRA FRAC INT
asinh : % -> % if Coef has ALGEBRA FRAC INT
atan : % -> % if Coef has ALGEBRA FRAC INT
atanh : % -> % if Coef has ALGEBRA FRAC INT
center : % -> Coef
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if Coef has CHARNZ
coefficient : (%,Fraction Integer) -> Coef
coerce : % -> % if Coef has INTDOM
coerce : Fraction Integer -> % if Coef has ALGEBRA FRAC INT
coerce : Coef -> % if Coef has COMRING
coerce : Integer -> %
coerce : % -> OutputForm
```

```
complete : % -> %
cos : % -> % if Coef has ALGEBRA FRAC INT
cosh : % -> % if Coef has ALGEBRA FRAC INT
cot : % -> % if Coef has ALGEBRA FRAC INT
coth : % -> % if Coef has ALGEBRA FRAC INT
csc : % -> % if Coef has ALGEBRA FRAC INT
csch : % -> % if Coef has ALGEBRA FRAC INT
D : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,NonNegativeInteger) -> %
  if Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,List Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,Symbol,NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
D : (%,List Symbol,List NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,List Symbol) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,Symbol,NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
  if Coef has PDRING SYMBOL
  and Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : % -> % if Coef has *: (Fraction Integer,Coef) -> Coef
differentiate : (%,NonNegativeInteger) -> %
  if Coef has *: (Fraction Integer,Coef) -> Coef
divide : (%,%) -> Record(quotient: %,remainder: %) if Coef has FIELD
euclideanSize : % -> NonNegativeInteger if Coef has FIELD
eval : (%,Coef) -> Stream Coef
  if Coef has **: (Coef,Fraction Integer) -> Coef
exp : % -> % if Coef has ALGEBRA FRAC INT
expressIdealMember : (List %,%) -> Union(List %,"failed")
  if Coef has FIELD
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
  if Coef has FIELD
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
  if Coef has FIELD
exquo : (%,%) -> Union(%,"failed") if Coef has INTDOM
extend : (%,Fraction Integer) -> %
```

```
factor : % -> Factored % if Coef has FIELD
gcd : (%,%) -> % if Coef has FIELD
gcd : List % -> % if Coef has FIELD
gcdPolynomial :
  (SparseUnivariatePolynomial %,
   SparseUnivariatePolynomial %) ->
     SparseUnivariatePolynomial %
       if Coef has FIELD
hash : % -> SingleInteger
integrate : (%,Symbol) -> %
  if Coef has ACFS INT
  and Coef has PRIMCAT
  and Coef has TRANFUN
  and Coef has ALGEBRA FRAC INT
  or Coef has variables: Coef -> List Symbol
  and Coef has integrate: (Coef,Symbol) -> Coef
  and Coef has ALGEBRA FRAC INT
integrate : % -> % if Coef has ALGEBRA FRAC INT
inv : % -> % if Coef has FIELD
latex : % -> String
lcm : (%,%) -> % if Coef has FIELD
lcm : List % -> % if Coef has FIELD
leadingCoefficient : % -> Coef
leadingMonomial : % -> %
log : % -> % if Coef has ALGEBRA FRAC INT
map : ((Coef -> Coef),%) -> %
monomial : (%,List SingletonAsOrderedSet,List Fraction Integer) -> %
monomial : (%,SingletonAsOrderedSet,Fraction Integer) -> %
monomial : (Coef,Fraction Integer) -> %
monomial? : % -> Boolean
multiEuclidean : (List %,%) -> Union(List %,"failed") if Coef has FIELD
multiplyExponents : (%,PositiveInteger) -> %
multiplyExponents : (%,Fraction Integer) -> %
nthRoot : (%,Integer) -> % if Coef has ALGEBRA FRAC INT
one? : % -> Boolean
order : (%,Fraction Integer) -> Fraction Integer
order : % -> Fraction Integer
pi : () -> % if Coef has ALGEBRA FRAC INT
pole? : % -> Boolean
prime? : % -> Boolean if Coef has FIELD
principalIdeal : List % -> Record(coef: List %,generator: %)
  if Coef has FIELD
recip : % -> Union(%,"failed")
reductum : % -> %
sample : () -> %
sec : % -> % if Coef has ALGEBRA FRAC INT
sech : % -> % if Coef has ALGEBRA FRAC INT
series :
  (NonNegativeInteger,Stream Record(k: Fraction Integer,c: Coef)) -> %
sin : % -> % if Coef has ALGEBRA FRAC INT
```

```
sinh : % -> % if Coef has ALGEBRA FRAC INT
sizeLess? : (%,%) -> Boolean if Coef has FIELD
sqrt : % -> % if Coef has ALGEBRA FRAC INT
squareFree : % -> Factored % if Coef has FIELD
squareFreePart : % -> % if Coef has FIELD
subtractIfCan : (%,%) -> Union(%,"failed")
tan : % -> % if Coef has ALGEBRA FRAC INT
tanh : % -> % if Coef has ALGEBRA FRAC INT
terms : % -> Stream Record(k: Fraction Integer,c: Coef)
truncate : (%,Fraction Integer,Fraction Integer) -> %
truncate : (%,Fraction Integer) -> %
unit? : % -> Boolean if Coef has INTDOM
unitCanonical : % -> % if Coef has INTDOM
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
  if Coef has INTDOM
variable : % -> Symbol
variables : % -> List SingletonAsOrderedSet
?**? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?**? : (%,Integer) -> % if Coef has FIELD
?^? : (%,Integer) -> % if Coef has FIELD
?/? : (%,%) -> % if Coef has FIELD
?**? : (%,%) -> % if Coef has ALGEBRA FRAC INT
?**? : (%,NonNegativeInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (NonNegativeInteger,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (%,%) -> %
?-? : (%,%) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,PositiveInteger) -> %
?*? : (Integer,%) -> %
?*? : (Coef,%) -> %
?*? : (%,Coef) -> %
?*? : (%,Fraction Integer) -> % if Coef has ALGEBRA FRAC INT
?*? : (Fraction Integer,%) -> % if Coef has ALGEBRA FRAC INT
?/? : (%,Coef) -> % if Coef has FIELD
-? : % -> %
?.? : (%,%) -> % if Fraction Integer has SGROUP
?.? : (%,Fraction Integer) -> Coef
?quo? : (%,%) -> % if Coef has FIELD
?rem? : (%,%) -> % if Coef has FIELD
```

⟨*category UPXSCCA UnivariatePuiseuxSeriesConstructorCategory*⟩≡

```
)abbrev category UPXSCCA UnivariatePuiseuxSeriesConstructorCategory
++ Author: Clifton J. Williamson
++ Date Created: 6 February 1990
++ Date Last Updated: 22 March 1990
```

```
++ Basic Operations:
++ Related Domains:
++ Also See:
++ AMS Classifications:
++ Keywords: series, Puiseux, Laurent
++ Examples:
++ References:
++ Description:
++   This is a category of univariate Puiseux series constructed
++   from univariate Laurent series.  A Puiseux series is represented
++   by a pair \spad{[r,f(x)]}, where r is a positive rational number and
++   \spad{f(x)} is a Laurent series.  This pair represents the Puiseux
++   series \spad{f(x^r)}.
UnivariatePuiseuxSeriesConstructorCategory(Coef,ULS):_
 Category == Definition where
  Coef : Ring
  ULS  : UnivariateLaurentSeriesCategory Coef
  I  ==> Integer
  RN ==> Fraction Integer

  Definition ==> Join(UnivariatePuiseuxSeriesCategory(Coef),_
                      RetractableTo ULS) with
    puiseux: (RN,ULS) -> %
      ++ \spad{puiseux(r,f(x))} returns \spad{f(x^r)}.
    rationalPower: % -> RN
      ++ \spad{rationalPower(f(x))} returns r where the Puiseux series
      ++ \spad{f(x) = g(x^r)}.
    laurentRep  : % -> ULS
      ++ \spad{laurentRep(f(x))} returns \spad{g(x)} where the Puiseux series
      ++ \spad{f(x) = g(x^r)} is represented by \spad{[r,g(x)]}.
    degree: % -> RN
      ++ \spad{degree(f(x))} returns the degree of the leading term of the
      ++ Puiseux series \spad{f(x)}, which may have zero as a coefficient.
    coerce: ULS -> %
      ++ \spad{coerce(f(x))} converts the Laurent series \spad{f(x)} to a
      ++ Puiseux series.
    laurent: % -> ULS
      ++ \spad{laurent(f(x))} converts the Puiseux series \spad{f(x)} to a
      ++ Laurent series if possible. Error: if this is not possible.
    laurentIfCan: % -> Union(ULS,"failed")
      ++ \spad{laurentIfCan(f(x))} converts the Puiseux series \spad{f(x)}
      ++ to a Laurent series if possible.
      ++ If this is not possible, "failed" is returned.

   add
```

```
zero? x == zero? laurentRep x

retract(x:%):ULS == laurent x

retractIfCan(x:%):Union(ULS,"failed") == laurentIfCan x
```

⟨*UPXSCCA.dotabb*⟩≡

```
"UPXSCCA"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPXSCCA"];
"UPXSCCA" -> "RETRACT"
"UPXSCCA" -> "UPXSCAT"
```

⟨*UPXSCCA.dotfull*⟩≡

```
"UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLaurentSeriesCategory(a))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPXSCCA"];
"UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLaurentSeriesCategory(a))"
  -> "RetractableTo(UnivariatePuiseuxSeriesCategory(Ring))"
"UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLaurentSeriesCategory(a))"
  -> "UnivariatePuiseuxSeriesCategory(a:Ring)"
```

⟨*UPXSCCA.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLaurentSeriesCateg
 [color=lightblue,href="bookvol10.2.pdf#nameddest=UPXSCCA"];
"UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLaurentSeriesCateg
  -> "RETRACT..."
"UnivariatePuiseuxSeriesConstructorCategory(a:Ring,b:UnivariateLaurentSeriesCateg
  -> "UnivariatePuiseuxSeriesCategory(a:Ring)"

"UnivariatePuiseuxSeriesCategory(a:Ring)" [color=lightblue];
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))"
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "TRANFUN..."
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "FIELD..."
"UnivariatePuiseuxSeriesCategory(a:Ring)" ->
    "RADCAT..."

"UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))" [color=seagreen];
"UnivariatePowerSeriesCategory(a:Ring,Fraction(Integer))" ->
    "UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"

"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)"
 [color=lightblue];
"UnivariatePowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid)" ->
 "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
 [color=seagreen];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:SingletonAsOrderedSet)"
  -> "PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"

"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)"
 [color=lightblue];
"PowerSeriesCategory(a:Ring,b:OrderedAbelianMonoid,c:OrderedSet)" ->
    "AMR..."

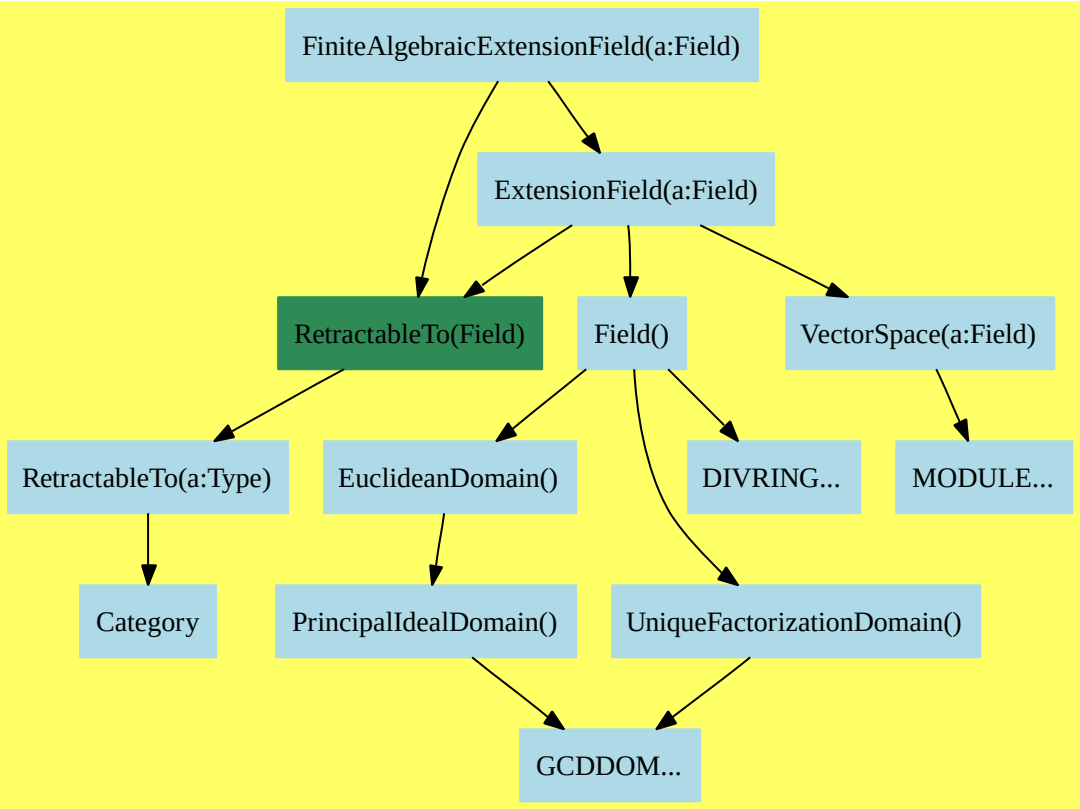
"RETRACT..." [color=lightblue];
"TRANFUN..." [color=lightblue];
"FIELD..." [color=lightblue];
"RADCAT..." [color=lightblue];
```

```
"AMR..." [color=lightblue];
}
```

Chapter 19

Category Layer 18

19.1 FiniteAlgebraicExtensionField (FAXF)



See:

⇐ "ExtensionField" (XF) 18.2 on page 1237
⇐ "RetractableTo" (RETRACT) 2.18 on page 44

Exports:

0	1	algebraic?
associates?	basis	characteristic
charthRoot	coerce	conditionP
coordinates	createNormalElement	createPrimitiveElement
D	definingPolynomial	degree
differentiate	dimension	discreteLog
divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	extensionDegree
factor	factorsOfCyclicGroupSize	Frobenius
gcd	gcdPolynomial	generator
hash	index	inGroundField?
init	inv	latex
lcm	linearAssociatedExp	linearAssociatedLog
linearAssociatedOrder	lookup	minimalPolynomial
multiEuclidean	nextItem	norm
normal?	normalElement	one?
order	prime?	primeFrobenius
primitive?	primitiveElement	principalIdeal
random	recip	representationType
represents	retract	retractIfCan
sample	size	sizeLess?
squareFree	squareFreePart	subtractIfCan
tableForDiscreteLogarithm	trace	transcendenceDegree
transcendent?	unit?	unitCanonical
unitNormal	zero?	?*?
?**?	?+?	?-?
-?	?/?	?=?
?^?	?quo?	?rem?
?~=?		

Attributes Exported:

- **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.

- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
basis : () -> Vector %
basis : PositiveInteger -> Vector %
coordinates : % -> Vector F
definingPolynomial : () -> SparseUnivariatePolynomial F
generator : () -> % if F has FINITE
minimalPolynomial : (%,PositiveInteger) ->
   SparseUnivariatePolynomial %
     if F has FINITE
normalElement : () -> % if F has FINITE
```

These are implemented by this category:

```
algebraic? : % -> Boolean
charthRoot : % -> Union(%,"failed")
    if F has CHARNZ or F has FINITE
coordinates : Vector % -> Matrix F
createNormalElement : () -> % if F has FINITE
degree : % -> PositiveInteger
dimension : () -> CardinalNumber
extensionDegree : () -> PositiveInteger
linearAssociatedExp : (%,SparseUnivariatePolynomial F) -> %
    if F has FINITE
linearAssociatedLog : (%,%) ->
   Union(SparseUnivariatePolynomial F,"failed")
     if F has FINITE
linearAssociatedLog : % -> SparseUnivariatePolynomial F
    if F has FINITE
linearAssociatedOrder : % -> SparseUnivariatePolynomial F
    if F has FINITE
minimalPolynomial : % -> SparseUnivariatePolynomial F
norm : % -> F
norm : (%,PositiveInteger) -> % if F has FINITE
normal? : % -> Boolean if F has FINITE
represents : Vector F -> %
size : () -> NonNegativeInteger if F has FINITE
trace : % -> F
trace : (%,PositiveInteger) -> % if F has FINITE
transcendenceDegree : () -> NonNegativeInteger
transcendent? : % -> Boolean
```

These exports come from (p1237) ExtensionField(F:Field):

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
characteristic : () -> NonNegativeInteger
coerce : F -> %
coerce : % -> %
coerce : Integer -> %
coerce : % -> OutputForm
coerce : Fraction Integer -> %
discreteLog : (%,%) ->
   Union(NonNegativeInteger,"failed")
     if F has CHARNZ or F has FINITE
divide : (%,%) -> Record(quotient: %,remainder: %)
euclideanSize : % -> NonNegativeInteger
expressIdealMember : (List %,%) -> Union(List %,"failed")
exquo : (%,%) -> Union(%,"failed")
extendedEuclidean : (%,%,%) ->
   Union(Record(coef1: %,coef2: %),"failed")
extendedEuclidean : (%,%) ->
   Record(coef1: %,coef2: %,generator: %)
factor : % -> Factored %
Frobenius : (%,NonNegativeInteger) -> % if F has FINITE
Frobenius : % -> % if F has FINITE
gcd : List % -> %
gcd : (%,%) -> %
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
hash : % -> SingleInteger
inGroundField? : % -> Boolean
inv : % -> %
latex : % -> String
lcm : List % -> %
lcm : (%,%) -> %
multiEuclidean : (List %,%) -> Union(List %,"failed")
one? : % -> Boolean
order : % -> OnePointCompletion PositiveInteger
     if F has CHARNZ or F has FINITE
prime? : % -> Boolean
primeFrobenius : % -> %
    if F has CHARNZ or F has FINITE
primeFrobenius : (%,NonNegativeInteger) -> %
    if F has CHARNZ or F has FINITE
principalIdeal : List % -> Record(coef: List %,generator: %)
recip : % -> Union(%,"failed")
retract : % -> F
retractIfCan : % -> Union(F,"failed")
sample : () -> %
```

```
squareFree : % -> Factored %
squareFreePart : % -> %
sizeLess? : (%,%) -> Boolean
subtractIfCan : (%,%) -> Union(%,"failed")
unit? : % -> Boolean
unitCanonical : % -> %
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
zero? : % -> Boolean
?/? : (%,%) -> %
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (Fraction Integer,%) -> %
?*? : (%,Fraction Integer) -> %
?*? : (F,%) -> %
?*? : (%,F) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,Integer) -> %
?^? : (%,Integer) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?quo? : (%,%) -> %
?rem? : (%,%) -> %
?/? : (%,F) -> %
```

These exports come from (p44) RetractableTo(F:Field):

These exports come from (p1244) FiniteFieldCategory():

```
charthRoot : % -> % if F has FINITE
conditionP : Matrix % -> Union(Vector %,"failed")
    if F has FINITE
createPrimitiveElement : () -> % if F has FINITE
D : % -> % if F has FINITE
D : (%,NonNegativeInteger) -> % if F has FINITE
differentiate : % -> % if F has FINITE
differentiate : (%,NonNegativeInteger) -> %
    if F has FINITE
discreteLog : % -> NonNegativeInteger if F has FINITE
factorsOfCyclicGroupSize : () ->
   List Record(factor: Integer,exponent: Integer)
```

```
     if F has FINITE
 index : PositiveInteger -> % if F has FINITE
 init : () -> % if F has FINITE
 lookup : % -> PositiveInteger if F has FINITE
 nextItem : % -> Union(%,"failed") if F has FINITE
 order : % -> PositiveInteger if F has FINITE
 primitive? : % -> Boolean if F has FINITE
 primitiveElement : () -> % if F has FINITE
 random : () -> % if F has FINITE
 representationType : () ->
    Union("prime",polynomial,normal,cyclic)
      if F has FINITE
 tableForDiscreteLogarithm : Integer ->
    Table(PositiveInteger,NonNegativeInteger)
      if F has FINITE
```

⟨*category FAXF FiniteAlgebraicExtensionField*⟩≡

```
 )abbrev category FAXF FiniteAlgebraicExtensionField
 ++ Author: J. Grabmeier, A. Scheerhorn
 ++ Date Created: 11 March 1991
 ++ Date Last Updated: 31 March 1991
 ++ Basic Operations: _+, _*, extensionDegree,
 ++ Related Constructors:
 ++ Also See:
 ++ AMS Classifications:
 ++ Keywords: field, extension field, algebraic extension, finite extension
 ++ References:
 ++  R.Lidl, H.Niederreiter: Finite Field, Encycoldia of Mathematics and
 ++  Its Applications, Vol. 20, Cambridge Univ. Press, 1983,
 ++  ISBN 0 521 30240 4 J. Grabmeier, A. Scheerhorn: Finite Fields in AXIOM.
 ++  AXIOM Technical Report Series, ATR/5 NP2522.
 ++ Description:
 ++  FiniteAlgebraicExtensionField {\em F} is the category of fields
 ++  which are finite algebraic extensions of the field {\em F}.
 ++  If {\em F} is finite then any finite algebraic extension of {\em F}
 ++  is finite, too. Let {\em K} be a finite algebraic extension of the
 ++  finite field {\em F}. The exponentiation of elements of {\em K}
 ++  defines a Z-module structure on the multiplicative group of {\em K}.
 ++  The additive group of {\em K} becomes a module over the ring of
 ++  polynomials over {\em F} via the operation
 ++  \spadfun{linearAssociatedExp}(a:K,f:SparseUnivariatePolynomial F)
 ++  which is linear over {\em F}, i.e. for elements {\em a} from {\em K},
 ++  {\em c,d} from {\em F} and {\em f,g} univariate polynomials over {\em F}
 ++  we have \spadfun{linearAssociatedExp}(a,cf+dg) equals {\em c} times
 ++  \spadfun{linearAssociatedExp}(a,f) plus {\em d} times
 ++  \spadfun{linearAssociatedExp}(a,g).
 ++  Therefore \spadfun{linearAssociatedExp} is defined completely by
```

```
++  its action on  monomials from {\em F[X]}:
++  \spadfun{linearAssociatedExp}(a,monomial(1,k)\$SUP(F)) is defined to be
++  \spadfun{Frobenius}(a,k) which is {\em a**(q**k)} where {\em q=size()\$F}.
++  The operations order and discreteLog associated with the multiplicative
++  exponentiation have additive analogues associated to the operation
++  \spadfun{linearAssociatedExp}. These are the functions
++  \spadfun{linearAssociatedOrder} and \spadfun{linearAssociatedLog},
++  respectively.

FiniteAlgebraicExtensionField(F : Field) : Category == _
  Join(ExtensionField F, RetractableTo F) with
  -- should be unified with algebras
  -- Join(ExtensionField F, FramedAlgebra F, RetractableTo F) with
    basis : () -> Vector $
      ++ basis() returns a fixed basis of \$ as \spad{F}-vectorspace.
    basis : PositiveInteger -> Vector $
      ++ basis(n) returns a fixed basis of a subfield of \$ as
      ++ \spad{F}-vectorspace.
    coordinates : $ -> Vector F
      ++ coordinates(a) returns the coordinates of \spad{a} with respect
      ++ to the fixed \spad{F}-vectorspace basis.
    coordinates : Vector $ -> Matrix F
      ++ coordinates([v1,...,vm]) returns the coordinates of the
      ++ vi's with to the fixed basis.  The coordinates of vi are
      ++ contained in the ith row of the matrix returned by this
      ++ function.
    represents:  Vector F -> $
      ++ represents([a1,..,an]) returns \spad{a1*v1 + ... + an*vn}, where
      ++ v1,...,vn are the elements of the fixed basis.
    minimalPolynomial: $ -> SparseUnivariatePolynomial F
      ++ minimalPolynomial(a) returns the minimal polynomial of an
      ++ element \spad{a} over the ground field F.
    definingPolynomial: () -> SparseUnivariatePolynomial F
      ++ definingPolynomial() returns the polynomial used to define
      ++ the field extension.
    extensionDegree : () ->  PositiveInteger
      ++ extensionDegree() returns the degree of field extension.
    degree : $ -> PositiveInteger
      ++ degree(a) returns the degree of the minimal polynomial of an
      ++ element \spad{a} over the ground field F.
    norm: $  -> F
      ++ norm(a) computes the norm of \spad{a} with respect to the
      ++ field considered as an algebra with 1 over the ground field F.
    trace: $ -> F
      ++ trace(a) computes the trace of \spad{a} with respect to
      ++ the field considered as an algebra with 1 over the ground field F.
```

```
if F has Finite then
  FiniteFieldCategory
  minimalPolynomial: ($,PositiveInteger) -> SparseUnivariatePolynomial $
    ++ minimalPolynomial(x,n) computes the minimal polynomial of x over
    ++ the field of extension degree n over the ground field F.
  norm: ($,PositiveInteger)  -> $
    ++ norm(a,d) computes the norm of \spad{a} with respect to the field
    ++ of extension degree d over the ground field of size.
    ++ Error: if d does not divide the extension degree of \spad{a}.
    ++ Note: norm(a,d) = reduce(*,[a**(q**(d*i)) for i in 0..n/d])
  trace: ($,PositiveInteger)   -> $
    ++ trace(a,d) computes the trace of \spad{a} with respect to the
    ++ field of extension degree d over the ground field of size q.
    ++ Error: if d does not divide the extension degree of \spad{a}.
    ++ Note: \spad{trace(a,d)=reduce(+,[a**(q**(d*i)) for i in 0..n/d])}.
  createNormalElement: () -> $
    ++ createNormalElement() computes a normal element over the ground
    ++ field F, that is,
    ++ \spad{a**(q**i), 0 <= i < extensionDegree()} is an F-basis,
    ++ where \spad{q = size()\$F}.
    ++ Reference: Such an element exists Lidl/Niederreiter: Theorem 2.35.
  normalElement: () -> $
    ++ normalElement() returns a element, normal over the ground field F,
    ++ i.e. \spad{a**(q**i), 0 <= i < extensionDegree()} is an F-basis,
    ++ where \spad{q = size()\$F}.
    ++ At the first call, the element is computed by
    ++ \spadfunFrom{createNormalElement}{FiniteAlgebraicExtensionField}
    ++ then cached in a global variable.
    ++ On subsequent calls, the element is retrieved by referencing the
    ++ global variable.
  normal?: $ -> Boolean
    ++ normal?(a) tests whether the element \spad{a} is normal over the
    ++ ground field F, i.e.
    ++ \spad{a**(q**i), 0 <= i <= extensionDegree()-1} is an F-basis,
    ++ where \spad{q = size()\$F}.
    ++ Implementation according to Lidl/Niederreiter: Theorem 2.39.
  generator: () -> $
    ++ generator() returns a root of the defining polynomial.
    ++ This element generates the field as an algebra over the ground
    ++ field.
  linearAssociatedExp:($,SparseUnivariatePolynomial F) -> $
    ++ linearAssociatedExp(a,f) is linear over {\em F}, i.e.
    ++ for elements {\em a} from {\em \$}, {\em c,d} form {\em F} and
    ++ {\em f,g} univariate polynomials over {\em F} we have
    ++ \spadfun{linearAssociatedExp}(a,cf+dg) equals {\em c} times
    ++ \spadfun{linearAssociatedExp}(a,f) plus {\em d} times
```

```
      ++ \spadfun{linearAssociatedExp}(a,g). Therefore
      ++ \spadfun{linearAssociatedExp} is defined completely by its
      ++ action on monomials from {\em F[X]}:
      ++ \spadfun{linearAssociatedExp}(a,monomial(1,k)\$SUP(F)) is
      ++ defined to be \spadfun{Frobenius}(a,k) which is {\em a**(q**k)},
      ++ where {\em q=size()\$F}.
    linearAssociatedOrder: $ -> SparseUnivariatePolynomial F
      ++ linearAssociatedOrder(a) retruns the monic polynomial {\em g} of
      ++ least degree, such that \spadfun{linearAssociatedExp}(a,g) is 0.
    linearAssociatedLog: $ -> SparseUnivariatePolynomial F
      ++ linearAssociatedLog(a) returns a polynomial {\em g}, such that
      ++ \spadfun{linearAssociatedExp}(normalElement(),g) equals {\em a}.
    linearAssociatedLog: ($,$) -> _
      Union(SparseUnivariatePolynomial F,"failed")
      ++ linearAssociatedLog(b,a) returns a polynomial {\em g}, such
      ++ that the \spadfun{linearAssociatedExp}(b,g) equals {\em a}.
      ++ If there is no such polynomial {\em g}, then
      ++ \spadfun{linearAssociatedLog} fails.
  add
    I   ==> Integer
    PI  ==> PositiveInteger
    NNI ==> NonNegativeInteger
    SUP ==> SparseUnivariatePolynomial
    DLP ==> DiscreteLogarithmPackage

    represents(v) ==
      a:$:=0
      b:=basis()
      for i in 1..extensionDegree()@PI repeat
        a:=a+(v.i)*(b.i)
      a

    transcendenceDegree() == 0$NNI

    dimension() == (#basis()) ::NonNegativeInteger::CardinalNumber

    coordinates(v:Vector $) ==
      m := new(#v, extensionDegree(), 0)$Matrix(F)
      for i in minIndex v .. maxIndex v for j in minRowIndex m .. repeat
        setRow_!(m, j, coordinates qelt(v, i))
      m

    algebraic? a == true

    transcendent? a == false
```

```
  -- This definition is a duplicate and has been removed
  --    extensionDegree():OnePointCompletion(PositiveInteger) ==
  --      (#basis()) :: PositiveInteger::OnePointCompletion(PositiveInteger)

      extensionDegree() == (#basis()) :: PositiveInteger

  -- These definitions are duplicates and have been removed
  --    degree(a):OnePointCompletion(PositiveInteger) ==
  --      degree(a)@PI::OnePointCompletion(PositiveInteger)

      -- degree a == degree(minimalPolynomial a)$SUP(F) :: PI

      trace a ==
        b := basis()
        abs : F := 0
        for i in 1..#b repeat
          abs := abs + coordinates(a*b.i).i
        abs

      norm a ==
        b := basis()
        m := new(#b,#b, 0)$Matrix(F)
        for i in 1..#b repeat
          setRow_!(m,i, coordinates(a*b.i))
        determinant(m)

      if F has Finite then
        linearAssociatedExp(x,f) ==
          erg:$:=0
          y:=x
          for i in 0..degree(f) repeat
            erg:=erg + coefficient(f,i) * y
            y:=Frobenius(y)
          erg

        linearAssociatedLog(b,x) ==
          x=0 => 0
          l:List List F:=[entries coordinates b]
          a:$:=b
          extdeg:NNI:=extensionDegree()@PI
          for i in 2..extdeg repeat
            a:=Frobenius(a)
            l:=concat(l,entries coordinates a)$(List List F)
          l:=concat(l,entries coordinates x)$(List List F)
          m1:=rowEchelon transpose matrix(l)$(Matrix F)
          v:=zero(extdeg)$(Vector F)
```

```
  rown:I:=1
  for i in 1..extdeg repeat
    if qelt(m1,rown,i) = 1$F then
      v.i:=qelt(m1,rown,extdeg+1)
      rown:=rown+1
  p:=+/[monomial(v.(i+1),i::NNI) for i in 0..(#v-1)]
  p=0 =>
   messagePrint("linearAssociatedLog: second argument not in_
                 group generated by first argument")$OutputForm
   "failed"
  p

linearAssociatedLog(x) == linearAssociatedLog(normalElement(),x) ::
                       SparseUnivariatePolynomial(F)

linearAssociatedOrder(x) ==
  x=0 => 0
  l:List List F:=[entries coordinates x]
  a:$:=x
  for i in 1..extensionDegree()@PI repeat
    a:=Frobenius(a)
    l:=concat(l,entries coordinates a)$(List List F)
  v:=first nullSpace transpose matrix(l)$(Matrix F)
  +/[monomial(v.(i+1),i::NNI) for i in 0..(#v-1)]

charthRoot(x):Union($,"failed") ==
  (charthRoot(x)@$)::Union($,"failed")
-- norm(e) == norm(e,1) pretend F
-- trace(e) == trace(e,1) pretend F

minimalPolynomial(a,n) ==
  extensionDegree()@PI rem n ^= 0 =>
    error "minimalPolynomial: 2. argument must divide extension degree"
  f:SUP $:=monomial(1,1)$(SUP $) - monomial(a,0)$(SUP $)
  u:$:=Frobenius(a,n)
  while not(u = a) repeat
    f:=f * (monomial(1,1)$(SUP $) - monomial(u,0)$(SUP $))
    u:=Frobenius(u,n)
  f

norm(e,s) ==
  qr := divide(extensionDegree(), s)
  zero?(qr.remainder) =>
    pow := (size()-1) quo (size()$F ** s - 1)
    e ** (pow::NonNegativeInteger)
  error "norm: second argument must divide degree of extension"
```

```
        trace(e,s) ==
          qr:=divide(extensionDegree(),s)
          q:=size()$F
          zero?(qr.remainder) =>
            a:$:=0
            for i in 0..qr.quotient-1 repeat
              a:=a + e**(q**(s*i))
            a
          error "trace: second argument must divide degree of extension"

        size() == size()$F ** extensionDegree()

        createNormalElement() ==
          characteristic() = size() => 1
          res : $
          for i in 1.. repeat
            res := index(i :: PI)
            not inGroundField? res =>
              normal? res => return res
          -- theorem: there exists a normal element, this theorem is
          -- unknown to the compiler
          res

        normal?(x:$) ==
          p:SUP $:=(monomial(1,extensionDegree()) - monomial(1,0))@(SUP $)
          f:SUP $:= +/[monomial(Frobenius(x,i),i)$(SUP $) _
                      for i in 0..extensionDegree()-1]
          gcd(p,f) = 1 => true
          false

        degree a ==
          y:$:=Frobenius a
          deg:PI:=1
          while y^=a repeat
            y := Frobenius(y)
            deg:=deg+1
          deg
```

⟨*FAXF.dotabb*⟩≡

```
"FAXF"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FAXF"];
"FAXF" -> "XF"
"FAXF" -> "RETRACT"
```

⟨*FAXF.dotfull*⟩≡
```
"FiniteAlgebraicExtensionField(a:Field)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FAXF"];
"FiniteAlgebraicExtensionField(a:Field)" -> "ExtensionField(a:Field)"
"FiniteAlgebraicExtensionField(a:Field)" -> "RetractableTo(a:Field)"
```

⟨*FAXF.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FiniteAlgebraicExtensionField(a:Field)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FAXF"];
"FiniteAlgebraicExtensionField(a:Field)" -> "ExtensionField(a:Field)"
"FiniteAlgebraicExtensionField(a:Field)" -> "RetractableTo(Field)"

"ExtensionField(a:Field)" [color=lightblue];
"ExtensionField(a:Field)" -> "Field()"
"ExtensionField(a:Field)" -> "RetractableTo(Field)"
"ExtensionField(a:Field)" -> "VectorSpace(a:Field)"

"Field()" [color=lightblue];
"Field()" -> "EuclideanDomain()"
"Field()" -> "UniqueFactorizationDomain()"
"Field()" -> "DIVRING..."

"EuclideanDomain()" [color=lightblue];
"EuclideanDomain()" -> "PrincipalIdealDomain()"

"UniqueFactorizationDomain()" [color=lightblue];
"UniqueFactorizationDomain()" -> "GCDDOM..."

"PrincipalIdealDomain()" [color=lightblue];
"PrincipalIdealDomain()" -> "GCDDOM..."

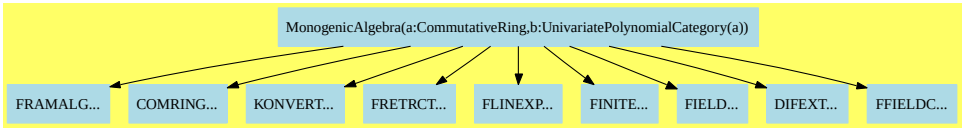
"RetractableTo(Field)" [color=seagreen];
"RetractableTo(Field)" -> "RetractableTo(a:Type)"

"RetractableTo(a:Type)" [color=lightblue];
"RetractableTo(a:Type)" -> "Category"

"VectorSpace(a:Field)" [color=lightblue];
"VectorSpace(a:Field)" -> "MODULE..."

"MODULE..." [color=lightblue];
"DIVRING..." [color=lightblue];
"GCDDOM..." [color=lightblue];
"Category" [color=lightblue];
}
```

19.2 MonogenicAlgebra (MONOGEN)



See:

⇒ “FunctionFieldCategory” (FFCAT) 20.2 on page 1334
⇐ “CommutativeRing” (COMRING) 10.4 on page 685
⇐ “ConvertibleTo” (KONVERT) 2.8 on page 19
⇐ “FramedAlgebra” (FRAMALG) 18.5 on page 1261
⇐ “FullyRetractableTo” (FRETRCT) 3.6 on page 71
⇐ “LinearlyExplicitRingOver” (LINEXP) 10.9 on page 707

Exports:

0	1	associates?
basis	characteristic	characteristicPolynomial
charthRoot	coerce	conditionP
convert	coordinates	createPrimitiveElement
D	definingPolynomial	derivationCoordinates
differentiate	discreteLog	discriminant
divide	euclideanSize	expressIdealMember
exquo	extendedEuclidean	factor
factorsOfCyclicGroupSize	generator	gcd
gcdPolynomial	hash	index
init	inv	latex
lcm	lift	lookup
minimalPolynomial	multiEuclidean	nextItem
norm	one?	order
prime?	primeFrobenius	primitive?
primitiveElement	principalIdeal	random
rank	recip	reduce
reducedSystem	regularRepresentation	represents
representationType	retract	retractIfCan
sample	size	sizeLess?
squareFree	squareFreePart	subtractIfCan
tableForDiscreteLogarithm	trace	traceMatrix
unit?	unitCanonical	unitNormal
zero?	?*?	?**?
?+?	?-?	-?
?=?	?^?	?~=?
?/?	?quo?	?rem?

Attributes Exported:

- if $ has Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if $ has Field then canonicalClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- if $ has Field then noZeroDivisors where **noZeroDivisors** is true if $x*y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
definingPolynomial : () -> UP
lift : % -> UP
reduce : UP -> %
```

These are implemented by this category:

```
basis : () -> Vector %
characteristicPolynomial : % -> UP
convert : % -> UP
convert : UP -> %
derivationCoordinates : (Vector %,(R -> R)) -> Matrix R
    if R has FIELD
differentiate : (%,(R -> R)) -> % if R has FIELD
generator : () -> %
norm : % -> R
random : () -> % if R has FINITE
recip : % -> Union(%,"failed")
reduce : Fraction UP -> Union(%,"failed") if R has FIELD
retract : % -> R
retractIfCan : % -> Union(R,"failed")
size : () -> NonNegativeInteger if R has FINITE
```

These exports come from (p1261) FramedAlgebra(R,UP)
where R:CommutativeRing and UP:UnivariatePolynomialCategory(a)

```
0 : () -> %
1 : () -> %
characteristic : () -> NonNegativeInteger
charthRoot : % -> Union(%,"failed") if R has CHARNZ
coerce : R -> %
coerce : Integer -> %
coerce : % -> OutputForm
convert : Vector R -> %
convert : % -> Vector R
coordinates : (%,Vector %) -> Vector R
coordinates : (Vector %,Vector %) -> Matrix R
coordinates : Vector % -> Matrix R
coordinates : % -> Vector R
discriminant : Vector % -> R
discriminant : () -> R
hash : % -> SingleInteger
latex : % -> String
minimalPolynomial : % -> UP if R has FIELD
one? : % -> Boolean
rank : () -> PositiveInteger
regularRepresentation : (%,Vector %) -> Matrix R
regularRepresentation : % -> Matrix R
represents : (Vector R,Vector %) -> %
represents : Vector R -> %
sample : () -> %
subtractIfCan : (%,%) -> Union(%,"failed")
trace : % -> R
traceMatrix : Vector % -> Matrix R
traceMatrix : () -> Matrix R
zero? : % -> Boolean
?+? : (%,%) -> %
?=? : (%,%) -> Boolean
?~=? : (%,%) -> Boolean
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (R,%) -> %
?*? : (%,R) -> %
?-? : (%,%) -> %
-? : % -> %
?**? : (%,PositiveInteger) -> %
?**? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
```

These exports come from (p71) FullyRetractableTo(R)
where R:CommutativeRing

```
coerce : Fraction Integer -> %
```

```
    if R has FIELD or R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retract : % -> Fraction Integer
    if R has RETRACT FRAC INT
retractIfCan : % -> Union(Fraction Integer,"failed")
    if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
    if R has RETRACT INT
```

These exports come from (p782) FullyLinearlyExplicitRingOver(R)
where R:CommutativeRing

```
reducedSystem : Matrix % -> Matrix R
reducedSystem :
  (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
reducedSystem :
  (Matrix %,Vector %) ->
     Record(mat: Matrix Integer,vec: Vector Integer)
       if R has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
```

These exports come from (p129) Finite()

```
index : PositiveInteger -> % if R has FINITE
lookup : % -> PositiveInteger if R has FINITE
```

These exports come from (p1005) Field()

```
associates? : (%,%) -> Boolean if R has FIELD
coerce : % -> % if R has FIELD
divide : (%,%) -> Record(quotient: %,remainder: %)
    if R has FIELD
euclideanSize : % -> NonNegativeInteger if R has FIELD
expressIdealMember : (List %,%) -> Union(List %,"failed")
    if R has FIELD
exquo : (%,%) -> Union(%,"failed") if R has FIELD
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %)
    if R has FIELD
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed")
    if R has FIELD
factor : % -> Factored % if R has FIELD
gcd : (%,%) -> % if R has FIELD
gcd : List % -> % if R has FIELD
gcdPolynomial :
   (SparseUnivariatePolynomial %,
    SparseUnivariatePolynomial %) ->
       SparseUnivariatePolynomial % if R has FIELD
inv : % -> % if R has FIELD
lcm : (%,%) -> % if R has FIELD
```

```
lcm : List % -> % if R has FIELD
multiEuclidean : (List %,%) -> Union(List %,"failed")
    if R has FIELD
prime? : % -> Boolean if R has FIELD
principalIdeal : List % -> Record(coef: List %,generator: %)
    if R has FIELD
sizeLess? : (%,%) -> Boolean if R has FIELD
squareFree : % -> Factored % if R has FIELD
squareFreePart : % -> % if R has FIELD
unit? : % -> Boolean if R has FIELD
unitCanonical : % -> % if R has FIELD
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if R has FIELD
?/? : (%,%) -> % if R has FIELD
?*? : (%,Fraction Integer) -> % if R has FIELD
?*? : (Fraction Integer,%) -> % if R has FIELD
?**? : (%,Integer) -> % if R has FIELD
?^? : (%,Integer) -> % if R has FIELD
?quo? : (%,%) -> % if R has FIELD
?rem? : (%,%) -> % if R has FIELD
```

These exports come from (p777) DifferentialExtension(R)
where R:CommutativeRing

```
D : (%,(R -> R)) -> % if R has FIELD
D : (%,(R -> R),NonNegativeInteger) -> % if R has FIELD
D : % -> %
    if and(has(R,DifferentialRing),has(R,Field))
    or R has FFIELDC
D : (%,NonNegativeInteger) -> %
    if and(has(R,DifferentialRing),has(R,Field))
    or R has FFIELDC
D : (%,List Symbol,List NonNegativeInteger) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
D : (%,Symbol,NonNegativeInteger) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
D : (%,List Symbol) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
D : (%,Symbol) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
differentiate : % -> %
    if and(has(R,DifferentialRing),has(R,Field))
    or R has FFIELDC
differentiate : (%,NonNegativeInteger) -> %
    if and(has(R,DifferentialRing),has(R,Field))
    or R has FFIELDC
differentiate : (%,List Symbol) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
```

```
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
differentiate : (%,(R -> R),NonNegativeInteger) -> %
    if R has FIELD
differentiate : (%,Symbol) -> %
    if and(has(R,PartialDifferentialRing Symbol),has(R,Field))
```

These exports come from (p1244) FiniteFieldCategory():

```
charthRoot : % -> % if R has FFIELDC
conditionP : Matrix % -> Union(Vector %,"failed")
    if R has FFIELDC
createPrimitiveElement : () -> % if R has FFIELDC
discreteLog : % -> NonNegativeInteger if R has FFIELDC
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
    if R has FFIELDC
factorsOfCyclicGroupSize : () ->
   List Record(factor: Integer,exponent: Integer)
     if R has FFIELDC
init : () -> % if R has FFIELDC
nextItem : % -> Union(%,"failed") if R has FFIELDC
order : % -> OnePointCompletion PositiveInteger
    if R has FFIELDC
order : % -> PositiveInteger if R has FFIELDC
primeFrobenius : % -> % if R has FFIELDC
primeFrobenius : (%,NonNegativeInteger) -> % if R has FFIELDC
primitive? : % -> Boolean if R has FFIELDC
primitiveElement : () -> % if R has FFIELDC
representationType : () ->
   Union("prime",polynomial,normal,cyclic)
     if R has FFIELDC
tableForDiscreteLogarithm :
   Integer -> Table(PositiveInteger,NonNegativeInteger)
     if R has FFIELDC
```

⟨*category MONOGEN MonogenicAlgebra*⟩≡

```
)abbrev category MONOGEN MonogenicAlgebra
++ Author: Barry Trager
++ Date Created:
++ Date Last Updated:
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords:
++ References:
++ Description:
++ A \spadtype{MonogenicAlgebra} is an algebra of finite rank which
++ can be generated by a single element.
```

```
MonogenicAlgebra(R:CommutativeRing, UP:UnivariatePolynomialCategory R):
 Category ==
    Join(FramedAlgebra(R, UP), CommutativeRing, ConvertibleTo UP,
              FullyRetractableTo R, FullyLinearlyExplicitRingOver R) with
      generator         : () -> %
        ++ generator() returns the generator for this domain.
      definingPolynomial: () -> UP
        ++ definingPolynomial() returns the minimal polynomial which
        ++ \spad{generator()} satisfies.
      reduce            : UP -> %
        ++ reduce(up) converts the univariate polynomial up to an algebra
        ++ element, reducing by the \spad{definingPolynomial()} if necessary.
      convert           : UP -> %
        ++ convert(up) converts the univariate polynomial up to an algebra
        ++ element, reducing by the \spad{definingPolynomial()} if necessary.
      lift              : % -> UP
        ++ lift(z) returns a minimal degree univariate polynomial up such that
        ++ \spad{z=reduce up}.
      if R has Finite then Finite
      if R has Field then
        Field
        DifferentialExtension R
        reduce               : Fraction UP -> Union(%, "failed")
          ++ reduce(frac) converts the fraction frac to an algebra element.
        derivationCoordinates: (Vector %, R -> R) -> Matrix R
          ++ derivationCoordinates(b, ') returns M such that \spad{b' = M b}.
      if R has FiniteFieldCategory then FiniteFieldCategory
  add
   convert(x:%):UP == lift x
   convert(p:UP):% == reduce p
   generator()     == reduce monomial(1, 1)$UP
   norm x          == resultant(definingPolynomial(), lift x)
   retract(x:%):R  == retract lift x
   retractIfCan(x:%):Union(R, "failed") == retractIfCan lift x

   basis() ==
     [reduce monomial(1,i)$UP for i in 0..(rank()-1)::NonNegativeInteger]

   characteristicPolynomial(x:%):UP ==
     characteristicPolynomial(x)$CharacteristicPolynomialInMonogenicalAlgebra(R,UP,%)

   if R has Finite then
     size()   == size()$R ** rank()
     random() == represents [random()$R for i in 1..rank()]$Vector(R)
```

```
    if R has Field then
      reduce(x:Fraction UP) == reduce(numer x) exquo reduce(denom x)

      differentiate(x:%, d:R -> R) ==
        p := definingPolynomial()
        yprime := - reduce(map(d, p)) / reduce(differentiate p)
        reduce(map(d, lift x)) + yprime * reduce differentiate lift x

      derivationCoordinates(b, d) ==
        coordinates(map(x +-> differentiate(x, d), b), b)

      recip x ==
        (bc := extendedEuclidean(lift x, definingPolynomial(), 1))
                                                  case "failed" => "failed"
        reduce(bc.coef1)
```

⟨*MONOGEN.dotabb*⟩≡

```
"MONOGEN"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONOGEN"];
"MONOGEN" -> "FRAMALG"
"MONOGEN" -> "COMRING"
"MONOGEN" -> "KONVERT"
"MONOGEN" -> "FRETRCT"
"MONOGEN" -> "FLINEXP"
"MONOGEN" -> "FINITE"
"MONOGEN" -> "FIELD"
"MONOGEN" -> "DIFEXT"
"MONOGEN" -> "FFIELDC"
```

⟨*MONOGEN.dotfull*⟩≡

```
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=MONOGEN"];
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FramedAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "CommutativeRing()"
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "ConvertibleTo(UnivariatePolynomialCategory(CommutativeRing))"
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FullyRetractableTo(a:CommutativeRing)"
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FullyLinearlyExplicitRingOver(a:CommutativeRing)"

"MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))"
 [color=seagreen,href="bookvol10.2.pdf#nameddest=MONOGEN"];
"MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))" ->
    "MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
```

⟨*MONOGEN.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

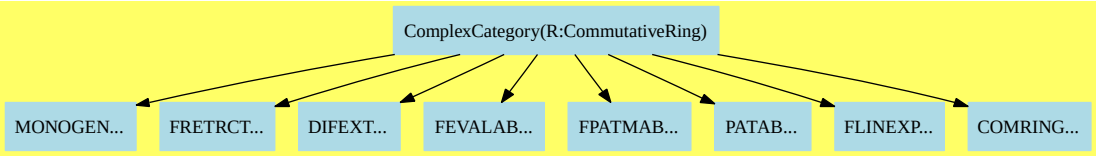
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue];
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FRAMALG..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "COMRING..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "KONVERT..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FRETRCT..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FLINEXP..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FINITE..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FIELD..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "DIFEXT..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FFIELDC..."

"FRAMALG..." [color=lightblue];
"COMRING..." [color=lightblue];
"KONVERT..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"FINITE..." [color=lightblue];
"FIELD..." [color=lightblue];
"DIFEXT..." [color=lightblue];
"FFIELDC..." [color=lightblue];
}
```

Chapter 20

Category Layer 19

20.1 ComplexCategory (COMPCAT)



See:

⇐ “CommutativeRing” (COMRING) 10.4 on page 685
⇐ “DifferentialExtension” (DIFEXT) 11.2 on page 777
⇐ “FullyEvalableOver” (FEVALAB) 4.7 on page 121
⇐ “FullyPatternMatchable” (FPATMAB) 3.7 on page 74
⇐ “FullyRetractableTo” (FRETRCT) 3.6 on page 71
⇐ “MonogenicAlgebra” (MONOGEN) 19.2 on page 1305
⇐ “Patternable” (PATAB) 2.15 on page 38
⇐ “FullyLinearlyExplicitRingOver” (FLINEXP) 11.3 on page 782

Exports:

0	1	abs
acos	acosh	acot
acoth	acsc	acsch
argument	asec	asech
asin	asinh	associates?
atan	atanh	basis
characteristic	characteristicPolynomial	charthRoot
coerce	complex	conditionP
conjugate	convert	coordinates
cos	cosh	cot
coth	createPrimitiveElement	csc
csch	D	definingPolynomial
derivationCoordinates	differentiate	discreteLog
discriminant	divide	euclideanSize
eval	exp	expressIdealMember
exquo	extendedEuclidean	factor
factorPolynomial	factorSquareFreePolynomial	factorsOfCyclicGroupSize
gcd	gcdPolynomial	generator
hash	imag	imaginary
index	init	inv
latex	lcm	lift
log	lookup	map
max	min	minimalPolynomial
multiEuclidean	nextItem	norm
nthRoot	one?	order
patternMatch	pi	polarCoordinates
prime?	primeFrobenius	primitive?
primitiveElement	principalIdeal	random
rank	rational	rational?
rationalIfCan	real	recip
reduce	reducedSystem	regularRepresentation
represents	representationType	retract
retractIfCan	sample	sec
sech	sin	sinh
size	sizeLess?	solveLinearPolynomialEquation
sqrt	squareFree	squareFreePart
squareFreePolynomial	subtractIfCan	tableForDiscreteLogarithm
tan	tanh	trace
traceMatrix	unit?	unitCanonical
unitNormal	zero?	?*?
?**?	?+?	?-?
-?	?=?	?^?
?~=?	?/?	?<?
?<=?	?>?	?>=?
?.?	?quo?	?rem?

Attributes Exported:

- if #1 has multiplicativeValuation then multiplicativeValuation where **multiplicativeValuation** implies `euclideanSize(a*b)=euclideanSize(a)*euclideanSize(b)`.
- if #1 has additiveValuation then additiveValuation where **additiveValuation** implies `euclideanSize(a*b)=euclideanSize(a)+euclideanSize(b)`.
- if #1 has Field then canonicalsClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- if #1 has Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if #1 has IntegralDomain or #1 has both EuclideanDomain and PolynomialFactorizationExplicit then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.
- **complex** means that this domain has $\sqrt{-1}$
- **nil**

These are directly exported but not implemented:

```
abs : % -> % if R has RNS
argument : % -> R if R has TRANFUN
complex : (R,R) -> %
conjugate : % -> %
exquo : (%,R) -> Union(%,"failed") if R has INTDOM
imag : % -> R
imaginary : () -> %
norm : % -> R
polarCoordinates : % -> Record(r: R,phi: R) if R has RNS and R has TRANFUN
rational : % -> Fraction Integer if R has INS
rational? : % -> Boolean if R has INS
rationalIfCan : % -> Union(Fraction Integer,"failed") if R has INS
real : % -> R
```

These are implemented by this category:

```
acos : % -> % if R has TRANFUN
acosh : % -> % if R has TRANFUN
asin : % -> % if R has TRANFUN
asinh : % -> % if R has TRANFUN
atan : % -> % if R has TRANFUN
atanh : % -> % if R has TRANFUN
characteristic : () -> NonNegativeInteger
characteristicPolynomial : % -> SparseUnivariatePolynomial R
coerce : % -> OutputForm
convert : % -> Complex DoubleFloat if R has REAL
convert : % -> Complex Float if R has REAL
convert : % -> InputForm if R has KONVERT INFORM
convert : % -> Pattern Float if R has KONVERT PATTERN FLOAT
convert : % -> Pattern Integer if R has KONVERT PATTERN INT
coordinates : % -> Vector R
coordinates : (%,Vector %) -> Vector R
cos : % -> % if R has TRANFUN
cosh : % -> % if R has TRANFUN
definingPolynomial : () -> SparseUnivariatePolynomial R
differentiate : (%,(R -> R)) -> %
discriminant : () -> R
divide : (%,%) -> Record(quotient: %,remainder: %) if R has EUCDOM
euclideanSize : % -> NonNegativeInteger if R has EUCDOM
exp : % -> % if R has TRANFUN
exquo : (%,%) -> Union(%,"failed") if R has INTDOM or R has EUCDOM and R has PFECAT
factorPolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial % if
factorSquareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolync
inv : % -> % if R has FIELD
lift : % -> SparseUnivariatePolynomial R
log : % -> % if R has TRANFUN
map : ((R -> R),%) -> %
minimalPolynomial : % -> SparseUnivariatePolynomial R if R has FIELD
patternMatch : (%,Pattern Integer,PatternMatchResult(Integer,%)) -> PatternMatchResult(Integ
patternMatch : (%,Pattern Float,PatternMatchResult(Float,%)) -> PatternMatchResult(Float,%)
pi : () -> % if R has TRANFUN
rank : () -> PositiveInteger
recip : % -> Union(%,"failed")
reduce : SparseUnivariatePolynomial R -> %
reducedSystem : Matrix % -> Matrix R
reducedSystem : Matrix % -> Matrix Integer if R has LINEXP INT
retract : % -> R
retractIfCan : % -> Union(R,"failed")
sin : % -> % if R has TRANFUN
sinh : % -> % if R has TRANFUN
solveLinearPolynomialEquation : (List SparseUnivariatePolynomial %,SparseUnivariatePolynomia
tan : % -> % if R has TRANFUN
tanh : % -> % if R has TRANFUN
trace : % -> R
```

```
unitNormal : % -> Record(unit: %,canonical: %,associate: %) if R has INTDOM or R has EUCDOM and R has P
?=? : (%,%) -> Boolean
?+? : (%,%) -> %
-? : % -> %
?*? : (%,%) -> %
?*? : (R,%) -> %
?*? : (Integer,%) -> %
?<? : (%,%) -> Boolean if R has ORDSET
?**? : (%,Fraction Integer) -> % if R has RADCAT and R has TRANFUN
?rem? : (%,%) -> % if R has EUCDOM
?quo? : (%,%) -> % if R has EUCDOM
```

These exports come from (p1305) MonogenicAlgebra(R,S)
where R:CommutativeRing and S:SparseUnivariatePolynomial(R):

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean if R has INTDOM or R has EUCDOM and R has PFECAT
basis : () -> Vector %
charthRoot : % -> Union(%,"failed") if and(has($,CharacteristicNonZero),AND(has(R,EuclideanDomain),has(
charthRoot : % -> % if R has FFIELDC
coerce : R -> %
coerce : % -> % if R has INTDOM or R has EUCDOM and R has PFECAT
coerce : Integer -> %
coerce : Fraction Integer -> % if R has FIELD or R has RETRACT FRAC INT
conditionP : Matrix % -> Union(Vector %,"failed") if and(has($,CharacteristicNonZero),AND(has(R,Euclide
convert : SparseUnivariatePolynomial R -> %
convert : Vector R -> %
convert : % -> Vector R
convert : % -> SparseUnivariatePolynomial R
coordinates : Vector % -> Matrix R
coordinates : (Vector %,Vector %) -> Matrix R
createPrimitiveElement : () -> % if R has FFIELDC
D : (%,(R -> R)) -> %
D : (%,(R -> R),NonNegativeInteger) -> %
D : % -> % if and(has(R,Field),has(R,DifferentialRing)) or R has DIFRING or and(has(R,DifferentialRing)
D : (%,NonNegativeInteger) -> % if and(has(R,Field),has(R,DifferentialRing)) or R has DIFRING or and(ha
D : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
D : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
D : (%,List Symbol) -> % if R has PDRING SYMBOL
D : (%,Symbol) -> % if R has PDRING SYMBOL
derivationCoordinates : (Vector %,(R -> R)) -> Matrix R if R has FIELD
differentiate : (%,NonNegativeInteger) -> % if and(has(R,Field),has(R,DifferentialRing)) or R has DIFRI
differentiate : (%,List Symbol) -> % if R has PDRING SYMBOL
differentiate : (%,Symbol,NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has PDRING SYMBOL
differentiate : (%,(R -> R),NonNegativeInteger) -> %
differentiate : % -> % if and(has(R,Field),has(R,DifferentialRing)) or R has DIFRING or and(has(R,Diffe
differentiate : (%,Symbol) -> % if R has PDRING SYMBOL
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed") if R has FFIELDC
```

```
discreteLog : % -> NonNegativeInteger if R has FFIELDC
discriminant : Vector % -> R
expressIdealMember : (List %,%) -> Union(List %,"failed") if R has EUCDOM
extendedEuclidean : (%,%) -> Record(coef1: %,coef2: %,generator: %) if R has EUCDOM
extendedEuclidean : (%,%,%) -> Union(Record(coef1: %,coef2: %),"failed") if R has EUCDOM
factor : % -> Factored % if R has EUCDOM and R has PFECAT or R has FIELD
factorsOfCyclicGroupSize : () -> List Record(factor: Integer,exponent: Integer) if R has FF
gcd : (%,%) -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
gcd : List % -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
gcdPolynomial : (SparseUnivariatePolynomial %,SparseUnivariatePolynomial %) -> SparseUnivar
generator : () -> %
hash : % -> SingleInteger
index : PositiveInteger -> % if R has FINITE
init : () -> % if R has FFIELDC
latex : % -> String
lcm : (%,%) -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
lcm : List % -> % if R has EUCDOM or R has EUCDOM and R has PFECAT
lookup : % -> PositiveInteger if R has FINITE
multiEuclidean : (List %,%) -> Union(List %,"failed") if R has EUCDOM
nextItem : % -> Union(%,"failed") if R has FFIELDC
one? : % -> Boolean
order : % -> OnePointCompletion PositiveInteger if R has FFIELDC
order : % -> PositiveInteger if R has FFIELDC
prime? : % -> Boolean if R has EUCDOM and R has PFECAT or R has FIELD
primeFrobenius : % -> % if R has FFIELDC
primeFrobenius : (%,NonNegativeInteger) -> % if R has FFIELDC
primitive? : % -> Boolean if R has FFIELDC
primitiveElement : () -> % if R has FFIELDC
principalIdeal : List % -> Record(coef: List %,generator: %) if R has EUCDOM
random : () -> % if R has FINITE
reduce : Fraction SparseUnivariatePolynomial R -> Union(%,"failed") if R has FIELD
reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix R,vec: Vector R)
reducedSystem : (Matrix %,Vector %) -> Record(mat: Matrix Integer,vec: Vector Integer) if R
regularRepresentation : % -> Matrix R
regularRepresentation : (%,Vector %) -> Matrix R
representationType : () -> Union("prime",polynomial,normal,cyclic) if R has FFIELDC
represents : (Vector R,Vector %) -> %
represents : Vector R -> %
retract : % -> Fraction Integer if R has RETRACT FRAC INT
retract : % -> Integer if R has RETRACT INT
retractIfCan : % -> Union(Fraction Integer,"failed") if R has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed") if R has RETRACT INT
sample : () -> %
size : () -> NonNegativeInteger if R has FINITE
sizeLess? : (%,%) -> Boolean if R has EUCDOM
subtractIfCan : (%,%) -> Union(%,"failed")
squareFree : % -> Factored % if R has EUCDOM and R has PFECAT or R has FIELD
squareFreePart : % -> % if R has EUCDOM and R has PFECAT or R has FIELD
tableForDiscreteLogarithm : Integer -> Table(PositiveInteger,NonNegativeInteger) if R has FI
traceMatrix : () -> Matrix R
```

```
traceMatrix : Vector % -> Matrix R
unit? : % -> Boolean if R has INTDOM or R has EUCDOM and R has PFECAT
unitCanonical : % -> % if R has INTDOM or R has EUCDOM and R has PFECAT
zero? : % -> Boolean
?/? : (%,%) -> % if R has FIELD
?*? : (%,Fraction Integer) -> % if R has FIELD
?*? : (Fraction Integer,%) -> % if R has FIELD
?**? : (%,Integer) -> % if R has FIELD
?^? : (%,Integer) -> % if R has FIELD
?*? : (PositiveInteger,%) -> %
?*? : (NonNegativeInteger,%) -> %
?*? : (%,R) -> %
?-? : (%,%) -> %
?**? : (%,NonNegativeInteger) -> %
?**? : (%,PositiveInteger) -> %
?^? : (%,NonNegativeInteger) -> %
?^? : (%,PositiveInteger) -> %
```

These exports come from (p121) FullyEvalableOver(R:CommutativeRing)

```
?.? : (%,R) -> % if R has ELTAB(R,R)
eval : (%,Equation R) -> % if R has EVALAB R
eval : (%,List Symbol,List R) -> % if R has IEVALAB(SYMBOL,R)
eval : (%,List Equation R) -> % if R has EVALAB R
eval : (%,R,R) -> % if R has EVALAB R
eval : (%,List R,List R) -> % if R has EVALAB R
eval : (%,Symbol,R) -> % if R has IEVALAB(SYMBOL,R)
```

These exports come from (p685) CommutativeRing():

```
?~=? : (%,%) -> Boolean
```

These exports come from (p168) OrderedSet():

```
max : (%,%) -> % if R has ORDSET
min : (%,%) -> % if R has ORDSET
?<=? : (%,%) -> Boolean if R has ORDSET
?>? : (%,%) -> Boolean if R has ORDSET
?>=? : (%,%) -> Boolean if R has ORDSET
```

These exports come from (p42) RadicalCategory():

```
nthRoot : (%,Integer) -> % if R has RADCAT and R has TRANFUN
sqrt : % -> % if R has RADCAT and R has TRANFUN
```

These exports come from (p94) TranscendentalFunctionCategory():

```
acot : % -> % if R has TRANFUN
acoth : % -> % if R has TRANFUN
```

```
acsc : % -> % if R has TRANFUN
acsch : % -> % if R has TRANFUN
asec : % -> % if R has TRANFUN
asech : % -> % if R has TRANFUN
cot : % -> % if R has TRANFUN
coth : % -> % if R has TRANFUN
csc : % -> % if R has TRANFUN
csch : % -> % if R has TRANFUN
sec : % -> % if R has TRANFUN
sech : % -> % if R has TRANFUN
?**? : (%,%) -> % if R has TRANFUN
```

These exports come from (p990) PolynomialFactorizationExplicit():

```
squareFreePolynomial : SparseUnivariatePolynomial % -> Factored SparseUnivariatePolynomial %
```

⟨*category COMPCAT ComplexCategory*⟩≡

```
)abbrev category COMPCAT ComplexCategory
++ Author:
++ Date Created:
++ Date Last Updated: 18 March 1994
++ Basic Functions:
++ Related Constructors:
++ Also See:
++ AMS Classifications:
++ Keywords: complex, gaussian
++ References:
++ Description:
++ This category represents the extension of a ring by a square
++ root of -1.
ComplexCategory(R:CommutativeRing): Category ==
  Join(MonogenicAlgebra(R, SparseUnivariatePolynomial R), FullyRetractableTo R,
   DifferentialExtension R, FullyEvalableOver R, FullyPatternMatchable(R),
    Patternable(R), FullyLinearlyExplicitRingOver R, CommutativeRing) with
     complex                      ++ indicates that % has sqrt(-1)
     imaginary:   () -> %         ++ imaginary() = sqrt(-1) = %i.
     conjugate:   % -> %          ++ conjugate(x + %i y) returns x - %i y.
     complex  :   (R, R) -> %     ++ complex(x,y) constructs x + %i*y.
     imag     :   % -> R          ++ imag(x) returns imaginary part of x.
     real     :   % -> R          ++ real(x) returns real part of x.
     norm     :   % -> R          ++ norm(x) returns x * conjugate(x)
     if R has OrderedSet then OrderedSet
     if R has IntegralDomain then
       IntegralDomain
       _exquo : (%,R) -> Union(%,"failed")
         ++ exquo(x, r) returns the exact quotient of x by r, or
         ++ "failed" if r does not divide x exactly.
     if R has EuclideanDomain then EuclideanDomain
```

```
      if R has multiplicativeValuation then multiplicativeValuation
      if R has additiveValuation then additiveValuation
      if R has Field then        -- this is a lie; we must know that
        Field                    -- x**2+1 is irreducible in R
      if R has ConvertibleTo InputForm then ConvertibleTo InputForm
      if R has CharacteristicZero then CharacteristicZero
      if R has CharacteristicNonZero then CharacteristicNonZero
      if R has RealConstant then
        ConvertibleTo Complex DoubleFloat
        ConvertibleTo Complex Float
      if R has RealNumberSystem then
        abs: % -> %
          ++ abs(x) returns the absolute value of x = sqrt(norm(x)).
      if R has TranscendentalFunctionCategory then
        TranscendentalFunctionCategory
        argument: % -> R
          ++ argument(x) returns the angle made by (0,1) and (0,x).
        if R has RadicalCategory then RadicalCategory
        if R has RealNumberSystem then
          polarCoordinates: % -> Record(r:R, phi:R)
            ++ polarCoordinates(x) returns (r, phi) such that
            ++ x = r * exp(%i * phi).
      if R has IntegerNumberSystem then
        rational?    : % -> Boolean
          ++ rational?(x) tests if x is a rational number.
        rational     : % -> Fraction Integer
          ++ rational(x) returns x as a rational number.
          ++ Error: if x is not a rational number.
        rationalIfCan: % -> Union(Fraction Integer, "failed")
          ++ rationalIfCan(x) returns x as a rational number, or
          ++ "failed" if x is not a rational number.
      if R has PolynomialFactorizationExplicit and R has EuclideanDomain then
         PolynomialFactorizationExplicit
  add
      import MatrixCategoryFunctions2(%, Vector %, Vector %, Matrix %,
                                      R, Vector R, Vector R, Matrix R)
      SUP ==> SparseUnivariatePolynomial
      characteristicPolynomial x ==
         v := monomial(1,1)$SUP(R)
         v**2 - trace(x)*v**1 + norm(x)*v**0
      if R has PolynomialFactorizationExplicit and R has EuclideanDomain then
         SupR ==> SparseUnivariatePolynomial R
         Sup ==> SparseUnivariatePolynomial %
         import FactoredFunctionUtilities Sup
         import UnivariatePolynomialCategoryFunctions2(R,SupR,%,Sup)
         import UnivariatePolynomialCategoryFunctions2(%,Sup,R,SupR)
```

```
      pp,qq:Sup
      if R has IntegerNumberSystem then
         myNextPrime: (%,NonNegativeInteger) -> %
         myNextPrime(x,n ) == -- prime is actually in R, and = 3(mod 4)
            xr:=real(x)-4::R
            while not prime? xr repeat
               xr:=xr-4::R
            complex(xr,0)
         --!TT:=InnerModularGcd(%,Sup,32719 :: %,myNextPrime)
         --!gcdPolynomial(pp,qq) == modularGcd(pp,qq)$TT
         solveLinearPolynomialEquation(lp:List Sup,p:Sup) ==
           solveLinearPolynomialEquation(lp,p)$ComplexIntegerSolveLinearPolyn
      normPolynomial: Sup -> SupR
      normPolynomial pp ==
          map(z+->retract(z@%)::R,pp * map(conjugate,pp))
      factorPolynomial pp ==
          refine(squareFree pp,factorSquareFreePolynomial)
      factorSquareFreePolynomial pp ==
          pnorm:=normPolynomial pp
          k:R:=0
          while degree gcd(pnorm,differentiate pnorm)>0 repeat
             k:=k+1
             pnorm:=normPolynomial
                      elt(pp,monomial(1,1)-monomial(complex(0,k),0))
          fR:=factorSquareFreePolynomial pnorm
          numberOfFactors fR = 1 =>
              makeFR(1,[["irred",pp,1]])
          lF:List Record(flg:Union("nil", "sqfr", "irred", "prime"),
                         fctr:Sup, xpnt:Integer):=[]
          for u in factorList fR repeat
              p1:=map((z:R):%+->z::%,u.fctr)
              if not zero? k then
                 p1:=elt(p1,monomial(1,1)+monomial(complex(0,k),0))
              p2:=gcd(p1,pp)
              lF:=cons(["irred",p2,1],lF)
              pp:=(pp exquo p2)::Sup
          makeFR(pp,lF)
rank()           == 2
discriminant()   == -4 :: R
norm x           == real(x)**2 + imag(x)**2
trace x          == 2 * real x
imaginary()      == complex(0, 1)
conjugate x      == complex(real x, - imag x)
characteristic() == characteristic()$R
map(fn, x)       == complex(fn real x, fn imag x)
x = y            == real(x) = real(y) and imag(x) = imag(y)
```

```
        x + y            == complex(real x + real y, imag x + imag y)
        - x              == complex(- real x, - imag x)
        r:R * x:%        == complex(r * real x, r * imag x)
        coordinates(x:%) == [real x, imag x]
        n:Integer * x:%  == complex(n * real x, n * imag x)
        differentiate(x:%, d:R -> R) == complex(d real x, d imag x)

        definingPolynomial() ==
          monomial(1,2)$(SUP R) + monomial(1,0)$(SUP R)

        reduce(pol:SUP R) ==
          part:= (monicDivide(pol,definingPolynomial())).remainder
          complex(coefficient(part,0),coefficient(part,1))

        lift(x) == monomial(real x,0)$(SUP R)+monomial(imag x,1)$(SUP R)

        minimalPolynomial x ==
          zero? imag x =>
            monomial(1, 1)$(SUP R) - monomial(real x, 0)$(SUP R)
          monomial(1, 2)$(SUP R) - monomial(trace x, 1)$(SUP R)
            + monomial(norm x, 0)$(SUP R)

        coordinates(x:%, v:Vector %):Vector(R) ==
          ra := real(a := v(minIndex v))
          rb := real(b := v(maxIndex v))
          (#v ^= 2) or
            ((d := recip(ra * (ib := imag b) - (ia := imag a) * rb))
              case "failed") =>error "coordinates: vector is not a basis"
          rx := real x
          ix := imag x
          [d::R * (rx * ib - ix * rb), d::R * (ra * ix - ia * rx)]

        coerce(x:%):OutputForm ==
          re := (r := real x)::OutputForm
          ie := (i := imag x)::OutputForm
          zero? i => re
          outi := "%i"::Symbol::OutputForm
          ip :=
--            one? i => outi
            (i = 1) => outi
--            one?(-i) => -outi
            ((-i) = 1) => -outi
            ie * outi
          zero? r => ip
          re + ip
```

```
retract(x:%):R ==
  not zero?(imag x) =>
    error "Imaginary part is nonzero. Cannot retract."
  real x

retractIfCan(x:%):Union(R, "failed") ==
  not zero?(imag x) => "failed"
  real x

x:% * y:% ==
  complex(real x * real y - imag x * imag y,
          imag x * real y + imag y * real x)

reducedSystem(m:Matrix %):Matrix R ==
  vertConcat(map(real, m), map(imag, m))

reducedSystem(m:Matrix %, v:Vector %):
 Record(mat:Matrix R, vec:Vector R) ==
  rh := reducedSystem(v::Matrix %)@Matrix(R)
  [reducedSystem(m)@Matrix(R), column(rh, minColIndex rh)]

if R has RealNumberSystem then
  abs(x:%):%        == (sqrt norm x)::%

if R has RealConstant then
  convert(x:%):Complex(DoubleFloat) ==
   complex(convert(real x)@DoubleFloat,convert(imag x)@DoubleFloat)

  convert(x:%):Complex(Float) ==
    complex(convert(real x)@Float, convert(imag x)@Float)

if R has ConvertibleTo InputForm then
  convert(x:%):InputForm ==
    convert([convert("complex"::Symbol), convert real x,
             convert imag x]$List(InputForm))@InputForm

if R has ConvertibleTo Pattern Integer then
  convert(x:%):Pattern Integer ==
     convert(x)$ComplexPattern(Integer, R, %)
if R has ConvertibleTo Pattern Float then
  convert(x:%):Pattern Float ==
     convert(x)$ComplexPattern(Float, R, %)

if R has PatternMatchable Integer then
  patternMatch(x:%, p:Pattern Integer,
   l:PatternMatchResult(Integer, %)) ==
```

```
            patternMatch(x, p, l)$ComplexPatternMatch(Integer, R, %)

        if R has PatternMatchable Float then
          patternMatch(x:%, p:Pattern Float,
           l:PatternMatchResult(Float, %)) ==
            patternMatch(x, p, l)$ComplexPatternMatch(Float, R, %)


        if R has OrderedSet then
          x < y ==
            real x = real y => imag x < imag y
            real x < real y

        if R has IntegerNumberSystem then
          rational? x == zero? imag x

          rational x ==
            zero? imag x => rational real x
            error "Not a rational number"

          rationalIfCan x ==
            zero? imag x => rational real x
            "failed"

        if R has Field then
          inv x ==
            zero? imag x => (inv real x)::%
            r := norm x
            complex(real(x) / r, - imag(x) / r)

        if R has IntegralDomain then
          _exquo(x:%, r:R) ==
--            one? r => x
            (r = 1) => x
            (r1 := real(x) exquo r) case "failed" => "failed"
            (r2 := imag(x) exquo r) case "failed" => "failed"
            complex(r1, r2)

          _exquo(x:%, y:%) ==
            zero? imag y => x exquo real y
            x * conjugate(y) exquo norm(y)

          recip(x:%) == 1 exquo x

          if R has OrderedRing then
            unitNormal x ==
```

```
        zero? x => [1,x,1]
        (u := recip x) case % => [x, 1, u]
        zero? real x =>
          c := unitNormal imag x
          [complex(0, c.unit), (c.associate * imag x)::%,
                                        complex(0, - c.associate)]
        c := unitNormal real x
        x := c.associate * x
        imag x < 0 =>
          x := complex(- imag x, real x)
          [- c.unit * imaginary(), x, c.associate * imaginary()]
        [c.unit ::%, x, c.associate ::%]
      else
       unitNormal x ==
        zero? x => [1,x,1]
        (u := recip x) case % => [x, 1, u]
        zero? real x =>
          c := unitNormal imag x
          [complex(0, c.unit), (c.associate * imag x)::%,
                                        complex(0, - c.associate)]
        c := unitNormal real x
        x := c.associate * x
        [c.unit ::%, x, c.associate ::%]

    if R has EuclideanDomain then
       if R has additiveValuation then
           euclideanSize x == max(euclideanSize real x,
                                  euclideanSize imag x)
       else
           euclideanSize x == euclideanSize(real(x)**2 + imag(x)**2)$R
       if R has IntegerNumberSystem then
         x rem y ==
           zero? imag y =>
             yr:=real y
             complex(symmetricRemainder(real(x), yr),
                     symmetricRemainder(imag(x), yr))
           divide(x, y).remainder
         x quo y ==
           zero? imag y =>
             yr:= real y
             xr:= real x
             xi:= imag x
             complex((xr-symmetricRemainder(xr,yr)) quo yr,
                     (xi-symmetricRemainder(xi,yr)) quo yr)
           divide(x, y).quotient
```

```
    else
      x rem y ==
        zero? imag y =>
          yr:=real y
          complex(real(x) rem yr,imag(x) rem yr)
        divide(x, y).remainder
      x quo y ==
        zero? imag y => complex(real x quo real y,imag x quo real y)
        divide(x, y).quotient

    divide(x, y) ==
      r := norm y
      y1 := conjugate y
      xx := x * y1
      x1 := real(xx) rem r
      a  := x1
      if x1^=0 and sizeLess?(r, 2 * x1) then
        a := x1 - r
        if sizeLess?(x1, a) then a := x1 + r
      x2 := imag(xx) rem r
      b  := x2
      if x2^=0 and sizeLess?(r, 2 * x2) then
        b := x2 - r
        if sizeLess?(x2, b) then b := x2 + r
      y1 := (complex(a, b) exquo y1)::%
      [((x - y1) exquo y)::%, y1]

  if R has TranscendentalFunctionCategory then
    half := recip(2::R)::R

    if R has RealNumberSystem then
      atan2loc(y: R, x: R): R ==
          pi1 := pi()$R
          pi2 := pi1 * half
          x = 0 => if y >= 0 then pi2 else -pi2

          -- Atan in (-pi/2,pi/2]
          theta := atan(y * recip(x)::R)
          while theta <= -pi2 repeat theta := theta + pi1
          while theta >   pi2 repeat theta := theta - pi1

          x >= 0 => theta      -- I or IV

          if y >= 0 then
              theta + pi1      -- II
          else
```

```
          theta - pi1      -- III

  argument x == atan2loc(imag x, real x)

else
  -- Not ordered so dictate two quadrants
  argument x ==
    zero? real x => pi()$R * half
    atan(imag(x) * recip(real x)::R)

pi()  == pi()$R :: %

if R is DoubleFloat then
   stoc ==> S_-TO_-C$Lisp
   ctos ==> C_-TO_-S$Lisp

   exp   x == ctos EXP(stoc x)$Lisp
   log   x == ctos LOG(stoc x)$Lisp

   sin   x == ctos SIN(stoc x)$Lisp
   cos   x == ctos COS(stoc x)$Lisp
   tan   x == ctos TAN(stoc x)$Lisp
   asin  x == ctos ASIN(stoc x)$Lisp
   acos  x == ctos ACOS(stoc x)$Lisp
   atan  x == ctos ATAN(stoc x)$Lisp

   sinh  x == ctos SINH(stoc x)$Lisp
   cosh  x == ctos COSH(stoc x)$Lisp
   tanh  x == ctos TANH(stoc x)$Lisp
   asinh x == ctos ASINH(stoc x)$Lisp
   acosh x == ctos ACOSH(stoc x)$Lisp
   atanh x == ctos ATANH(stoc x)$Lisp

else
  atan x ==
    ix := imaginary()*x
    - imaginary() * half * (log(1 + ix) - log(1 - ix))

  log x ==
    complex(log(norm x) * half, argument x)

  exp x ==
    e := exp real x
    complex(e * cos imag x, e * sin imag x)

  cos x ==
```

```
      e := exp(imaginary() * x)
      half * (e + recip(e)::%)

    sin x ==
      e := exp(imaginary() * x)
      - imaginary() * half * (e - recip(e)::%)

  if R has RealNumberSystem then
    polarCoordinates x ==
      [sqrt norm x, (negative?(t := argument x) => t + 2 * pi(); t)]

    x:% ** q:Fraction(Integer) ==
      zero? q =>
        zero? x => error "0 ** 0 is undefined"
        1
      zero? x => 0
      rx := real x
      zero? imag x and positive? rx => (rx ** q)::%
      zero? imag x and denom q = 2 => complex(0, (-rx)**q)
      ax := sqrt(norm x) ** q
      tx := q::R * argument x
      complex(ax * cos tx, ax * sin tx)

  else if R has RadicalCategory then
    x:% ** q:Fraction(Integer) ==
      zero? q =>
        zero? x => error "0 ** 0 is undefined"
        1
      r := real x
      zero?(i := imag x) => (r ** q)::%
      t := numer(q) * recip(denom(q)::R)::R * argument x
      e:R :=
        zero? r => i ** q
        norm(x) ** (q / (2::Fraction(Integer)))
      complex(e * cos t, e * sin t)
```

⟨*COMPCAT.dotabb*⟩≡
```
"COMPCAT"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=COMPCAT"];
"COMPCAT" -> "COMRING"
"COMPCAT" -> "DIFEXT"
"COMPCAT" -> "FEVALAB"
"COMPCAT" -> "FPATMAB"
"COMPCAT" -> "FRETRCT"
"COMPCAT" -> "MONOGEN"
"COMPCAT" -> "PATAB"
"COMPCAT" -> "FLINEXP"
"COMPCAT" -> "ORDSET"
```

⟨*COMPCAT.dotfull*⟩≡
```
"ComplexCategory(R:CommutativeRing)"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=COMPCAT"];
"ComplexCategory(R:CommutativeRing)" ->
  "MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
"ComplexCategory(R:CommutativeRing)" ->
  "FullyRetractableTo(a:CommutativeRing)"
"ComplexCategory(R:CommutativeRing)" ->
  "DifferentialExtension(CommutativeRing)"
"ComplexCategory(R:CommutativeRing)" ->
  "FullyEvalableOver(CommutativeRing)"
"ComplexCategory(R:CommutativeRing)" ->
  "FullyPatternMatchable(CommutativeRing)"
"ComplexCategory(R:CommutativeRing)" ->
  "Patternable(CommutativeRing)"
"ComplexCategory(R:CommutativeRing)" ->
  "FullyLinearlyExplicitRingOver(a:CommutativeRing)"
"ComplexCategory(R:CommutativeRing)" ->
  "CommutativeRing()"
"ComplexCategory(R:CommutativeRing)" ->
  "OrderedSet()"
```

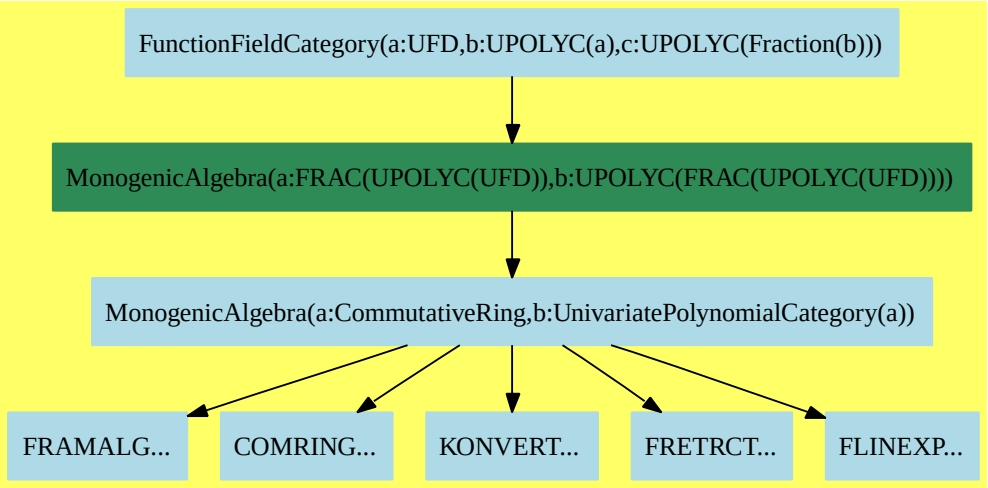
⟨*COMPCAT.dotpic*⟩≡

```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"ComplexCategory(R:CommutativeRing)" [color=lightblue];
"ComplexCategory(R:CommutativeRing)" -> "MONOGEN..."
"ComplexCategory(R:CommutativeRing)" -> "FRETRCT..."
"ComplexCategory(R:CommutativeRing)" -> "DIFEXT..."
"ComplexCategory(R:CommutativeRing)" -> "FEVALAB..."
"ComplexCategory(R:CommutativeRing)" -> "FPATMAB..."
"ComplexCategory(R:CommutativeRing)" -> "PATAB..."
"ComplexCategory(R:CommutativeRing)" -> "FLINEXP..."
"ComplexCategory(R:CommutativeRing)" -> "COMRING..."
"ComplexCategory(R:CommutativeRing)" -> "ORDSET..."

"MONOGEN..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"DIFEXT..." [color=lightblue];
"FEVALAB..." [color=lightblue];
"COMRING..." [color=lightblue];
"FPATMAB..." [color=lightblue];
"PATAB..." [color=lightblue];
"FLINEXP..." [color=lightblue];
"ORDSET..." [color=lightblue];
}
```

20.2 FunctionFieldCategory (FFCAT)



See:

⇐ "MonogenicAlgebra" (MONOGEN) 19.2 on page 1305

Exports:

0	1
absolutelyIrreducible?	algSplitSimple
associates?	basis
branchPoint?	branchPointAtInfinity?
characteristic	characteristicPolynomial
charthRoot	coerce
complementaryBasis	conditionP
convert	coordinates
createPrimitiveElement	D
definingPolynomial	derivationCoordinates
differentiate	discreteLog
discriminant	divide
elliptic	elt
euclideanSize	expressIdealMember
exquo	extendedEuclidean
factor	factorsOfCyclicGroupSize
gcd	gcdPolynomial
generator	genus
hash	hyperelliptic
index	init
integral?	integralAtInfinity?
integralBasis	integralBasisAtInfinity
integralCoordinates	integralDerivationMatrix
integralMatrix	integralMatrixAtInfinity
integralRepresents	inv
inverseIntegralMatrix	inverseIntegralMatrixAtInfinity
latex	lcm
lift	lookup
minimalPolynomial	multiEuclidean
nextItem	nonSingularModel
norm	normalizeAtInfinity
numberOfComponents	one?
order	prime?
primeFrobenius	primitive?
primitiveElement	primitivePart
principalIdeal	ramified?
ramifiedAtInfinity?	rank
random	rationalPoints
rationalPoint?	recip
reduce	reduceBasisAtInfinity
reducedSystem	regularRepresentation
representationType	represents
retract	retractIfCan
sample	singular?
singularAtInfinity?	size
sizeLess?	squareFree
squareFreePart	subtractIfCan
tableForDiscreteLogarithm	trace
traceMatrix	unit?
unitCanonical	unitNormal
yCoordinates	zero?
?*?	?**?
?+?	?-?
-?	?=?
?^?	?~=?
?/?	?quo?
?rem?	

Attributes Exported:

- if $ has Fraction(UPOLYC(UFD)) and Field then noZeroDivisors where **noZeroDivisors** is true if $x * y \neq 0$ implies both x and y are non-zero.
- if $ has Fraction(UPOLYC(UFD)) and Field then canonicalUnitNormal where **canonicalUnitNormal** is true if we can choose a canonical representative for each class of associate elements, that is `associates?(a,b)` returns true if and only if `unitCanonical(a) = unitCanonical(b)`.
- if $ has Fraction(UPOLYC(UFD)) and Field then canonicalsClosed where **canonicalsClosed** is true if `unitCanonical(a)*unitCanonical(b) = unitCanonical(a*b)`.
- **commutative("*")** is true if it has an operation " $*$ " : $(D, D) -> D$ which is commutative.
- **unitsKnown** is true if a monoid (a multiplicative semigroup with a 1) has unitsKnown means that the operation `recip` can only return "failed" if its argument is not a unit.
- **leftUnitary** is true if $1 * x = x$ for all x.
- **rightUnitary** is true if $x * 1 = x$ for all x.

These are directly exported but not implemented:

```
branchPointAtInfinity? : () -> Boolean
branchPoint? : UP -> Boolean
branchPoint? : F -> Boolean
integralBasis : () -> Vector %
integralBasisAtInfinity : () -> Vector %
ramifiedAtInfinity? : () -> Boolean
ramified? : UP -> Boolean
ramified? : F -> Boolean
singularAtInfinity? : () -> Boolean
singular? : F -> Boolean
singular? : UP -> Boolean
```

These are implemented by this category:

```
absolutelyIrreducible? : () -> Boolean
algSplitSimple : (%,(UP -> UP)) ->
     Record(num: %,den: UP,derivden: UP,gd: UP)
complementaryBasis : Vector % -> Vector %
differentiate : (%,(UP -> UP)) -> %
elliptic : () -> Union(UP,"failed")
elt : (%,F,F) -> F
genus : () -> NonNegativeInteger
```

```
hyperelliptic : () -> Union(UP,"failed")
integral? : % -> Boolean
integral? : (%,F) -> Boolean
integral? : (%,UP) -> Boolean
integralAtInfinity? : % -> Boolean
normalizeAtInfinity : Vector % -> Vector %
numberOfComponents : () -> NonNegativeInteger
primitivePart : % -> %
rationalPoint? : (F,F) -> Boolean
rationalPoints : () -> List List F if F has FINITE
reduceBasisAtInfinity : Vector % -> Vector %
represents : (Vector UP,UP) -> %
yCoordinates : % -> Record(num: Vector UP,den: UP)
```

These exports come from (p1305) MonogenicAlgebra(RF, UPUP)
where RF:Fraction UP, UP:UnivariatePolynomialCategory F
F:UniqueFactorizationDomain, and
UPUP:UnivariatePolynomialCategory Fraction UP

```
0 : () -> %
1 : () -> %
associates? : (%,%) -> Boolean
    if Fraction UP has FIELD
basis : () -> Vector %
characteristic : () -> NonNegativeInteger
characteristicPolynomial : % -> UPUP
charthRoot : % -> Union(%,"failed")
    if Fraction UP has CHARNZ
charthRoot : % -> %
    if Fraction UP has FFIELDC
coerce : % -> %
    if Fraction UP has FIELD
coerce : Fraction Integer -> %
    if Fraction UP has FIELD
    or Fraction UP has RETRACT FRAC INT
coerce : Fraction UP -> %
coerce : Integer -> %
coerce : % -> OutputForm
conditionP : Matrix % -> Union(Vector %,"failed")
    if Fraction UP has FFIELDC
convert : UPUP -> %
convert : % -> UPUP
convert : Vector Fraction UP -> %
convert : % -> Vector Fraction UP
coordinates : Vector % -> Matrix Fraction UP
coordinates : % -> Vector Fraction UP
coordinates : (Vector %,Vector %) -> Matrix Fraction UP
coordinates : (%,Vector %) -> Vector Fraction UP
createPrimitiveElement : () -> %
    if Fraction UP has FFIELDC
```

```
D : % -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,DifferentialRing))
     or
       and(
         has(Fraction UP,DifferentialRing),
         has(Fraction UP,Field))
     or Fraction UP has FFIELDC
D : (%,NonNegativeInteger) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,DifferentialRing))
     or
       and(
         has(Fraction UP,DifferentialRing),
         has(Fraction UP,Field))
     or Fraction UP has FFIELDC
D : (%,Symbol) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,PartialDifferentialRing Symbol))
     or
       and(
         has(Fraction UP,PartialDifferentialRing Symbol),
         has(Fraction UP,Field))
D : (%,List Symbol) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,PartialDifferentialRing Symbol))
     or
       and(
         has(Fraction UP,PartialDifferentialRing Symbol),
         has(Fraction UP,Field))
D : (%,Symbol,NonNegativeInteger) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,PartialDifferentialRing Symbol))
     or
       and(
         has(Fraction UP,PartialDifferentialRing Symbol),
         has(Fraction UP,Field))
D : (%,List Symbol,List NonNegativeInteger) -> %
    if
       and(
```

```
          has(Fraction UP,Field),
          has(Fraction UP,PartialDifferentialRing Symbol))
      or
        and(
          has(Fraction UP,PartialDifferentialRing Symbol),
          has(Fraction UP,Field))
D : (%,(Fraction UP -> Fraction UP)) -> %
    if Fraction UP has FIELD
D : (%,(Fraction UP -> Fraction UP),NonNegativeInteger) -> %
    if Fraction UP has FIELD
definingPolynomial : () -> UPUP
derivationCoordinates : (Vector %,(Fraction UP -> Fraction UP))
  -> Matrix Fraction UP
    if Fraction UP has FIELD
differentiate : % -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,DifferentialRing))
      or
        and(
          has(Fraction UP,DifferentialRing),
          has(Fraction UP,Field))
      or Fraction UP has FFIELDC
differentiate : (%,NonNegativeInteger) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,DifferentialRing))
      or
        and(
          has(Fraction UP,DifferentialRing),
          has(Fraction UP,Field))
      or Fraction UP has FFIELDC
differentiate : (%,Symbol) -> %
    if
      and(
        has(Fraction UP,Field),
        has(Fraction UP,PartialDifferentialRing Symbol))
    or
     and(
       has(Fraction UP,PartialDifferentialRing Symbol),
       has(Fraction UP,Field))
differentiate : (%,List Symbol) -> %
    if
      and(
        has(Fraction UP,Field),
        has(Fraction UP,PartialDifferentialRing Symbol))
      or
       and(
```

```
        has(Fraction UP,PartialDifferentialRing Symbol),
        has(Fraction UP,Field))
differentiate : (%,Symbol,NonNegativeInteger) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,PartialDifferentialRing Symbol))
     or
       and(
         has(Fraction UP,PartialDifferentialRing Symbol),
         has(Fraction UP,Field))
differentiate : (%,List Symbol,List NonNegativeInteger) -> %
    if
       and(
         has(Fraction UP,Field),
         has(Fraction UP,PartialDifferentialRing Symbol))
      or
       and(
         has(Fraction UP,PartialDifferentialRing Symbol),
         has(Fraction UP,Field))
differentiate : (%,(Fraction UP -> Fraction UP)) -> %
    if Fraction UP has FIELD
differentiate :
  (%,(Fraction UP -> Fraction UP),NonNegativeInteger) -> %
    if Fraction UP has FIELD
discreteLog : (%,%) -> Union(NonNegativeInteger,"failed")
    if Fraction UP has FFIELDC
discreteLog : % -> NonNegativeInteger
    if Fraction UP has FFIELDC
discriminant : Vector % -> Fraction UP
discriminant : () -> Fraction UP
divide : (%,%) -> Record(quotient: %,remainder: %)
    if Fraction UP has FIELD
euclideanSize : % -> NonNegativeInteger
    if Fraction UP has FIELD
expressIdealMember : (List %,%) -> Union(List %,"failed")
    if Fraction UP has FIELD
exquo : (%,%) -> Union(%,"failed")
    if Fraction UP has FIELD
extendedEuclidean : (%,%) ->
   Record(coef1: %,coef2: %,generator: %)
    if Fraction UP has FIELD
extendedEuclidean : (%,%,%) ->
   Union(Record(coef1: %,coef2: %),"failed")
    if Fraction UP has FIELD
factor : % -> Factored %
    if Fraction UP has FIELD
factorsOfCyclicGroupSize : () ->
   List Record(factor: Integer,exponent: Integer)
    if Fraction UP has FFIELDC
```

```
gcd : (%,%) -> %
    if Fraction UP has FIELD
gcd : List % -> %
    if Fraction UP has FIELD
gcdPolynomial : (SparseUnivariatePolynomial %,
                 SparseUnivariatePolynomial %) ->
                    SparseUnivariatePolynomial %
    if Fraction UP has FIELD
generator : () -> %
hash : % -> SingleInteger
index : PositiveInteger -> %
    if Fraction UP has FINITE
init : () -> %
    if Fraction UP has FFIELDC
integralCoordinates : % ->
   Record(num: Vector UP,den: UP)
integralDerivationMatrix : (UP -> UP) ->
   Record(num: Matrix UP,den: UP)
integralMatrix : () -> Matrix Fraction UP
integralMatrixAtInfinity : () -> Matrix Fraction UP
integralRepresents : (Vector UP,UP) -> %
inv : % -> %
    if Fraction UP has FIELD
inverseIntegralMatrix : () -> Matrix Fraction UP
inverseIntegralMatrixAtInfinity : () ->
   Matrix Fraction UP
latex : % -> String
lcm : (%,%) -> %
    if Fraction UP has FIELD
lcm : List % -> %
    if Fraction UP has FIELD
lift : % -> UPUP
lookup : % -> PositiveInteger
    if Fraction UP has FINITE
minimalPolynomial : % -> UPUP
    if Fraction UP has FIELD
multiEuclidean : (List %,%) -> Union(List %,"failed")
    if Fraction UP has FIELD
nextItem : % -> Union(%,"failed")
    if Fraction UP has FFIELDC
nonSingularModel : Symbol -> List Polynomial F
    if F has FIELD
norm : % -> Fraction UP
one? : % -> Boolean
order : % -> OnePointCompletion PositiveInteger
    if Fraction UP has FFIELDC
order : % -> PositiveInteger
    if Fraction UP has FFIELDC
prime? : % -> Boolean
    if Fraction UP has FIELD
```

```
primeFrobenius : % -> %
    if Fraction UP has FFIELDC
primeFrobenius : (%,NonNegativeInteger) -> %
    if Fraction UP has FFIELDC
primitive? : % -> Boolean
    if Fraction UP has FFIELDC
primitiveElement : () -> %
    if Fraction UP has FFIELDC
principalIdeal : List % ->
  Record(coef: List %,generator: %)
    if Fraction UP has FIELD
rank : () -> PositiveInteger
random : () -> %
    if Fraction UP has FINITE
recip : % -> Union(%,"failed")
reduce : UPUP -> %
reduce : Fraction UPUP -> Union(%,"failed")
    if Fraction UP has FIELD
reducedSystem : Matrix % -> Matrix Fraction UP
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Fraction UP,vec: Vector Fraction UP)
reducedSystem : (Matrix %,Vector %) ->
  Record(mat: Matrix Integer,vec: Vector Integer)
    if Fraction UP has LINEXP INT
reducedSystem : Matrix % -> Matrix Integer
    if Fraction UP has LINEXP INT
regularRepresentation : % -> Matrix Fraction UP
regularRepresentation : (%,Vector %) -> Matrix Fraction UP
representationType : () ->
  Union("prime",polynomial,normal,cyclic)
    if Fraction UP has FFIELDC
represents : Vector Fraction UP -> %
represents : (Vector Fraction UP,Vector %) -> %
retract : % -> Fraction Integer
    if Fraction UP has RETRACT FRAC INT
retract : % -> Integer
    if Fraction UP has RETRACT INT
retract : % -> Fraction UP
retractIfCan : % -> Union(Fraction UP,"failed")
retractIfCan : % -> Union(Fraction Integer,"failed")
    if Fraction UP has RETRACT FRAC INT
retractIfCan : % -> Union(Integer,"failed")
    if Fraction UP has RETRACT INT
sample : () -> %
size : () -> NonNegativeInteger
    if Fraction UP has FINITE
sizeLess? : (%,%) -> Boolean
    if Fraction UP has FIELD
squareFree : % -> Factored %
    if Fraction UP has FIELD
```

```
squareFreePart : % -> %
    if Fraction UP has FIELD
subtractIfCan : (%,%) -> Union(%,"failed")
tableForDiscreteLogarithm : Integer ->
  Table(PositiveInteger,NonNegativeInteger)
    if Fraction UP has FFIELDC
trace : % -> Fraction UP
traceMatrix : () -> Matrix Fraction UP
traceMatrix : Vector % -> Matrix Fraction UP
unit? : % -> Boolean
    if Fraction UP has FIELD
unitCanonical : % -> %
    if Fraction UP has FIELD
unitNormal : % -> Record(unit: %,canonical: %,associate: %)
    if Fraction UP has FIELD
zero? : % -> Boolean
?*? : (Fraction UP,%) -> %
?*? : (%,Fraction UP) -> %
?*? : (%,%) -> %
?*? : (Integer,%) -> %
?*? : (PositiveInteger,%) -> %
?**? : (%,PositiveInteger) -> %
?+? : (%,%) -> %
?-? : (%,%) -> %
-? : % -> %
?=? : (%,%) -> Boolean
?^? : (%,PositiveInteger) -> %
?~=? : (%,%) -> Boolean
?*? : (%,Fraction Integer) -> % if Fraction UP has FIELD
?*? : (Fraction Integer,%) -> % if Fraction UP has FIELD
?*? : (NonNegativeInteger,%) -> %
?**? : (%,Integer) -> % if Fraction UP has FIELD
?**? : (%,NonNegativeInteger) -> %
?/? : (%,%) -> % if Fraction UP has FIELD
?^? : (%,Integer) -> % if Fraction UP has FIELD
?^? : (%,NonNegativeInteger) -> %
?quo? : (%,%) -> % if Fraction UP has FIELD
?rem? : (%,%) -> % if Fraction UP has FIELD
```

⟨*category FFCAT FunctionFieldCategory*⟩≡

```
)abbrev category FFCAT FunctionFieldCategory
++ Function field of a curve
++ Author: Manuel Bronstein
++ Date Created: 1987
++ Date Last Updated: 19 Mai 1993
++ Description: This category is a model for the function field of a
++ plane algebraic curve.
++ Keywords: algebraic, curve, function, field.
FunctionFieldCategory(F, UP, UPUP): Category == Definition where
```

```
F   : UniqueFactorizationDomain
UP  : UnivariatePolynomialCategory F
UPUP: UnivariatePolynomialCategory Fraction UP

Z   ==> Integer
Q   ==> Fraction F
P   ==> Polynomial F
RF  ==> Fraction UP
QF  ==> Fraction UPUP
SY  ==> Symbol
REC ==> Record(num:$, den:UP, derivden:UP, gd:UP)

Definition ==> MonogenicAlgebra(RF, UPUP) with
  numberOfComponents     : () -> NonNegativeInteger
    ++ numberOfComponents() returns the number of absolutely irreducible
    ++ components.
    ++
    ++X P0 := UnivariatePolynomial(x, Integer)
    ++X P1 := UnivariatePolynomial(y, Fraction P0)
    ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
    ++X numberOfComponents()$R
  genus                  : () -> NonNegativeInteger
    ++ genus() returns the genus of one absolutely irreducible component
    ++
    ++X P0 := UnivariatePolynomial(x, Integer)
    ++X P1 := UnivariatePolynomial(y, Fraction P0)
    ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
    ++X genus()$R
  absolutelyIrreducible? : () -> Boolean
    ++ absolutelyIrreducible?() tests if the curve absolutely irreducible?
    ++
    ++X P0 := UnivariatePolynomial(x, Integer)
    ++X P1 := UnivariatePolynomial(y, Fraction P0)
    ++X R2 := RadicalFunctionField(INT, P0, P1, 2 * x**2, 4)
    ++X absolutelyIrreducible?()$R2
  rationalPoint?         : (F, F) -> Boolean
    ++ rationalPoint?(a, b) tests if \spad{(x=a,y=b)} is on the curve.
    ++
    ++X P0 := UnivariatePolynomial(x, Integer)
    ++X P1 := UnivariatePolynomial(y, Fraction P0)
    ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
    ++X rationalPoint?(0,0)$R
    ++X R2 := RadicalFunctionField(INT, P0, P1, 2 * x**2, 4)
    ++X rationalPoint?(0,0)$R2
  branchPointAtInfinity? : () -> Boolean
    ++ branchPointAtInfinity?() tests if there is a branch point
```

```
  ++ at infinity.
  ++
  ++X P0 := UnivariatePolynomial(x, Integer)
  ++X P1 := UnivariatePolynomial(y, Fraction P0)
  ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
  ++X branchPointAtInfinity?()$R
  ++X R2 := RadicalFunctionField(INT, P0, P1, 2 * x**2, 4)
  ++X branchPointAtInfinity?()$R
branchPoint?           : F -> Boolean
  ++ branchPoint?(a) tests whether \spad{x = a} is a branch point.
branchPoint?           : UP -> Boolean
  ++ branchPoint?(p) tests whether \spad{p(x) = 0} is a branch point.
singularAtInfinity?    : () -> Boolean
  ++ singularAtInfinity?() tests if there is a singularity at infinity.
singular?              : F -> Boolean
  ++ singular?(a) tests whether \spad{x = a} is singular.
singular?              : UP -> Boolean
  ++ singular?(p) tests whether \spad{p(x) = 0} is singular.
ramifiedAtInfinity?    : () -> Boolean
  ++ ramifiedAtInfinity?() tests if infinity is ramified.
ramified?              : F -> Boolean
  ++ ramified?(a) tests whether \spad{x = a} is ramified.
ramified?              : UP -> Boolean
  ++ ramified?(p) tests whether \spad{p(x) = 0} is ramified.
integralBasis          : () -> Vector $
  ++ integralBasis() returns the integral basis for the curve.
  ++
  ++X P0 := UnivariatePolynomial(x, Integer)
  ++X P1 := UnivariatePolynomial(y, Fraction P0)
  ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
  ++X integralBasis()$R
integralBasisAtInfinity: () -> Vector $
  ++ integralBasisAtInfinity() returns the local integral basis
  ++ at infinity
  ++
  ++X P0 := UnivariatePolynomial(x, Integer)
  ++X P1 := UnivariatePolynomial(y, Fraction P0)
  ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
  ++X integralBasisAtInfinity()$R
integralAtInfinity?    : $  -> Boolean
  ++ integralAtInfinity?() tests if f is locally integral at infinity.
integral?              : $  -> Boolean
  ++ integral?() tests if f is integral over \spad{k[x]}.
complementaryBasis     : Vector $ -> Vector $
  ++ complementaryBasis(b1,...,bn) returns the complementary basis
  ++ \spad{(b1',...,bn')} of \spad{(b1,...,bn)}.
```

```
normalizeAtInfinity    : Vector $ -> Vector $
  ++ normalizeAtInfinity(v) makes v normal at infinity.
reduceBasisAtInfinity  : Vector $ -> Vector $
  ++ reduceBasisAtInfinity(b1,...,bn) returns \spad{(x**i * bj)}
  ++ for all i,j such that \spad{x**i*bj} is locally integral
  ++ at infinity.
integralMatrix         : () -> Matrix RF
  ++ integralMatrix() returns M such that
  ++ \spad{(w1,...,wn) = M (1, y, ..., y**(n-1))},
  ++ where \spad{(w1,...,wn)} is the integral basis of
  ++ \spadfunFrom{integralBasis}{FunctionFieldCategory}.
  ++
  ++X P0 := UnivariatePolynomial(x, Integer)
  ++X P1 := UnivariatePolynomial(y, Fraction P0)
  ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
  ++X integralMatrix()$R
inverseIntegralMatrix  : () -> Matrix RF
  ++ inverseIntegralMatrix() returns M such that
  ++ \spad{M (w1,...,wn) = (1, y, ..., y**(n-1))}
  ++ where \spad{(w1,...,wn)} is the integral basis of
  ++ \spadfunFrom{integralBasis}{FunctionFieldCategory}.
  ++
  ++X P0 := UnivariatePolynomial(x, Integer)
  ++X P1 := UnivariatePolynomial(y, Fraction P0)
  ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
  ++X inverseIntegralMatrix()$R
integralMatrixAtInfinity       : () -> Matrix RF
  ++ integralMatrixAtInfinity() returns M such that
  ++ \spad{(v1,...,vn) = M (1, y, ..., y**(n-1))}
  ++ where \spad{(v1,...,vn)} is the local integral basis at infinity
  ++ returned by \spad{infIntBasis()}.
  ++
  ++X P0 := UnivariatePolynomial(x, Integer)
  ++X P1 := UnivariatePolynomial(y, Fraction P0)
  ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
  ++X integralMatrixAtInfinity()$R
inverseIntegralMatrixAtInfinity: () -> Matrix RF
  ++ inverseIntegralMatrixAtInfinity() returns M such
  ++ that \spad{M (v1,...,vn) = (1, y, ..., y**(n-1))}
  ++ where \spad{(v1,...,vn)} is the local integral basis at infinity
  ++ returned by \spad{infIntBasis()}.
  ++
  ++X P0 := UnivariatePolynomial(x, Integer)
  ++X P1 := UnivariatePolynomial(y, Fraction P0)
  ++X R := RadicalFunctionField(INT, P0, P1, 1 - x**20, 20)
  ++X inverseIntegralMatrixAtInfinity()$R
```

```
yCoordinates           : $ -> Record(num:Vector(UP), den:UP)
  ++ yCoordinates(f) returns \spad{[[A1,...,An], D]} such that
  ++ \spad{f = (A1 + A2 y +...+ An y**(n-1)) / D}.
represents             : (Vector UP, UP) -> $
  ++ represents([A0,...,A(n-1)],D) returns
  ++ \spad{(A0 + A1 y +...+ A(n-1)*y**(n-1))/D}.
integralCoordinates    : $ -> Record(num:Vector(UP), den:UP)
  ++ integralCoordinates(f) returns \spad{[[A1,...,An], D]} such that
  ++ \spad{f = (A1 w1 +...+ An wn) / D}  where \spad{(w1,...,wn)} is the
  ++ integral basis returned by \spad{integralBasis()}.
integralRepresents     : (Vector UP, UP) -> $
  ++ integralRepresents([A1,...,An], D) returns
  ++ \spad{(A1 w1+...+An wn)/D}
  ++ where \spad{(w1,...,wn)} is the integral
  ++ basis of \spad{integralBasis()}.
integralDerivationMatrix:(UP -> UP) -> Record(num:Matrix(UP),den:UP)
  ++ integralDerivationMatrix(d) extends the derivation d from UP to $
  ++ and returns (M, Q) such that the i^th row of M divided by Q form
  ++ the coordinates of \spad{d(wi)} with respect to \spad{(w1,...,wn)}
  ++ where \spad{(w1,...,wn)} is the integral basis returned
  ++ by integralBasis().
integral?              : ($,  F) -> Boolean
  ++ integral?(f, a) tests whether f is locally integral at \spad{x = a}.
integral?              : ($, UP) -> Boolean
  ++ integral?(f, p) tests whether f is locally integral at
  ++ \spad{p(x) = 0}
differentiate          : ($, UP -> UP) -> $
  ++ differentiate(x, d) extends the derivation d from UP to $ and
  ++ applies it to x.
represents             : (Vector UP, UP) -> $
  ++ represents([A0,...,A(n-1)],D) returns
  ++ \spad{(A0 + A1 y +...+ A(n-1)*y**(n-1))/D}.
primitivePart          : $ -> $
  ++ primitivePart(f) removes the content of the denominator and
  ++ the common content of the numerator of f.
elt                    : ($, F, F) -> F
  ++ elt(f,a,b) or f(a, b) returns the value of f
  ++ at the point \spad{(x = a, y = b)}
  ++ if it is not singular.
elliptic               : () -> Union(UP, "failed")
  ++ elliptic() returns \spad{p(x)} if the curve is the elliptic
  ++ defined by \spad{y**2 = p(x)}, "failed" otherwise.
hyperelliptic          : () -> Union(UP, "failed")
  ++ hyperelliptic() returns \spad{p(x)} if the curve is the
  ++ hyperelliptic
  ++ defined by \spad{y**2 = p(x)}, "failed" otherwise.
```

```
    algSplitSimple         : ($, UP -> UP) -> REC
      ++ algSplitSimple(f, D) returns \spad{[h,d,d',g]} such that
      ++ \spad{f=h/d},
      ++ \spad{h} is integral at all the normal places w.r.t. \spad{D},
      ++ \spad{d' = Dd}, \spad{g = gcd(d, discriminant())} and \spad{D}
      ++ is the derivation to use. \spad{f} must have at most simple finite
      ++ poles.
    if F has Field then
      nonSingularModel: SY -> List Polynomial F
        ++ nonSingularModel(u) returns the equations in u1,...,un of
        ++ an affine non-singular model for the curve.
    if F has Finite then
      rationalPoints: () -> List List F
        ++ rationalPoints() returns the list of all the affine
        ++rational points.
  add
    import InnerCommonDenominator(UP, RF, Vector UP, Vector RF)
    import UnivariatePolynomialCommonDenominator(UP, RF, UPUP)

    repOrder: (Matrix RF, Z) -> Z
    Q2RF    : Q  -> RF
    infOrder: RF -> Z
    infValue: RF -> Fraction F
    intvalue: (Vector UP, F, F) -> F
    rfmonom : Z  -> RF
    kmin    : (Matrix RF,Vector Q) -> Union(Record(pos:Z,km:Z),"failed")

    Q2RF q                 == numer(q)::UP / denom(q)::UP
    infOrder f             == (degree denom f)::Z - (degree numer f)::Z
    integral? f            == ground?(integralCoordinates(f).den)
    integral?(f:$, a:F)    == (integralCoordinates(f).den)(a) ^= 0
--    absolutelyIrreducible? == one? numberOfComponents()
    absolutelyIrreducible? == numberOfComponents() = 1
    yCoordinates f         == splitDenominator coordinates f

    hyperelliptic() ==
      degree(f := definingPolynomial()) ^= 2 => "failed"
      (u:=retractIfCan(reductum f)@Union(RF,"failed"))
        case "failed" => "failed"
      (v:=retractIfCan(-(u::RF) / leadingCoefficient f)@Union(UP, "failed"))
        case "failed" => "failed"
      odd? degree(p := v::UP) => p
      "failed"

    algSplitSimple(f, derivation) ==
      cd := splitDenominator lift f
```

```
        dd := (cd.den exquo (g := gcd(cd.den, derivation(cd.den))))::UP
        [reduce(inv(g::RF) * cd.num), dd, derivation dd,
                                    gcd(dd, retract(discriminant())@UP)]

      elliptic() ==
        (u := hyperelliptic()) case "failed" => "failed"
        degree(p := u::UP) = 3 => p
        "failed"

      rationalPoint?(x, y)   ==
        zero?((definingPolynomial() (y::UP::RF)) (x::UP::RF))

      if F has Field then
        import PolyGroebner(F)
        import MatrixCommonDenominator(UP, RF)

        UP2P  : (UP,   P)    -> P
        UPUP2P: (UPUP, P, P) -> P

        UP2P(p, x) ==
          (map((s:F):P +-> s::P, p)_
            $UnivariatePolynomialCategoryFunctions2(F, UP,
                                    P, SparseUnivariatePolynomial P)) x

        UPUP2P(p, x, y) ==
          (map((s:RF):P +-> UP2P(retract(s)@UP, x),p)_
            $UnivariatePolynomialCategoryFunctions2(RF, UPUP,
                                    P, SparseUnivariatePolynomial P)) y

        nonSingularModel u ==
          d    := commonDenominator(coordinates(w := integralBasis()))::RF
          vars := [concat(string u, string i)::SY for i in 1..(n := #w)]
          x    := "%%dummy1"::SY
          y    := "%%dummy2"::SY
          select_!(s+->zero?(degree(s, x)) and zero?(degree(s, y)),
                   lexGroebner([v::P - UPUP2P(lift(d * w.i), x::P, y::P)
                      for v in vars for i in 1..n], concat([x, y], vars)))

      if F has Finite then
        ispoint: (UPUP, F, F) -> List F

  -- must use the 'elt function explicitely or the compiler takes 45 mins
  -- on that function    MB 5/90
  -- still takes ages : I split the expression up. JHD 6/Aug/90
        ispoint(p, x, y) ==
          jhd:RF:=p(y::UP::RF)
```

```
      zero?(jhd (x::UP::RF)) => [x, y]
      empty()

    rationalPoints() ==
      p := definingPolynomial()
      concat [[pt for y in 1..size()$F | not empty?(pt :=
        ispoint(p, index(x::PositiveInteger)$F,
                   index(y::PositiveInteger)$F))]$List(List F)
                              for x in 1..size()$F]$List(List(List F))

  intvalue(v, x, y) ==
    singular? x => error "Point is singular"
    mini := minIndex(w := integralBasis())
    rec := yCoordinates(+/[qelt(v, i)::RF * qelt(w, i)
                           for i in mini .. maxIndex w])
    n   := +/[(qelt(rec.num, i) x) *
              (y ** ((i - mini)::NonNegativeInteger))
                           for i in mini .. maxIndex w]
    zero?(d := (rec.den) x) =>
      zero? n => error "0/0 -- cannot compute value yet"
      error "Shouldn't happen"
    (n exquo d)::F

  elt(f, x, y) ==
    rec := integralCoordinates f
    n   := intvalue(rec.num, x, y)
    zero?(d := (rec.den) x) =>
      zero? n => error "0/0 -- cannot compute value yet"
      error "Function has a pole at the given point"
    (n exquo d)::F

  primitivePart f ==
    cd := yCoordinates f
    d  := gcd([content qelt(cd.num, i)
               for i in minIndex(cd.num) .. maxIndex(cd.num)]$List(F))
                 * primitivePart(cd.den)
    represents [qelt(cd.num, i) / d
             for i in minIndex(cd.num) .. maxIndex(cd.num)]$Vector(RF)

  reduceBasisAtInfinity b ==
    x := monomial(1, 1)$UP ::RF
    concat([[f for j in 0.. while
              integralAtInfinity?(f := x**j * qelt(b, i))]$Vector($)
                    for i in minIndex b .. maxIndex b]$List(Vector $))

  complementaryBasis b ==
```

```
    m := inverse(traceMatrix b)::Matrix(RF)
    [represents row(m, i) for i in minRowIndex m .. maxRowIndex m]

  integralAtInfinity? f ==
    not any?(s +-> infOrder(s) < 0,
       coordinates(f) * inverseIntegralMatrixAtInfinity())$Vector(RF)

  numberOfComponents() ==
    count(integralAtInfinity?, integralBasis())$Vector($)

  represents(v:Vector UP, d:UP) ==
    represents
      [qelt(v, i) / d for i in minIndex v .. maxIndex v]$Vector(RF)

  genus() ==
    ds := discriminant()
    d  := degree(retract(ds)@UP) + infOrder(ds * determinant(
          integralMatrixAtInfinity() * inverseIntegralMatrix()) ** 2)
    dd := (((d exquo 2)::Z - rank()) exquo numberOfComponents())::Z
    (dd + 1)::NonNegativeInteger

  repOrder(m, i) ==
    nostart:Boolean := true
    ans:Z := 0
    r := row(m, i)
    for j in minIndex r .. maxIndex r | qelt(r, j) ^= 0 repeat
      ans :=
        nostart => (nostart := false; infOrder qelt(r, j))
        min(ans, infOrder qelt(r,j))
    nostart => error "Null row"
    ans

  infValue f ==
    zero? f => 0
    (n := infOrder f) > 0 => 0
    zero? n =>
      (leadingCoefficient numer f) / (leadingCoefficient denom f)
    error "f not locally integral at infinity"

  rfmonom n ==
    n < 0 => inv(monomial(1, (-n)::NonNegativeInteger)$UP :: RF)
    monomial(1, n::NonNegativeInteger)$UP :: RF

  kmin(m, v) ==
    nostart:Boolean := true
    k:Z := 0
```

```
        ii  := minRowIndex m - (i0  := minIndex v)
        for i in minIndex v .. maxIndex v | qelt(v, i) ^= 0 repeat
          nk := repOrder(m, i + ii)
          if nostart then (nostart := false; k := nk; i0 := i)
          else
            if nk < k then (k := nk; i0 := i)
        nostart => "failed"
        [i0, k]

      normalizeAtInfinity w ==
        ans   := copy w
        infm  := inverseIntegralMatrixAtInfinity()
        mhat  := zero(rank(), rank())$Matrix(RF)
        ii    := minIndex w - minRowIndex mhat
        repeat
          m := coordinates(ans) * infm
          r := [rfmonom repOrder(m, i)
                       for i in minRowIndex m .. maxRowIndex m]$Vector(RF)
          for i in minRowIndex m .. maxRowIndex m repeat
            for j in minColIndex m .. maxColIndex m repeat
              qsetelt_!(mhat, i, j, qelt(r, i + ii) * qelt(m, i, j))
          sol := first nullSpace transpose map(infValue,
                  mhat)$MatrixCategoryFunctions2(RF, Vector RF, Vector RF,
                                 Matrix RF, Q, Vector Q, Vector Q, Matrix Q)
          (pr := kmin(m, sol)) case "failed" => return ans
          qsetelt_!(ans, pr.pos,
           +/[Q2RF(qelt(sol, i)) * rfmonom(repOrder(m, i - ii) - pr.km)
                    * qelt(ans, i) for i in minIndex sol .. maxIndex sol])

      integral?(f:$, p:UP) ==
        (r:=retractIfCan(p)@Union(F,"failed")) case F => integral?(f,r::F)
        (integralCoordinates(f).den exquo p) case "failed"

      differentiate(f:$, d:UP -> UP) ==
        differentiate(f, x +-> differentiate(x, d)$RF)
```

⟨*FFCAT.dotabb*⟩≡

```
  "FFCAT"
   [color=lightblue,href="bookvol10.2.pdf#nameddest=FFCAT"];
  "FFCAT" -> "MONOGEN"
```

⟨*FFCAT.dotfull*⟩≡
```
"FunctionFieldCategory(a:UFD,b:UPOLYC(a),c:UPOLYC(Fraction(b)))"
 [color=lightblue,href="bookvol10.2.pdf#nameddest=FFCAT"];
"FunctionFieldCategory(a:UFD,b:UPOLYC(a),c:UPOLYC(Fraction(b)))"
   -> "MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))"
```

⟨*FFCAT.dotpic*⟩≡
```
digraph pic {
 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];

"FunctionFieldCategory(a:UFD,b:UPOLYC(a),c:UPOLYC(Fraction(b)))"
 [color=lightblue];
"FunctionFieldCategory(a:UFD,b:UPOLYC(a),c:UPOLYC(Fraction(b)))"
   -> "MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))"

"MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))"
 [color=seagreen];
"MonogenicAlgebra(a:FRAC(UPOLYC(UFD)),b:UPOLYC(FRAC(UPOLYC(UFD))))" ->
    "MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"

"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))"
 [color=lightblue];
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FRAMALG..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "COMRING..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "KONVERT..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FRETRCT..."
"MonogenicAlgebra(a:CommutativeRing,b:UnivariatePolynomialCategory(a))" ->
    "FLINEXP..."

"FRAMALG..." [color=lightblue];
"COMRING..." [color=lightblue];
"KONVERT..." [color=lightblue];
"FRETRCT..." [color=lightblue];
"FLINEXP..." [color=lightblue];
}
```

Chapter 21

The bootstrap code

21.1 ABELGRP.lsp BOOTSTRAP

ABELGRP depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ABELGRP** category which we can write into the **MID** directory. We compile the lisp code and copy the **ABELGRP.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ABELGRP.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |AbelianGroup;AL| (QUOTE NIL))

(DEFUN |AbelianGroup| NIL
  (LET (#:G82664)
    (COND
      (|AbelianGroup;AL|)
      (T (SETQ |AbelianGroup;AL| (|AbelianGroup;|))))))

(DEFUN |AbelianGroup;| NIL
  (PROG (#1=#:G82662)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|CancellationAbelianMonoid|)
            (|mkCategory|
```

```
                (QUOTE |domain|)
                (QUOTE (
                  ((|-| (|$| |$|)) T)
                  ((|-| (|$| |$| |$|)) T)
                  ((|*| (|$| (|Integer|) |$|)) T)))
                NIL
                (QUOTE ((|Integer|)))
                NIL))
              |AbelianGroup|)
            (SETELT #1# 0 (QUOTE (|AbelianGroup|)))))))

  (MAKEPROP (QUOTE |AbelianGroup|) (QUOTE NILADIC) T)
```

21.2 ABELGRP-.lsp BOOTSTRAP

ABELGRP- depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ABELGRP-** category which we can write into the **MID** directory. We compile the lisp code and copy the **ABELGRP-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ABELGRP-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |ABELGRP-;-;3S;1| (|x| |y| |$|)
  (SPADCALL |x| (SPADCALL |y| (QREFELT |$| 7)) (QREFELT |$| 8)))

(DEFUN |ABELGRP-;subtractIfCan;2SU;2| (|x| |y| |$|)
  (CONS 0 (SPADCALL |x| |y| (QREFELT |$| 10))))

(DEFUN |ABELGRP-;*;Nni2S;3| (|n| |x| |$|)
  (SPADCALL |n| |x| (QREFELT |$| 14)))

(DEFUN |ABELGRP-;*;I2S;4| (|n| |x| |$|)
  (COND
    ((ZEROP |n|) (|spadConstant| |$| 17))
    ((|<| 0 |n|) (SPADCALL |n| |x| (QREFELT |$| 20)))
    ((QUOTE T)
      (SPADCALL (|-| |n|) (SPADCALL |x| (QREFELT |$| 7)) (QREFELT |$| 20)))))

(DEFUN |AbelianGroup&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|AbelianGroup&|))
        (LETT |dv$| (LIST (QUOTE |AbelianGroup&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 22) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        (COND
          ((|HasCategory| |#1| (QUOTE (|Ring|))))
          ((QUOTE T)
            (QSETREFV |$| 21
              (CONS (|dispatchFunction| |ABELGRP-;*;I2S;4|) |$|))))
        |$|))))
```

```
(MAKEPROP
  (QUOTE |AbelianGroup&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (0 . |-|)
        (5 . |+|)
        |ABELGRP-;-;3S;1|
        (11 . |-|)
        (|Union| |$| (QUOTE "failed"))
        |ABELGRP-;subtractIfCan;2SU;2|
        (|Integer|)
        (17 . |*|)
        (|NonNegativeInteger|)
        |ABELGRP-;*;Nni2S;3|
        (23 . |Zero|)
        (|PositiveInteger|)
        (|RepeatedDoubling| 6)
        (27 . |double|)
        (33 . |*|)))
    (QUOTE #(|subtractIfCan| 39 |-| 45 |*| 51))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 21
            (QUOTE (1 6 0 0 7 2 6 0 0 0 8 2 6 0 0 0 10 2 6 0 13 0 14 0 6 0 17
                    2 19 6 18 6 20 2 0 0 13 0 21 2 0 11 0 0 12 2 0 0 0 0 9 2
                    0 0 13 0 21 2 0 0 15 0 16))))))
    (QUOTE |lookupComplete|)))
```

21.3 ABELMON.lsp BOOTSTRAP

ABELMON which needs **ABELSG** which needs **SETCAT** which needs **SINT** which needs **UFD** which needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON**. We break this chain with **ABELMON.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ABELMON** category which we can write into the **MID** directory. We compile the lisp code and copy the **ABELMON.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ABELMON.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |AbelianMonoid;AL| (QUOTE NIL))

(DEFUN |AbelianMonoid| NIL
  (LET (#:G82597)
    (COND
      (|AbelianMonoid;AL|)
      (T (SETQ |AbelianMonoid;AL| (|AbelianMonoid;|))))))

(DEFUN |AbelianMonoid;| NIL
  (PROG (#1=#:G82595)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|AbelianSemiGroup|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|Zero| (|$|) |constant|) T)
                ((|sample| (|$|) |constant|) T)
                ((|zero?| ((|Boolean|) |$|)) T)
                ((|*| (|$| (|NonNegativeInteger|) |$|)) T)))
              NIL
              (QUOTE ((|NonNegativeInteger|) (|Boolean|)))
              NIL))
            |AbelianMonoid|)
        (SETELT #1# 0 (QUOTE (|AbelianMonoid|)))))))

(MAKEPROP (QUOTE |AbelianMonoid|) (QUOTE NILADIC) T)
```

21.4 ABELMON-.lsp BOOTSTRAP

ABELMON- which needs **ABELSG** which needs **SETCAT** which needs **SINT** which needs **UFD** which needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON-**. We break this chain with **ABELMON-.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ABELMON-** category which we can write into the **MID** directory. We compile the lisp code and copy the **ABELMON-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ABELMON-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |ABELMON-;zero?;SB;1| (|x| |$|)
  (SPADCALL |x| (|spadConstant| |$| 7) (QREFELT |$| 9)))

(DEFUN |ABELMON-;*;Pi2S;2| (|n| |x| |$|)
  (SPADCALL |n| |x| (QREFELT |$| 12)))

(DEFUN |ABELMON-;sample;S;3| (|$|)
  (|spadConstant| |$| 7))

(DEFUN |ABELMON-;*;Nni2S;4| (|n| |x| |$|)
  (COND
    ((ZEROP |n|) (|spadConstant| |$| 7))
    ((QUOTE T) (SPADCALL |n| |x| (QREFELT |$| 17)))))

(DEFUN |AbelianMonoid&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|AbelianMonoid&|))
        (LETT |dv$| (LIST (QUOTE |AbelianMonoid&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 19) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        (COND
          ((|HasCategory| |#1| (QUOTE (|Ring|))))
          ((QUOTE T)
            (QSETREFV |$| 18
```

```
              (CONS (|dispatchFunction| |ABELMON-;*;Nni2S;4|) |$|)))) |$|))))

(MAKEPROP
  (QUOTE |AbelianMonoid&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (0 . |Zero|)
        (|Boolean|)
        (4 . |=|)
        |ABELMON-;zero?;SB;1|
        (|NonNegativeInteger|)
        (10 . |*|)
        (|PositiveInteger|)
        |ABELMON-;*;Pi2S;2|
        |ABELMON-;sample;S;3|
        (|RepeatedDoubling| 6)
        (16 . |double|)
        (22 . |*|)))
    (QUOTE #(|zero?| 28 |sample| 33 |*| 37))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 18
            (QUOTE (0 6 0 7 2 6 8 0 0 9 2 6 0 11 0 12 2 16 6 13 6 17 2 0 0 11
                    0 18 1 0 8 0 10 0 0 0 15 2 0 0 11 0 18 2 0 0 13 0 14))))))
    (QUOTE |lookupComplete|)))
```

21.5 ABELSG.lsp BOOTSTRAP

ABELSG needs **SETCAT** which needs **SINT** which needs **UFD** which needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON** which needs **ABELSG**. We break this chain with **ABELSG.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ABELSG** category which we can write into the **MID** directory. We compile the lisp code and copy the **ABELSG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ABELSG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |AbelianSemiGroup;AL| (QUOTE NIL))

(DEFUN |AbelianSemiGroup| NIL
  (LET (#:G82568)
    (COND
      (|AbelianSemiGroup;AL|)
      (T (SETQ |AbelianSemiGroup;AL| (|AbelianSemiGroup;|))))))

(DEFUN |AbelianSemiGroup;| NIL
  (PROG (#1=#:G82566)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|SetCategory|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|+| (|$| |$| |$|)) T)
                ((|*| (|$| (|PositiveInteger|) |$|)) T)))
              NIL
              (QUOTE ((|PositiveInteger|)))
              NIL))
            |AbelianSemiGroup|)
        (SETELT #1# 0 (QUOTE (|AbelianSemiGroup|)))))))

(MAKEPROP (QUOTE |AbelianSemiGroup|) (QUOTE NILADIC) T)
```

21.6 ABELSG-.lsp BOOTSTRAP

ABELSG- needs **SETCAT** which needs **SINT** which needs **UFD** which needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON** which needs **ABELSG-**. We break this chain with **ABELSG-.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ABELSG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **ABELSG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ABELSG-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |ABELSG-;*;Pi2S;1| (|n| |x| |$|) (SPADCALL |n| |x| (QREFELT |$| 9)))

(DEFUN |AbelianSemiGroup&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|AbelianSemiGroup&|))
        (LETT |dv$| (LIST (QUOTE |AbelianSemiGroup&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 11) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        (COND
          ((|HasCategory| |#1| (QUOTE (|Ring|))))
          ((QUOTE T)
            (QSETREFV |$| 10
              (CONS (|dispatchFunction| |ABELSG-;*;Pi2S;1|) |$|))))
        |$|))))

(MAKEPROP
  (QUOTE |AbelianSemiGroup&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (|PositiveInteger|)
        (|RepeatedDoubling| 6)
        (0 . |double|)
```

```
    (6 . |*|)))
(QUOTE #(|*| 12))
(QUOTE NIL)
(CONS
  (|makeByteWordVec2| 1 (QUOTE NIL))
  (CONS
    (QUOTE #())
    (CONS
      (QUOTE #())
      (|makeByteWordVec2| 10
        (QUOTE (2 8 6 7 6 9 2 0 0 7 0 10 2 0 0 7 0 10))))))
(QUOTE |lookupComplete|)))
```

21.7 ALAGG.lsp BOOTSTRAP

ALAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ALAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **ALAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ALAGG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |AssociationListAggregate;CAT| (QUOTE NIL))

(SETQ |AssociationListAggregate;AL| (QUOTE NIL))

(DEFUN |AssociationListAggregate|
 (|&REST| #1=#:G88404 |&AUX| #2=#:G88402)
  (DSETQ #2# #1#)
  (LET (#3=#:G88403)
   (COND
    ((SETQ #3# (|assoc| (|devaluateList| #2#) |AssociationListAggregate;AL|))
      (CDR #3#))
    (T
      (SETQ |AssociationListAggregate;AL|
       (|cons5|
        (CONS
         (|devaluateList| #2#)
         (SETQ #3# (APPLY (FUNCTION |AssociationListAggregate;|) #2#)))
        |AssociationListAggregate;AL|)) #3#))))

(DEFUN |AssociationListAggregate;| (|t#1| |t#2|)
 (PROG (#1=#:G88401)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR
       (QUOTE (|t#1| |t#2|)) (LIST (|devaluate| |t#1|) (|devaluate| |t#2|)))
      (|sublisV|
       (PAIR
        (QUOTE (#2=#:G88400))
        (LIST (QUOTE (|Record| (|:| |key| |t#1|) (|:| |entry| |t#2|)))))
       (COND
        (|AssociationListAggregate;CAT|)
```

```
      ((QUOTE T)
       (LETT |AssociationListAggregate;CAT|
        (|Join|
         (|TableAggregate| (QUOTE |t#1|) (QUOTE |t#2|))
         (|ListAggregate| (QUOTE #2#))
         (|mkCategory|
          (QUOTE |domain|)
          (QUOTE
           (((|assoc|
              ((|Union|
                (|Record| (|:| |key| |t#1|) (|:| |entry| |t#2|)) "failed")
               |t#1| |$|))
              T)))
          NIL (QUOTE NIL) NIL))
         . #3=(|AssociationListAggregate|))))))
     . #3#)
    (SETELT #1# 0
     (LIST
      (QUOTE |AssociationListAggregate|)
      (|devaluate| |t#1|)
      (|devaluate| |t#2|)))))))
```

21.8 CABMON.lsp BOOTSTRAP

CABMON which needs **ABELMON** which needs **ABELSG** which needs **SETCAT** which needs **SINT** which needs **UFD** which needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON**. We break this chain with **CABMON.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **CABMON** category which we can write into the **MID** directory. We compile the lisp code and copy the **CABMON.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*CABMON.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |CancellationAbelianMonoid;AL| (QUOTE NIL))

(DEFUN |CancellationAbelianMonoid| NIL
  (LET (#:G82646)
    (COND
      (|CancellationAbelianMonoid;AL|)
      (T
        (SETQ
          |CancellationAbelianMonoid;AL|
          (|CancellationAbelianMonoid;|))))))

(DEFUN |CancellationAbelianMonoid;| NIL
  (PROG (#1=#:G82644)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|AbelianMonoid|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE
               ((((|subtractIfCan| ((|Union| |$| "failed") |$| |$|)) T)))
              NIL
             (QUOTE NIL)
             NIL))
           |CancellationAbelianMonoid|)
        (SETELT #1# 0 (QUOTE (|CancellationAbelianMonoid|)))))))

(MAKEPROP (QUOTE |CancellationAbelianMonoid|) (QUOTE NILADIC) T)
```

21.9 CLAGG.lsp BOOTSTRAP

CLAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **CLAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **CLAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*CLAGG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |Collection;CAT| (QUOTE NIL))

(SETQ |Collection;AL| (QUOTE NIL))

(DEFUN |Collection| (#1=#:G82618)
 (LET (#2=#:G82619)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |Collection;AL|)) (CDR #2#))
   (T
    (SETQ |Collection;AL|
     (|cons5|
      (CONS
       (|devaluate| #1#)
       (SETQ #2# (|Collection;| #1#)))
      |Collection;AL|))
     #2#))))

(DEFUN |Collection;| (|t#1|)
 (PROG (#1=#:G82617)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|Collection;CAT|)
       ((QUOTE T)
        (LETT |Collection;CAT|
         (|Join|
          (|HomogeneousAggregate| (QUOTE |t#1|))
          (|mkCategory|
           (QUOTE |domain|)
           (QUOTE (
```

```
          ((|construct| (|$| (|List| |t#1|))) T)
          ((|find| ((|Union| |t#1| "failed")
                    (|Mapping| (|Boolean|) |t#1|) |$|))
            T)
          ((|reduce| (|t#1| (|Mapping| |t#1| |t#1| |t#1|) |$|))
           (|has| |$| (ATTRIBUTE |finiteAggregate|)))
          ((|reduce| (|t#1| (|Mapping| |t#1| |t#1| |t#1|) |$| |t#1|))
           (|has| |$| (ATTRIBUTE |finiteAggregate|)))
          ((|remove| (|$| (|Mapping| (|Boolean|) |t#1|) |$|))
           (|has| |$| (ATTRIBUTE |finiteAggregate|)))
          ((|select| (|$| (|Mapping| (|Boolean|) |t#1|) |$|))
           (|has| |$| (ATTRIBUTE |finiteAggregate|)))
          ((|reduce| (|t#1| (|Mapping| |t#1| |t#1| |t#1|) |$| |t#1| |t#1|))
           (AND
            (|has| |t#1| (|SetCategory|))
            (|has| |$| (ATTRIBUTE |finiteAggregate|))))
          ((|remove| (|$| |t#1| |$|))
           (AND
            (|has| |t#1| (|SetCategory|))
            (|has| |$| (ATTRIBUTE |finiteAggregate|))))
          ((|removeDuplicates| (|$| |$|))
           (AND
            (|has| |t#1| (|SetCategory|))
            (|has| |$| (ATTRIBUTE |finiteAggregate|))))))
         (QUOTE ((((|ConvertibleTo| (|InputForm|))
                   (|has| |t#1| (|ConvertibleTo| (|InputForm|))))))
         (QUOTE ((|List| |t#1|))) NIL))
       . #2=(|Collection|))))))
  . #2#)
(SETELT #1# 0 (LIST (QUOTE |Collection|) (|devaluate| |t#1|)))))))
```

21.10 CLAGG-.lsp BOOTSTRAP

CLAGG- depends on **CLAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **CLAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **CLAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*CLAGG-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |CLAGG-;#;ANni;1| (|c| |$|) (LENGTH (SPADCALL |c| (QREFELT |$| 9))))

(DEFUN |CLAGG-;count;MANni;2| (|f| |c| |$|)
 (PROG (|x| #1=#:G82637 #2=#:G82634 #3=#:G82632 #4=#:G82633)
  (RETURN
   (SEQ
    (PROGN
     (LETT #4# NIL |CLAGG-;count;MANni;2|)
     (SEQ
      (LETT |x| NIL |CLAGG-;count;MANni;2|)
      (LETT #1# (SPADCALL |c| (QREFELT |$| 9)) |CLAGG-;count;MANni;2|)
      G190
      (COND
       ((OR (ATOM #1#) (PROGN (LETT |x| (CAR #1#) |CLAGG-;count;MANni;2|) NIL))
        (GO G191)))
      (SEQ
       (EXIT
        (COND
         ((SPADCALL |x| |f|)
          (PROGN
           (LETT #2# 1 |CLAGG-;count;MANni;2|)
           (COND
            (#4# (LETT #3# (|+| #3# #2#) |CLAGG-;count;MANni;2|))
            ((QUOTE T)
             (PROGN
              (LETT #3# #2# |CLAGG-;count;MANni;2|)
              (LETT #4# (QUOTE T) |CLAGG-;count;MANni;2|)))))))))
      (LETT #1# (CDR #1#) |CLAGG-;count;MANni;2|)
      (GO G190)
      G191
      (EXIT NIL))
     (COND (#4# #3#) ((QUOTE T) 0)))))))
```

```
(DEFUN |CLAGG-;any?;MAB;3| (|f| |c| |$|)
 (PROG (|x| #1=#:G82642 #2=#:G82640 #3=#:G82638 #4=#:G82639)
  (RETURN
   (SEQ
    (PROGN
     (LETT #4# NIL |CLAGG-;any?;MAB;3|)
     (SEQ
      (LETT |x| NIL |CLAGG-;any?;MAB;3|)
      (LETT #1# (SPADCALL |c| (QREFELT |$| 9)) |CLAGG-;any?;MAB;3|)
      G190
      (COND
       ((OR (ATOM #1#) (PROGN (LETT |x| (CAR #1#) |CLAGG-;any?;MAB;3|) NIL))
        (GO G191)))
      (SEQ
       (EXIT
        (PROGN
         (LETT #2# (SPADCALL |x| |f|) |CLAGG-;any?;MAB;3|)
         (COND
          (#4#
           (LETT #3#
            (COND (#3# (QUOTE T)) ((QUOTE T) #2#))
            |CLAGG-;any?;MAB;3|))
          ((QUOTE T)
           (PROGN
            (LETT #3# #2# |CLAGG-;any?;MAB;3|)
            (LETT #4# (QUOTE T) |CLAGG-;any?;MAB;3|)))))))
      (LETT #1# (CDR #1#) |CLAGG-;any?;MAB;3|)
      (GO G190)
      G191
      (EXIT NIL))
     (COND (#4# #3#) ((QUOTE T) (QUOTE NIL)))))))))

(DEFUN |CLAGG-;every?;MAB;4| (|f| |c| |$|)
 (PROG (|x| #1=#:G82647 #2=#:G82645 #3=#:G82643 #4=#:G82644)
  (RETURN
   (SEQ
    (PROGN
     (LETT #4# NIL |CLAGG-;every?;MAB;4|)
     (SEQ
      (LETT |x| NIL |CLAGG-;every?;MAB;4|)
      (LETT #1# (SPADCALL |c| (QREFELT |$| 9)) |CLAGG-;every?;MAB;4|)
      G190
      (COND
       ((OR (ATOM #1#) (PROGN (LETT |x| (CAR #1#) |CLAGG-;every?;MAB;4|) NIL))
        (GO G191)))
      (SEQ
```

```
        (EXIT
         (PROGN
          (LETT #2# (SPADCALL |x| |f|) |CLAGG-;every?;MAB;4|)
          (COND
           (#4#
            (LETT #3#
             (COND (#3# #2#) ((QUOTE T) (QUOTE NIL)))
             |CLAGG-;every?;MAB;4|))
           ((QUOTE T)
            (PROGN
             (LETT #3# #2# |CLAGG-;every?;MAB;4|)
             (LETT #4# (QUOTE T) |CLAGG-;every?;MAB;4|)))))))
       (LETT #1# (CDR #1#) |CLAGG-;every?;MAB;4|)
       (GO G190)
       G191
       (EXIT NIL))
      (COND (#4# #3#) ((QUOTE T) (QUOTE T))))))))

(DEFUN |CLAGG-;find;MAU;5| (|f| |c| |$|)
 (SPADCALL |f| (SPADCALL |c| (QREFELT |$| 9)) (QREFELT |$| 18)))

(DEFUN |CLAGG-;reduce;MAS;6| (|f| |x| |$|)
 (SPADCALL |f| (SPADCALL |x| (QREFELT |$| 9)) (QREFELT |$| 21)))

(DEFUN |CLAGG-;reduce;MA2S;7| (|f| |x| |s| |$|)
 (SPADCALL |f| (SPADCALL |x| (QREFELT |$| 9)) |s| (QREFELT |$| 23)))

(DEFUN |CLAGG-;remove;M2A;8| (|f| |x| |$|)
 (SPADCALL
  (SPADCALL |f| (SPADCALL |x| (QREFELT |$| 9)) (QREFELT |$| 25))
  (QREFELT |$| 26)))

(DEFUN |CLAGG-;select;M2A;9| (|f| |x| |$|)
 (SPADCALL
  (SPADCALL |f| (SPADCALL |x| (QREFELT |$| 9)) (QREFELT |$| 28))
  (QREFELT |$| 26)))

(DEFUN |CLAGG-;remove;S2A;10| (|s| |x| |$|)
 (SPADCALL
  (CONS (FUNCTION |CLAGG-;remove;S2A;10!0|) (VECTOR |$| |s|))
  |x|
  (QREFELT |$| 31)))

(DEFUN |CLAGG-;remove;S2A;10!0| (|#1| |$$|)
 (SPADCALL |#1| (QREFELT |$$| 1) (QREFELT (QREFELT |$$| 0) 30)))
```

```
(DEFUN |CLAGG-;reduce;MA3S;11| (|f| |x| |s1| |s2| |$|)
 (SPADCALL |f| (SPADCALL |x| (QREFELT |$| 9)) |s1| |s2| (QREFELT |$| 33)))

(DEFUN |CLAGG-;removeDuplicates;2A;12| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 9)) (QREFELT |$| 35))
  (QREFELT |$| 26)))

(DEFUN |Collection&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|Collection&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |Collection&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (GETREFV 37) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST
        (|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|InputForm|))))
        (|HasCategory| |#2| (QUOTE (|SetCategory|)))
        (|HasAttribute| |#1| (QUOTE |finiteAggregate|))))
      . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|testBitVector| |pv$| 3)
      (PROGN
       (QSETREFV |$| 11 (CONS (|dispatchFunction| |CLAGG-;#;ANni;1|) |$|))
       (QSETREFV |$| 13 (CONS (|dispatchFunction| |CLAGG-;count;MANni;2|) |$|))
       (QSETREFV |$| 15 (CONS (|dispatchFunction| |CLAGG-;any?;MAB;3|) |$|))
       (QSETREFV |$| 16 (CONS (|dispatchFunction| |CLAGG-;every?;MAB;4|) |$|))
       (QSETREFV |$| 19 (CONS (|dispatchFunction| |CLAGG-;find;MAU;5|) |$|))
       (QSETREFV |$| 22 (CONS (|dispatchFunction| |CLAGG-;reduce;MAS;6|) |$|))
       (QSETREFV |$| 24 (CONS (|dispatchFunction| |CLAGG-;reduce;MA2S;7|) |$|))
       (QSETREFV |$| 27 (CONS (|dispatchFunction| |CLAGG-;remove;M2A;8|) |$|))
       (QSETREFV |$| 29 (CONS (|dispatchFunction| |CLAGG-;select;M2A;9|) |$|))
       (COND
        ((|testBitVector| |pv$| 2)
         (PROGN
          (QSETREFV |$| 32
           (CONS (|dispatchFunction| |CLAGG-;remove;S2A;10|) |$|))
          (QSETREFV |$| 34
```

```
            (CONS (|dispatchFunction| |CLAGG-;reduce;MA3S;11|) |$|))
           (QSETREFV |$| 36
            (CONS (|dispatchFunction| |CLAGG-;removeDuplicates;2A;12|)
                  |$|)))))))))
    |$|))))

(MAKEPROP
 (QUOTE |Collection&|)
 (QUOTE |infovec|)
 (LIST (QUOTE
  #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|List| 7)
   (0 . |parts|) (|NonNegativeInteger|) (5 . |#|) (|Mapping| 14 7)
   (10 . |count|) (|Boolean|) (16 . |any?|) (22 . |every?|)
   (|Union| 7 (QUOTE "failed")) (28 . |find|) (34 . |find|)
   (|Mapping| 7 7 7) (40 . |reduce|) (46 . |reduce|) (52 . |reduce|)
   (59 . |reduce|) (66 . |remove|) (72 . |construct|) (77 . |remove|)
   (83 . |select|) (89 . |select|) (95 . |=|) (101 . |remove|)
   (107 . |remove|) (113 . |reduce|) (121 . |reduce|)
   (129 . |removeDuplicates|) (134 . |removeDuplicates|)))
   (QUOTE #(|select| 139 |removeDuplicates| 145 |remove| 150 |reduce|
   162 |find| 183 |every?| 189 |count| 195 |any?| 201 |#| 207))
   (QUOTE NIL)
   (CONS
    (|makeByteWordVec2| 1 (QUOTE NIL))
    (CONS
     (QUOTE #())
     (CONS
      (QUOTE #())
      (|makeByteWordVec2| 36
       (QUOTE (1 6 8 0 9 1 0 10 0 11 2 0 10 12 0 13 2 0 14 12 0 15 2 0 14
        12 0 16 2 8 17 12 0 18 2 0 17 12 0 19 2 8 7 20 0 21 2 0 7 20 0 22
        3 8 7 20 0 7 23 3 0 7 20 0 7 24 2 8 0 12 0 25 1 6 0 8 26 2 0 0 12
        0 27 2 8 0 12 0 28 2 0 0 12 0 29 2 7 14 0 0 30 2 6 0 12 0 31 2 0 0
        7 0 32 4 8 7 20 0 7 7 33 4 0 7 20 0 7 7 34 1 8 0 0 35 1 0 0 0 36 2
        0 0 12 0 29 1 0 0 0 36 2 0 0 7 0 32 2 0 0 12 0 27 4 0 7 20 0 7 7 34
        3 0 7 20 0 7 24 2 0 7 20 0 22 2 0 17 12 0 19 2 0 14 12 0 16 2 0 10
        12 0 13 2 0 14 12 0 15 1 0 10 0 11))))))
    (QUOTE |lookupComplete|)))
```

21.11 COMRING.lsp BOOTSTRAP

COMRING depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **COMRING** category which we can write into the **MID** directory. We compile the lisp code and copy the **COMRING.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*COMRING.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |CommutativeRing;AL| (QUOTE NIL))

(DEFUN |CommutativeRing| NIL
  (LET (#:G82892)
    (COND
      (|CommutativeRing;AL|)
      (T (SETQ |CommutativeRing;AL| (|CommutativeRing;|))))))

(DEFUN |CommutativeRing;| NIL
  (PROG (#1=#:G82890)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|Ring|)
            (|BiModule| (QUOTE |$|) (QUOTE |$|))
            (|mkCategory|
              (QUOTE |package|)
              NIL
              (QUOTE ((((|commutative| "*") T)))
              (QUOTE NIL)
              NIL))
           |CommutativeRing|)
        (SETELT #1# 0 (QUOTE (|CommutativeRing|)))))))

(MAKEPROP (QUOTE |CommutativeRing|) (QUOTE NILADIC) T)
```

21.12 DIFRING.lsp BOOTSTRAP

DIFRING needs **INT** which needs **DIFRING**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **DIFRING** category which we can write into the **MID** directory. We compile the lisp code and copy the **DIFRING.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*DIFRING.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |DifferentialRing;AL| (QUOTE NIL))

(DEFUN |DifferentialRing| NIL
  (LET (#:G84565)
    (COND
      (|DifferentialRing;AL|)
      (T (SETQ |DifferentialRing;AL| (|DifferentialRing;|))))))

(DEFUN |DifferentialRing;| NIL
  (PROG (#1=#:G84563)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|Ring|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE
                ((((|differentiate| (|$| |$|)) T)
                  ((D (|$| |$|)) T)
                  ((|differentiate| (|$| |$| (|NonNegativeInteger|))) T)
                  ((D (|$| |$| (|NonNegativeInteger|))) T)))
              NIL
              (QUOTE ((|NonNegativeInteger|)))
              NIL))
            |DifferentialRing|)
        (SETELT #1# 0 (QUOTE (|DifferentialRing|)))))))

(MAKEPROP (QUOTE |DifferentialRing|) (QUOTE NILADIC) T)
```

21.13 DIFRING-.lsp BOOTSTRAP

DIFRING- needs **DIFRING**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **DIFRING-** category which we can write into the **MID** directory. We compile the lisp code and copy the **DIFRING-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*DIFRING-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |DIFRING-;D;2S;1| (|r| |$|)
  (SPADCALL |r| (QREFELT |$| 7)))

(DEFUN |DIFRING-;differentiate;SNniS;2| (|r| |n| |$|)
  (PROG (|i|)
    (RETURN
      (SEQ
        (SEQ
          (LETT |i| 1 |DIFRING-;differentiate;SNniS;2|)
          G190
          (COND ((QSGREATERP |i| |n|) (GO G191)))
          (SEQ
            (EXIT
              (LETT |r|
                (SPADCALL |r| (QREFELT |$| 7))
                |DIFRING-;differentiate;SNniS;2|)))
          (LETT |i| (QSADD1 |i|) |DIFRING-;differentiate;SNniS;2|)
          (GO G190)
          G191
          (EXIT NIL))
        (EXIT |r|)))))

(DEFUN |DIFRING-;D;SNniS;3| (|r| |n| |$|)
  (SPADCALL |r| |n| (QREFELT |$| 11)))

(DEFUN |DifferentialRing&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|DifferentialRing&|))
        (LETT |dv$| (LIST (QUOTE |DifferentialRing&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 13) . #1#)
        (QSETREFV |$| 0 |dv$|)
```

```
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|)))))

(MAKEPROP
  (QUOTE |DifferentialRing&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (0 . |differentiate|)
        |DIFRING-;D;2S;1|
        (|NonNegativeInteger|)
        |DIFRING-;differentiate;SNniS;2|
        (5 . |differentiate|)
        |DIFRING-;D;SNniS;3|))
    (QUOTE #(|differentiate| 11 D 17))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 12
            (QUOTE
             (1 6 0 0 7 2 6 0 0 9 11 2 0 0 0 9 10 2 0 0 0 9 12 1 0 0 0 8))))))
    (QUOTE |lookupComplete|)))
```

21.14 DIVRING.lsp BOOTSTRAP

DIVRING depends on **QFCAT** which eventually depends on **DIVRING**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **DIVRING** category which we can write into the **MID** directory. We compile the lisp code and copy the **DIVRING.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*DIVRING.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |DivisionRing;AL| (QUOTE NIL))

(DEFUN |DivisionRing| NIL
  (LET (#:G84035)
    (COND
      (|DivisionRing;AL|)
      (T (SETQ |DivisionRing;AL| (|DivisionRing;|))))))

(DEFUN |DivisionRing;| NIL
  (PROG (#1=#:G84033)
    (RETURN
      (PROG1
        (LETT #1#
          (|sublisV|
            (PAIR
              (QUOTE (#2=#:G84032))
              (LIST (QUOTE (|Fraction| (|Integer|)))))
            (|Join|
              (|EntireRing|)
              (|Algebra| (QUOTE #2#))
              (|mkCategory|
                (QUOTE |domain|)
                (QUOTE (
                  ((|**| (|$| |$| (|Integer|))) T)
                  ((|^| (|$| |$| (|Integer|))) T)
                  ((|inv| (|$| |$|)) T)))
                NIL
                (QUOTE ((|Integer|)))
                NIL)))
            |DivisionRing|)
        (SETELT #1# 0 (QUOTE (|DivisionRing|)))))))

(MAKEPROP (QUOTE |DivisionRing|) (QUOTE NILADIC) T)
```

21.15 DIVRING-.lsp BOOTSTRAP

DIVRING- depends on **DIVRING**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **DIVRING-** category which we can write into the **MID** directory. We compile the lisp code and copy the **DIVRING-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*DIVRING-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |DIVRING-;^;SIS;1| (|x| |n| |$|)
  (SPADCALL |x| |n| (QREFELT |$| 8)))

(DEFUN |DIVRING-;**;SIS;2| (|x| |n| |$|)
  (COND
    ((ZEROP |n|) (|spadConstant| |$| 10))
    ((SPADCALL |x| (QREFELT |$| 12))
      (COND
        ((|<| |n| 0) (|error| "division by zero"))
        ((QUOTE T) |x|)))
    ((|<| |n| 0)
      (SPADCALL (SPADCALL |x| (QREFELT |$| 14)) (|-| |n|) (QREFELT |$| 17)))
    ((QUOTE T) (SPADCALL |x| |n| (QREFELT |$| 17)))))

(DEFUN |DIVRING-;*;F2S;3| (|q| |x| |$|)
  (SPADCALL
    (SPADCALL
      (SPADCALL |q| (QREFELT |$| 20))
      (SPADCALL
        (SPADCALL (SPADCALL |q| (QREFELT |$| 21)) (QREFELT |$| 22))
        (QREFELT |$| 14))
      (QREFELT |$| 23))
    |x|
    (QREFELT |$| 24)))

(DEFUN |DivisionRing&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|DivisionRing&|))
        (LETT |dv$| (LIST (QUOTE |DivisionRing&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 27) . #1#)
        (QSETREFV |$| 0 |dv$|)
```

```
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(MAKEPROP
  (QUOTE |DivisionRing&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (|Integer|)
        (0 . |**|)
        |DIVRING-;^;SIS;1|
        (6 . |One|)
        (|Boolean|)
        (10 . |zero?|)
        (15 . |Zero|)
        (19 . |inv|)
        (|PositiveInteger|)
        (|RepeatedSquaring| 6)
        (24 . |expt|)
        |DIVRING-;**;SIS;2|
        (|Fraction| 7)
        (30 . |numer|)
        (35 . |denom|)
        (40 . |coerce|)
        (45 . |*|)
        (51 . |*|)
        |DIVRING-;*;F2S;3|
        (|NonNegativeInteger|)))
    (QUOTE #(|^| 57 |**| 63 |*| 69))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 25
            (QUOTE
              (2 6 0 0 7 8 0 6 0 10 1 6 11 0 12 0 6 0 13 1 6 0 0 14 2 16 6
               6 15 17 1 19 7 0 20 1 19 7 0 21 1 6 0 7 22 2 6 0 7 0 23 2 6
               0 0 0 24 2 0 0 0 7 9 2 0 0 0 7 18 2 0 0 19 0 25))))))
    (QUOTE |lookupComplete|)))
```

21.16 ES.lsp BOOTSTRAP

ES depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ES** category which we can write into the **MID** directory. We compile the lisp code and copy the **ES.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ES.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |ExpressionSpace;AL| (QUOTE NIL))

(DEFUN |ExpressionSpace| NIL
 (LET (#:G82344)
  (COND
   (|ExpressionSpace;AL|)
   (T (SETQ |ExpressionSpace;AL| (|ExpressionSpace;|))))))

(DEFUN |ExpressionSpace;| NIL
 (PROG (#1=#:G82342)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR
       (QUOTE (#2=#:G82340 #3=#:G82341))
       (LIST (QUOTE (|Kernel| |$|)) (QUOTE (|Kernel| |$|))))
      (|Join|
       (|OrderedSet|)
       (|RetractableTo| (QUOTE #2#))
       (|InnerEvalable| (QUOTE #3#) (QUOTE |$|))
       (|Evalable| (QUOTE |$|))
       (|mkCategory|
        (QUOTE |domain|)
        (QUOTE (
         ((|elt| (|$| (|BasicOperator|) |$|)) T)
         ((|elt| (|$| (|BasicOperator|) |$| |$|)) T)
         ((|elt| (|$| (|BasicOperator|) |$| |$| |$|)) T)
         ((|elt| (|$| (|BasicOperator|) |$| |$| |$| |$|)) T)
         ((|elt| (|$| (|BasicOperator|) (|List| |$|))) T)
         ((|subst| (|$| |$| (|Equation| |$|))) T)
         ((|subst| (|$| |$| (|List| (|Equation| |$|)))) T)
         ((|subst| (|$| |$| (|List| (|Kernel| |$|)) (|List| |$|))) T)
         ((|box| (|$| |$|)) T)
```

```
 ((|box| (|$| (|List| |$|))) T)
 ((|paren| (|$| |$|)) T)
 ((|paren| (|$| (|List| |$|))) T)
 ((|distribute| (|$| |$|)) T)
 ((|distribute| (|$| |$| |$|)) T)
 ((|height| ((|NonNegativeInteger|) |$|)) T)
 ((|mainKernel| ((|Union| (|Kernel| |$|) "failed") |$|)) T)
 ((|kernels| ((|List| (|Kernel| |$|)) |$|)) T)
 ((|tower| ((|List| (|Kernel| |$|)) |$|)) T)
 ((|operators| ((|List| (|BasicOperator|)) |$|)) T)
 ((|operator| ((|BasicOperator|) (|BasicOperator|))) T)
 ((|belong?| ((|Boolean|) (|BasicOperator|))) T)
 ((|is?| ((|Boolean|) |$| (|BasicOperator|))) T)
 ((|is?| ((|Boolean|) |$| (|Symbol|))) T)
 ((|kernel| (|$| (|BasicOperator|) |$|)) T)
 ((|kernel| (|$| (|BasicOperator|) (|List| |$|))) T)
 ((|map| (|$| (|Mapping| |$| |$|) (|Kernel| |$|))) T)
 ((|freeOf?| ((|Boolean|) |$| |$|)) T)
 ((|freeOf?| ((|Boolean|) |$| (|Symbol|))) T)
 ((|eval| (|$| |$| (|List| (|Symbol|)) (|List| (|Mapping| |$| |$|))))
    T)
 ((|eval|
   (|$| |$| (|List| (|Symbol|)) (|List| (|Mapping| |$| (|List| |$|)))))
   T)
 ((|eval| (|$| |$| (|Symbol|) (|Mapping| |$| (|List| |$|)))) T)
 ((|eval| (|$| |$| (|Symbol|) (|Mapping| |$| |$|))) T)
 ((|eval|
   (|$| |$| (|List| (|BasicOperator|)) (|List| (|Mapping| |$| |$|))))
   T)
 ((|eval|
   (|$| |$| (|List| (|BasicOperator|))
     (|List| (|Mapping| |$| (|List| |$|)))))
   T)
 ((|eval| (|$| |$| (|BasicOperator|) (|Mapping| |$| (|List| |$|)))) T)
 ((|eval| (|$| |$| (|BasicOperator|) (|Mapping| |$| |$|))) T)
 ((|minPoly|
   ((|SparseUnivariatePolynomial| |$|) (|Kernel| |$|)))
   (|has| |$| (|Ring|)))
 ((|definingPolynomial| (|$| |$|)) (|has| |$| (|Ring|)))
 ((|even?|
   ((|Boolean|) |$|)) (|has| |$| (|RetractableTo| (|Integer|))))
 ((|odd?|
   ((|Boolean|) |$|)) (|has| |$| (|RetractableTo| (|Integer|)))))
NIL
(QUOTE (
 (|Boolean|)
```

```
          (|SparseUnivariatePolynomial| |$|)
          (|Kernel| |$|)
          (|BasicOperator|)
          (|List| (|BasicOperator|))
          (|List| (|Mapping| |$| (|List| |$|)))
          (|List| (|Mapping| |$| |$|))
          (|Symbol|)
          (|List| (|Symbol|))
          (|List| |$|)
          (|List| (|Kernel| |$|))
          (|NonNegativeInteger|)
          (|List| (|Equation| |$|))
          (|Equation| |$|)))
         NIL)))
      |ExpressionSpace|)
    (SETELT #1# 0 (QUOTE (|ExpressionSpace|)))))))

(MAKEPROP (QUOTE |ExpressionSpace|) (QUOTE NILADIC) T)
```

21.17 ES-.lsp BOOTSTRAP

ES- depends on **ES**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ES-** category which we can write into the **MID** directory. We compile the lisp code and copy the **ES-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ES-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |ES-;box;2S;1| (|x| |$|)
 (SPADCALL (LIST |x|) (QREFELT |$| 16)))

(DEFUN |ES-;paren;2S;2| (|x| |$|)
 (SPADCALL (LIST |x|) (QREFELT |$| 18)))

(DEFUN |ES-;belong?;BoB;3| (|op| |$|)
 (COND
  ((SPADCALL |op| (QREFELT |$| 13) (QREFELT |$| 21)) (QUOTE T))
  ((QUOTE T) (SPADCALL |op| (QREFELT |$| 14) (QREFELT |$| 21)))))

(DEFUN |ES-;listk| (|f| |$|)
 (SPADCALL (|ES-;allKernels| |f| |$|) (QREFELT |$| 25)))

(DEFUN |ES-;tower;SL;5| (|f| |$|)
 (SPADCALL (|ES-;listk| |f| |$|) (QREFELT |$| 26)))

(DEFUN |ES-;allk| (|l| |$|)
 (PROG (#1=#:G82361 |f| #2=#:G82362)
  (RETURN
   (SEQ
    (SPADCALL
     (ELT |$| 30)
     (PROGN
      (LETT #1# NIL |ES-;allk|)
      (SEQ
       (LETT |f| NIL |ES-;allk|)
       (LETT #2# |l| |ES-;allk|)
       G190
       (COND
        ((OR (ATOM #2#)
             (PROGN (LETT |f| (CAR #2#) |ES-;allk|) NIL))
         (GO G191)))
       (SEQ (EXIT (LETT #1# (CONS (|ES-;allKernels| |f| |$|) #1#) |ES-;allk|)))
```

```
        (LETT #2# (CDR #2#) |ES-;allk|) (GO G190) G191 (EXIT (NREVERSE0 #1#))))
      (SPADCALL NIL (QREFELT |$| 29))
      (QREFELT |$| 33))))))

(DEFUN |ES-;operators;SL;7| (|f| |$|)
 (PROG (#1=#:G82365 |k| #2=#:G82366)
  (RETURN
   (SEQ
    (PROGN
     (LETT #1# NIL |ES-;operators;SL;7|)
     (SEQ
      (LETT |k| NIL |ES-;operators;SL;7|)
      (LETT #2# (|ES-;listk| |f| |$|) |ES-;operators;SL;7|)
      G190
      (COND
       ((OR (ATOM #2#) (PROGN (LETT |k| (CAR #2#) |ES-;operators;SL;7|) NIL))
        (GO G191)))
      (SEQ
       (EXIT
        (LETT #1#
         (CONS (SPADCALL |k| (QREFELT |$| 35)) #1#) |ES-;operators;SL;7|)))
      (LETT #2# (CDR #2#) |ES-;operators;SL;7|)
      (GO G190)
      G191
      (EXIT (NREVERSE0 #1#)))))))))

(DEFUN |ES-;height;SNni;8| (|f| |$|)
 (PROG (#1=#:G82371 |k| #2=#:G82372)
  (RETURN
   (SEQ
    (SPADCALL
     (ELT |$| 41)
     (PROGN
      (LETT #1# NIL |ES-;height;SNni;8|)
      (SEQ
       (LETT |k| NIL |ES-;height;SNni;8|)
       (LETT #2# (SPADCALL |f| (QREFELT |$| 38)) |ES-;height;SNni;8|)
       G190
       (COND
        ((OR (ATOM #2#) (PROGN (LETT |k| (CAR #2#) |ES-;height;SNni;8|) NIL))
         (GO G191)))
       (SEQ
        (EXIT
         (LETT #1#
          (CONS (SPADCALL |k| (QREFELT |$| 40)) #1#) |ES-;height;SNni;8|)))
       (LETT #2# (CDR #2#) |ES-;height;SNni;8|)
```

```
        (GO G190)
        G191
        (EXIT (NREVERSE0 #1#))))
       0
      (QREFELT |$| 44))))))

(DEFUN |ES-;freeOf?;SSB;9| (|x| |s| |$|)
 (PROG (#1=#:G82377 |k| #2=#:G82378)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |s|
      (PROGN
       (LETT #1# NIL |ES-;freeOf?;SSB;9|)
       (SEQ
        (LETT |k| NIL |ES-;freeOf?;SSB;9|)
        (LETT #2# (|ES-;listk| |x| |$|) |ES-;freeOf?;SSB;9|)
        G190
        (COND
         ((OR (ATOM #2#) (PROGN (LETT |k| (CAR #2#) |ES-;freeOf?;SSB;9|) NIL))
           (GO G191)))
        (SEQ
         (EXIT
          (LETT #1#
           (CONS (SPADCALL |k| (QREFELT |$| 46)) #1#) |ES-;freeOf?;SSB;9|)))
        (LETT #2# (CDR #2#) |ES-;freeOf?;SSB;9|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #1#))))
       (QREFELT |$| 48))
      (QUOTE NIL))
     ((QUOTE T) (QUOTE T)))))))

(DEFUN |ES-;distribute;2S;10| (|x| |$|)
 (PROG (#1=#:G82381 |k| #2=#:G82382)
  (RETURN
   (SEQ
    (|ES-;unwrap|
     (PROGN (LETT #1# NIL |ES-;distribute;2S;10|)
      (SEQ
       (LETT |k| NIL |ES-;distribute;2S;10|)
       (LETT #2# (|ES-;listk| |x| |$|) |ES-;distribute;2S;10|)
       G190
       (COND
        ((OR
          (ATOM #2#)
```

```
           (PROGN (LETT |k| (CAR #2#) |ES-;distribute;2S;10|) NIL))
            (GO G191)))
        (SEQ
         (EXIT
          (COND
           ((SPADCALL |k| (QREFELT |$| 13) (QREFELT |$| 50))
            (LETT #1# (CONS |k| #1#) |ES-;distribute;2S;10|)))))
        (LETT #2# (CDR #2#) |ES-;distribute;2S;10|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #1#))))
      |x|
      |$|)))))

(DEFUN |ES-;box;LS;11| (|l| |$|)
 (SPADCALL (QREFELT |$| 14) |l| (QREFELT |$| 52)))

(DEFUN |ES-;paren;LS;12| (|l| |$|)
 (SPADCALL (QREFELT |$| 13) |l| (QREFELT |$| 52)))

(DEFUN |ES-;freeOf?;2SB;13| (|x| |k| |$|)
 (COND
  ((SPADCALL
    (SPADCALL |k| (QREFELT |$| 56))
    (|ES-;listk| |x| |$|)
    (QREFELT |$| 57))
     (QUOTE NIL))
  ((QUOTE T) (QUOTE T))))

(DEFUN |ES-;kernel;Bo2S;14| (|op| |arg| |$|)
 (SPADCALL |op| (LIST |arg|) (QREFELT |$| 59)))

(DEFUN |ES-;elt;Bo2S;15| (|op| |x| |$|)
 (SPADCALL |op| (LIST |x|) (QREFELT |$| 52)))

(DEFUN |ES-;elt;Bo3S;16| (|op| |x| |y| |$|)
 (SPADCALL |op| (LIST |x| |y|) (QREFELT |$| 52)))

(DEFUN |ES-;elt;Bo4S;17| (|op| |x| |y| |z| |$|)
 (SPADCALL |op| (LIST |x| |y| |z|) (QREFELT |$| 52)))

(DEFUN |ES-;elt;Bo5S;18| (|op| |x| |y| |z| |t| |$|)
 (SPADCALL |op| (LIST |x| |y| |z| |t|) (QREFELT |$| 52)))

(DEFUN |ES-;eval;SSMS;19| (|x| |s| |f| |$|)
 (SPADCALL |x| (LIST |s|) (LIST |f|) (QREFELT |$| 67)))
```

```
(DEFUN |ES-;eval;SBoMS;20| (|x| |s| |f| |$|)
 (SPADCALL
  |x|
  (LIST (SPADCALL |s| (QREFELT |$| 69)))
  (LIST |f|)
  (QREFELT |$| 67)))

(DEFUN |ES-;eval;SSMS;21| (|x| |s| |f| |$|)
 (SPADCALL
  |x|
  (LIST |s|)
  (LIST (CONS (FUNCTION |ES-;eval;SSMS;21!0|) (VECTOR |f| |$|)))
  (QREFELT |$| 67)))

(DEFUN |ES-;eval;SSMS;21!0| (|#1| |$$|)
 (SPADCALL (SPADCALL |#1| (QREFELT (QREFELT |$$| 1) 72)) (QREFELT |$$| 0)))

(DEFUN |ES-;eval;SBoMS;22| (|x| |s| |f| |$|)
 (SPADCALL
  |x|
  (LIST |s|)
  (LIST (CONS (FUNCTION |ES-;eval;SBoMS;22!0|) (VECTOR |f| |$|)))
  (QREFELT |$| 75)))

(DEFUN |ES-;eval;SBoMS;22!0| (|#1| |$$|)
 (SPADCALL (SPADCALL |#1| (QREFELT (QREFELT |$$| 1) 72)) (QREFELT |$$| 0)))

(DEFUN |ES-;subst;SES;23| (|x| |e| |$|)
 (SPADCALL |x| (LIST |e|) (QREFELT |$| 78)))

(DEFUN |ES-;eval;SLLS;24| (|x| |ls| |lf| |$|)
 (PROG (#1=#:G82403 |f| #2=#:G82404)
  (RETURN
   (SEQ
    (SPADCALL
     |x|
     |ls|
     (PROGN
      (LETT #1# NIL |ES-;eval;SLLS;24|)
      (SEQ
       (LETT |f| NIL |ES-;eval;SLLS;24|)
       (LETT #2# |lf| |ES-;eval;SLLS;24|)
       G190
       (COND
        ((OR (ATOM #2#) (PROGN (LETT |f| (CAR #2#) |ES-;eval;SLLS;24|) NIL))
```

```
          (GO G191)))
        (SEQ
         (EXIT
          (LETT #1#
           (CONS (CONS (FUNCTION |ES-;eval;SLLS;24!0|) (VECTOR |f| |$|)) #1#)
           |ES-;eval;SLLS;24|)))
        (LETT #2# (CDR #2#) |ES-;eval;SLLS;24|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #1#))))
     (QREFELT |$| 75))))))

(DEFUN |ES-;eval;SLLS;24!0| (|#1| |$$|)
 (SPADCALL (SPADCALL |#1| (QREFELT (QREFELT |$$| 1) 72)) (QREFELT |$$| 0)))

(DEFUN |ES-;eval;SLLS;25| (|x| |ls| |lf| |$|)
 (PROG (#1=#:G82407 |f| #2=#:G82408)
  (RETURN
   (SEQ
    (SPADCALL
     |x|
     |ls|
     (PROGN
      (LETT #1# NIL |ES-;eval;SLLS;25|)
      (SEQ
       (LETT |f| NIL |ES-;eval;SLLS;25|)
       (LETT #2# |lf| |ES-;eval;SLLS;25|)
       G190
       (COND
        ((OR (ATOM #2#) (PROGN (LETT |f| (CAR #2#) |ES-;eval;SLLS;25|) NIL))
          (GO G191)))
        (SEQ
         (EXIT
          (LETT #1#
           (CONS (CONS (FUNCTION |ES-;eval;SLLS;25!0|) (VECTOR |f| |$|)) #1#)
           |ES-;eval;SLLS;25|)))
        (LETT #2# (CDR #2#) |ES-;eval;SLLS;25|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #1#))))
     (QREFELT |$| 67))))))

(DEFUN |ES-;eval;SLLS;25!0| (|#1| |$$|)
 (SPADCALL (SPADCALL |#1| (QREFELT (QREFELT |$$| 1) 72)) (QREFELT |$$| 0)))

(DEFUN |ES-;eval;SLLS;26| (|x| |ls| |lf| |$|)
```

```
  (PROG (#1=#:G82412 |s| #2=#:G82413)
   (RETURN
    (SEQ
     (SPADCALL
      |x|
      (PROGN
       (LETT #1# NIL |ES-;eval;SLLS;26|)
       (SEQ
        (LETT |s| NIL |ES-;eval;SLLS;26|)
        (LETT #2# |ls| |ES-;eval;SLLS;26|)
        G190
        (COND
         ((OR (ATOM #2#) (PROGN (LETT |s| (CAR #2#) |ES-;eval;SLLS;26|) NIL))
           (GO G191)))
        (SEQ
         (EXIT
          (LETT #1#
           (CONS (SPADCALL |s| (QREFELT |$| 69)) #1#) |ES-;eval;SLLS;26|)))
        (LETT #2# (CDR #2#) |ES-;eval;SLLS;26|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #1#))))
      |lf|
      (QREFELT |$| 67))))))

(DEFUN |ES-;map;MKS;27| (|fn| |k| |$|)
 (PROG (#1=#:G82428 |x| #2=#:G82429 |l|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL
       (LETT |l|
        (PROGN
         (LETT #1# NIL |ES-;map;MKS;27|)
         (SEQ
          (LETT |x| NIL |ES-;map;MKS;27|)
          (LETT #2# (SPADCALL |k| (QREFELT |$| 85)) |ES-;map;MKS;27|)
          G190
          (COND
           ((OR (ATOM #2#) (PROGN (LETT |x| (CAR #2#) |ES-;map;MKS;27|) NIL))
             (GO G191)))
          (SEQ
           (EXIT
            (LETT #1# (CONS (SPADCALL |x| |fn|) #1#) |ES-;map;MKS;27|)))
          (LETT #2# (CDR #2#) |ES-;map;MKS;27|)
          (GO G190)
```

```
          G191
          (EXIT (NREVERSE0 #1#))))
         |ES-;map;MKS;27|)
         (SPADCALL |k| (QREFELT |$| 85)) (QREFELT |$| 86))
        (SPADCALL |k| (QREFELT |$| 87)))
       ((QUOTE T)
        (SPADCALL (SPADCALL |k| (QREFELT |$| 35)) |l| (QREFELT |$| 52))))))))

(DEFUN |ES-;operator;2Bo;28| (|op| |$|)
 (COND
  ((SPADCALL |op| (SPADCALL "%paren" (QREFELT |$| 9)) (QREFELT |$| 89))
    (QREFELT |$| 13))
  ((SPADCALL |op| (SPADCALL "%box" (QREFELT |$| 9)) (QREFELT |$| 89))
    (QREFELT |$| 14))
  ((QUOTE T) (|error| "Unknown operator"))))

(DEFUN |ES-;mainKernel;SU;29| (|x| |$|)
 (PROG (|l| |kk| #1=#:G82445 |n| |k|)
  (RETURN
   (SEQ
    (COND
     ((NULL (LETT |l| (SPADCALL |x| (QREFELT |$| 38)) |ES-;mainKernel;SU;29|))
       (CONS 1 "failed"))
     ((QUOTE T)
      (SEQ
       (LETT |n|
        (SPADCALL
         (LETT |k| (|SPADfirst| |l|) |ES-;mainKernel;SU;29|) (QREFELT |$| 40))
        |ES-;mainKernel;SU;29|)
       (SEQ
        (LETT |kk| NIL |ES-;mainKernel;SU;29|)
        (LETT #1# (CDR |l|) |ES-;mainKernel;SU;29|)
        G190
        (COND
         ((OR
           (ATOM #1#)
           (PROGN (LETT |kk| (CAR #1#) |ES-;mainKernel;SU;29|) NIL))
            (GO G191)))
        (SEQ
         (EXIT
          (COND
           ((|<| |n| (SPADCALL |kk| (QREFELT |$| 40)))
            (SEQ
             (LETT |n| (SPADCALL |kk| (QREFELT |$| 40)) |ES-;mainKernel;SU;29|)
             (EXIT (LETT |k| |kk| |ES-;mainKernel;SU;29|)))))))
        (LETT #1# (CDR #1#) |ES-;mainKernel;SU;29|) (GO G190) G191 (EXIT NIL))
```

```
        (EXIT (CONS 0 |k|)))))))))

(DEFUN |ES-;allKernels| (|f| |$|)
 (PROG (|l| |k| #1=#:G82458 |u| |s0| |n| |arg| |t| |s|)
  (RETURN
   (SEQ
    (LETT |s|
     (SPADCALL
      (LETT |l|
       (SPADCALL |f| (QREFELT |$| 38))
       |ES-;allKernels|)
      (QREFELT |$| 29))
     |ES-;allKernels|)
    (SEQ
     (LETT |k| NIL |ES-;allKernels|)
     (LETT #1# |l| |ES-;allKernels|)
     G190
     (COND
      ((OR (ATOM #1#) (PROGN (LETT |k| (CAR #1#) |ES-;allKernels|) NIL))
       (GO G191)))
     (SEQ
      (LETT |t|
       (SEQ
        (LETT |u|
         (SPADCALL
          (SPADCALL |k| (QREFELT |$| 35))
          "%dummyVar"
          (QREFELT |$| 94))
         |ES-;allKernels|)
        (EXIT
         (COND
          ((QEQCAR |u| 0)
           (SEQ
            (LETT |arg| (SPADCALL |k| (QREFELT |$| 85)) |ES-;allKernels|)
            (LETT |s0|
             (SPADCALL
              (SPADCALL
               (SPADCALL |arg| (QREFELT |$| 95))
               (QREFELT |$| 56))
              (|ES-;allKernels| (|SPADfirst| |arg|) |$|)
              (QREFELT |$| 96))
             |ES-;allKernels|)
            (LETT |arg| (CDR (CDR |arg|)) |ES-;allKernels|)
            (LETT |n| (QCDR |u|) |ES-;allKernels|)
            (COND ((|<| 1 |n|) (LETT |arg| (CDR |arg|) |ES-;allKernels|)))
            (EXIT (SPADCALL |s0| (|ES-;allk| |arg| |$|) (QREFELT |$| 30)))))))
       |ES-;allKernels|)
```

```
           ((QUOTE T) (|ES-;allk| (SPADCALL |k| (QREFELT |$| 85)) |$|)))))
         |ES-;allKernels|)
        (EXIT
         (LETT |s| (SPADCALL |s| |t| (QREFELT |$| 30)) |ES-;allKernels|)))
       (LETT #1# (CDR #1#) |ES-;allKernels|)
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT |s|)))))

(DEFUN |ES-;kernel;BoLS;31| (|op| |args| |$|)
 (COND
  ((NULL (SPADCALL |op| (QREFELT |$| 97))) (|error| "Unknown operator"))
  ((QUOTE T) (|ES-;okkernel| |op| |args| |$|))))

(DEFUN |ES-;okkernel| (|op| |l| |$|)
 (PROG (#1=#:G82465 |f| #2=#:G82466)
  (RETURN
   (SEQ
    (SPADCALL
     (SPADCALL |op| |l|
      (|+| 1
       (SPADCALL
        (ELT |$| 41)
        (PROGN
         (LETT #1# NIL |ES-;okkernel|)
         (SEQ
          (LETT |f| NIL |ES-;okkernel|)
          (LETT #2# |l| |ES-;okkernel|)
          G190
          (COND
           ((OR (ATOM #2#) (PROGN (LETT |f| (CAR #2#) |ES-;okkernel|) NIL))
             (GO G191)))
          (SEQ
           (EXIT
            (LETT #1#
             (CONS (SPADCALL |f| (QREFELT |$| 99)) #1#) |ES-;okkernel|)))
          (LETT #2# (CDR #2#) |ES-;okkernel|)
          (GO G190)
          G191
          (EXIT (NREVERSE0 #1#))))
        0
        (QREFELT |$| 44)))
      (QREFELT |$| 100))
     (QREFELT |$| 87))))))
```

```
(DEFUN |ES-;elt;BoLS;33| (|op| |args| |$|)
 (PROG (|u| #1=#:G82482 |v|)
  (RETURN
   (SEQ
    (EXIT
     (COND
      ((NULL (SPADCALL |op| (QREFELT |$| 97))) (|error| "Unknown operator"))
      ((QUOTE T)
       (SEQ
        (SEQ
         (LETT |u| (SPADCALL |op| (QREFELT |$| 102)) |ES-;elt;BoLS;33|)
         (EXIT
          (COND
           ((QEQCAR |u| 0)
            (COND
             ((NULL (EQL (LENGTH |args|) (QCDR |u|)))
              (PROGN
               (LETT #1#
                (|error| "Wrong number of arguments")
                |ES-;elt;BoLS;33|)
                (GO #1#))))))))
        (LETT |v| (SPADCALL |op| |args| (QREFELT |$| 105)) |ES-;elt;BoLS;33|)
        (EXIT
         (COND
          ((QEQCAR |v| 0) (QCDR |v|))
          ((QUOTE T) (|ES-;okkernel| |op| |args| |$|)))))))))
    #1#
    (EXIT #1#)))))

(DEFUN |ES-;retract;SK;34| (|f| |$|)
 (PROG (|k|)
  (RETURN
   (SEQ
    (LETT |k| (SPADCALL |f| (QREFELT |$| 107)) |ES-;retract;SK;34|)
    (EXIT
     (COND
      ((OR
        (QEQCAR |k| 1)
        (NULL
         (SPADCALL
          (SPADCALL (QCDR |k|) (QREFELT |$| 87))
          |f|
          (QREFELT |$| 108))))
        (|error| "not a kernel"))
      ((QUOTE T) (QCDR |k|)))))))))
```

```
(DEFUN |ES-;retractIfCan;SU;35| (|f| |$|)
 (PROG (|k|)
  (RETURN
   (SEQ
    (LETT |k| (SPADCALL |f| (QREFELT |$| 107)) |ES-;retractIfCan;SU;35|)
    (EXIT
     (COND
      ((OR
        (QEQCAR |k| 1)
        (NULL
         (SPADCALL
          (SPADCALL (QCDR |k|) (QREFELT |$| 87))
          |f|
          (QREFELT |$| 108))))
        (CONS 1 "failed"))
       ((QUOTE T) |k|))))))))

(DEFUN |ES-;is?;SSB;36| (|f| |s| |$|)
 (PROG (|k|)
  (RETURN
   (SEQ
    (LETT |k| (SPADCALL |f| (QREFELT |$| 111)) |ES-;is?;SSB;36|)
    (EXIT
     (COND
      ((QEQCAR |k| 1) (QUOTE NIL))
      ((QUOTE T) (SPADCALL (QCDR |k|) |s| (QREFELT |$| 112)))))))))

(DEFUN |ES-;is?;SBoB;37| (|f| |op| |$|)
 (PROG (|k|)
  (RETURN
   (SEQ
    (LETT |k| (SPADCALL |f| (QREFELT |$| 111)) |ES-;is?;SBoB;37|)
    (EXIT
     (COND
      ((QEQCAR |k| 1) (QUOTE NIL))
      ((QUOTE T) (SPADCALL (QCDR |k|) |op| (QREFELT |$| 50)))))))))

(DEFUN |ES-;unwrap| (|l| |x| |$|)
 (PROG (|k| #1=#:G82507)
  (RETURN
   (SEQ
    (SEQ
     (LETT |k| NIL |ES-;unwrap|)
     (LETT #1# (NREVERSE |l|) |ES-;unwrap|)
     G190
     (COND
```

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        ((OR (ATOM #1#) (PROGN (LETT |k| (CAR #1#) |ES-;unwrap|) NIL))
          (GO G191)))
       (SEQ
        (EXIT
         (LETT |x|
          (SPADCALL |x| |k|
           (|SPADfirst| (SPADCALL |k| (QREFELT |$| 85)))
           (QREFELT |$| 115))
          |ES-;unwrap|)))
       (LETT #1# (CDR #1#) |ES-;unwrap|)
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT |x|)))))

(DEFUN |ES-;distribute;3S;39| (|x| |y| |$|)
 (PROG (|ky| #1=#:G82512 |k| #2=#:G82513)
  (RETURN
   (SEQ
    (LETT |ky| (SPADCALL |y| (QREFELT |$| 56)) |ES-;distribute;3S;39|)
    (EXIT
     (|ES-;unwrap|
      (PROGN
       (LETT #1# NIL |ES-;distribute;3S;39|)
       (SEQ
        (LETT |k| NIL |ES-;distribute;3S;39|)
        (LETT #2# (|ES-;listk| |x| |$|) |ES-;distribute;3S;39|)
        G190
        (COND
         ((OR
           (ATOM #2#)
           (PROGN (LETT |k| (CAR #2#) |ES-;distribute;3S;39|) NIL))
          (GO G191)))
        (SEQ
         (EXIT
          (COND
           ((COND
             ((SPADCALL |k|
               (SPADCALL "%paren" (QREFELT |$| 9))
                (QREFELT |$| 112))
               (SPADCALL |ky|
                (|ES-;listk| (SPADCALL |k| (QREFELT |$| 87)) |$|)
                (QREFELT |$| 57)))
             ((QUOTE T) (QUOTE NIL)))
            (LETT #1# (CONS |k| #1#) |ES-;distribute;3S;39|)))))
        (LETT #2# (CDR #2#) |ES-;distribute;3S;39|)
```

```
          (GO G190)
          G191
          (EXIT (NREVERSE0 #1#))))
        |x|
        |$|)))))))

(DEFUN |ES-;eval;SLS;40| (|f| |leq| |$|)
 (PROG (|rec|)
  (RETURN
   (SEQ
    (LETT |rec| (|ES-;mkKerLists| |leq| |$|) |ES-;eval;SLS;40|)
    (EXIT (SPADCALL |f| (QCAR |rec|) (QCDR |rec|) (QREFELT |$| 117)))))))

(DEFUN |ES-;subst;SLS;41| (|f| |leq| |$|)
 (PROG (|rec|)
  (RETURN
   (SEQ
    (LETT |rec| (|ES-;mkKerLists| |leq| |$|) |ES-;subst;SLS;41|)
    (EXIT (SPADCALL |f| (QCAR |rec|) (QCDR |rec|) (QREFELT |$| 119)))))))

(DEFUN |ES-;mkKerLists| (|leq| |$|)
 (PROG (|eq| #1=#:G82530 |k| |lk| |lv|)
  (RETURN
   (SEQ
    (LETT |lk| NIL |ES-;mkKerLists|)
    (LETT |lv| NIL |ES-;mkKerLists|)
    (SEQ
     (LETT |eq| NIL |ES-;mkKerLists|)
     (LETT #1# |leq| |ES-;mkKerLists|)
     G190
     (COND
      ((OR (ATOM #1#) (PROGN (LETT |eq| (CAR #1#) |ES-;mkKerLists|) NIL))
        (GO G191)))
     (SEQ
      (LETT |k|
       (SPADCALL (SPADCALL |eq| (QREFELT |$| 122)) (QREFELT |$| 111))
       |ES-;mkKerLists|)
      (EXIT
       (COND
        ((QEQCAR |k| 1) (|error| "left hand side must be a single kernel"))
        ((NULL (SPADCALL (QCDR |k|) |lk| (QREFELT |$| 57)))
         (SEQ
          (LETT |lk| (CONS (QCDR |k|) |lk|) |ES-;mkKerLists|)
          (EXIT
           (LETT |lv|
            (CONS (SPADCALL |eq| (QREFELT |$| 123)) |lv|)
```

```
             |ES-;mkKerLists|)))))))
       (LETT #1# (CDR #1#) |ES-;mkKerLists|)
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT (CONS |lk| |lv|))))))

(DEFUN |ES-;even?;SB;43| (|x| |$|)
 (|ES-;intpred?| |x| (ELT |$| 125) |$|))

(DEFUN |ES-;odd?;SB;44| (|x| |$|)
 (|ES-;intpred?| |x| (ELT |$| 127) |$|))

(DEFUN |ES-;intpred?| (|x| |pred?| |$|)
 (PROG (|u|)
  (RETURN
   (SEQ
    (LETT |u| (SPADCALL |x| (QREFELT |$| 130)) |ES-;intpred?|)
    (EXIT
     (COND
      ((QEQCAR |u| 0) (SPADCALL (QCDR |u|) |pred?|))
      ((QUOTE T) (QUOTE NIL))))))))

(DEFUN |ExpressionSpace&| (|#1|)
 (PROG (|DV$1| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|ExpressionSpace&|))
    (LETT |dv$| (LIST (QUOTE |ExpressionSpace&|) |DV$1|) . #1#)
    (LETT |$| (GETREFV 131) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST
        (|HasCategory| |#1| (QUOTE (|RetractableTo| (|Integer|))))
        (|HasCategory| |#1| (QUOTE (|Ring|))))) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 13
     (SPADCALL (SPADCALL "%paren" (QREFELT |$| 9)) (QREFELT |$| 12)))
    (QSETREFV |$| 14
     (SPADCALL (SPADCALL "%box" (QREFELT |$| 9)) (QREFELT |$| 12)))
    (COND
     ((|testBitVector| |pv$| 1)
      (PROGN
```

```
        (QSETREFV |$| 126 (CONS (|dispatchFunction| |ES-;even?;SB;43|) |$|))
        (QSETREFV |$| 128 (CONS (|dispatchFunction| |ES-;odd?;SB;44|) |$|)))))
     |$|)))))

(MAKEPROP
 (QUOTE |ExpressionSpace&|)
 (QUOTE |infovec|)
 (LIST
  (QUOTE #(
   NIL NIL NIL NIL NIL NIL (|local| |#1|) (|String|) (|Symbol|)
   (0 . |coerce|) (|BasicOperator|) (|CommonOperators|) (5 . |operator|)
   (QUOTE |oppren|) (QUOTE |opbox|) (|List| |$|) (10 . |box|)
   |ES-;box;2S;1| (15 . |paren|) |ES-;paren;2S;2| (|Boolean|)
   (20 . |=|) |ES-;belong?;BoB;3| (|List| 34) (|Set| 34)
   (26 . |parts|) (31 . |sort!|) (|List| 55) |ES-;tower;SL;5|
   (36 . |brace|) (41 . |union|) (|Mapping| 24 24 24) (|List| 24)
   (47 . |reduce|) (|Kernel| 6) (54 . |operator|) (|List| 10)
   |ES-;operators;SL;7| (59 . |kernels|) (|NonNegativeInteger|)
   (64 . |height|) (69 . |max|) (|Mapping| 39 39 39) (|List| 39)
   (75 . |reduce|) |ES-;height;SNni;8| (82 . |name|) (|List| 8)
   (87 . |member?|) |ES-;freeOf?;SSB;9| (93 . |is?|)
   |ES-;distribute;2S;10| (99 . |elt|) |ES-;box;LS;11|
   |ES-;paren;LS;12| (|Kernel| |$|) (105 . |retract|)
   (110 . |member?|) |ES-;freeOf?;2SB;13| (116 . |kernel|)
   |ES-;kernel;Bo2S;14| |ES-;elt;Bo2S;15| |ES-;elt;Bo3S;16|
   |ES-;elt;Bo4S;17| |ES-;elt;Bo5S;18| (|Mapping| |$| 15)
   (|List| 65) (122 . |eval|) |ES-;eval;SSMS;19| (129 . |name|)
   |ES-;eval;SBoMS;20| (|List| 6) (134 . |first|)
   (|Mapping| |$| |$|) |ES-;eval;SSMS;21| (139 . |eval|)
   |ES-;eval;SBoMS;22| (|List| 79) (146 . |subst|) (|Equation| |$|)
   |ES-;subst;SES;23| (|List| 73) |ES-;eval;SLLS;24|
   |ES-;eval;SLLS;25| |ES-;eval;SLLS;26| (152 . |argument|)
   (157 . |=|) (163 . |coerce|) |ES-;map;MKS;27| (168 . |is?|)
   |ES-;operator;2Bo;28| (|Union| 55 (QUOTE "failed"))
   |ES-;mainKernel;SU;29| (|Union| (|None|) (QUOTE "failed"))
   (174 . |property|) (180 . |second|) (185 . |remove!|)
   (191 . |belong?|) |ES-;kernel;BoLS;31| (196 . |height|)
   (201 . |kernel|) (|Union| 39 (QUOTE "failed")) (208 . |arity|)
   (|Union| 6 (QUOTE "failed")) (|BasicOperatorFunctions1| 6)
   (213 . |evaluate|) |ES-;elt;BoLS;33| (219 . |mainKernel|)
   (224 . |=|) |ES-;retract;SK;34| |ES-;retractIfCan;SU;35|
   (230 . |retractIfCan|) (235 . |is?|) |ES-;is?;SSB;36|
   |ES-;is?;SBoB;37| (241 . |eval|) |ES-;distribute;3S;39|
   (248 . |eval|) |ES-;eval;SLS;40| (255 . |subst|)
   |ES-;subst;SLS;41| (|Equation| 6) (262 . |lhs|) (267 . |rhs|)
   (|Integer|) (272 . |even?|) (277 . |even?|) (282 . |odd?|)
```

```
 (287 . |odd?|) (|Union| 124 (QUOTE "failed")) (292 . |retractIfCan|)))
(QUOTE #(
 |tower| 297 |subst| 302 |retractIfCan| 314 |retract| 319 |paren| 324
 |operators| 334 |operator| 339 |odd?| 344 |map| 349 |mainKernel| 355
 |kernel| 360 |is?| 372 |height| 384 |freeOf?| 389 |even?| 401 |eval| 406
 |elt| 461 |distribute| 497 |box| 508 |belong?| 518))
(QUOTE NIL)
(CONS
 (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS
  (QUOTE #())
  (CONS
   (QUOTE #())
   (|makeByteWordVec2| 130 (QUOTE
    (1 8 0 7 9 1 11 10 8 12 1 6 0 15 16 1 6 0 15 18 2 10 20 0 0 21 1 24
     23 0 25 1 23 0 0 26 1 24 0 23 29 2 24 0 0 0 30 3 32 24 31 0 24 33
     1 34 10 0 35 1 6 27 0 38 1 34 39 0 40 2 39 0 0 0 41 3 43 39 42 0
     39 44 1 34 8 0 46 2 47 20 8 0 48 2 34 20 0 10 50 2 6 0 10 15 52 1
     6 55 0 56 2 23 20 34 0 57 2 6 0 10 15 59 3 6 0 0 47 66 67 1 10 8
     0 69 1 71 6 0 72 3 6 0 0 36 66 75 2 6 0 0 77 78 1 34 71 0 85 2 71
     20 0 0 86 1 6 0 55 87 2 10 20 0 8 89 2 10 93 0 7 94 1 71 6 0 95
     2 24 0 34 0 96 1 6 20 10 97 1 6 39 0 99 3 34 0 10 71 39 100 1 10
     101 0 102 2 104 103 10 71 105 1 6 91 0 107 2 6 20 0 0 108 1 6 91
     0 111 2 34 20 0 8 112 3 6 0 0 55 0 115 3 6 0 0 27 15 117 3 6 0 0
     27 15 119 1 121 6 0 122 1 121 6 0 123 1 124 20 0 125 1 0 20 0 126
     1 124 20 0 127 1 0 20 0 128 1 6 129 0 130 1 0 27 0 28 2 0 0 0 77
     120 2 0 0 0 79 80 1 0 91 0 110 1 0 55 0 109 1 0 0 0 19 1 0 0 15
     54 1 0 36 0 37 1 0 10 10 90 1 0 20 0 128 2 0 0 73 55 88 1 0 91 0
     92 2 0 0 10 15 98 2 0 0 10 0 60 2 0 20 0 8 113 2 0 20 0 10 114 1
     0 39 0 45 2 0 20 0 8 49 2 0 20 0 0 58 1 0 20 0 126 3 0 0 0 10 73
     76 3 0 0 0 36 66 84 3 0 0 0 10 65 70 3 0 0 0 36 81 82 3 0 0 0 8
     65 68 3 0 0 0 8 73 74 3 0 0 0 47 81 83 2 0 0 0 77 118 2 0 0 10
     15 106 5 0 0 10 0 0 0 0 64 3 0 0 10 0 0 62 4 0 0 10 0 0 0 63 2
     0 0 10 0 61 2 0 0 0 0 116 1 0 0 0 51 1 0 0 15 53 1 0 0 0 17 1 0
     20 10 22))))))
(QUOTE |lookupComplete|)))
```

21.18 EUCDOM.lsp BOOTSTRAP

EUCDOM depends on **INT** which depends on **EUCDOM**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **EUCDOM** category which we can write into the **MID** directory. We compile the lisp code and copy the **EUCDOM.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

21.18.1 The Lisp Implementation

EUCDOM;VersionCheck

This implements the bootstrap code for **EuclideanDomain**. The call to **VERSIONCHECK** is a legacy check to ensure that we did not load algebra code from a previous system version (which would not run due to major surgical changes in the system) without recompiling.

⟨*EUCDOM;VersionCheck*⟩≡
```
(|/VERSIONCHECK| 2)
```

The Domain Cache Variable

We create a variable which is formed by concatenating the string "**;AL**" to the domain name forming, in this case, "**EuclideanDomain;AL**". The variable has the initial value at load time of a list of one element, **NIL**. This list is a data structure that will be modified to hold an executable function. This function is created the first time the domain is used which it replaces the **NIL**.

⟨*EuclideanDomain;AL*⟩≡
```
(SETQ |EuclideanDomain;AL| (QUOTE NIL))
```

The Domain Function

When you call a domain the code is pretty simple at the top level. This code will check to see if this domain has ever been used. It does this by checking the value of the cached domain variable (which is the domain name **EuclideanDomain** concatenated with the string "**;AL**" to form the cache variable name which is **EuclideanDomain;AL**).

If this value is NIL we have never executed this function before. If it is not NIL we have executed this function before and we need only return the cached function which was stored in the cache variable.

If this is the first time this function is called, the cache variable is NIL and we execute the other branch of the conditional. This calls a function which

1. creates a procedure
2. returns the procedure as a value.

This procedure replaces the cached variable **EuclideanDomain;AL** value so it will be non-NIL the second time this domain is used. Thus the work of building the domain only happens once.

If this function has never been called before we call the

⟨*EuclideanDomain*⟩≡

```
(DEFUN |EuclideanDomain| NIL
  (LET (#:G83585)
    (COND
      (|EuclideanDomain;AL|)
      (T (SETQ |EuclideanDomain;AL| (|EuclideanDomain;|))))))
```

The First Call Domain Function

⟨*EuclideanDomain;*⟩≡

```
(DEFUN |EuclideanDomain;| NIL
  (PROG (#1=#:G83583)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|PrincipalIdealDomain|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|sizeLess?| ((|Boolean|) |$| |$|)) T)
                ((|euclideanSize| ((|NonNegativeInteger|) |$|)) T)
                ((|divide|
                  ((|Record|
                    (|:| |quotient| |$|)
                    (|:| |remainder| |$|))
                  |$| |$|)) T)
                ((|quo| (|$| |$| |$|)) T)
                ((|rem| (|$| |$| |$|)) T)
                ((|extendedEuclidean|
                  ((|Record|
                    (|:| |coef1| |$|)
                    (|:| |coef2| |$|)
                    (|:| |generator| |$|))
                  |$| |$|)) T)
                ((|extendedEuclidean|
                  ((|Union|
                      (|Record| (|:| |coef1| |$|) (|:| |coef2| |$|))
                      "failed")
                    |$| |$| |$|)) T)
                ((|multiEuclidean|
                  ((|Union|
                      (|List| |$|)
                      "failed")
                   (|List| |$|) |$|)) T)))
              NIL
              (QUOTE ((|List| |$|) (|NonNegativeInteger|) (|Boolean|)))
              NIL))
            |EuclideanDomain|)
          (SETELT #1# 0 (QUOTE (|EuclideanDomain|)))))))
```

EUCDOM;MAKEPROP

⟨*EUCDOM;MAKEPROP*⟩≡
```
(MAKEPROP (QUOTE |EuclideanDomain|) (QUOTE NILADIC) T)
```

⟨*EUCDOM.lsp BOOTSTRAP*⟩≡
⟨*EUCDOM;VersionCheck*⟩
⟨*EuclideanDomain;AL*⟩
⟨*EuclideanDomain*⟩
⟨*EuclideanDomain;*⟩
⟨*EUCDOM;MAKEPROP*⟩

21.19 EUCDOM-.lsp BOOTSTRAP

EUCDOM- depends on **EUCDOM**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **EUCDOM-** category which we can write into the **MID** directory. We compile the lisp code and copy the **EUCDOM-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

21.19.1 The Lisp Implementation

EUCDOM-;VersionCheck

This implements the bootstrap code for **EuclideanDomain**. The call to **VERSIONCHECK** is a legacy check to ensure that we did not load algebra code from a previous system version (which would not run due to major surgical changes in the system) without recompiling.

⟨*EUCDOM-;VersionCheck*⟩≡
```
(|/VERSIONCHECK| 2)
```

EUCDOM-;sizeLess?;2SB;1

⟨*EUCDOM-;sizeLess?;2SB;1*⟩≡
```
(DEFUN |EUCDOM-;sizeLess?;2SB;1| (|x| |y| $)
 (COND
  ((SPADCALL |y| (QREFELT $ 8)) (QUOTE NIL))
  ((SPADCALL |x| (QREFELT $ 8)) (QUOTE T))
  ((QUOTE T)
   (< (SPADCALL |x| (QREFELT $ 10)) (SPADCALL |y| (QREFELT $ 10))))))
```

EUCDOM-;quo;3S;2

⟨*EUCDOM-;quo;3S;2*⟩≡
```
(DEFUN |EUCDOM-;quo;3S;2| (|x| |y| $)
 (QCAR (SPADCALL |x| |y| (QREFELT $ 13))))
```

EUCDOM-;rem;3S;3

⟨*EUCDOM-;rem;3S;3*⟩≡
```
(DEFUN |EUCDOM-;rem;3S;3| (|x| |y| $)
 (QCDR (SPADCALL |x| |y| (QREFELT $ 13))))
```

EUCDOM-;exquo;2SU;4

⟨*EUCDOM-;exquo;2SU;4*⟩≡

```
(DEFUN |EUCDOM-;exquo;2SU;4| (|x| |y| $)
 (PROG (|qr|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |x| (QREFELT $ 8)) (CONS 0 (|spadConstant| $ 16)))
     ((SPADCALL |y| (QREFELT $ 8)) (CONS 1 "failed"))
     ((QUOTE T)
      (SEQ
       (LETT |qr|
        (SPADCALL |x| |y| (QREFELT $ 13))
        |EUCDOM-;exquo;2SU;4|)
       (EXIT
        (COND
         ((SPADCALL (QCDR |qr|) (QREFELT $ 8)) (CONS 0 (QCAR |qr|)))
         ((QUOTE T) (CONS 1 "failed")))))))))))
```

EUCDOM-;gcd;3S;5

⟨*EUCDOM-;gcd;3S;5*⟩≡

```
(DEFUN |EUCDOM-;gcd;3S;5| (|x| |y| $)
 (PROG (|#G13| |#G14|)
  (RETURN
   (SEQ
    (LETT |x| (SPADCALL |x| (QREFELT $ 19)) |EUCDOM-;gcd;3S;5|)
    (LETT |y| (SPADCALL |y| (QREFELT $ 19)) |EUCDOM-;gcd;3S;5|)
    (SEQ G190
     (COND
      ((NULL
       (COND
        ((SPADCALL |y| (QREFELT $ 8)) (QUOTE NIL))
        ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (PROGN
       (LETT |#G13| |y| |EUCDOM-;gcd;3S;5|)
       (LETT |#G14| (SPADCALL |x| |y| (QREFELT $ 20)) |EUCDOM-;gcd;3S;5|)
       (LETT |x| |#G13| |EUCDOM-;gcd;3S;5|)
       (LETT |y| |#G14| |EUCDOM-;gcd;3S;5|))
      (EXIT
       (LETT |y| (SPADCALL |y| (QREFELT $ 19)) |EUCDOM-;gcd;3S;5|)))
     NIL
     (GO G190)
    G191
    (EXIT NIL))
   (EXIT |x|)))))
```

EUCDOM-;unitNormalizeIdealElt

⟨*EUCDOM-;unitNormalizeIdealElt*⟩≡

```
(DEFUN |EUCDOM-;unitNormalizeIdealElt| (|s| $)
 (PROG (|#G16| |u| |c| |a|)
  (RETURN
   (SEQ
    (PROGN
     (LETT |#G16|
       (SPADCALL (QVELT |s| 2) (QREFELT $ 23))
       |EUCDOM-;unitNormalizeIdealElt|)
     (LETT |u| (QVELT |#G16| 0) |EUCDOM-;unitNormalizeIdealElt|)
     (LETT |c| (QVELT |#G16| 1) |EUCDOM-;unitNormalizeIdealElt|)
     (LETT |a| (QVELT |#G16| 2) |EUCDOM-;unitNormalizeIdealElt|)
     |#G16|)
    (EXIT
     (COND
      ((SPADCALL |a| (|spadConstant| $ 24) (QREFELT $ 25)) |s|)
      ((QUOTE T)
       (VECTOR
        (SPADCALL |a| (QVELT |s| 0) (QREFELT $ 26))
        (SPADCALL |a| (QVELT |s| 1) (QREFELT $ 26))
        |c|))))))))
```

EUCDOM-;extendedEuclidean;2SR;7

⟨*EUCDOM-;extendedEuclidean;2SR;7*⟩≡

```
(DEFUN |EUCDOM-;extendedEuclidean;2SR;7| (|x| |y| $)
 (PROG (|s3| |s2| |qr| |s1|)
  (RETURN
   (SEQ
    (LETT |s1|
     (|EUCDOM-;unitNormalizeIdealElt|
      (VECTOR (|spadConstant| $ 24) (|spadConstant| $ 16) |x|)
       $)
     |EUCDOM-;extendedEuclidean;2SR;7|)
    (LETT |s2|
     (|EUCDOM-;unitNormalizeIdealElt|
      (VECTOR (|spadConstant| $ 16) (|spadConstant| $ 24) |y|)
      $)
      |EUCDOM-;extendedEuclidean;2SR;7|)
    (EXIT
     (COND
      ((SPADCALL |y| (QREFELT $ 8)) |s1|)
      ((SPADCALL |x| (QREFELT $ 8)) |s2|)
      ((QUOTE T)
       (SEQ
        (SEQ
         G190
         (COND
          ((NULL
            (COND
             ((SPADCALL (QVELT |s2| 2) (QREFELT $ 8)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (LETT |qr|
           (SPADCALL (QVELT |s1| 2) (QVELT |s2| 2) (QREFELT $ 13))
           |EUCDOM-;extendedEuclidean;2SR;7|)
          (LETT |s3|
           (VECTOR
            (SPADCALL (QVELT |s1| 0)
             (SPADCALL (QCAR |qr|) (QVELT |s2| 0) (QREFELT $ 26))
             (QREFELT $ 27))
            (SPADCALL (QVELT |s1| 1)
             (SPADCALL (QCAR |qr|) (QVELT |s2| 1) (QREFELT $ 26))
             (QREFELT $ 27))
            (QCDR |qr|))
           |EUCDOM-;extendedEuclidean;2SR;7|)
          (LETT |s1| |s2| |EUCDOM-;extendedEuclidean;2SR;7|)
```

```
  (EXIT
   (LETT |s2|
    (|EUCDOM-;unitNormalizeIdealElt| |s3| $)
    |EUCDOM-;extendedEuclidean;2SR;7|)))
 NIL
 (GO G190)
 G191
 (EXIT NIL))
(COND
 ((NULL (SPADCALL (QVELT |s1| 0) (QREFELT $ 8)))
   (COND
    ((NULL (SPADCALL (QVELT |s1| 0) |y| (QREFELT $ 28)))
     (SEQ
      (LETT |qr|
       (SPADCALL (QVELT |s1| 0) |y| (QREFELT $ 13))
       |EUCDOM-;extendedEuclidean;2SR;7|)
      (QSETVELT |s1| 0 (QCDR |qr|))
      (QSETVELT |s1| 1
       (SPADCALL (QVELT |s1| 1)
        (SPADCALL (QCAR |qr|) |x| (QREFELT $ 26)) (QREFELT $ 29)))
      (EXIT
       (LETT |s1|
        (|EUCDOM-;unitNormalizeIdealElt| |s1| $)
        |EUCDOM-;extendedEuclidean;2SR;7|)))))))
(EXIT |s1|)))))))))
```

EUCDOM-;extendedEuclidean;3SU;8

⟨*EUCDOM-;extendedEuclidean;3SU;8*⟩≡

```
(DEFUN |EUCDOM-;extendedEuclidean;3SU;8| (|x| |y| |z| $)
 (PROG (|s| |w| |qr|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |z| (QREFELT $ 8))
       (CONS 0 (CONS (|spadConstant| $ 16) (|spadConstant| $ 16))))
     ((QUOTE T)
       (SEQ
        (LETT |s|
         (SPADCALL |x| |y| (QREFELT $ 32))
         |EUCDOM-;extendedEuclidean;3SU;8|)
        (LETT |w|
         (SPADCALL |z| (QVELT |s| 2) (QREFELT $ 33))
         |EUCDOM-;extendedEuclidean;3SU;8|)
        (EXIT
         (COND
          ((QEQCAR |w| 1) (CONS 1 "failed"))
          ((SPADCALL |y| (QREFELT $ 8))
            (CONS 0
             (CONS (SPADCALL (QVELT |s| 0) (QCDR |w|) (QREFELT $ 26))
              (SPADCALL (QVELT |s| 1) (QCDR |w|) (QREFELT $ 26)))))
          ((QUOTE T)
           (SEQ
            (LETT |qr|
             (SPADCALL
              (SPADCALL (QVELT |s| 0) (QCDR |w|) (QREFELT $ 26))
              |y|
              (QREFELT $ 13))
             |EUCDOM-;extendedEuclidean;3SU;8|)
            (EXIT
             (CONS 0
              (CONS (QCDR |qr|)
               (SPADCALL
                (SPADCALL (QVELT |s| 1) (QCDR |w|) (QREFELT $ 26))
                (SPADCALL (QCAR |qr|) |x| (QREFELT $ 26))
                (QREFELT $ 29))))))))))))))))
```

EUCDOM-;principalIdeal;LR;9

⟨*EUCDOM-;principalIdeal;LR;9*⟩≡

```
(DEFUN |EUCDOM-;principalIdeal;LR;9| (|l| $)
 (PROG (|uca| |v| |u| #0=#:G1497 |vv| #1=#:G1498)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |l| NIL (QREFELT $ 38))
       (|error| "empty list passed to principalIdeal"))
     ((SPADCALL (CDR |l|) NIL (QREFELT $ 38))
       (SEQ
        (LETT |uca|
         (SPADCALL (|SPADfirst| |l|) (QREFELT $ 23))
         |EUCDOM-;principalIdeal;LR;9|)
        (EXIT (CONS (LIST (QVELT |uca| 0)) (QVELT |uca| 1)))))
     ((SPADCALL (CDR (CDR |l|)) NIL (QREFELT $ 38))
       (SEQ
        (LETT |u|
         (SPADCALL (|SPADfirst| |l|)
          (SPADCALL |l| (QREFELT $ 39)) (QREFELT $ 32))
         |EUCDOM-;principalIdeal;LR;9|)
        (EXIT (CONS (LIST (QVELT |u| 0) (QVELT |u| 1)) (QVELT |u| 2)))))
     ((QUOTE T)
       (SEQ
        (LETT |v|
         (SPADCALL (CDR |l|) (QREFELT $ 42))
         |EUCDOM-;principalIdeal;LR;9|)
        (LETT |u|
         (SPADCALL (|SPADfirst| |l|) (QCDR |v|) (QREFELT $ 32))
         |EUCDOM-;principalIdeal;LR;9|)
        (EXIT
         (CONS
          (CONS (QVELT |u| 0)
           (PROGN
            (LETT #0# NIL |EUCDOM-;principalIdeal;LR;9|)
            (SEQ
             (LETT |vv| NIL |EUCDOM-;principalIdeal;LR;9|)
             (LETT #1# (QCAR |v|) |EUCDOM-;principalIdeal;LR;9|)
             G190
             (COND
              ((OR (ATOM #1#)
                   (PROGN
                    (LETT |vv| (CAR #1#) |EUCDOM-;principalIdeal;LR;9|) NIL))
               (GO G191)))
             (SEQ
```

```
     (EXIT
      (LETT #0#
       (CONS (SPADCALL (QVELT |u| 1) |vv| (QREFELT $ 26))
          #0#)
       |EUCDOM-;principalIdeal;LR;9|)))
    (LETT #1# (CDR #1#)
     |EUCDOM-;principalIdeal;LR;9|)
    (GO G190)
    G191
    (EXIT (NREVERSE0 #0#)))))
(QVELT |u| 2))))))))))
```

EUCDOM-;expressIdealMember;LSU;10

⟨*EUCDOM-;expressIdealMember;LSU;10*⟩≡

```
(DEFUN |EUCDOM-;expressIdealMember;LSU;10| (|l| |z| $)
 (PROG (#0=#:G1513 #1=#:G1514 |pid| |q| #2=#:G1515 |v| #3=#:G1516)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |z| (|spadConstant| $ 16) (QREFELT $ 25))
      (CONS 0
       (PROGN
        (LETT #0# NIL |EUCDOM-;expressIdealMember;LSU;10|)
        (SEQ
         (LETT |v| NIL |EUCDOM-;expressIdealMember;LSU;10|)
         (LETT #1# |l| |EUCDOM-;expressIdealMember;LSU;10|)
         G190
         (COND
          ((OR (ATOM #1#)
               (PROGN
                (LETT |v| (CAR #1#) |EUCDOM-;expressIdealMember;LSU;10|) NIL))
           (GO G191)))
         (SEQ
          (EXIT
           (LETT #0#
            (CONS (|spadConstant| $ 16) #0#)
            |EUCDOM-;expressIdealMember;LSU;10|)))
         (LETT #1# (CDR #1#) |EUCDOM-;expressIdealMember;LSU;10|)
         (GO G190)
         G191
         (EXIT (NREVERSE0 #0#))))))
     ((QUOTE T)
      (SEQ
       (LETT |pid|
        (SPADCALL |l| (QREFELT $ 42))
        |EUCDOM-;expressIdealMember;LSU;10|)
       (LETT |q|
        (SPADCALL |z| (QCDR |pid|) (QREFELT $ 33))
        |EUCDOM-;expressIdealMember;LSU;10|)
       (EXIT
        (COND
         ((QEQCAR |q| 1) (CONS 1 "failed"))
         ((QUOTE T)
          (CONS 0
           (PROGN
            (LETT #2# NIL |EUCDOM-;expressIdealMember;LSU;10|)
            (SEQ
```

```
(LETT |v| NIL |EUCDOM-;expressIdealMember;LSU;10|)
(LETT #3# (QCAR |pid|) |EUCDOM-;expressIdealMember;LSU;10|)
G190
(COND
 ((OR (ATOM #3#)
      (PROGN
       (LETT |v| (CAR #3#) |EUCDOM-;expressIdealMember;LSU;10|)
       NIL))
  (GO G191)))
(SEQ
 (EXIT
 (LETT #2#
  (CONS (SPADCALL (QCDR |q|) |v| (QREFELT $ 26))
     #2#)
  |EUCDOM-;expressIdealMember;LSU;10|)))
(LETT #3# (CDR #3#) |EUCDOM-;expressIdealMember;LSU;10|)
(GO G190)
G191
(EXIT (NREVERSE0 #2#)))))))))))))))
```

EUCDOM-;multiEuclidean;LSU;11

⟨*EUCDOM-;multiEuclidean;LSU;11*⟩≡

```
(DEFUN |EUCDOM-;multiEuclidean;LSU;11| (|l| |z| $)
 (PROG (|n| |l1| |l2| #0=#:G1405 #1=#:G1535 #2=#:G1522 #3=#:G1520
        #4=#:G1521 #5=#:G1406 #6=#:G1536 #7=#:G1525 #8=#:G1523 #9=#:G1524
        |u| |v1| |v2|)
  (RETURN
   (SEQ
    (LETT |n| (LENGTH |l|) |EUCDOM-;multiEuclidean;LSU;11|)
    (EXIT
     (COND
      ((ZEROP |n|) (|error| "empty list passed to multiEuclidean"))
      ((EQL |n| 1) (CONS 0 (LIST |z|)))
      ((QUOTE T)
       (SEQ
        (LETT |l1|
         (SPADCALL |l| (QREFELT $ 46)) |EUCDOM-;multiEuclidean;LSU;11|)
        (LETT |l2|
         (SPADCALL |l1| (QUOTIENT2 |n| 2) (QREFELT $ 48))
         |EUCDOM-;multiEuclidean;LSU;11|)
        (LETT |u|
         (SPADCALL
          (PROGN
           (LETT #4# NIL |EUCDOM-;multiEuclidean;LSU;11|)
           (SEQ
            (LETT #0# NIL |EUCDOM-;multiEuclidean;LSU;11|)
            (LETT #1# |l1| |EUCDOM-;multiEuclidean;LSU;11|)
            G190
            (COND
             ((OR (ATOM #1#)
                  (PROGN
                   (LETT #0# (CAR #1#) |EUCDOM-;multiEuclidean;LSU;11|)
                   NIL))
              (GO G191)))
            (SEQ
             (EXIT
              (PROGN
               (LETT #2# #0# |EUCDOM-;multiEuclidean;LSU;11|)
               (COND
                (#4#
                 (LETT #3#
                  (SPADCALL #3# #2# (QREFELT $ 26))
                  |EUCDOM-;multiEuclidean;LSU;11|))
                ((QUOTE T)
                 (PROGN
```

```
                (LETT #3# #2# |EUCDOM-;multiEuclidean;LSU;11|)
                (LETT #4# (QUOTE T) |EUCDOM-;multiEuclidean;LSU;11|)))))))
           (LETT #1# (CDR #1#) |EUCDOM-;multiEuclidean;LSU;11|)
           (GO G190)
           G191
           (EXIT NIL))
          (COND (#4# #3#) ((QUOTE T) (|spadConstant| $ 24))))
         (PROGN
          (LETT #9# NIL |EUCDOM-;multiEuclidean;LSU;11|)
          (SEQ
           (LETT #5# NIL |EUCDOM-;multiEuclidean;LSU;11|)
           (LETT #6# |l2| |EUCDOM-;multiEuclidean;LSU;11|)
           G190
           (COND
            ((OR (ATOM #6#)
                 (PROGN
                  (LETT #5# (CAR #6#) |EUCDOM-;multiEuclidean;LSU;11|)
                  NIL))
             (GO G191)))
           (SEQ
            (EXIT
             (PROGN
              (LETT #7# #5# |EUCDOM-;multiEuclidean;LSU;11|)
              (COND
               (#9#
                (LETT #8#
                 (SPADCALL #8# #7# (QREFELT $ 26))
                 |EUCDOM-;multiEuclidean;LSU;11|))
               ((QUOTE T)
                (PROGN
                 (LETT #8# #7# |EUCDOM-;multiEuclidean;LSU;11|)
                 (LETT #9# (QUOTE T) |EUCDOM-;multiEuclidean;LSU;11|)))))))
           (LETT #6# (CDR #6#) |EUCDOM-;multiEuclidean;LSU;11|)
           (GO G190)
           G191
           (EXIT NIL))
          (COND (#9# #8#) ((QUOTE T) (|spadConstant| $ 24))))
         |z| (QREFELT $ 49))
        |EUCDOM-;multiEuclidean;LSU;11|)
       (EXIT
        (COND
         ((QEQCAR |u| 1) (CONS 1 "failed"))
         ((QUOTE T)
          (SEQ
           (LETT |v1|
            (SPADCALL |l1| (QCDR (QCDR |u|)) (QREFELT $ 50))
```

```
 |EUCDOM-;multiEuclidean;LSU;11|)
(EXIT
 (COND
  ((QEQCAR |v1| 1) (CONS 1 "failed"))
  ((QUOTE T)
   (SEQ
    (LETT |v2|
     (SPADCALL |l2| (QCAR (QCDR |u|)) (QREFELT $ 50))
     |EUCDOM-;multiEuclidean;LSU;11|)
    (EXIT
     (COND
      ((QEQCAR |v2| 1) (CONS 1 "failed"))
      ((QUOTE T)
       (CONS 0
        (SPADCALL
         (QCDR |v1|)
         (QCDR |v2|)
         (QREFELT $ 51))))))))))))))))))))))))
```

EuclideanDomain&

⟨*EuclideanDomainAmp*⟩≡

```
(DEFUN |EuclideanDomain&| (|#1|)
 (PROG (DV$1 |dv$| $ |pv$|)
  (RETURN
   (PROGN
    (LETT DV$1 (|devaluate| |#1|) . #0=(|EuclideanDomain&|))
    (LETT |dv$| (LIST (QUOTE |EuclideanDomain&|) DV$1) . #0#)
    (LETT $ (GETREFV 53) . #0#)
    (QSETREFV $ 0 |dv$|)
    (QSETREFV $ 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #0#))
    (|stuffDomainSlots| $)
    (QSETREFV $ 6 |#1|)
    $))))
```

EUCDOM-;MAKEPROP

⟨*EUCDOM-;MAKEPROP*⟩≡

```
(MAKEPROP
 (QUOTE |EuclideanDomain&|)
 (QUOTE |infovec|)
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|Boolean|) (0 . |zero?|)
  (|NonNegativeInteger|) (5 . |euclideanSize|) |EUCDOM-;sizeLess?;2SB;1|
  (|Record| (|:| |quotient| $) (|:| |remainder| $)) (10 . |divide|)
  |EUCDOM-;quo;3S;2| |EUCDOM-;rem;3S;3| (16 . |Zero|)
  (|Union| $ (QUOTE "failed")) |EUCDOM-;exquo;2SU;4| (20 . |unitCanonical|)
  (25 . |rem|) |EUCDOM-;gcd;3S;5|
  (|Record| (|:| |unit| $) (|:| |canonical| $) (|:| |associate| $))
  (31 . |unitNormal|) (36 . |One|) (40 . =) (46 . *) (52 . -)
  (58 . |sizeLess?|) (64 . +)
  (|Record| (|:| |coef1| $) (|:| |coef2| $) (|:| |generator| $))
  |EUCDOM-;extendedEuclidean;2SR;7|
  (70 . |extendedEuclidean|) (76 . |exquo|)
  (|Record| (|:| |coef1| $) (|:| |coef2| $))
  (|Union| 34 (QUOTE "failed")) |EUCDOM-;extendedEuclidean;3SU;8|
  (|List| 6) (82 . =) (88 . |second|)
  (|Record| (|:| |coef| 41) (|:| |generator| $))
  (|List| $) (93 . |principalIdeal|) |EUCDOM-;principalIdeal;LR;9|
  (|Union| 41 (QUOTE "failed")) |EUCDOM-;expressIdealMember;LSU;10|
  (98 . |copy|) (|Integer|) (103 . |split!|) (109 . |extendedEuclidean|)
  (116 . |multiEuclidean|) (122 . |concat|) |EUCDOM-;multiEuclidean;LSU;11|))
  (QUOTE
   #(|sizeLess?| 128 |rem| 134 |quo| 140 |principalIdeal| 146
     |multiEuclidean| 151 |gcd| 157 |extendedEuclidean| 163
     |exquo| 176 |expressIdealMember| 182))
  (QUOTE NIL)
  (CONS (|makeByteWordVec2| 1 (QUOTE NIL))
   (CONS (QUOTE #())
    (CONS (QUOTE #())
     (|makeByteWordVec2| 52 (QUOTE (1 6 7 0 8 1 6 9 0 10 2 6 12 0 0 13 0
       6 0 16 1 6 0 0 19 2 6 0 0 0 20 1 6 22 0 23 0 6 0 24 2 6 7 0 0 25 2 6 0
       0 0 26 2 6 0 0 0 27 2 6 7 0 0 28 2 6 0 0 0 29 2 6 30 0 0 32 2 6 17 0 0
       33 2 37 7 0 0 38 1 37 6 0 39 1 6 40 41 42 1 37 0 0 46 2 37 0 0 47 48 3
       6 35 0 0 0 49 2 6 44 41 0 50 2 37 0 0 0 51 2 0 7 0 0 11 2 0 0 0 0 0 15 2
       0 0 0 0 0 14 1 0 40 41 43 2 0 44 41 0 52 2 0 0 0 0 21 3 0 35 0 0 0 36 2 0
       30 0 0 31 2 0 17 0 0 18 2 0 44 41 0 45))))))
  (QUOTE |lookupComplete|)))
```

⟨*EUCDOM-.lsp BOOTSTRAP*⟩≡

⟨*EUCDOM-;VersionCheck*⟩
⟨*EUCDOM-;sizeLess?;2SB;1*⟩
⟨*EUCDOM-;quo;3S;2*⟩
⟨*EUCDOM-;rem;3S;3*⟩
⟨*EUCDOM-;exquo;2SU;4*⟩
⟨*EUCDOM-;gcd;3S;5*⟩
⟨*EUCDOM-;unitNormalizeIdealElt*⟩
⟨*EUCDOM-;extendedEuclidean;2SR;7*⟩
⟨*EUCDOM-;extendedEuclidean;3SU;8*⟩
⟨*EUCDOM-;principalIdeal;LR;9*⟩
⟨*EUCDOM-;expressIdealMember;LSU;10*⟩
⟨*EUCDOM-;multiEuclidean;LSU;11*⟩
⟨*EuclideanDomainAmp*⟩
⟨*EUCDOM-;MAKEPROP*⟩

21.20 ENTIRER.lsp BOOTSTRAP

ENTIRER depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ENTIRER** category which we can write into the **MID** directory. We compile the lisp code and copy the **ENTIRER.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ENTIRER.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |EntireRing;AL| (QUOTE NIL))

(DEFUN |EntireRing| NIL
  (LET (#:G82841)
    (COND
      (|EntireRing;AL|)
      (T (SETQ |EntireRing;AL| (|EntireRing;|))))))

(DEFUN |EntireRing;| NIL
  (PROG (#1=#:G82839)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|Ring|)
            (|BiModule| (QUOTE |$|) (QUOTE |$|))
            (|mkCategory|
              (QUOTE |package|)
              NIL
              (QUOTE ((|noZeroDivisors| T)))
              (QUOTE NIL)
              NIL))
          |EntireRing|)
        (SETELT #1# 0 (QUOTE (|EntireRing|)))))))

(MAKEPROP (QUOTE |EntireRing|) (QUOTE NILADIC) T)
```

21.21 FFIELDC.lsp BOOTSTRAP

FFIELDC depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **FFIELDC** category which we can write into the **MID** directory. We compile the lisp code and copy the **FFIELDC.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*FFIELDC.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |FiniteFieldCategory;AL| (QUOTE NIL))

(DEFUN |FiniteFieldCategory| NIL
 (LET (#:G83129)
  (COND
   (|FiniteFieldCategory;AL|)
   (T (SETQ |FiniteFieldCategory;AL| (|FiniteFieldCategory;|))))))

(DEFUN |FiniteFieldCategory;| NIL
 (PROG (#1=#:G83127)
  (RETURN
   (PROG1
    (LETT #1#
     (|Join|
      (|FieldOfPrimeCharacteristic|)
      (|Finite|)
      (|StepThrough|)
      (|DifferentialRing|)
      (|mkCategory|
       (QUOTE |domain|)
       (QUOTE (
        ((|charthRoot| (|$| |$|)) T)
        ((|conditionP| ((|Union| (|Vector| |$|) "failed") (|Matrix| |$|))) T)
        ((|factorsOfCyclicGroupSize|
         ((|List| (|Record|
                   (|:| |factor| (|Integer|))
                   (|:| |exponent| (|Integer|))))))
          T)
        ((|tableForDiscreteLogarithm|
          ((|Table| (|PositiveInteger|) (|NonNegativeInteger|))
           (|Integer|))) T)
        ((|createPrimitiveElement| (|$|)) T)
        ((|primitiveElement| (|$|)) T)
```

```
         ((|primitive?| ((|Boolean|) |$|)) T)
         ((|discreteLog| ((|NonNegativeInteger|) |$|)) T)
         ((|order| ((|PositiveInteger|) |$|)) T)
         ((|representationType|
           ((|Union| "prime" "polynomial" "normal" "cyclic"))) T)))
        NIL
        (QUOTE (
         (|PositiveInteger|)
         (|NonNegativeInteger|)
         (|Boolean|)
         (|Table| (|PositiveInteger|) (|NonNegativeInteger|))
         (|Integer|)
         (|List|
          (|Record| (|:| |factor| (|Integer|)) (|:| |exponent| (|Integer|))))
         (|Matrix| |$|)))
        NIL))
      |FiniteFieldCategory|)
     (SETELT #1# 0 (QUOTE (|FiniteFieldCategory|)))))))

(MAKEPROP (QUOTE |FiniteFieldCategory|) (QUOTE NILADIC) T)
```

21.22 FFIELDC-.lsp BOOTSTRAP

FFIELDC- depends on **FFIELDC**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **FFIELDC-** category which we can write into the **MID** directory. We compile the lisp code and copy the **FFIELDC-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*FFIELDC-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |FFIELDC-;differentiate;2S;1| (|x| |$|) (|spadConstant| |$| 7))

(DEFUN |FFIELDC-;init;S;2| (|$|) (|spadConstant| |$| 7))

(DEFUN |FFIELDC-;nextItem;SU;3| (|a| |$|)
 (COND
  ((SPADCALL
    (LETT |a|
     (SPADCALL (|+| (SPADCALL |a| (QREFELT |$| 11)) 1) (QREFELT |$| 12))
     |FFIELDC-;nextItem;SU;3|)
    (QREFELT |$| 14))
     (CONS 1 "failed"))
  ((QUOTE T) (CONS 0 |a|))))

(DEFUN |FFIELDC-;order;SOpc;4| (|e| |$|)
 (SPADCALL (SPADCALL |e| (QREFELT |$| 17)) (QREFELT |$| 20)))

(DEFUN |FFIELDC-;conditionP;MU;5| (|mat| |$|)
 (PROG (|l|)
  (RETURN
   (SEQ
    (LETT |l| (SPADCALL |mat| (QREFELT |$| 24)) |FFIELDC-;conditionP;MU;5|)
    (COND
     ((OR
       (NULL |l|)
       (SPADCALL (ELT |$| 14) (|SPADfirst| |l|) (QREFELT |$| 27)))
        (EXIT (CONS 1 "failed"))))
    (EXIT
     (CONS 0
      (SPADCALL (ELT |$| 28) (|SPADfirst| |l|) (QREFELT |$| 30))))))))

(DEFUN |FFIELDC-;charthRoot;2S;6| (|x| |$|)
 (SPADCALL |x|
```

```
   (QUOTIENT2 (SPADCALL (QREFELT |$| 35)) (SPADCALL (QREFELT |$| 36)))
   (QREFELT |$| 37)))

 (DEFUN |FFIELDC-;charthRoot;SU;7| (|x| |$|)
  (CONS 0 (SPADCALL |x| (QREFELT |$| 28))))

 (DEFUN |FFIELDC-;createPrimitiveElement;S;8| (|$|)
  (PROG (|sm1| |start| |i| #1=#:G83175 |e| |found|)
   (RETURN
    (SEQ
     (LETT |sm1|
      (|-| (SPADCALL (QREFELT |$| 35)) 1)
      |FFIELDC-;createPrimitiveElement;S;8|)
     (LETT |start|
      (COND
       ((SPADCALL
         (SPADCALL (QREFELT |$| 42))
         (CONS 1 "polynomial")
         (QREFELT |$| 43))
        (SPADCALL (QREFELT |$| 36)))
       ((QUOTE T) 1))
      |FFIELDC-;createPrimitiveElement;S;8|)
     (LETT |found| (QUOTE NIL) |FFIELDC-;createPrimitiveElement;S;8|)
     (SEQ
      (LETT |i| |start| |FFIELDC-;createPrimitiveElement;S;8|)
      G190
      (COND
       ((NULL (COND (|found| (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
         (GO G191)))
      (SEQ
       (LETT |e|
        (SPADCALL
         (PROG1
          (LETT #1# |i| |FFIELDC-;createPrimitiveElement;S;8|)
          (|check-subtype| (|>| #1# 0) (QUOTE (|PositiveInteger|)) #1#))
         (QREFELT |$| 12))
        |FFIELDC-;createPrimitiveElement;S;8|)
       (EXIT
        (LETT |found|
         (EQL (SPADCALL |e| (QREFELT |$| 17)) |sm1|)
         |FFIELDC-;createPrimitiveElement;S;8|)))
      (LETT |i| (|+| |i| 1) |FFIELDC-;createPrimitiveElement;S;8|)
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT |e|)))))
```

```
(DEFUN |FFIELDC-;primitive?;SB;9| (|a| |$|)
 (PROG (|explist| |q| |exp| #1=#:G83187 |equalone|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |a| (QREFELT |$| 14)) (QUOTE NIL))
     ((QUOTE T)
      (SEQ
       (LETT |explist|
        (SPADCALL (QREFELT |$| 47)) |FFIELDC-;primitive?;SB;9|)
       (LETT |q|
        (|-| (SPADCALL (QREFELT |$| 35)) 1) |FFIELDC-;primitive?;SB;9|)
       (LETT |equalone| (QUOTE NIL) |FFIELDC-;primitive?;SB;9|)
       (SEQ
        (LETT |exp| NIL |FFIELDC-;primitive?;SB;9|)
        (LETT #1# |explist| |FFIELDC-;primitive?;SB;9|)
        G190
        (COND
         ((OR
           (ATOM #1#)
           (PROGN (LETT |exp| (CAR #1#) |FFIELDC-;primitive?;SB;9|) NIL)
           (NULL (COND (|equalone| (QUOTE NIL)) ((QUOTE T) (QUOTE T)))))
          (GO G191)))
        (SEQ
         (EXIT
          (LETT |equalone|
           (SPADCALL
            (SPADCALL |a| (QUOTIENT2 |q| (QCAR |exp|)) (QREFELT |$| 48))
            (QREFELT |$| 49))
           |FFIELDC-;primitive?;SB;9|)))
        (LETT #1# (CDR #1#) |FFIELDC-;primitive?;SB;9|)
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT (COND (|equalone| (QUOTE NIL)) ((QUOTE T) (QUOTE T))))))))))

(DEFUN |FFIELDC-;order;SPi;10| (|e| |$|)
 (PROG (|lof| |rec| #1=#:G83195 |primeDivisor|
        |j| #2=#:G83196 |a| |goon| |ord|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |e| (|spadConstant| |$| 7) (QREFELT |$| 51))
       (|error| "order(0) is not defined "))
     ((QUOTE T)
```

```
(SEQ
 (LETT |ord|
  (|-| (SPADCALL (QREFELT |$| 35)) 1) |FFIELDC-;order;SPi;10|)
 (LETT |a| 0 |FFIELDC-;order;SPi;10|)
 (LETT |lof| (SPADCALL (QREFELT |$| 47)) |FFIELDC-;order;SPi;10|)
 (SEQ
  (LETT |rec| NIL |FFIELDC-;order;SPi;10|)
  (LETT #1# |lof| |FFIELDC-;order;SPi;10|)
  G190
  (COND
   ((OR
     (ATOM #1#)
     (PROGN (LETT |rec| (CAR #1#) |FFIELDC-;order;SPi;10|) NIL))
    (GO G191)))
  (SEQ
   (LETT |a|
    (QUOTIENT2 |ord|
     (LETT |primeDivisor| (QCAR |rec|) |FFIELDC-;order;SPi;10|))
    |FFIELDC-;order;SPi;10|)
   (LETT |goon|
    (SPADCALL (SPADCALL |e| |a| (QREFELT |$| 48)) (QREFELT |$| 49))
    |FFIELDC-;order;SPi;10|)
   (SEQ
    (LETT |j| 0 |FFIELDC-;order;SPi;10|)
    (LETT #2# (|-| (QCDR |rec|) 2) |FFIELDC-;order;SPi;10|)
    G190
    (COND ((OR (QSGREATERP |j| #2#) (NULL |goon|)) (GO G191)))
    (SEQ
     (LETT |ord| |a| |FFIELDC-;order;SPi;10|)
     (LETT |a|
      (QUOTIENT2 |ord| |primeDivisor|)
      |FFIELDC-;order;SPi;10|)
     (EXIT
      (LETT |goon|
       (SPADCALL (SPADCALL |e| |a| (QREFELT |$| 48)) (QREFELT |$| 49))
       |FFIELDC-;order;SPi;10|)))
    (LETT |j| (QSADD1 |j|) |FFIELDC-;order;SPi;10|)
    (GO G190)
    G191
    (EXIT NIL))
   (EXIT (COND (|goon| (LETT |ord| |a| |FFIELDC-;order;SPi;10|)))))
  (LETT #1# (CDR #1#) |FFIELDC-;order;SPi;10|)
  (GO G190)
  G191
  (EXIT NIL))
 (EXIT |ord|)))))))
```

```
(DEFUN |FFIELDC-;discreteLog;SNni;11| (|b| |$|)
 (PROG (|faclist| |gen| |groupord| |f| #1=#:G83216 |fac| |t| #2=#:G83217
        |exp| |exptable| |n| |end| |i| |rho| |found| |disc1| |c| |mult|
        |disclog| |a|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |b| (QREFELT |$| 14))
       (|error| "discreteLog: logarithm of zero"))
     ((QUOTE T)
      (SEQ
       (LETT |faclist|
        (SPADCALL (QREFELT |$| 47))
        |FFIELDC-;discreteLog;SNni;11|)
       (LETT |a| |b| |FFIELDC-;discreteLog;SNni;11|)
       (LETT |gen|
        (SPADCALL (QREFELT |$| 53))
        |FFIELDC-;discreteLog;SNni;11|)
       (EXIT
        (COND
         ((SPADCALL |b| |gen| (QREFELT |$| 51)) 1)
         ((QUOTE T)
          (SEQ
           (LETT |disclog| 0 |FFIELDC-;discreteLog;SNni;11|)
           (LETT |mult| 1 |FFIELDC-;discreteLog;SNni;11|)
           (LETT |groupord|
            (|-| (SPADCALL (QREFELT |$| 35)) 1)
            |FFIELDC-;discreteLog;SNni;11|)
           (LETT |exp| |groupord| |FFIELDC-;discreteLog;SNni;11|)
           (SEQ
            (LETT |f| NIL |FFIELDC-;discreteLog;SNni;11|)
            (LETT #1# |faclist| |FFIELDC-;discreteLog;SNni;11|)
            G190
            (COND
             ((OR
               (ATOM #1#)
               (PROGN
                (LETT |f| (CAR #1#) |FFIELDC-;discreteLog;SNni;11|)
                NIL))
              (GO G191)))
             (SEQ
              (LETT |fac| (QCAR |f|) |FFIELDC-;discreteLog;SNni;11|)
              (EXIT
               (SEQ
                (LETT |t| 0 |FFIELDC-;discreteLog;SNni;11|)
```

```
(LETT #2# (|-| (QCDR |f|) 1) |FFIELDC-;discreteLog;SNni;11|)
G190
(COND ((QSGREATERP |t| #2#) (GO G191)))
(SEQ
 (LETT |exp|
  (QUOTIENT2 |exp| |fac|)
  |FFIELDC-;discreteLog;SNni;11|)
 (LETT |exptable|
  (SPADCALL |fac| (QREFELT |$| 55))
  |FFIELDC-;discreteLog;SNni;11|)
 (LETT |n|
  (SPADCALL |exptable| (QREFELT |$| 56))
  |FFIELDC-;discreteLog;SNni;11|)
 (LETT |c|
  (SPADCALL |a| |exp| (QREFELT |$| 48))
  |FFIELDC-;discreteLog;SNni;11|)
 (LETT |end|
  (QUOTIENT2 (|-| |fac| 1) |n|)
  |FFIELDC-;discreteLog;SNni;11|)
 (LETT |found| (QUOTE NIL) |FFIELDC-;discreteLog;SNni;11|)
 (LETT |disc1| 0 |FFIELDC-;discreteLog;SNni;11|)
 (SEQ
  (LETT |i| 0 |FFIELDC-;discreteLog;SNni;11|)
  G190
  (COND
   ((OR
     (QSGREATERP |i| |end|)
     (NULL
      (COND (|found| (QUOTE NIL)) ((QUOTE T) (QUOTE T)))))
    (GO G191)))
  (SEQ
   (LETT |rho|
    (SPADCALL
     (SPADCALL |c| (QREFELT |$| 11))
     |exptable|
     (QREFELT |$| 58))
    |FFIELDC-;discreteLog;SNni;11|)
   (EXIT
    (COND
     ((QEQCAR |rho| 0)
      (SEQ
       (LETT |found| (QUOTE T) |FFIELDC-;discreteLog;SNni;11|)
       (EXIT
        (LETT |disc1|
         (|*| (|+| (|*| |n| |i|) (QCDR |rho|)) |mult|)
         |FFIELDC-;discreteLog;SNni;11|))))
```

```
                    ((QUOTE T)
                     (LETT |c|
                      (SPADCALL |c|
                       (SPADCALL |gen|
                        (|*| (QUOTIENT2 |groupord| |fac|) (|-| |n|))
                        (QREFELT |$| 48))
                       (QREFELT |$| 59))
                       |FFIELDC-;discreteLog;SNni;11|)))))
                   (LETT |i| (QSADD1 |i|) |FFIELDC-;discreteLog;SNni;11|)
                   (GO G190)
                   G191
                   (EXIT NIL))
                  (EXIT
                   (COND
                    (|found|
                     (SEQ
                      (LETT |mult|
                       (|*| |mult| |fac|)
                       |FFIELDC-;discreteLog;SNni;11|)
                      (LETT |disclog|
                       (|+| |disclog| |disc1|)
                       |FFIELDC-;discreteLog;SNni;11|)
                      (EXIT
                       (LETT |a|
                        (SPADCALL |a|
                         (SPADCALL |gen| (|-| |disc1|) (QREFELT |$| 48))
                         (QREFELT |$| 59))
                        |FFIELDC-;discreteLog;SNni;11|))))
                     ((QUOTE T)
                      (|error| "discreteLog: ?? discrete logarithm")))))
                  (LETT |t|
                   (QSADD1 |t|)
                   |FFIELDC-;discreteLog;SNni;11|)
                  (GO G190)
                  G191
                  (EXIT NIL))))
                (LETT #1#
                 (CDR #1#)
                 |FFIELDC-;discreteLog;SNni;11|)
                (GO G190)
                G191
                (EXIT NIL))
               (EXIT |disclog|))))))))))))

(DEFUN |FFIELDC-;discreteLog;2SU;12| (|logbase| |b| |$|)
 (PROG (|groupord| |faclist| |f| #1=#:G83235 |fac| |primroot|
```

```
          |t| #2=#:G83236 |exp| |rhoHelp| #3=#:G83234 |rho| |disclog|
          |mult| |a|)
    (RETURN
     (SEQ
      (EXIT
       (COND
        ((SPADCALL |b| (QREFELT |$| 14))
         (SEQ
          (SPADCALL "discreteLog: logarithm of zero" (QREFELT |$| 64))
          (EXIT (CONS 1 "failed"))))
        ((SPADCALL |logbase| (QREFELT |$| 14))
         (SEQ
          (SPADCALL "discreteLog: logarithm to base zero" (QREFELT |$| 64))
          (EXIT (CONS 1 "failed"))))
        ((SPADCALL |b| |logbase| (QREFELT |$| 51)) (CONS 0 1))
        ((QUOTE T)
         (COND
          ((NULL
            (ZEROP
             (REMAINDER2
              (LETT |groupord|
               (SPADCALL |logbase| (QREFELT |$| 17))
               |FFIELDC-;discreteLog;2SU;12|)
              (SPADCALL |b| (QREFELT |$| 17)))))
           (SEQ
            (SPADCALL
  "discreteLog: second argument not in cyclic group generated by first argument"
             (QREFELT |$| 64))
            (EXIT (CONS 1 "failed"))))
          ((QUOTE T)
           (SEQ
            (LETT |faclist|
             (SPADCALL (SPADCALL |groupord| (QREFELT |$| 66)) (QREFELT |$| 68))
             |FFIELDC-;discreteLog;2SU;12|)
            (LETT |a| |b| |FFIELDC-;discreteLog;2SU;12|)
            (LETT |disclog| 0 |FFIELDC-;discreteLog;2SU;12|)
            (LETT |mult| 1 |FFIELDC-;discreteLog;2SU;12|)
            (LETT |exp| |groupord| |FFIELDC-;discreteLog;2SU;12|)
            (SEQ
             (LETT |f| NIL |FFIELDC-;discreteLog;2SU;12|)
             (LETT #1# |faclist| |FFIELDC-;discreteLog;2SU;12|)
             G190
             (COND
              ((OR
                (ATOM #1#)
                (PROGN (LETT |f| (CAR #1#) |FFIELDC-;discreteLog;2SU;12|) NIL))
```

```
  (GO G191)))
 (SEQ
  (LETT |fac| (QCAR |f|) |FFIELDC-;discreteLog;2SU;12|)
  (LETT |primroot|
   (SPADCALL |logbase|
    (QUOTIENT2 |groupord| |fac|)
    (QREFELT |$| 48))
   |FFIELDC-;discreteLog;2SU;12|)
  (EXIT
   (SEQ
    (LETT |t| 0 |FFIELDC-;discreteLog;2SU;12|)
    (LETT #2# (|-| (QCDR |f|) 1) |FFIELDC-;discreteLog;2SU;12|)
    G190
    (COND ((QSGREATERP |t| #2#) (GO G191)))
    (SEQ
     (LETT |exp|
      (QUOTIENT2 |exp| |fac|)
      |FFIELDC-;discreteLog;2SU;12|)
     (LETT |rhoHelp|
      (SPADCALL |primroot|
       (SPADCALL |a| |exp| (QREFELT |$| 48))
       |fac|
       (QREFELT |$| 70))
       |FFIELDC-;discreteLog;2SU;12|)
     (EXIT
      (COND
       ((QEQCAR |rhoHelp| 1)
        (PROGN
         (LETT #3# (CONS 1 "failed") |FFIELDC-;discreteLog;2SU;12|)
         (GO #3#)))
       ((QUOTE T)
        (SEQ
         (LETT |rho|
          (|*| (QCDR |rhoHelp|) |mult|)
          |FFIELDC-;discreteLog;2SU;12|)
         (LETT |disclog|
          (|+| |disclog| |rho|)
          |FFIELDC-;discreteLog;2SU;12|)
         (LETT |mult|
          (|*| |mult| |fac|)
          |FFIELDC-;discreteLog;2SU;12|)
         (EXIT
          (LETT |a|
           (SPADCALL |a|
            (SPADCALL |logbase| (|-| |rho|) (QREFELT |$| 48))
            (QREFELT |$| 59))
```

```
                    |FFIELDC-;discreteLog;2SU;12|)))))))
               (LETT |t| (QSADD1 |t|) |FFIELDC-;discreteLog;2SU;12|)
               (GO G190)
               G191
               (EXIT NIL))))
            (LETT #1# (CDR #1#) |FFIELDC-;discreteLog;2SU;12|)
            (GO G190)
            G191
            (EXIT NIL))
           (EXIT (CONS 0 |disclog|))))))))
    #3#
    (EXIT #3#)))))

(DEFUN |FFIELDC-;squareFreePolynomial| (|f| |$|)
 (SPADCALL |f| (QREFELT |$| 75)))

(DEFUN |FFIELDC-;factorPolynomial| (|f| |$|)
 (SPADCALL |f| (QREFELT |$| 77)))

(DEFUN |FFIELDC-;factorSquareFreePolynomial| (|f| |$|)
 (PROG (|flist| |u| #1=#:G83248 #2=#:G83245 #3=#:G83243 #4=#:G83244)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |f| (|spadConstant| |$| 78) (QREFELT |$| 79))
       (|spadConstant| |$| 80))
     ((QUOTE T)
      (SEQ
       (LETT |flist|
        (SPADCALL |f| (QUOTE T) (QREFELT |$| 83))
        |FFIELDC-;factorSquareFreePolynomial|)
       (EXIT
        (SPADCALL
         (SPADCALL (QCAR |flist|) (QREFELT |$| 84))
         (PROGN
          (LETT #4# NIL |FFIELDC-;factorSquareFreePolynomial|)
          (SEQ
           (LETT |u| NIL |FFIELDC-;factorSquareFreePolynomial|)
           (LETT #1# (QCDR |flist|) |FFIELDC-;factorSquareFreePolynomial|)
           G190
           (COND
            ((OR
              (ATOM #1#)
              (PROGN
               (LETT |u| (CAR #1#) |FFIELDC-;factorSquareFreePolynomial|)
               NIL))
```

```
              (GO G191)))
             (SEQ
              (EXIT
               (PROGN
                (LETT #2#
                 (SPADCALL (QCAR |u|) (QCDR |u|) (QREFELT |$| 85))
                 |FFIELDC-;factorSquareFreePolynomial|)
                (COND
                 (#4#
                  (LETT #3#
                   (SPADCALL #3# #2# (QREFELT |$| 86))
                   |FFIELDC-;factorSquareFreePolynomial|))
                 ((QUOTE T)
                  (PROGN
                   (LETT #3# #2# |FFIELDC-;factorSquareFreePolynomial|)
                   (LETT #4#
                    (QUOTE T)
                    |FFIELDC-;factorSquareFreePolynomial|)))))))
              (LETT #1# (CDR #1#) |FFIELDC-;factorSquareFreePolynomial|)
              (GO G190)
              G191
             (EXIT NIL))
             (COND (#4# #3#) ((QUOTE T) (|spadConstant| |$| 87))))
            (QREFELT |$| 88)))))))))

(DEFUN |FFIELDC-;gcdPolynomial;3Sup;16| (|f| |g| |$|)
 (SPADCALL |f| |g| (QREFELT |$| 90)))

(DEFUN |FiniteFieldCategory&| (|#1|)
 (PROG (|DV$1| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|FiniteFieldCategory&|))
    (LETT |dv$| (LIST (QUOTE |FiniteFieldCategory&|) |DV$1|) . #1#)
    (LETT |$| (GETREFV 93) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|) |$|))))

(MAKEPROP
 (QUOTE |FiniteFieldCategory&|)
 (QUOTE |infovec|)
 (LIST
  (QUOTE
  #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (0 . |Zero|)
```

```
  |FFIELDC-;differentiate;2S;1| |FFIELDC-;init;S;2| (|PositiveInteger|)
  (4 . |lookup|) (9 . |index|) (|Boolean|) (14 . |zero?|)
  (|Union| |$| (QUOTE "failed")) |FFIELDC-;nextItem;SU;3| (19 . |order|)
  (|Integer|) (|OnePointCompletion| 10) (24 . |coerce|)
  |FFIELDC-;order;SOpc;4| (|List| 26) (|Matrix| 6) (29 . |nullSpace|)
  (|Mapping| 13 6) (|Vector| 6) (34 . |every?|) (40 . |charthRoot|)
  (|Mapping| 6 6) (45 . |map|) (|Union| (|Vector| |$|) (QUOTE "failed"))
  (|Matrix| |$|) |FFIELDC-;conditionP;MU;5| (|NonNegativeInteger|)
  (51 . |size|) (55 . |characteristic|) (59 . |**|)
  |FFIELDC-;charthRoot;2S;6| |FFIELDC-;charthRoot;SU;7| (65 . |One|)
  (|Union| (QUOTE "prime") (QUOTE "polynomial") (QUOTE "normal")
  (QUOTE "cyclic")) (69 . |representationType|) (73 . |=|)
  |FFIELDC-;createPrimitiveElement;S;8| (|Record| (|:| |factor| 18)
  (|:| |exponent| 18)) (|List| 45) (79 . |factorsOfCyclicGroupSize|)
  (83 . |**|) (89 . |one?|) |FFIELDC-;primitive?;SB;9| (94 . |=|)
  |FFIELDC-;order;SPi;10| (100 . |primitiveElement|) (|Table| 10 34)
  (104 . |tableForDiscreteLogarithm|) (109 . |#|)
  (|Union| 34 (QUOTE "failed")) (114 . |search|) (120 . |*|)
  |FFIELDC-;discreteLog;SNni;11| (|Void|) (|String|) (|OutputForm|)
  (126 . |messagePrint|) (|Factored| |$|) (131 . |factor|)
  (|Factored| 18) (136 . |factors|) (|DiscreteLogarithmPackage| 6)
  (141 . |shanksDiscLogAlgorithm|) |FFIELDC-;discreteLog;2SU;12|
  (|Factored| 73) (|SparseUnivariatePolynomial| 6)
  (|UnivariatePolynomialSquareFree| 6 73) (148 . |squareFree|)
  (|DistinctDegreeFactorize| 6 73) (153 . |factor|) (158 . |Zero|)
  (162 . |=|) (168 . |Zero|) (|Record| (|:| |irr| 73) (|:| |pow| 18))
  (|Record| (|:| |cont| 6) (|:| |factors| (|List| 81)))
  (172 . |distdfact|) (178 . |coerce|) (183 . |primeFactor|)
  (189 . |*|) (195 . |One|) (199 . |*|) (|EuclideanDomain&| 73)
  (205 . |gcd|) (|SparseUnivariatePolynomial| |$|)
  |FFIELDC-;gcdPolynomial;3Sup;16|))
 (QUOTE
  #(|primitive?| 211 |order| 216 |nextItem| 226 |init| 231
    |gcdPolynomial| 235 |discreteLog| 241 |differentiate| 252
    |createPrimitiveElement| 257 |conditionP| 261 |charthRoot| 266))
(QUOTE NIL)
(CONS
 (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS
  (QUOTE #())
  (CONS
   (QUOTE #())
   (|makeByteWordVec2| 92
    (QUOTE
     (0 6 0 7 1 6 10 0 11 1 6 0 10 12 1 6 13 0 14 1 6 10 0 17 1 19
      0 18 20 1 23 22 0 24 2 26 13 25 0 27 1 6 0 0 28 2 26 0 29 0 30
```

```
        0 6 34 35 0 6 34 36 2 6 0 0 34 37 0 6 0 40 0 6 41 42 2 41 13
        0 0 43 0 6 46 47 2 6 0 0 18 48 1 6 13 0 49 2 6 13 0 0 51 0 6 0
        53 1 6 54 18 55 1 54 34 0 56 2 54 57 10 0 58 2 6 0 0 0 59 1
        63 61 62 64 1 18 65 0 66 1 67 46 0 68 3 69 57 6 6 34 70 1 74
        72 73 75 1 76 72 73 77 0 73 0 78 2 73 13 0 0 79 0 72 0 80 2
        76 82 73 13 83 1 73 0 6 84 2 72 0 73 18 85 2 72 0 0 0 86 0 72
        0 87 2 72 0 73 0 88 2 89 0 0 0 90 1 0 13 0 50 1 0 10 0 52 1 0
        19 0 21 1 0 15 0 16 0 0 0 9 2 0 91 91 91 92 1 0 34 0 60 2 0 57
        0 0 71 1 0 0 0 8 0 0 0 44 1 0 31 32 33 1 0 0 0 38 1 0 15 0 39))))))
    (QUOTE |lookupComplete|)))
```

21.23 FPS.lsp BOOTSTRAP

FPS depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **FPS** category which we can write into the **MID** directory. We compile the lisp code and copy the **FPS.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*FPS.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |FloatingPointSystem;AL| (QUOTE NIL))

(DEFUN |FloatingPointSystem| NIL
  (LET (#:G105645)
    (COND
      (|FloatingPointSystem;AL|)
      (T (SETQ |FloatingPointSystem;AL| (|FloatingPointSystem;|))))))

(DEFUN |FloatingPointSystem;| NIL
  (PROG (#1=#:G105643)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|RealNumberSystem|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|float| (|$| (|Integer|) (|Integer|))) T)
                ((|float| (|$| (|Integer|) (|Integer|) (|PositiveInteger|))) T)
                ((|order| ((|Integer|) |$|)) T)
                ((|base| ((|PositiveInteger|))) T)
                ((|exponent| ((|Integer|) |$|)) T)
                ((|mantissa| ((|Integer|) |$|)) T)
                ((|bits| ((|PositiveInteger|))) T)
                ((|digits| ((|PositiveInteger|))) T)
                ((|precision| ((|PositiveInteger|))) T)
                ((|bits| ((|PositiveInteger|) (|PositiveInteger|)))
                  (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
                ((|digits| ((|PositiveInteger|) (|PositiveInteger|)))
                  (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
                ((|precision| ((|PositiveInteger|) (|PositiveInteger|)))
                  (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
```

```
                ((|increasePrecision| ((|PositiveInteger|) (|Integer|)))
                  (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
                ((|decreasePrecision| ((|PositiveInteger|) (|Integer|)))
                  (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
                ((|min| (|$|))
                  (AND
                    (|not| (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
                    (|not| (|has| |$| (ATTRIBUTE |arbitraryExponent|)))))
                ((|max| (|$|))
                  (AND
                    (|not| (|has| |$| (ATTRIBUTE |arbitraryPrecision|)))
                    (|not| (|has| |$| (ATTRIBUTE |arbitraryExponent|))))))))
              (QUOTE ((|approximate| T)))
              (QUOTE ((|PositiveInteger|) (|Integer|)))
              NIL))
            |FloatingPointSystem|)
          (SETELT #1# 0 (QUOTE (|FloatingPointSystem|)))))))

(MAKEPROP (QUOTE |FloatingPointSystem|) (QUOTE NILADIC) T)
```

21.24 FPS-.lsp BOOTSTRAP

FPS- depends **FPS**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **FPS-** category which we can write into the **MID** directory. We compile the lisp code and copy the **FPS-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*FPS-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |FPS-;float;2IS;1| (|ma| |ex| |$|)
  (SPADCALL |ma| |ex| (SPADCALL (QREFELT |$| 8)) (QREFELT |$| 10)))

(DEFUN |FPS-;digits;Pi;2| (|$|)
  (PROG (#1=#:G105654)
    (RETURN
      (PROG1
        (LETT #1#
          (MAX 1
            (QUOTIENT2
              (SPADCALL 4004
                (|-| (SPADCALL (QREFELT |$| 13)) 1)
                (QREFELT |$| 14))
              13301))
          |FPS-;digits;Pi;2|)
        (|check-subtype| (|>| #1# 0) (QUOTE (|PositiveInteger|)) #1#)))))

(DEFUN |FloatingPointSystem&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|FloatingPointSystem&|))
        (LETT |dv$| (LIST (QUOTE |FloatingPointSystem&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 17) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3
          (LETT |pv$|
            (|buildPredVector| 0 0
              (LIST
                (|HasAttribute| |#1| (QUOTE |arbitraryExponent|))
                (|HasAttribute| |#1| (QUOTE |arbitraryPrecision|)))) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))
```

```
(MAKEPROP
  (QUOTE |FloatingPointSystem&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (|PositiveInteger|)
        (0 . |base|)
        (|Integer|)
        (4 . |float|)
        |FPS-;float;2IS;1|
        (11 . |One|)
        (15 . |bits|)
        (19 . |*|)
        (25 . |max|)
        |FPS-;digits;Pi;2|))
    (QUOTE #(|float| 29 |digits| 35))
    (QUOTE NIL)
    (CONS
      (|makeByteWordVec2| 1 (QUOTE NIL))
      (CONS
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 16
            (QUOTE
              (0 6 7 8 3 6 0 9 9 7 10 0 6 0 12 0 6 7 13 2 9 0 7 0 14 0 6 0 15
               2 0 0 9 9 11 0 0 7 16))))))
     (QUOTE |lookupComplete|)))
```

21.25 GCDDOM.lsp BOOTSTRAP

GCDDOM needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON** which needs **ABELSG** which needs **SETCAT** which needs **SINT** which needs **UFD** which needs **GCDDOM**. We break this chain with **GCDDOM.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **GCDDOM** category which we can write into the **MID** directory. We compile the lisp code and copy the **GCDDOM.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*GCDDOM.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |GcdDomain;AL| (QUOTE NIL))

(DEFUN |GcdDomain| NIL
  (LET (#:G83171)
    (COND
      (|GcdDomain;AL|)
      (T (SETQ |GcdDomain;AL| (|GcdDomain;|))))))

(DEFUN |GcdDomain;| NIL
  (PROG (#1=#:G83169)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|IntegralDomain|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|gcd| (|$| |$| |$|)) T)
                ((|gcd| (|$| (|List| |$|))) T)
                ((|lcm| (|$| |$| |$|)) T)
                ((|lcm| (|$| (|List| |$|))) T)
                ((|gcdPolynomial|
                  ((|SparseUnivariatePolynomial| |$|)
                   (|SparseUnivariatePolynomial| |$|)
                   (|SparseUnivariatePolynomial| |$|)))
                  T)))
              NIL
              (QUOTE ((|SparseUnivariatePolynomial| |$|) (|List| |$|)))
              NIL))
```

```
            |GcdDomain|)
         (SETELT #1# 0 (QUOTE (|GcdDomain|)))))))

  (MAKEPROP (QUOTE |GcdDomain|) (QUOTE NILADIC) T)
```

21.26 GCDDOM-.lsp BOOTSTRAP

GCDDOM- depends on **GCDDOM**. We break this chain with **GCDDOM-.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **GCDDOM-** category which we can write into the **MID** directory. We compile the lisp code and copy the **GCDDOM-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*GCDDOM-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |GCDDOM-;lcm;3S;1| (|x| |y| |$|)
  (PROG (LCM)
    (RETURN
      (SEQ
        (COND
          ((OR
              (SPADCALL |y| (|spadConstant| |$| 7) (QREFELT |$| 9))
              (SPADCALL |x| (|spadConstant| |$| 7) (QREFELT |$| 9)))
            (|spadConstant| |$| 7))
          ((QUOTE T)
            (SEQ
              (LETT LCM
                (SPADCALL |y|
                  (SPADCALL |x| |y| (QREFELT |$| 10))
                  (QREFELT |$| 12))
                |GCDDOM-;lcm;3S;1|)
              (EXIT
                (COND
                  ((QEQCAR LCM 0) (SPADCALL |x| (QCDR LCM) (QREFELT |$| 13)))
                  ((QUOTE T) (|error| "bad gcd in lcm computation")))))))))))

(DEFUN |GCDDOM-;lcm;LS;2| (|l| |$|)
  (SPADCALL
    (ELT |$| 15)
    |l|
    (|spadConstant| |$| 16)
    (|spadConstant| |$| 7)
    (QREFELT |$| 19)))

(DEFUN |GCDDOM-;gcd;LS;3| (|l| |$|)
  (SPADCALL
    (ELT |$| 10)
```

```
      |l|
      (|spadConstant| |$| 7)
      (|spadConstant| |$| 16)
      (QREFELT |$| 19)))

  (DEFUN |GCDDOM-;gcdPolynomial;3Sup;4| (|p1| |p2| |$|)
    (PROG (|e2| |e1| |c1| |p| |c2| #1=#:G83191)
      (RETURN
        (SEQ
          (COND
            ((SPADCALL |p1| (QREFELT |$| 24)) (SPADCALL |p2| (QREFELT |$| 25)))
            ((SPADCALL |p2| (QREFELT |$| 24)) (SPADCALL |p1| (QREFELT |$| 25)))
            ((QUOTE T)
              (SEQ
                (LETT |c1|
                  (SPADCALL |p1| (QREFELT |$| 26))
                  |GCDDOM-;gcdPolynomial;3Sup;4|)
                (LETT |c2|
                  (SPADCALL |p2| (QREFELT |$| 26))
                  |GCDDOM-;gcdPolynomial;3Sup;4|)
                (LETT |p1|
                  (PROG2
                    (LETT #1#
                      (SPADCALL |p1| |c1| (QREFELT |$| 27))
                      |GCDDOM-;gcdPolynomial;3Sup;4|)
                    (QCDR #1#)
                    (|check-union|
                      (QEQCAR #1# 0)
                      (|SparseUnivariatePolynomial| (QREFELT |$| 6))
                      #1#))
                  |GCDDOM-;gcdPolynomial;3Sup;4|)
                (LETT |p2|
                  (PROG2
                    (LETT #1#
                      (SPADCALL |p2| |c2| (QREFELT |$| 27))
                      |GCDDOM-;gcdPolynomial;3Sup;4|)
                    (QCDR #1#)
                    (|check-union|
                      (QEQCAR #1# 0)
                      (|SparseUnivariatePolynomial| (QREFELT |$| 6))
                      #1#))
                  |GCDDOM-;gcdPolynomial;3Sup;4|)
                (SEQ
                  (LETT |e1|
                    (SPADCALL |p1| (QREFELT |$| 29))
                    |GCDDOM-;gcdPolynomial;3Sup;4|)
```

```
  (EXIT
    (COND
      ((|<| 0 |e1|)
        (LETT |p1|
          (PROG2
            (LETT #1#
              (SPADCALL |p1|
                (SPADCALL
                 (|spadConstant| |$| 16) |e1| (QREFELT |$| 32))
                (QREFELT |$| 33))
              |GCDDOM-;gcdPolynomial;3Sup;4|)
            (QCDR #1#)
            (|check-union|
              (QEQCAR #1# 0)
              (|SparseUnivariatePolynomial| (QREFELT |$| 6))
              #1#))
          |GCDDOM-;gcdPolynomial;3Sup;4|)))))
(SEQ
  (LETT |e2|
    (SPADCALL |p2| (QREFELT |$| 29))
    |GCDDOM-;gcdPolynomial;3Sup;4|)
  (EXIT
    (COND
      ((|<| 0 |e2|)
        (LETT |p2|
          (PROG2
            (LETT #1#
              (SPADCALL |p2|
                (SPADCALL
                  (|spadConstant| |$| 16)
                  |e2|
                  (QREFELT |$| 32))
                (QREFELT |$| 33))
              |GCDDOM-;gcdPolynomial;3Sup;4|)
            (QCDR #1#)
            (|check-union|
              (QEQCAR #1# 0)
              (|SparseUnivariatePolynomial| (QREFELT |$| 6))
              #1#))
          |GCDDOM-;gcdPolynomial;3Sup;4|)))))
(LETT |e1|
  (MIN |e1| |e2|)
  |GCDDOM-;gcdPolynomial;3Sup;4|)
(LETT |c1|
  (SPADCALL |c1| |c2| (QREFELT |$| 10))
  |GCDDOM-;gcdPolynomial;3Sup;4|)
```

```
(LETT |p1|
  (COND
    ((OR
         (EQL (SPADCALL |p1| (QREFELT |$| 34)) 0)
         (EQL (SPADCALL |p2| (QREFELT |$| 34)) 0))
       (SPADCALL |c1| 0 (QREFELT |$| 32)))
    ((QUOTE T)
      (SEQ
        (LETT |p|
          (SPADCALL |p1| |p2| (QREFELT |$| 35))
          |GCDDOM-;gcdPolynomial;3Sup;4|)
        (EXIT
          (COND
            ((EQL (SPADCALL |p| (QREFELT |$| 34)) 0)
              (SPADCALL |c1| 0 (QREFELT |$| 32)))
            ((QUOTE T)
              (SEQ
                (LETT |c2|
                  (SPADCALL
                    (SPADCALL |p1| (QREFELT |$| 36))
                    (SPADCALL |p2| (QREFELT |$| 36))
                    (QREFELT |$| 10))
                  |GCDDOM-;gcdPolynomial;3Sup;4|)
                (EXIT
                  (SPADCALL
                    (SPADCALL |c1|
                      (SPADCALL
                        (PROG2
                          (LETT #1#
                            (SPADCALL
                              (SPADCALL
                                |c2|
                                |p|
                                (QREFELT |$| 37))
                              (SPADCALL |p| (QREFELT |$| 36))
                              (QREFELT |$| 27))
                            |GCDDOM-;gcdPolynomial;3Sup;4|)
                          (QCDR #1#)
                          (|check-union|
                            (QEQCAR #1# 0)
                            (|SparseUnivariatePolynomial|
                              (QREFELT |$| 6))
                            #1#))
                        (QREFELT |$| 38))
                      (QREFELT |$| 37))
                    (QREFELT |$| 25)))))))))))
```

```
                |GCDDOM-;gcdPolynomial;3Sup;4|)
               (EXIT
                (COND
                 ((ZEROP |e1|) |p1|)
                 ((QUOTE T)
                   (SPADCALL
                    (SPADCALL (|spadConstant| |$| 16) |e1| (QREFELT |$| 32))
                    |p1| (QREFELT |$| 39)))))))))))

(DEFUN |GcdDomain&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|GcdDomain&|))
        (LETT |dv$| (LIST (QUOTE |GcdDomain&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 42) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(MAKEPROP
  (QUOTE |GcdDomain&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (0 . |Zero|)
        (|Boolean|)
        (4 . |=|)
        (10 . |gcd|)
        (|Union| |$| (QUOTE "failed"))
        (16 . |exquo|)
        (22 . |*|)
        |GCDDOM-;lcm;3S;1|
        (28 . |lcm|)
        (34 . |One|)
        (|Mapping| 6 6 6)
        (|List| 6)
        (38 . |reduce|)
        (|List| |$|)
        |GCDDOM-;lcm;LS;2|
        |GCDDOM-;gcd;LS;3|
        (|SparseUnivariatePolynomial| 6)
```

```
      (46 . |zero?|)
      (51 . |unitCanonical|)
      (56 . |content|)
      (61 . |exquo|)
      (|NonNegativeInteger|)
      (67 . |minimumDegree|)
      (72 . |Zero|)
      (76 . |One|)
      (80 . |monomial|)
      (86 . |exquo|)
      (92 . |degree|)
      (97 . |subResultantGcd|)
      (103 . |leadingCoefficient|)
      (108 . |*|)
      (114 . |primitivePart|)
      (119 . |*|)
      (|SparseUnivariatePolynomial| |$|)
      |GCDDOM-;gcdPolynomial;3Sup;4|))
  (QUOTE #(|lcm| 125 |gcdPolynomial| 136 |gcd| 142))
  (QUOTE NIL)
  (CONS
    (|makeByteWordVec2| 1 (QUOTE NIL))
    (CONS
      (QUOTE #())
      (CONS
        (QUOTE #())
          (|makeByteWordVec2| 41
           (QUOTE (0 6 0 7 2 6 8 0 0 9 2 6 0 0 0 10 2 6 11 0 0 12 2 6 0 0 0
                   13 2 6 0 0 0 15 0 6 0 16 4 18 6 17 0 6 6 19 1 23 8 0 24
                   1 23 0 0 25 1 23 6 0 26 2 23 11 0 6 27 1 23 28 0 29 0 23
                   0 30 0 23 0 31 2 23 0 6 28 32 2 23 11 0 0 33 1 23 28 0
                   34 2 23 0 0 0 35 1 23 6 0 36 2 23 0 6 0 37 1 23 0 0 38 2
                   23 0 0 0 39 1 0 0 20 21 2 0 0 0 0 14 2 0 40 40 40 41 1 0
                   0 20 22))))))
  (QUOTE |lookupComplete|)))
```

21.27 HOAGG.lsp BOOTSTRAP

HOAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **HOAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **HOAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*HOAGG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |HomogeneousAggregate;CAT| (QUOTE NIL))

(SETQ |HomogeneousAggregate;AL| (QUOTE NIL))

(DEFUN |HomogeneousAggregate| (#1=#:G82375)
  (LET (#2=#:G82376)
    (COND
      ((SETQ #2# (|assoc| (|devaluate| #1#) |HomogeneousAggregate;AL|))
        (CDR #2#))
      (T
        (SETQ |HomogeneousAggregate;AL|
          (|cons5|
            (CONS (|devaluate| #1#) (SETQ #2# (|HomogeneousAggregate;| #1#)))
            |HomogeneousAggregate;AL|))
        #2#))))

(DEFUN |HomogeneousAggregate;| (|t#1|)
  (PROG (#1=#:G82374)
    (RETURN
      (PROG1
        (LETT #1#
          (|sublisV|
            (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
            (COND
              (|HomogeneousAggregate;CAT|)
              ((QUOTE T)
                (LETT |HomogeneousAggregate;CAT|
                  (|Join|
                    (|Aggregate|)
                    (|mkCategory|
                      (QUOTE |domain|)
                      (QUOTE (
                        ((|map| (|$| (|Mapping| |t#1| |t#1|) |$|)) T)
```

```
                    ((|map!| (|$| (|Mapping| |t#1| |t#1|) |$|))
                      (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
                    ((|any?|
                       ((|Boolean|) (|Mapping| (|Boolean|) |t#1|) |$|))
                      (|has| |$| (ATTRIBUTE |finiteAggregate|)))
                    ((|every?|
                       ((|Boolean|) (|Mapping| (|Boolean|) |t#1|) |$|))
                      (|has| |$| (ATTRIBUTE |finiteAggregate|)))
                    ((|count|
                       ((|NonNegativeInteger|)
                        (|Mapping| (|Boolean|) |t#1|) |$|))
                      (|has| |$| (ATTRIBUTE |finiteAggregate|)))
                    ((|parts| ((|List| |t#1|) |$|))
                      (|has| |$| (ATTRIBUTE |finiteAggregate|)))
                    ((|members| ((|List| |t#1|) |$|))
                      (|has| |$| (ATTRIBUTE |finiteAggregate|)))
                    ((|count| ((|NonNegativeInteger|) |t#1| |$|))
                      (AND
                        (|has| |t#1| (|SetCategory|))
                        (|has| |$| (ATTRIBUTE |finiteAggregate|))))
                    ((|member?| ((|Boolean|) |t#1| |$|))
                      (AND
                        (|has| |t#1| (|SetCategory|))
                        (|has| |$| (ATTRIBUTE |finiteAggregate|))))))
                   (QUOTE (
                    ((|SetCategory|) (|has| |t#1| (|SetCategory|)))
                    ((|Evalable| |t#1|)
                      (AND
                        (|has| |t#1| (|Evalable| |t#1|))
                        (|has| |t#1| (|SetCategory|))))))
                  (QUOTE (
                    (|Boolean|)
                    (|NonNegativeInteger|)
                    (|List| |t#1|)))
                  NIL))
          . #2=(|HomogeneousAggregate|))))) . #2#)
(SETELT #1# 0
  (LIST (QUOTE |HomogeneousAggregate|) (|devaluate| |t#1|)))))))
```

21.28 HOAGG-.lsp BOOTSTRAP

HOAGG- depends on **HOAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **HOAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **HOAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*HOAGG-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |HOAGG-;eval;ALA;1| (|u| |l| |$|)
 (SPADCALL
  (CONS (FUNCTION |HOAGG-;eval;ALA;1!0|) (VECTOR |$| |l|))
  |u|
  (QREFELT |$| 11)))

(DEFUN |HOAGG-;eval;ALA;1!0| (|#1| |$$|)
 (SPADCALL |#1| (QREFELT |$$| 1) (QREFELT (QREFELT |$$| 0) 9)))

(DEFUN |HOAGG-;#;ANni;2| (|c| |$|)
 (LENGTH (SPADCALL |c| (QREFELT |$| 14))))

(DEFUN |HOAGG-;any?;MAB;3| (|f| |c| |$|)
 (PROG (|x| #1=#:G82396 #2=#:G82393 #3=#:G82391 #4=#:G82392)
  (RETURN
   (SEQ
    (PROGN
     (LETT #4# NIL |HOAGG-;any?;MAB;3|)
     (SEQ
      (LETT |x| NIL |HOAGG-;any?;MAB;3|)
      (LETT #1# (SPADCALL |c| (QREFELT |$| 14)) |HOAGG-;any?;MAB;3|)
      G190
      (COND
       ((OR (ATOM #1#) (PROGN (LETT |x| (CAR #1#) |HOAGG-;any?;MAB;3|) NIL))
        (GO G191)))
      (SEQ
       (EXIT
        (PROGN
         (LETT #2# (SPADCALL |x| |f|) |HOAGG-;any?;MAB;3|)
         (COND
          (#4#
           (LETT #3#
             (COND
```

```
               (#3# (QUOTE T))
               ((QUOTE T) #2#))
              |HOAGG-;any?;MAB;3|))
           ((QUOTE T)
            (PROGN
             (LETT #3# #2# |HOAGG-;any?;MAB;3|)
             (LETT #4# (QUOTE T) |HOAGG-;any?;MAB;3|)))))))
      (LETT #1# (CDR #1#) |HOAGG-;any?;MAB;3|) (GO G190) G191 (EXIT NIL))
     (COND (#4# #3#) ((QUOTE T) (QUOTE NIL))))))))

(DEFUN |HOAGG-;every?;MAB;4| (|f| |c| |$|)
 (PROG (|x| #1=#:G82401 #2=#:G82399 #3=#:G82397 #4=#:G82398)
  (RETURN
   (SEQ
    (PROGN
     (LETT #4# NIL |HOAGG-;every?;MAB;4|)
     (SEQ
      (LETT |x| NIL |HOAGG-;every?;MAB;4|)
      (LETT #1# (SPADCALL |c| (QREFELT |$| 14)) |HOAGG-;every?;MAB;4|)
      G190
      (COND
        ((OR (ATOM #1#) (PROGN (LETT |x| (CAR #1#) |HOAGG-;every?;MAB;4|) NIL))
         (GO G191)))
      (SEQ
       (EXIT
        (PROGN
         (LETT #2# (SPADCALL |x| |f|) |HOAGG-;every?;MAB;4|)
         (COND
          (#4#
           (LETT #3#
            (COND (#3# #2#) ((QUOTE T) (QUOTE NIL)))
            |HOAGG-;every?;MAB;4|))
          ((QUOTE T)
           (PROGN
            (LETT #3# #2# |HOAGG-;every?;MAB;4|)
            (LETT #4# (QUOTE T) |HOAGG-;every?;MAB;4|)))))))
      (LETT #1# (CDR #1#) |HOAGG-;every?;MAB;4|)
      (GO G190)
      G191
      (EXIT NIL))
     (COND (#4# #3#) ((QUOTE T) (QUOTE T))))))))

(DEFUN |HOAGG-;count;MANni;5| (|f| |c| |$|)
 (PROG (|x| #1=#:G82406 #2=#:G82404 #3=#:G82402 #4=#:G82403)
  (RETURN
   (SEQ
```

```
     (PROGN
      (LETT #4# NIL |HOAGG-;count;MANni;5|)
      (SEQ
       (LETT |x| NIL |HOAGG-;count;MANni;5|)
       (LETT #1# (SPADCALL |c| (QREFELT |$| 14)) |HOAGG-;count;MANni;5|)
       G190
       (COND
        ((OR (ATOM #1#) (PROGN (LETT |x| (CAR #1#) |HOAGG-;count;MANni;5|) NIL))
         (GO G191)))
       (SEQ
        (EXIT
         (COND
          ((SPADCALL |x| |f|)
           (PROGN
            (LETT #2# 1 |HOAGG-;count;MANni;5|)
            (COND
             (#4# (LETT #3# (|+| #3# #2#) |HOAGG-;count;MANni;5|))
             ((QUOTE T)
              (PROGN
               (LETT #3# #2# |HOAGG-;count;MANni;5|)
               (LETT #4# (QUOTE T) |HOAGG-;count;MANni;5|)))))))))
        (LETT #1# (CDR #1#) |HOAGG-;count;MANni;5|)
        (GO G190)
        G191
        (EXIT NIL))
       (COND (#4# #3#) ((QUOTE T) 0)))))))

(DEFUN |HOAGG-;members;AL;6| (|x| |$|) (SPADCALL |x| (QREFELT |$| 14)))

(DEFUN |HOAGG-;count;SANni;7| (|s| |x| |$|)
 (SPADCALL
  (CONS (FUNCTION |HOAGG-;count;SANni;7!0|) (VECTOR |$| |s|))
  |x|
  (QREFELT |$| 24)))

(DEFUN |HOAGG-;count;SANni;7!0| (|#1| |$$|)
 (SPADCALL (QREFELT |$$| 1) |#1| (QREFELT (QREFELT |$$| 0) 23)))

(DEFUN |HOAGG-;member?;SAB;8| (|e| |c| |$|)
 (SPADCALL
  (CONS (FUNCTION |HOAGG-;member?;SAB;8!0|) (VECTOR |$| |e|))
  |c|
  (QREFELT |$| 26)))

(DEFUN |HOAGG-;member?;SAB;8!0| (|#1| |$$|)
 (SPADCALL (QREFELT |$$| 1) |#1| (QREFELT (QREFELT |$$| 0) 23)))
```

```
(DEFUN |HOAGG-;=;2AB;9| (|x| |y| |$|)
 (PROG (|b| #1=#:G82416 |a| #2=#:G82415 #3=#:G82412 #4=#:G82410 #5=#:G82411)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |x| (SPADCALL |y| (QREFELT |$| 28)) (QREFELT |$| 29))
      (PROGN
       (LETT #5# NIL |HOAGG-;=;2AB;9|)
       (SEQ
        (LETT |b| NIL |HOAGG-;=;2AB;9|)
        (LETT #1# (SPADCALL |y| (QREFELT |$| 14)) |HOAGG-;=;2AB;9|)
        (LETT |a| NIL |HOAGG-;=;2AB;9|)
        (LETT #2# (SPADCALL |x| (QREFELT |$| 14)) |HOAGG-;=;2AB;9|)
        G190
        (COND
         ((OR
           (ATOM #2#)
           (PROGN (LETT |a| (CAR #2#) |HOAGG-;=;2AB;9|) NIL)
           (ATOM #1#)
           (PROGN (LETT |b| (CAR #1#) |HOAGG-;=;2AB;9|) NIL))
          (GO G191)))
        (SEQ
         (EXIT
          (PROGN
           (LETT #3# (SPADCALL |a| |b| (QREFELT |$| 23)) |HOAGG-;=;2AB;9|)
           (COND
            (#5#
             (LETT #4#
              (COND (#4# #3#) ((QUOTE T) (QUOTE NIL)))
              |HOAGG-;=;2AB;9|))
            ((QUOTE T)
             (PROGN
              (LETT #4# #3# |HOAGG-;=;2AB;9|)
              (LETT #5# (QUOTE T) |HOAGG-;=;2AB;9|)))))))
        (LETT #2#
         (PROG1
          (CDR #2#)
          (LETT #1# (CDR #1#) |HOAGG-;=;2AB;9|))
         |HOAGG-;=;2AB;9|)
        (GO G190)
        G191
        (EXIT NIL))
       (COND (#5# #4#) ((QUOTE T) (QUOTE T)))))
     ((QUOTE T) (QUOTE NIL)))))))
```

```
(DEFUN |HOAGG-;coerce;AOf;10| (|x| |$|)
 (PROG (#1=#:G82420 |a| #2=#:G82421)
  (RETURN
   (SEQ
    (SPADCALL
     (SPADCALL
      (PROGN
       (LETT #1# NIL |HOAGG-;coerce;AOf;10|)
       (SEQ
        (LETT |a| NIL |HOAGG-;coerce;AOf;10|)
        (LETT #2# (SPADCALL |x| (QREFELT |$| 14)) |HOAGG-;coerce;AOf;10|)
        G190
        (COND
         ((OR
           (ATOM #2#)
           (PROGN (LETT |a| (CAR #2#) |HOAGG-;coerce;AOf;10|) NIL))
          (GO G191)))
        (SEQ
         (EXIT
          (LETT #1#
           (CONS (SPADCALL |a| (QREFELT |$| 32)) #1#)
           |HOAGG-;coerce;AOf;10|)))
        (LETT #2# (CDR #2#) |HOAGG-;coerce;AOf;10|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #1#))))
      (QREFELT |$| 34))
     (QREFELT |$| 35))))))

(DEFUN |HomogeneousAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|HomogeneousAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |HomogeneousAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (GETREFV 38) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST
        (|HasAttribute| |#1| (QUOTE |finiteAggregate|))
        (|HasAttribute| |#1| (QUOTE |shallowlyMutable|))
        (|HasCategory| |#2| (LIST (QUOTE |Evalable|) (|devaluate| |#2|)))
        (|HasCategory| |#2| (QUOTE (|SetCategory|)))))))
```

```
      . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|testBitVector| |pv$| 3)
       (QSETREFV |$| 12 (CONS (|dispatchFunction| |HOAGG-;eval;ALA;1|) |$|))))
    (COND
     ((|testBitVector| |pv$| 1)
      (PROGN
       (QSETREFV |$| 16 (CONS (|dispatchFunction| |HOAGG-;#;ANni;2|) |$|))
       (QSETREFV |$| 19 (CONS (|dispatchFunction| |HOAGG-;any?;MAB;3|) |$|))
       (QSETREFV |$| 20 (CONS (|dispatchFunction| |HOAGG-;every?;MAB;4|) |$|))
       (QSETREFV |$| 21 (CONS (|dispatchFunction| |HOAGG-;count;MANni;5|) |$|))
       (QSETREFV |$| 22 (CONS (|dispatchFunction| |HOAGG-;members;AL;6|) |$|))
       (COND
        ((|testBitVector| |pv$| 4)
         (PROGN
          (QSETREFV |$| 25
           (CONS (|dispatchFunction| |HOAGG-;count;SANni;7|) |$|))
          (QSETREFV |$| 27
           (CONS (|dispatchFunction| |HOAGG-;member?;SAB;8|) |$|))
          (QSETREFV |$| 30
           (CONS (|dispatchFunction| |HOAGG-;=;2AB;9|) |$|))
          (QSETREFV |$| 36
           (CONS (|dispatchFunction| |HOAGG-;coerce;AOf;10|) |$|)))))))
    |$|))))

(MAKEPROP
 (QUOTE |HomogeneousAggregate&|)
 (QUOTE |infovec|)
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|List| 37)
   (0 . |eval|) (|Mapping| 7 7) (6 . |map|) (12 . |eval|) (|List| 7)
   (18 . |parts|) (|NonNegativeInteger|) (23 . |#|) (|Boolean|)
   (|Mapping| 17 7) (28 . |any?|) (34 . |every?|) (40 . |count|)
   (46 . |members|) (51 . |=|) (57 . |count|) (63 . |count|) (69 . |any?|)
   (75 . |member?|) (81 . |#|) (86 . |size?|) (92 . |=|) (|OutputForm|)
   (98 . |coerce|) (|List| |$|) (103 . |commaSeparate|) (108 . |bracket|)
   (113 . |coerce|) (|Equation| 7)))
  (QUOTE #(|members| 118 |member?| 123 |every?| 129 |eval| 135 |count| 141
   |coerce| 153 |any?| 158 |=| 164 |#| 170))
  (QUOTE NIL)
  (CONS
   (|makeByteWordVec2| 1 (QUOTE NIL))
   (CONS
```

```
      (QUOTE #())
      (CONS
       (QUOTE #())
       (|makeByteWordVec2| 36
        (QUOTE (2 7 0 0 8 9 2 6 0 10 0 11 2 0 0 0 8 12 1 6 13 0 14 1 0 15 0
          16 2 0 17 18 0 19 2 0 17 18 0 20 2 0 15 18 0 21 1 0 13 0 22 2 7 17
          0 0 23 2 6 15 18 0 24 2 0 15 7 0 25 2 6 17 18 0 26 2 0 17 7 0 27 1
          6 15 0 28 2 6 17 0 15 29 2 0 17 0 0 30 1 7 31 0 32 1 31 0 33 34 1
          31 0 0 35 1 0 31 0 36 1 0 13 0 22 2 0 17 7 0 27 2 0 17 18 0 20 2 0
          0 0 8 12 2 0 15 7 0 25 2 0 15 18 0 21 1 0 31 0 36 2 0 17 18 0 19 2
          0 17 0 0 30 1 0 15 0 16))))))
    (QUOTE |lookupComplete|)))
```

21.29 INS.lsp BOOTSTRAP

INS depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **INS** category which we can write into the **MID** directory. We compile the lisp code and copy the **INS.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

⟨*INS.lsp BOOTSTRAP*⟩≡

```
(/VERSIONCHECK 2)

(SETQ |IntegerNumberSystem;AL| (QUOTE NIL))

(DEFUN |IntegerNumberSystem| NIL
  (LET (#:G1068)
    (COND
      (|IntegerNumberSystem;AL|)
      (T (SETQ |IntegerNumberSystem;AL| (|IntegerNumberSystem;|))))))

(DEFUN |IntegerNumberSystem;| NIL (PROG (#0=#:G1066)
  (RETURN
    (PROG1
      (LETT #0#
        (|sublisV|
          (PAIR
            (QUOTE (#1=#:G1060 #2=#:G1061 #3=#:G1062
                    #4=#:G1063 #5=#:G1064 #6=#:G1065))
            (LIST
              (QUOTE (|Integer|))
              (QUOTE (|Integer|))
              (QUOTE (|Integer|))
              (QUOTE (|InputForm|))
              (QUOTE (|Pattern| (|Integer|)))
              (QUOTE (|Integer|))))
          (|Join|
            (|UniqueFactorizationDomain|)
            (|EuclideanDomain|)
            (|OrderedIntegralDomain|)
            (|DifferentialRing|)
            (|ConvertibleTo| (QUOTE #1#))
            (|RetractableTo| (QUOTE #2#))
            (|LinearlyExplicitRingOver| (QUOTE #3#))
            (|ConvertibleTo| (QUOTE #4#))
            (|ConvertibleTo| (QUOTE #5#))
            (|PatternMatchable| (QUOTE #6#))
            (|CombinatorialFunctionCategory|)
```

```
                (|RealConstant|)
                (|CharacteristicZero|)
                (|StepThrough|)
                (|mkCategory|
                  (QUOTE |domain|)
                  (QUOTE (
                    ((|odd?| ((|Boolean|) $)) T)
                    ((|even?| ((|Boolean|) $)) T)
                    ((|base| ($)) T)
                    ((|length| ($ $)) T)
                    ((|shift| ($ $ $)) T)
                    ((|bit?| ((|Boolean|) $ $)) T)
                    ((|positiveRemainder| ($ $ $)) T)
                    ((|symmetricRemainder| ($ $ $)) T)
                    ((|rational?| ((|Boolean|) $)) T)
                    ((|rational| ((|Fraction| (|Integer|)) $)) T)
                    ((|rationalIfCan|
                      ((|Union| (|Fraction| (|Integer|)) "failed") $)) T)
                    ((|random| ($)) T)
                    ((|random| ($ $)) T)
                    ((|hash| ($ $)) T)
                    ((|copy| ($ $)) T)
                    ((|inc| ($ $)) T)
                    ((|dec| ($ $)) T)
                    ((|mask| ($ $)) T)
                    ((|addmod| ($ $ $ $)) T)
                    ((|submod| ($ $ $ $)) T)
                    ((|mulmod| ($ $ $ $)) T)
                    ((|powmod| ($ $ $ $)) T)
                    ((|invmod| ($ $ $)) T)))
                  (QUOTE ((|multiplicativeValuation| T) (|canonicalUnitNormal| T)))
                  (QUOTE ((|Fraction| (|Integer|)) (|Boolean|))) NIL)))
              |IntegerNumberSystem|)
            (SETELT #0# 0 (QUOTE (|IntegerNumberSystem|)))))))

(MAKEPROP (QUOTE |IntegerNumberSystem|) (QUOTE NILADIC) T)
```

21.30 INS-.lsp BOOTSTRAP

INS- depends on **INS**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **INS-** category which we can write into the **MID** directory. We compile the lisp code and copy the **INS-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

⟨*INS-.lsp BOOTSTRAP*⟩≡

```
(/VERSIONCHECK 2)

(PUT
  (QUOTE |INS-;characteristic;Nni;1|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL 0)))

(DEFUN |INS-;characteristic;Nni;1| ($) 0)

(DEFUN |INS-;differentiate;2S;2| (|x| $)
  (|spadConstant| $ 9))

(DEFUN |INS-;even?;SB;3| (|x| $)
 (COND
   ((SPADCALL |x| (QREFELT $ 12)) (QUOTE NIL))
   ((QUOTE T) (QUOTE T))))

(DEFUN |INS-;positive?;SB;4| (|x| $)
  (SPADCALL (|spadConstant| $ 9) |x| (QREFELT $ 14)))

(PUT
  (QUOTE |INS-;copy;2S;5|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|x|) |x|)))

(DEFUN |INS-;copy;2S;5| (|x| $) |x|)

(DEFUN |INS-;bit?;2SB;6| (|x| |i| $)
  (SPADCALL
    (SPADCALL |x|
      (SPADCALL |i| (QREFELT $ 17))
      (QREFELT $ 18))
    (QREFELT $ 12)))

(DEFUN |INS-;mask;2S;7| (|n| $)
  (SPADCALL
    (SPADCALL (|spadConstant| $ 20) |n| (QREFELT $ 18))
```

```
    (QREFELT $ 21)))

(PUT
  (QUOTE |INS-;rational?;SB;8|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|x|) (QUOTE T))))

(DEFUN |INS-;rational?;SB;8| (|x| $)
  (QUOTE T))

(DEFUN |INS-;euclideanSize;SNni;9| (|x| $)
  (PROG (#0=#:G1078 #1=#:G1079)
    (RETURN
      (COND
        ((SPADCALL |x| (|spadConstant| $ 9) (QREFELT $ 24))
          (|error| "euclideanSize called on zero"))
        ((SPADCALL |x| (|spadConstant| $ 9) (QREFELT $ 14))
          (PROG1
            (LETT #0#
              (- (SPADCALL |x| (QREFELT $ 26)))
              |INS-;euclideanSize;SNni;9|)
            (|check-subtype|
              (>= #0# 0)
              (QUOTE (|NonNegativeInteger|))
              #0#)))
        ((QUOTE T)
          (PROG1
            (LETT #1#
              (SPADCALL |x| (QREFELT $ 26))
              |INS-;euclideanSize;SNni;9|)
            (|check-subtype|
              (>= #1# 0)
              (QUOTE (|NonNegativeInteger|))
              #1#)))))))

(DEFUN |INS-;convert;SF;10| (|x| $)
  (SPADCALL (SPADCALL |x| (QREFELT $ 26)) (QREFELT $ 29)))

(DEFUN |INS-;convert;SDf;11| (|x| $)
  (FLOAT (SPADCALL |x| (QREFELT $ 26)) MOST-POSITIVE-LONG-FLOAT))

(DEFUN |INS-;convert;SIf;12| (|x| $)
  (SPADCALL (SPADCALL |x| (QREFELT $ 26)) (QREFELT $ 34)))

(DEFUN |INS-;retract;SI;13| (|x| $)
  (SPADCALL |x| (QREFELT $ 26)))
```

```
(DEFUN |INS-;convert;SP;14| (|x| $)
  (SPADCALL (SPADCALL |x| (QREFELT $ 26)) (QREFELT $ 38)))

(DEFUN |INS-;factor;SF;15| (|x| $)
  (SPADCALL |x| (QREFELT $ 42)))

(DEFUN |INS-;squareFree;SF;16| (|x| $)
  (SPADCALL |x| (QREFELT $ 45)))

(DEFUN |INS-;prime?;SB;17| (|x| $)
  (SPADCALL |x| (QREFELT $ 48)))

(DEFUN |INS-;factorial;2S;18| (|x| $)
  (SPADCALL |x| (QREFELT $ 51)))

(DEFUN |INS-;binomial;3S;19| (|n| |m| $)
  (SPADCALL |n| |m| (QREFELT $ 53)))

(DEFUN |INS-;permutation;3S;20| (|n| |m| $)
  (SPADCALL |n| |m| (QREFELT $ 55)))

(DEFUN |INS-;retractIfCan;SU;21| (|x| $)
  (CONS 0 (SPADCALL |x| (QREFELT $ 26))))

(DEFUN |INS-;init;S;22| ($)
  (|spadConstant| $ 9))

(DEFUN |INS-;nextItem;SU;23| (|n| $)
  (COND
    ((SPADCALL |n| (QREFELT $ 60))
       (CONS 0 (|spadConstant| $ 20)))
    ((SPADCALL (|spadConstant| $ 9) |n| (QREFELT $ 14))
       (CONS 0 (SPADCALL |n| (QREFELT $ 17))))
    ((QUOTE T)
       (CONS 0 (SPADCALL (|spadConstant| $ 20) |n| (QREFELT $ 61))))))

(DEFUN |INS-;patternMatch;SP2Pmr;24| (|x| |p| |l| $)
  (SPADCALL |x| |p| |l| (QREFELT $ 66)))

(DEFUN |INS-;rational;SF;25| (|x| $)
  (SPADCALL (SPADCALL |x| (QREFELT $ 26)) (QREFELT $ 70)))

(DEFUN |INS-;rationalIfCan;SU;26| (|x| $)
  (CONS 0 (SPADCALL (SPADCALL |x| (QREFELT $ 26)) (QREFELT $ 70))))
```

```
(DEFUN |INS-;symmetricRemainder;3S;27| (|x| |n| $)
  (PROG (|r|)
    (RETURN
      (SEQ
        (LETT |r|
          (SPADCALL |x| |n| (QREFELT $ 74))
          |INS-;symmetricRemainder;3S;27|)
        (EXIT
          (COND
            ((SPADCALL |r| (|spadConstant| $ 9) (QREFELT $ 24)) |r|)
            ((QUOTE T)
              (SEQ
                (COND
                  ((SPADCALL |n| (|spadConstant| $ 9) (QREFELT $ 14))
                    (LETT |n|
                      (SPADCALL |n| (QREFELT $ 17))
                      |INS-;symmetricRemainder;3S;27|)))
                (EXIT
                  (COND
                    ((SPADCALL (|spadConstant| $ 9) |r| (QREFELT $ 14))
                      (COND
                        ((SPADCALL |n|
                            (SPADCALL 2 |r| (QREFELT $ 76))
                            (QREFELT $ 14))
                          (SPADCALL |r| |n| (QREFELT $ 61)))
                        ((QUOTE T) |r|)))
                    ((NULL
                      (SPADCALL
                         (|spadConstant| $ 9)
                         (SPADCALL
                           (SPADCALL 2 |r| (QREFELT $ 76))
                           |n|
                           (QREFELT $ 77))
                         (QREFELT $ 14)))
                       (SPADCALL |r| |n| (QREFELT $ 77)))
                    ((QUOTE T) |r|)))))))))))

(DEFUN |INS-;invmod;3S;28| (|a| |b| $)
  (PROG (|q| |r| |r1| |c| |c1| |d| |d1|)
    (RETURN
      (SEQ
        (COND
          ((SPADCALL |a| (QREFELT $ 79))
            (LETT |a| (SPADCALL |a| |b| (QREFELT $ 80)) |INS-;invmod;3S;28|)))
        (LETT |c| |a| |INS-;invmod;3S;28|)
        (LETT |c1| (|spadConstant| $ 20) |INS-;invmod;3S;28|)
```

```
          (LETT |d| |b| |INS-;invmod;3S;28|)
          (LETT |d1| (|spadConstant| $ 9) |INS-;invmod;3S;28|)
          (SEQ G190
            (COND
              ((NULL
                (COND
                  ((SPADCALL |d| (QREFELT $ 60)) (QUOTE NIL))
                  ((QUOTE T) (QUOTE T))))
                (GO G191)))
            (SEQ
              (LETT |q| (SPADCALL |c| |d| (QREFELT $ 81)) |INS-;invmod;3S;28|)
              (LETT |r|
                (SPADCALL |c| (SPADCALL |q| |d| (QREFELT $ 82)) (QREFELT $ 61))
                |INS-;invmod;3S;28|)
              (LETT |r1|
                (SPADCALL |c1| (SPADCALL |q| |d1| (QREFELT $ 82)) (QREFELT $ 61))
                |INS-;invmod;3S;28|)
              (LETT |c| |d| |INS-;invmod;3S;28|)
              (LETT |c1| |d1| |INS-;invmod;3S;28|)
              (LETT |d| |r| |INS-;invmod;3S;28|)
              (EXIT (LETT |d1| |r1| |INS-;invmod;3S;28|)))
            NIL
            (GO G190)
            G191
            (EXIT NIL))
          (COND
            ((NULL (SPADCALL |c| (QREFELT $ 83)))
              (EXIT (|error| "inverse does not exist"))))
          (EXIT
            (COND
              ((SPADCALL |c1| (QREFELT $ 79)) (SPADCALL |c1| |b| (QREFELT $ 77)))
              ((QUOTE T) |c1|)))))))

(DEFUN |INS-;powmod;4S;29| (|x| |n| |p| $)
  (PROG (|y| #0=#:G1137 |z|)
    (RETURN
      (SEQ
        (EXIT
          (SEQ
            (COND
              ((SPADCALL |x| (QREFELT $ 79))
                (LETT |x|
                  (SPADCALL |x| |p| (QREFELT $ 80))
                  |INS-;powmod;4S;29|)))
             (EXIT
               (COND
```

```
                ((SPADCALL |x| (QREFELT $ 60)) (|spadConstant| $ 9))
                ((SPADCALL |n| (QREFELT $ 60)) (|spadConstant| $ 20))
                ((QUOTE T)
                  (SEQ
                    (LETT |y| (|spadConstant| $ 20) |INS-;powmod;4S;29|)
                    (LETT |z| |x| |INS-;powmod;4S;29|)
                    (EXIT
                      (SEQ G190
                        NIL
                        (SEQ
                          (COND
                            ((SPADCALL |n| (QREFELT $ 12))
                              (LETT |y|
                                (SPADCALL |y| |z| |p| (QREFELT $ 85))
                                |INS-;powmod;4S;29|)))
                          (EXIT
                            (COND
                              ((SPADCALL
                                (LETT |n|
                                  (SPADCALL |n|
                                    (SPADCALL
                                      (|spadConstant| $ 20)
                                      (QREFELT $ 17))
                                    (QREFELT $ 18))
                                   |INS-;powmod;4S;29|)
                                (QREFELT $ 60))
                               (PROGN
                                 (LETT #0# |y| |INS-;powmod;4S;29|)
                                 (GO #0#)))
                              ((QUOTE T)
                                (LETT |z|
                                  (SPADCALL |z| |z| |p| (QREFELT $ 85))
                                  |INS-;powmod;4S;29|)))))
                        NIL
                        (GO G190)
                        G191
                        (EXIT NIL)))))))
          #0#
          (EXIT #0#)))))

(DEFUN |IntegerNumberSystem&| (|#1|)
  (PROG (DV$1 |dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT DV$1 (|devaluate| |#1|) . #0=(|IntegerNumberSystem&|))
        (LETT |dv$| (LIST (QUOTE |IntegerNumberSystem&|) DV$1) . #0#)
```

```
          (LETT $ (GETREFV 87) . #0#)
          (QSETREFV $ 0 |dv$|)
          (QSETREFV $ 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #0#))
          (|stuffDomainSlots| $)
          (QSETREFV $ 6 |#1|)
          $))))

  (MAKEPROP
    (QUOTE |IntegerNumberSystem&|)
    (QUOTE |infovec|)
    (LIST
      (QUOTE
        #(NIL NIL NIL NIL NIL NIL
          (|local| |#1|)
          (|NonNegativeInteger|)
          |INS-;characteristic;Nni;1|
          (0 . |Zero|)
          |INS-;differentiate;2S;2|
          (|Boolean|)
          (4 . |odd?|)
          |INS-;even?;SB;3|
          (9 . <)
          |INS-;positive?;SB;4|
          |INS-;copy;2S;5|
          (15 . -)
          (20 . |shift|)
          |INS-;bit?;2SB;6|
          (26 . |One|)
          (30 . |dec|)
          |INS-;mask;2S;7|
          |INS-;rational?;SB;8|
          (35 . =)
          (|Integer|)
          (41 . |convert|)
          |INS-;euclideanSize;SNni;9|
          (|Float|)
          (46 . |coerce|)
          |INS-;convert;SF;10|
          (|DoubleFloat|)
          |INS-;convert;SDf;11|
          (|InputForm|)
          (51 . |convert|)
          |INS-;convert;SIf;12|
          |INS-;retract;SI;13|
          (|Pattern| 25)
          (56 . |coerce|)
```

```
|INS-;convert;SP;14|
(|Factored| 6)
(|IntegerFactorizationPackage| 6)
(61 . |factor|)
(|Factored| $)
|INS-;factor;SF;15|
(66 . |squareFree|)
|INS-;squareFree;SF;16|
(|IntegerPrimesPackage| 6)
(71 . |prime?|)
|INS-;prime?;SB;17|
(|IntegerCombinatoricFunctions| 6)
(76 . |factorial|)
|INS-;factorial;2S;18|
(81 . |binomial|)
|INS-;binomial;3S;19|
(87 . |permutation|)
|INS-;permutation;3S;20|
(|Union| 25 (QUOTE "failed"))
|INS-;retractIfCan;SU;21|
|INS-;init;S;22|
(93 . |zero?|)
(98 . -)
(|Union| $ (QUOTE "failed"))
|INS-;nextItem;SU;23|
(|PatternMatchResult| 25 6)
(|PatternMatchIntegerNumberSystem| 6)
(104 . |patternMatch|)
(|PatternMatchResult| 25 $)
|INS-;patternMatch;SP2Pmr;24|
(|Fraction| 25)
(111 . |coerce|)
|INS-;rational;SF;25|
(|Union| 69 (QUOTE "failed"))
|INS-;rationalIfCan;SU;26|
(116 . |rem|)
(|PositiveInteger|)
(122 . *)
(128 . +)
|INS-;symmetricRemainder;3S;27|
(134 . |negative?|)
(139 . |positiveRemainder|)
(145 . |quo|)
(151 . *)
(157 . |one?|)
|INS-;invmod;3S;28|
```

```
    (162 . |mulmod|)
    |INS-;powmod;4S;29|))
  (QUOTE
    #(|symmetricRemainder| 169 |squareFree| 175 |retractIfCan| 180
      |retract| 185 |rationalIfCan| 190 |rational?| 195 |rational| 200
      |prime?| 205 |powmod| 210 |positive?| 217 |permutation| 222
      |patternMatch| 228 |nextItem| 235 |mask| 240 |invmod| 245 |init| 251
      |factorial| 255 |factor| 260 |even?| 265 |euclideanSize| 270
      |differentiate| 275 |copy| 280 |convert| 285 |characteristic| 305
      |bit?| 309 |binomial| 315))
  (QUOTE NIL)
  (CONS
    (|makeByteWordVec2| 1 (QUOTE NIL))
    (CONS
      (QUOTE #())
      (CONS
        (QUOTE #())
        (|makeByteWordVec2| 86
          (QUOTE
            (0 6 0 9 1 6 11 0 12 2 6 11 0 0 14 1 6 0 0 17 2 6 0 0 0 18 0 6
             0 20 1 6 0 0 21 2 6 11 0 0 24 1 6 25 0 26 1 28 0 25 29 1 33 0
             25 34 1 37 0 25 38 1 41 40 6 42 1 41 40 6 45 1 47 11 6 48 1 50
             6 6 51 2 50 6 6 6 53 2 50 6 6 6 55 1 6 11 0 60 2 6 0 0 0 61 3
             65 64 6 37 64 66 1 69 0 25 70 2 6 0 0 0 74 2 6 0 75 0 76 2 6 0
             0 0 77 1 6 11 0 79 2 6 0 0 0 80 2 6 0 0 0 81 2 6 0 0 0 82 1 6
             11 0 83 3 6 0 0 0 0 85 2 0 0 0 0 78 1 0 43 0 46 1 0 57 0 58 1
             0 25 0 36 1 0 72 0 73 1 0 11 0 23 1 0 69 0 71 1 0 11 0 49 3 0
             0 0 0 0 86 1 0 11 0 15 2 0 0 0 0 56 3 0 67 0 37 67 68 1 0 62
             0 63 1 0 0 0 22 2 0 0 0 0 84 0 0 0 59 1 0 0 0 52 1 0 43 0 44
             1 0 11 0 13 1 0 7 0 27 1 0 0 0 10 1 0 0 0 16 1 0 31 0 32 1 0
             28 0 30 1 0 37 0 39 1 0 33 0 35 0 0 7 8 2 0 11 0 0 19 2 0 0
             0 0 54))))))
(QUOTE |lookupComplete|)))
```

21.31 INTDOM.lsp BOOTSTRAP

INTDOM depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **INTDOM** category which we can write into the **MID** directory. We compile the lisp code and copy the **INTDOM.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*INTDOM.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |IntegralDomain;AL| (QUOTE NIL))

(DEFUN |IntegralDomain| NIL
  (LET (#:G83060)
    (COND
      (|IntegralDomain;AL|)
      (T (SETQ |IntegralDomain;AL| (|IntegralDomain;|))))))

(DEFUN |IntegralDomain;| NIL
  (PROG (#1=#:G83058)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|CommutativeRing|)
            (|Algebra| (QUOTE |$|))
            (|EntireRing|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|exquo| ((|Union| |$| "failed") |$| |$|)) T)
                ((|unitNormal|
                  ((|Record|
                     (|:| |unit| |$|)
                     (|:| |canonical| |$|)
                     (|:| |associate| |$|)) |$|)) T)
                ((|unitCanonical| (|$| |$|)) T)
                ((|associates?| ((|Boolean|) |$| |$|)) T)
                ((|unit?| ((|Boolean|) |$|)) T)))
              NIL
              (QUOTE ((|Boolean|)))
              NIL))
            |IntegralDomain|)
```

```
          (SETELT #1# 0 (QUOTE (|IntegralDomain|)))))))

  (MAKEPROP (QUOTE |IntegralDomain|) (QUOTE NILADIC) T)
```

21.32 INTDOM-.lsp BOOTSTRAP

INTDOM- depends on **INTDOM**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **INTDOM-** category which we can write into the **MID** directory. We compile the lisp code and copy the **INTDOM-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*INTDOM-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |INTDOM-;unitNormal;SR;1| (|x| |$|)
  (VECTOR (|spadConstant| |$| 7) |x| (|spadConstant| |$| 7)))

(DEFUN |INTDOM-;unitCanonical;2S;2| (|x| |$|)
  (QVELT (SPADCALL |x| (QREFELT |$| 10)) 1))

(DEFUN |INTDOM-;recip;SU;3| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 13)) (CONS 1 "failed"))
    ((QUOTE T) (SPADCALL (|spadConstant| |$| 7) |x| (QREFELT |$| 15)))))

(DEFUN |INTDOM-;unit?;SB;4| (|x| |$|)
  (COND
    ((QEQCAR (SPADCALL |x| (QREFELT |$| 17)) 1) (QUOTE NIL))
    ((QUOTE T) (QUOTE T))))

(DEFUN |INTDOM-;associates?;2SB;5| (|x| |y| |$|)
  (SPADCALL
    (QVELT (SPADCALL |x| (QREFELT |$| 10)) 1)
    (QVELT (SPADCALL |y| (QREFELT |$| 10)) 1)
    (QREFELT |$| 19)))

(DEFUN |INTDOM-;associates?;2SB;6| (|x| |y| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 13)) (SPADCALL |y| (QREFELT |$| 13)))
    ((OR
        (SPADCALL |y| (QREFELT |$| 13))
        (OR
          (QEQCAR (SPADCALL |x| |y| (QREFELT |$| 15)) 1)
          (QEQCAR (SPADCALL |y| |x| (QREFELT |$| 15)) 1)))
      (QUOTE NIL))
    ((QUOTE T) (QUOTE T))))
```

```
(DEFUN |IntegralDomain&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|IntegralDomain&|))
        (LETT |dv$| (LIST (QUOTE |IntegralDomain&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 21) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        (COND
          ((|HasCategory| |#1| (QUOTE (|Field|))))
          ((QUOTE T)
            (QSETREFV |$| 9
              (CONS (|dispatchFunction| |INTDOM-;unitNormal;SR;1|) |$|))))
        (COND
          ((|HasAttribute| |#1| (QUOTE |canonicalUnitNormal|))
            (QSETREFV |$| 20
              (CONS (|dispatchFunction| |INTDOM-;associates?;2SB;5|) |$|)))
          ((QUOTE T)
            (QSETREFV |$| 20
              (CONS (|dispatchFunction| |INTDOM-;associates?;2SB;6|) |$|))))
        |$|))))

(MAKEPROP
  (QUOTE |IntegralDomain&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (0 . |One|)
        (|Record|
          (|:| |unit| |$|)
          (|:| |canonical| |$|)
          (|:| |associate| |$|))
        (4 . |unitNormal|)
        (9 . |unitNormal|)
        |INTDOM-;unitCanonical;2S;2|
        (|Boolean|)
        (14 . |zero?|)
        (|Union| |$| (QUOTE "failed"))
        (19 . |exquo|)
        |INTDOM-;recip;SU;3|
        (25 . |recip|)
```

```
      |INTDOM-;unit?;SB;4|
      (30 . |=|)
      (36 . |associates?|)))
  (QUOTE
    #(|unitNormal| 42 |unitCanonical| 47 |unit?| 52 |recip| 57
      |associates?| 62))
  (QUOTE NIL)
  (CONS
    (|makeByteWordVec2| 1 (QUOTE NIL))
    (CONS
      (QUOTE #())
      (CONS
        (QUOTE #())
        (|makeByteWordVec2| 20
          (QUOTE
            (0 6 0 7 1 0 8 0 9 1 6 8 0 10 1 6 12 0 13 2 6 14 0 0 15 1 6 14
             0 17 2 6 12 0 0 19 2 0 12 0 0 20 1 0 8 0 9 1 0 0 0 11 1 0 12 0
             18 1 0 14 0 16 2 0 12 0 0 20))))))
(QUOTE |lookupComplete|)))
```

21.33 LNAGG.lsp BOOTSTRAP

LNAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **LNAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **LNAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*LNAGG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |LinearAggregate;CAT| (QUOTE NIL))

(SETQ |LinearAggregate;AL| (QUOTE NIL))

(DEFUN |LinearAggregate| (#1=#:G85818)
 (LET (#2=#:G85819)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |LinearAggregate;AL|)) (CDR #2#))
   (T
    (SETQ |LinearAggregate;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|LinearAggregate;| #1#)))
      |LinearAggregate;AL|))
    #2#))))

(DEFUN |LinearAggregate;| (|t#1|)
 (PROG (#1=#:G85817)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (|sublisV|
       (PAIR (QUOTE (#2=#:G85816)) (LIST (QUOTE (|Integer|))))
       (COND
        (|LinearAggregate;CAT|)
        ((QUOTE T)
         (LETT |LinearAggregate;CAT|
          (|Join|
           (|IndexedAggregate| (QUOTE #2#) (QUOTE |t#1|))
           (|Collection| (QUOTE |t#1|))
           (|mkCategory|
            (QUOTE |domain|)
```

```
            (QUOTE (
             ((|new| (|$| (|NonNegativeInteger|) |t#1|)) T)
             ((|concat| (|$| |$| |t#1|)) T)
             ((|concat| (|$| |t#1| |$|)) T)
             ((|concat| (|$| |$| |$|)) T)
             ((|concat| (|$| (|List| |$|))) T)
             ((|map| (|$| (|Mapping| |t#1| |t#1| |t#1|) |$| |$|)) T)
             ((|elt| (|$| |$| (|UniversalSegment| (|Integer|)))) T)
             ((|delete| (|$| |$| (|Integer|))) T)
             ((|delete| (|$| |$| (|UniversalSegment| (|Integer|)))) T)
             ((|insert| (|$| |t#1| |$| (|Integer|))) T)
             ((|insert| (|$| |$| |$| (|Integer|))) T)
             ((|setelt| (|t#1| |$| (|UniversalSegment| (|Integer|)) |t#1|))
              (|has| |$| (ATTRIBUTE |shallowlyMutable|)))))
            NIL
            (QUOTE (
              (|UniversalSegment| (|Integer|))
              (|Integer|)
              (|List| |$|)
              (|NonNegativeInteger|)))
             NIL))
          . #3=(|LinearAggregate|))))))
 . #3#)
(SETELT #1# 0 (LIST (QUOTE |LinearAggregate|) (|devaluate| |t#1|)))))))
```

21.34 LNAGG-.lsp BOOTSTRAP

LNAGG- depends on **LNAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **LNAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **LNAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*LNAGG-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |LNAGG-;indices;AL;1| (|a| |$|)
 (PROG (#1=#:G85833 |i| #2=#:G85834)
  (RETURN
   (SEQ
    (PROGN
     (LETT #1# NIL |LNAGG-;indices;AL;1|)
     (SEQ
      (LETT |i| (SPADCALL |a| (QREFELT |$| 9)) |LNAGG-;indices;AL;1|)
      (LETT #2# (SPADCALL |a| (QREFELT |$| 10)) |LNAGG-;indices;AL;1|)
      G190
      (COND ((|>| |i| #2#) (GO G191)))
      (SEQ (EXIT (LETT #1# (CONS |i| #1#) |LNAGG-;indices;AL;1|)))
      (LETT |i| (|+| |i| 1) |LNAGG-;indices;AL;1|)
      (GO G190)
      G191
      (EXIT (NREVERSE0 #1#))))))))

(DEFUN |LNAGG-;index?;IAB;2| (|i| |a| |$|)
 (COND
  ((OR
     (|<| |i| (SPADCALL |a| (QREFELT |$| 9)))
     (|<| (SPADCALL |a| (QREFELT |$| 10)) |i|))
    (QUOTE NIL))
  ((QUOTE T) (QUOTE T))))

(DEFUN |LNAGG-;concat;ASA;3| (|a| |x| |$|)
 (SPADCALL |a| (SPADCALL 1 |x| (QREFELT |$| 16)) (QREFELT |$| 17)))

(DEFUN |LNAGG-;concat;S2A;4| (|x| |y| |$|)
 (SPADCALL (SPADCALL 1 |x| (QREFELT |$| 16)) |y| (QREFELT |$| 17)))

(DEFUN |LNAGG-;insert;SAIA;5| (|x| |a| |i| |$|)
 (SPADCALL (SPADCALL 1 |x| (QREFELT |$| 16)) |a| |i| (QREFELT |$| 20)))
```

```
(DEFUN |LNAGG-;maxIndex;AI;6| (|l| |$|)
 (|+| (|-| (SPADCALL |l| (QREFELT |$| 22)) 1) (SPADCALL |l| (QREFELT |$| 9))))

(DEFUN |LinearAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|LinearAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |LinearAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (GETREFV 25) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST (|HasAttribute| |#1| (QUOTE |shallowlyMutable|))))
      . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|HasAttribute| |#1| (QUOTE |finiteAggregate|))
      (QSETREFV |$| 23
       (CONS (|dispatchFunction| |LNAGG-;maxIndex;AI;6|) |$|))))
    |$|))))

(MAKEPROP
 (QUOTE |LinearAggregate&|)
 (QUOTE |infovec|)
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|Integer|)
    (0 . |minIndex|) (5 . |maxIndex|) (|List| 8) |LNAGG-;indices;AL;1|
    (|Boolean|) |LNAGG-;index?;IAB;2| (|NonNegativeInteger|) (10 . |new|)
    (16 . |concat|) |LNAGG-;concat;ASA;3| |LNAGG-;concat;S2A;4|
    (22 . |insert|) |LNAGG-;insert;SAIA;5| (29 . |#|) (34 . |maxIndex|)
    (|List| |$|)))
  (QUOTE #(|maxIndex| 39 |insert| 44 |indices| 51 |index?| 56 |concat| 62))
  (QUOTE NIL)
  (CONS
   (|makeByteWordVec2| 1 (QUOTE NIL))
   (CONS
    (QUOTE #())
    (CONS
     (QUOTE #())
     (|makeByteWordVec2| 23 (QUOTE (1 6 8 0 9 1 6 8 0 10 2 6 0 15 7
```

```
        16 2 6 0 0 0 17 3 6 0 0 0 8 20 1 6 15 0 22 1 0 8 0 23 1 0 8 0 23 3 0
        0 7 0 8 21 1 0 11 0 12 2 0 13 8 0 14 2 0 0 0 7 18 2 0 0 7 0 19))))))
    (QUOTE |lookupComplete|)))
```

21.35 LSAGG.lsp BOOTSTRAP

LSAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **LSAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **LSAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*LSAGG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |ListAggregate;CAT| (QUOTE NIL))

(SETQ |ListAggregate;AL| (QUOTE NIL))

(DEFUN |ListAggregate| (#1=#:G87500)
 (LET (#2=#:G87501)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |ListAggregate;AL|)) (CDR #2#))
   (T
    (SETQ |ListAggregate;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|ListAggregate;| #1#)))
      |ListAggregate;AL|))
    #2#))))

(DEFUN |ListAggregate;| (|t#1|)
 (PROG (#1=#:G87499)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|ListAggregate;CAT|)
       ((QUOTE T)
        (LETT |ListAggregate;CAT|
         (|Join|
          (|StreamAggregate| (QUOTE |t#1|))
          (|FiniteLinearAggregate| (QUOTE |t#1|))
          (|ExtensibleLinearAggregate| (QUOTE |t#1|))
          (|mkCategory|
           (QUOTE |domain|)
           (QUOTE (((|list| (|$| |t#1|)) T)))
```

```
            NIL
            (QUOTE NIL)
             NIL))
           . #2=(|ListAggregate|)))))
 . #2#)
(SETELT #1# 0 (LIST (QUOTE |ListAggregate|) (|devaluate| |t#1|)))))))
```

21.36 LSAGG-.lsp BOOTSTRAP

LSAGG- depends on **LSAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **LSAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **LSAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*LSAGG-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |LSAGG-;sort!;M2A;1| (|f| |l| |$|)
 (|LSAGG-;mergeSort| |f| |l| (SPADCALL |l| (QREFELT |$| 9)) |$|))

(DEFUN |LSAGG-;list;SA;2| (|x| |$|)
 (SPADCALL |x| (SPADCALL (QREFELT |$| 12)) (QREFELT |$| 13)))

(DEFUN |LSAGG-;reduce;MAS;3| (|f| |x| |$|)
 (COND
  ((SPADCALL |x| (QREFELT |$| 16))
    (|error| "reducing over an empty list needs the 3 argument form"))
  ((QUOTE T)
    (SPADCALL |f|
     (SPADCALL |x| (QREFELT |$| 17))
     (SPADCALL |x| (QREFELT |$| 18))
     (QREFELT |$| 20)))))

(DEFUN |LSAGG-;merge;M3A;4| (|f| |p| |q| |$|)
 (SPADCALL |f|
  (SPADCALL |p| (QREFELT |$| 22))
  (SPADCALL |q| (QREFELT |$| 22))
  (QREFELT |$| 23)))

(DEFUN |LSAGG-;select!;M2A;5| (|f| |x| |$|)
 (PROG (|y| |z|)
  (RETURN
   (SEQ
    (SEQ G190
     (COND
      ((NULL
        (COND
         ((OR
           (SPADCALL |x| (QREFELT |$| 16))
           (SPADCALL (SPADCALL |x| (QREFELT |$| 18)) |f|))
```

```
            (QUOTE NIL))
           ((QUOTE T) (QUOTE T))))
         (GO G191)))
       (SEQ
        (EXIT
         (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;select!;M2A;5|)))
       NIL
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT
       (COND
        ((SPADCALL |x| (QREFELT |$| 16)) |x|)
        ((QUOTE T)
         (SEQ
          (LETT |y| |x| |LSAGG-;select!;M2A;5|)
          (LETT |z| (SPADCALL |y| (QREFELT |$| 17)) |LSAGG-;select!;M2A;5|)
          (SEQ
           G190
           (COND
            ((NULL
              (COND
               ((SPADCALL |z| (QREFELT |$| 16)) (QUOTE NIL))
               ((QUOTE T) (QUOTE T))))
             (GO G191)))
           (SEQ
            (EXIT
             (COND
              ((SPADCALL (SPADCALL |z| (QREFELT |$| 18)) |f|)
               (SEQ
                (LETT |y| |z| |LSAGG-;select!;M2A;5|)
                (EXIT
                 (LETT |z|
                  (SPADCALL |z| (QREFELT |$| 17))
                  |LSAGG-;select!;M2A;5|))))
              ((QUOTE T)
               (SEQ
                (LETT |z| (SPADCALL |z| (QREFELT |$| 17)) |LSAGG-;select!;M2A;5|)
                (EXIT (SPADCALL |y| |z| (QREFELT |$| 25)))))))))
           NIL
           (GO G190)
           G191
           (EXIT NIL))
          (EXIT |x|)))))))))

(DEFUN |LSAGG-;merge!;M3A;6| (|f| |p| |q| |$|)
```

```
(PROG (|r| |t|)
 (RETURN
  (SEQ
   (COND
    ((SPADCALL |p| (QREFELT |$| 16)) |q|)
    ((SPADCALL |q| (QREFELT |$| 16)) |p|)
    ((SPADCALL |p| |q| (QREFELT |$| 28))
      (|error| "cannot merge a list into itself"))
    ((QUOTE T)
     (SEQ
      (COND
       ((SPADCALL
         (SPADCALL |p| (QREFELT |$| 18))
         (SPADCALL |q| (QREFELT |$| 18))
         |f|)
        (SEQ
         (LETT |r| (LETT |t| |p| |LSAGG-;merge!;M3A;6|) |LSAGG-;merge!;M3A;6|)
         (EXIT
          (LETT |p| (SPADCALL |p| (QREFELT |$| 17)) |LSAGG-;merge!;M3A;6|))))
       ((QUOTE T)
        (SEQ
         (LETT |r| (LETT |t| |q| |LSAGG-;merge!;M3A;6|) |LSAGG-;merge!;M3A;6|)
         (EXIT
          (LETT |q| (SPADCALL |q| (QREFELT |$| 17)) |LSAGG-;merge!;M3A;6|)))))
      (SEQ
       G190
       (COND
        ((NULL
           (COND
            ((OR
              (SPADCALL |p| (QREFELT |$| 16))
              (SPADCALL |q| (QREFELT |$| 16)))
             (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
         (GO G191)))
        (SEQ
         (EXIT
          (COND
           ((SPADCALL
              (SPADCALL |p| (QREFELT |$| 18))
              (SPADCALL |q| (QREFELT |$| 18))
              |f|)
            (SEQ
             (SPADCALL |t| |p| (QREFELT |$| 25))
             (LETT |t| |p| |LSAGG-;merge!;M3A;6|)
             (EXIT
```

```
               (LETT |p|
                (SPADCALL |p| (QREFELT |$| 17))
                |LSAGG-;merge!;M3A;6|))))
             ((QUOTE T)
              (SEQ
               (SPADCALL |t| |q| (QREFELT |$| 25))
               (LETT |t| |q| |LSAGG-;merge!;M3A;6|)
               (EXIT
                (LETT |q|
                 (SPADCALL |q| (QREFELT |$| 17))
                 |LSAGG-;merge!;M3A;6|)))))))
          NIL
          (GO G190)
          G191
          (EXIT NIL))
         (SPADCALL |t|
          (COND
           ((SPADCALL |p| (QREFELT |$| 16)) |q|)
           ((QUOTE T) |p|))
          (QREFELT |$| 25))
         (EXIT |r|))))))))

(DEFUN |LSAGG-;insert!;SAIA;7| (|s| |x| |i| |$|)
 (PROG (|m| #1=#:G87547 |y| |z|)
  (RETURN
   (SEQ
    (LETT |m| (SPADCALL |x| (QREFELT |$| 31)) |LSAGG-;insert!;SAIA;7|)
    (EXIT
     (COND
      ((|<| |i| |m|) (|error| "index out of range"))
      ((EQL |i| |m|) (SPADCALL |s| |x| (QREFELT |$| 13)))
      ((QUOTE T)
       (SEQ
        (LETT |y|
         (SPADCALL |x|
          (PROG1
           (LETT #1# (|-| (|-| |i| 1) |m|) |LSAGG-;insert!;SAIA;7|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 32))
         |LSAGG-;insert!;SAIA;7|)
        (LETT |z| (SPADCALL |y| (QREFELT |$| 17)) |LSAGG-;insert!;SAIA;7|)
        (SPADCALL |y| (SPADCALL |s| |z| (QREFELT |$| 13)) (QREFELT |$| 25))
        (EXIT |x|)))))))))

(DEFUN |LSAGG-;insert!;2AIA;8| (|w| |x| |i| |$|)
 (PROG (|m| #1=#:G87551 |y| |z|)
```

```
   (RETURN
    (SEQ
     (LETT |m| (SPADCALL |x| (QREFELT |$| 31)) |LSAGG-;insert!;2AIA;8|)
     (EXIT
      (COND
       ((|<| |i| |m|) (|error| "index out of range"))
       ((EQL |i| |m|) (SPADCALL |w| |x| (QREFELT |$| 34)))
       ((QUOTE T)
        (SEQ
         (LETT |y|
          (SPADCALL |x|
           (PROG1
            (LETT #1# (|-| (|-| |i| 1) |m|) |LSAGG-;insert!;2AIA;8|)
            (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
           (QREFELT |$| 32))
          |LSAGG-;insert!;2AIA;8|)
         (LETT |z| (SPADCALL |y| (QREFELT |$| 17)) |LSAGG-;insert!;2AIA;8|)
         (SPADCALL |y| |w| (QREFELT |$| 25))
         (SPADCALL |y| |z| (QREFELT |$| 34))
         (EXIT |x|)))))))

 (DEFUN |LSAGG-;remove!;M2A;9| (|f| |x| |$|)
  (PROG (|p| |q|)
   (RETURN
    (SEQ
     (SEQ
      G190
      (COND
       ((NULL
         (COND
          ((SPADCALL |x| (QREFELT |$| 16)) (QUOTE NIL))
          ((QUOTE T) (SPADCALL (SPADCALL |x| (QREFELT |$| 18)) |f|))))
        (GO G191)))
      (SEQ
       (EXIT
        (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;remove!;M2A;9|)))
       NIL
       (GO G190)
       G191
       (EXIT NIL))
     (EXIT
      (COND
       ((SPADCALL |x| (QREFELT |$| 16)) |x|)
       ((QUOTE T)
        (SEQ
         (LETT |p| |x| |LSAGG-;remove!;M2A;9|)
```

```
          (LETT |q| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;remove!;M2A;9|)
          (SEQ
           G190
           (COND
            ((NULL
               (COND
                ((SPADCALL |q| (QREFELT |$| 16)) (QUOTE NIL))
                ((QUOTE T) (QUOTE T))))
              (GO G191)))
           (SEQ
            (EXIT
             (COND
              ((SPADCALL (SPADCALL |q| (QREFELT |$| 18)) |f|)
                (LETT |q|
                 (SPADCALL |p| (SPADCALL |q| (QREFELT |$| 17)) (QREFELT |$| 25))
                 |LSAGG-;remove!;M2A;9|))
              ((QUOTE T)
                (SEQ
                 (LETT |p| |q| |LSAGG-;remove!;M2A;9|)
                 (EXIT
                  (LETT |q|
                    (SPADCALL |q| (QREFELT |$| 17))
                    |LSAGG-;remove!;M2A;9|)))))))
           NIL
           (GO G190)
           G191
           (EXIT NIL))
          (EXIT |x|)))))))))

(DEFUN |LSAGG-;delete!;AIA;10| (|x| |i| |$|)
 (PROG (|m| #1=#:G87564 |y|)
  (RETURN
   (SEQ
    (LETT |m| (SPADCALL |x| (QREFELT |$| 31)) |LSAGG-;delete!;AIA;10|)
    (EXIT
     (COND
      ((|<| |i| |m|) (|error| "index out of range"))
      ((EQL |i| |m|) (SPADCALL |x| (QREFELT |$| 17)))
      ((QUOTE T)
       (SEQ
        (LETT |y|
         (SPADCALL |x|
          (PROG1
           (LETT #1# (|-| (|-| |i| 1) |m|) |LSAGG-;delete!;AIA;10|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 32))
```

```
          |LSAGG-;delete!;AIA;10|)
         (SPADCALL |y| (SPADCALL |y| 2 (QREFELT |$| 32)) (QREFELT |$| 25))
         (EXIT |x|))))))))))

(DEFUN |LSAGG-;delete!;AUsA;11| (|x| |i| |$|)
 (PROG (|l| |m| |h| #1=#:G87569 #2=#:G87570 |t| #3=#:G87571)
  (RETURN
   (SEQ
    (LETT |l| (SPADCALL |i| (QREFELT |$| 39)) |LSAGG-;delete!;AUsA;11|)
    (LETT |m| (SPADCALL |x| (QREFELT |$| 31)) |LSAGG-;delete!;AUsA;11|)
    (EXIT
     (COND
      ((|<| |l| |m|) (|error| "index out of range"))
      ((QUOTE T)
       (SEQ
        (LETT |h|
         (COND
          ((SPADCALL |i| (QREFELT |$| 40)) (SPADCALL |i| (QREFELT |$| 41)))
          ((QUOTE T) (SPADCALL |x| (QREFELT |$| 42))))
         |LSAGG-;delete!;AUsA;11|)
        (EXIT
         (COND
          ((|<| |h| |l|) |x|)
          ((EQL |l| |m|)
           (SPADCALL |x|
            (PROG1
             (LETT #1# (|-| (|+| |h| 1) |m|) |LSAGG-;delete!;AUsA;11|)
             (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
            (QREFELT |$| 32)))
          ((QUOTE T)
           (SEQ
            (LETT |t|
             (SPADCALL |x|
              (PROG1
               (LETT #2# (|-| (|-| |l| 1) |m|) |LSAGG-;delete!;AUsA;11|)
               (|check-subtype| (|>=| #2# 0)
                 (QUOTE (|NonNegativeInteger|)) #2#))
              (QREFELT |$| 32))
             |LSAGG-;delete!;AUsA;11|)
            (SPADCALL |t|
             (SPADCALL |t|
              (PROG1
               (LETT #3# (|+| (|-| |h| |l|) 2) |LSAGG-;delete!;AUsA;11|)
               (|check-subtype| (|>=| #3# 0)
                (QUOTE (|NonNegativeInteger|)) #3#))
              (QREFELT |$| 32))
```

```
              (QREFELT |$| 25))
             (EXIT |x|)))))))))))))

(DEFUN |LSAGG-;find;MAU;12| (|f| |x| |$|)
 (SEQ
  (SEQ
   G190
   (COND
    ((NULL
     (COND
      ((OR
         (SPADCALL |x| (QREFELT |$| 16))
         (SPADCALL (SPADCALL |x| (QREFELT |$| 18)) |f|))
        (QUOTE NIL))
      ((QUOTE T) (QUOTE T))))
     (GO G191)))
   (SEQ (EXIT (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;find;MAU;12|)))
   NIL
   (GO G190)
   G191
   (EXIT NIL))
  (EXIT
   (COND
    ((SPADCALL |x| (QREFELT |$| 16)) (CONS 1 "failed"))
    ((QUOTE T) (CONS 0 (SPADCALL |x| (QREFELT |$| 18))))))))

(DEFUN |LSAGG-;position;MAI;13| (|f| |x| |$|)
 (PROG (|k|)
  (RETURN
   (SEQ
    (SEQ
     (LETT |k| (SPADCALL |x| (QREFELT |$| 31)) |LSAGG-;position;MAI;13|)
     G190
     (COND
      ((NULL
        (COND
         ((OR
           (SPADCALL |x| (QREFELT |$| 16))
           (SPADCALL (SPADCALL |x| (QREFELT |$| 18)) |f|))
          (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (EXIT
       (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;position;MAI;13|)))
     (LETT |k| (|+| |k| 1) |LSAGG-;position;MAI;13|)
```

```
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT
      (COND
       ((SPADCALL |x| (QREFELT |$| 16)) (|-| (SPADCALL |x| (QREFELT |$| 31)) 1))
       ((QUOTE T) |k|)))))))

(DEFUN |LSAGG-;mergeSort| (|f| |p| |n| |$|)
 (PROG (#1=#:G87593 |l| |q|)
  (RETURN
   (SEQ
    (COND
     ((EQL |n| 2)
      (COND
       ((SPADCALL
         (SPADCALL (SPADCALL |p| (QREFELT |$| 17)) (QREFELT |$| 18))
         (SPADCALL |p| (QREFELT |$| 18)) |f|)
        (LETT |p| (SPADCALL |p| (QREFELT |$| 47)) |LSAGG-;mergeSort|)))))
    (EXIT
     (COND
      ((|<| |n| 3) |p|)
      ((QUOTE T)
       (SEQ
        (LETT |l|
         (PROG1
          (LETT #1# (QUOTIENT2 |n| 2) |LSAGG-;mergeSort|)
          (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
         |LSAGG-;mergeSort|)
        (LETT |q| (SPADCALL |p| |l| (QREFELT |$| 48)) |LSAGG-;mergeSort|)
        (LETT |p| (|LSAGG-;mergeSort| |f| |p| |l| |$|) |LSAGG-;mergeSort|)
        (LETT |q|
          (|LSAGG-;mergeSort| |f| |q| (|-| |n| |l|) |$|)
          |LSAGG-;mergeSort|)
        (EXIT (SPADCALL |f| |p| |q| (QREFELT |$| 23)))))))))))

(DEFUN |LSAGG-;sorted?;MAB;15| (|f| |l| |$|)
 (PROG (#1=#:G87603 |p|)
  (RETURN
   (SEQ
    (EXIT
     (COND
      ((SPADCALL |l| (QREFELT |$| 16)) (QUOTE T))
      ((QUOTE T)
       (SEQ
        (LETT |p| (SPADCALL |l| (QREFELT |$| 17)) |LSAGG-;sorted?;MAB;15|)
```

```
          (SEQ
           G190
           (COND
            ((NULL
              (COND
               ((SPADCALL |p| (QREFELT |$| 16)) (QUOTE NIL))
               ((QUOTE T) (QUOTE T))))
             (GO G191)))
           (SEQ
            (EXIT
             (COND
              ((NULL
                (SPADCALL
                 (SPADCALL |l| (QREFELT |$| 18))
                 (SPADCALL |p| (QREFELT |$| 18))
                 |f|))
                (PROGN (LETT #1# (QUOTE NIL) |LSAGG-;sorted?;MAB;15|) (GO #1#)))
               ((QUOTE T)
                (LETT |p|
                 (SPADCALL
                  (LETT |l| |p| |LSAGG-;sorted?;MAB;15|)
                  (QREFELT |$| 17))
                 |LSAGG-;sorted?;MAB;15|))))))
            NIL
            (GO G190)
            G191
            (EXIT NIL))
           (EXIT (QUOTE T))))))
       #1#
       (EXIT #1#)))))

(DEFUN |LSAGG-;reduce;MA2S;16| (|f| |x| |i| |$|)
 (PROG (|r|)
  (RETURN
   (SEQ
    (LETT |r| |i| |LSAGG-;reduce;MA2S;16|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |x| (QREFELT |$| 16)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (LETT |r|
```

```
        (SPADCALL |r| (SPADCALL |x| (QREFELT |$| 18)) |f|)
        |LSAGG-;reduce;MA2S;16|)
       (EXIT
        (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;reduce;MA2S;16|)))
      NIL
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT |r|)))))

(DEFUN |LSAGG-;reduce;MA3S;17| (|f| |x| |i| |a| |$|)
 (PROG (|r|)
  (RETURN
   (SEQ
    (LETT |r| |i| |LSAGG-;reduce;MA3S;17|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((OR
            (SPADCALL |x| (QREFELT |$| 16))
            (SPADCALL |r| |a| (QREFELT |$| 51)))
           (QUOTE NIL))
         ((QUOTE T) (QUOTE T)))) (GO G191)))
     (SEQ
      (LETT |r|
       (SPADCALL |r| (SPADCALL |x| (QREFELT |$| 18)) |f|)
       |LSAGG-;reduce;MA3S;17|)
      (EXIT
       (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;reduce;MA3S;17|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |r|)))))

(DEFUN |LSAGG-;new;NniSA;18| (|n| |s| |$|)
 (PROG (|k| |l|)
  (RETURN
   (SEQ
    (LETT |l| (SPADCALL (QREFELT |$| 12)) |LSAGG-;new;NniSA;18|)
    (SEQ
     (LETT |k| 1 |LSAGG-;new;NniSA;18|)
     G190
     (COND ((QSGREATERP |k| |n|) (GO G191)))
```

```
      (SEQ
       (EXIT
        (LETT |l| (SPADCALL |s| |l| (QREFELT |$| 13)) |LSAGG-;new;NniSA;18|)))
      (LETT |k| (QSADD1 |k|) |LSAGG-;new;NniSA;18|)
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT |l|)))))

(DEFUN |LSAGG-;map;M3A;19| (|f| |x| |y| |$|)
 (PROG (|z|)
  (RETURN
   (SEQ
    (LETT |z| (SPADCALL (QREFELT |$| 12)) |LSAGG-;map;M3A;19|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((OR (SPADCALL |x| (QREFELT |$| 16)) (SPADCALL |y| (QREFELT |$| 16)))
          (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (LETT |z|
       (SPADCALL
        (SPADCALL
         (SPADCALL |x| (QREFELT |$| 18))
         (SPADCALL |y| (QREFELT |$| 18))
         |f|)
        |z|
        (QREFELT |$| 13))
       |LSAGG-;map;M3A;19|)
      (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;map;M3A;19|)
      (EXIT (LETT |y| (SPADCALL |y| (QREFELT |$| 17)) |LSAGG-;map;M3A;19|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (SPADCALL |z| (QREFELT |$| 47)))))))

(DEFUN |LSAGG-;reverse!;2A;20| (|x| |$|)
 (PROG (|z| |y|)
  (RETURN
   (SEQ
    (COND
```

```
       ((OR
         (SPADCALL |x| (QREFELT |$| 16))
         (SPADCALL
          (LETT |y| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;reverse!;2A;20|)
          (QREFELT |$| 16)))
        |x|)
       ((QUOTE T)
        (SEQ
         (SPADCALL |x| (SPADCALL (QREFELT |$| 12)) (QREFELT |$| 25))
         (SEQ
          G190
          (COND
           ((NULL
             (COND
              ((SPADCALL |y| (QREFELT |$| 16)) (QUOTE NIL))
              ((QUOTE T) (QUOTE T))))
            (GO G191)))
          (SEQ
           (LETT |z| (SPADCALL |y| (QREFELT |$| 17)) |LSAGG-;reverse!;2A;20|)
           (SPADCALL |y| |x| (QREFELT |$| 25))
           (LETT |x| |y| |LSAGG-;reverse!;2A;20|)
           (EXIT (LETT |y| |z| |LSAGG-;reverse!;2A;20|)))
           NIL
           (GO G190)
           G191
           (EXIT NIL))
         (EXIT |x|)))))))

(DEFUN |LSAGG-;copy;2A;21| (|x| |$|)
 (PROG (|k| |y|)
  (RETURN
   (SEQ
    (LETT |y| (SPADCALL (QREFELT |$| 12)) |LSAGG-;copy;2A;21|)
    (SEQ
     (LETT |k| 0 |LSAGG-;copy;2A;21|)
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |x| (QREFELT |$| 16)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (COND
       ((EQL |k| 1000)
        (COND
```

```
            ((SPADCALL |x| (QREFELT |$| 56)) (EXIT (|error| "cyclic list"))))))
       (LETT |y|
        (SPADCALL (SPADCALL |x| (QREFELT |$| 18)) |y| (QREFELT |$| 13))
        |LSAGG-;copy;2A;21|)
       (EXIT (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;copy;2A;21|)))
      (LETT |k| (QSADD1 |k|) |LSAGG-;copy;2A;21|)
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT (SPADCALL |y| (QREFELT |$| 47)))))))

(DEFUN |LSAGG-;copyInto!;2AIA;22| (|y| |x| |s| |$|)
 (PROG (|m| #1=#:G87636 |z|)
  (RETURN
   (SEQ
    (LETT |m| (SPADCALL |y| (QREFELT |$| 31)) |LSAGG-;copyInto!;2AIA;22|)
    (EXIT
     (COND
      ((|<| |s| |m|) (|error| "index out of range"))
      ((QUOTE T)
       (SEQ
        (LETT |z|
         (SPADCALL |y|
          (PROG1
           (LETT #1# (|-| |s| |m|) |LSAGG-;copyInto!;2AIA;22|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 32))
         |LSAGG-;copyInto!;2AIA;22|)
        (SEQ
         G190
         (COND
          ((NULL
            (COND
             ((OR
               (SPADCALL |z| (QREFELT |$| 16))
               (SPADCALL |x| (QREFELT |$| 16)))
              (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (SPADCALL |z| (SPADCALL |x| (QREFELT |$| 18)) (QREFELT |$| 58))
          (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;copyInto!;2AIA;22|)
          (EXIT
           (LETT |z|
            (SPADCALL |z| (QREFELT |$| 17))
            |LSAGG-;copyInto!;2AIA;22|)))
```

```
          NIL
          (GO G190)
          G191
          (EXIT NIL))
         (EXIT |y|)))))))))

(DEFUN |LSAGG-;position;SA2I;23| (|w| |x| |s| |$|)
 (PROG (|m| #1=#:G87644 |k|)
  (RETURN
   (SEQ
    (LETT |m| (SPADCALL |x| (QREFELT |$| 31)) |LSAGG-;position;SA2I;23|)
    (EXIT
     (COND
      ((|<| |s| |m|) (|error| "index out of range"))
      ((QUOTE T)
       (SEQ
        (LETT |x|
         (SPADCALL |x|
          (PROG1
           (LETT #1# (|-| |s| |m|) |LSAGG-;position;SA2I;23|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 32))
         |LSAGG-;position;SA2I;23|)
        (SEQ
         (LETT |k| |s| |LSAGG-;position;SA2I;23|)
         G190
         (COND
          ((NULL
            (COND
             ((OR
               (SPADCALL |x| (QREFELT |$| 16))
               (SPADCALL |w| (SPADCALL |x| (QREFELT |$| 18)) (QREFELT |$| 51)))
              (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (EXIT
           (LETT |x|
            (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;position;SA2I;23|)))
         (LETT |k| (|+| |k| 1) |LSAGG-;position;SA2I;23|)
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT
         (COND
          ((SPADCALL |x| (QREFELT |$| 16))
```

```
            (|-| (SPADCALL |x| (QREFELT |$| 31)) 1))
           ((QUOTE T) |k|))))))))))))

(DEFUN |LSAGG-;removeDuplicates!;2A;24| (|l| |$|)
 (PROG (|p|)
  (RETURN
   (SEQ
    (LETT |p| |l| |LSAGG-;removeDuplicates!;2A;24|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |p| (QREFELT |$| 16)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (EXIT
       (LETT |p|
        (SPADCALL |p|
         (SPADCALL
          (CONS
           (FUNCTION |LSAGG-;removeDuplicates!;2A;24!0|)
           (VECTOR |$| |p|))
          (SPADCALL |p| (QREFELT |$| 17))
          (QREFELT |$| 61))
         (QREFELT |$| 25))
        |LSAGG-;removeDuplicates!;2A;24|)))
      NIL
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT |l|)))))

(DEFUN |LSAGG-;removeDuplicates!;2A;24!0| (|#1| |$$|)
 (PROG (|$|)
  (LETT |$| (QREFELT |$$| 0) |LSAGG-;removeDuplicates!;2A;24|)
  (RETURN
   (PROGN
    (SPADCALL |#1|
     (SPADCALL (QREFELT |$$| 1) (QREFELT |$| 18))
     (QREFELT |$| 51))))))

(DEFUN |LSAGG-;<;2AB;25| (|x| |y| |$|)
 (PROG (#1=#:G87662)
  (RETURN
```

```
(SEQ
 (EXIT
  (SEQ
   (SEQ
    G190
    (COND
     ((NULL
       (COND
        ((OR
          (SPADCALL |x| (QREFELT |$| 16))
          (SPADCALL |y| (QREFELT |$| 16)))
         (QUOTE NIL))
        ((QUOTE T) (QUOTE T))))
      (GO G191)))
    (SEQ
     (EXIT
      (COND
       ((NULL
         (SPADCALL
          (SPADCALL |x| (QREFELT |$| 18))
          (SPADCALL |y| (QREFELT |$| 18))
          (QREFELT |$| 51)))
        (PROGN
         (LETT #1#
           (SPADCALL
            (SPADCALL |x| (QREFELT |$| 18))
            (SPADCALL |y| (QREFELT |$| 18))
           (QREFELT |$| 63))
          |LSAGG-;<;2AB;25|)
         (GO #1#)))
       ((QUOTE T)
        (SEQ
         (LETT |x| (SPADCALL |x| (QREFELT |$| 17)) |LSAGG-;<;2AB;25|)
         (EXIT
          (LETT |y| (SPADCALL |y| (QREFELT |$| 17)) |LSAGG-;<;2AB;25|)))))))
    NIL
    (GO G190)
    G191
    (EXIT NIL))
   (EXIT
    (COND
     ((SPADCALL |x| (QREFELT |$| 16))
       (COND
        ((SPADCALL |y| (QREFELT |$| 16)) (QUOTE NIL))
        ((QUOTE T) (QUOTE T))))
     ((QUOTE T) (QUOTE NIL))))))
```

```
     #1#
     (EXIT #1#)))))

(DEFUN |ListAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|ListAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |ListAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (GETREFV 66) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|HasCategory| |#2| (QUOTE (|SetCategory|)))
       (QSETREFV |$| 52
         (CONS (|dispatchFunction| |LSAGG-;reduce;MA3S;17|) |$|))))
    (COND
     ((|HasCategory| |#2| (QUOTE (|SetCategory|)))
       (PROGN
        (QSETREFV |$| 60
         (CONS (|dispatchFunction| |LSAGG-;position;SA2I;23|) |$|))
        (QSETREFV |$| 62
         (CONS (|dispatchFunction| |LSAGG-;removeDuplicates!;2A;24|) |$|)))))
    (COND
     ((|HasCategory| |#2| (QUOTE (|OrderedSet|)))
       (QSETREFV |$| 64 (CONS (|dispatchFunction| |LSAGG-;<;2AB;25|) |$|))))
    |$|))))

(MAKEPROP
 (QUOTE |ListAggregate&|)
 (QUOTE |infovec|)
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
   (|NonNegativeInteger|) (0 . |#|) (|Mapping| 15 7 7) |LSAGG-;sort!;M2A;1|
   (5 . |empty|) (9 . |concat|) |LSAGG-;list;SA;2| (|Boolean|)
   (15 . |empty?|) (20 . |rest|) (25 . |first|) (|Mapping| 7 7 7)
   (30 . |reduce|) |LSAGG-;reduce;MAS;3| (37 . |copy|) (42 . |merge!|)
   |LSAGG-;merge;M3A;4| (49 . |setrest!|) (|Mapping| 15 7)
   |LSAGG-;select!;M2A;5| (55 . |eq?|) |LSAGG-;merge!;M3A;6|
   (|Integer|) (61 . |minIndex|) (66 . |rest|) |LSAGG-;insert!;SAIA;7|
   (72 . |concat!|) |LSAGG-;insert!;2AIA;8| |LSAGG-;remove!;M2A;9|
   |LSAGG-;delete!;AIA;10| (|UniversalSegment| 30) (78 . |lo|)
```

```
(83 . |hasHi|) (88 . |hi|) (93 . |maxIndex|) |LSAGG-;delete!;AUsA;11|
(|Union| 7 (QUOTE "failed")) |LSAGG-;find;MAU;12|
|LSAGG-;position;MAI;13| (98 . |reverse!|) (103 . |split!|)
|LSAGG-;sorted?;MAB;15| |LSAGG-;reduce;MA2S;16| (109 . |=|)
(115 . |reduce|) |LSAGG-;new;NniSA;18| |LSAGG-;map;M3A;19|
|LSAGG-;reverse!;2A;20| (123 . |cyclic?|) |LSAGG-;copy;2A;21|
(128 . |setfirst!|) |LSAGG-;copyInto!;2AIA;22| (134 . |position|)
(141 . |remove!|) (147 . |removeDuplicates!|) (152 . |<|) (158 . |<|)
(|Mapping| 7 7)))
(QUOTE #(|sorted?| 164 |sort!| 170 |select!| 176 |reverse!| 182
 |removeDuplicates!| 187 |remove!| 192 |reduce| 198 |position| 219
 |new| 232 |merge!| 238 |merge| 245 |map| 252 |list| 259 |insert!|
 264 |find| 278 |delete!| 284 |copyInto!| 296 |copy| 303 |<| 308))
(QUOTE NIL)
(CONS
 (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS
  (QUOTE #())
  (CONS
   (QUOTE #())
   (|makeByteWordVec2| 64 (QUOTE (1 6 8 0 9 0 6 0 12 2 6 0 7 0 13 1 6
    15 0 16 1 6 0 0 17 1 6 7 0 18 3 6 7 19 0 7 20 1 6 0 0 22 3 6 0 10
    0 0 23 2 6 0 0 0 25 2 6 15 0 0 28 1 6 30 0 31 2 6 0 0 8 32 2 6 0 0
    0 34 1 38 30 0 39 1 38 15 0 40 1 38 30 0 41 1 6 30 0 42 1 6 0 0 47
    2 6 0 0 30 48 2 7 15 0 0 51 4 0 7 19 0 7 7 52 1 6 15 0 56 2 6 7 0
    7 58 3 0 30 7 0 30 60 2 6 0 26 0 61 1 0 0 0 62 2 7 15 0 0 63 2 0 15
    0 0 64 2 0 15 10 0 49 2 0 0 10 0 11 2 0 0 26 0 27 1 0 0 0 55 1 0 0
    0 62 2 0 0 26 0 36 3 0 7 19 0 7 50 4 0 7 19 0 7 7 52 2 0 7 19 0 21
    2 0 30 26 0 46 3 0 30 7 0 30 60 2 0 0 8 7 53 3 0 0 10 0 0 29 3 0 0
    10 0 0 24 3 0 0 19 0 0 54 1 0 0 7 14 3 0 0 7 0 30 33 3 0 0 0 0 30
    35 2 0 44 26 0 45 2 0 0 0 38 43 2 0 0 0 30 37 3 0 0 0 0 30 59 1 0
    0 0 57 2 0 15 0 0 64))))))
(QUOTE |lookupComplete|)))
```

21.37 MONOID.lsp BOOTSTRAP

MONOID depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **MONOID** category which we can write into the **MID** directory. We compile the lisp code and copy the **MONOID.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*MONOID.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |Monoid;AL| (QUOTE NIL))

(DEFUN |Monoid| NIL
  (LET (#:G82434)
    (COND
      (|Monoid;AL|)
      (T (SETQ |Monoid;AL| (|Monoid;|))))))

(DEFUN |Monoid;| NIL
  (PROG (#1=#:G82432)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|SemiGroup|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|One| (|$|) |constant|) T)
                ((|sample| (|$|) |constant|) T)
                ((|one?| ((|Boolean|) |$|)) T)
                ((|**| (|$| |$| (|NonNegativeInteger|))) T)
                ((|^| (|$| |$| (|NonNegativeInteger|))) T)
                ((|recip| ((|Union| |$| "failed") |$|)) T)))
              NIL
              (QUOTE ((|NonNegativeInteger|) (|Boolean|)))
              NIL))
            |Monoid|)
          (SETELT #1# 0 (QUOTE (|Monoid|)))))))

(MAKEPROP (QUOTE |Monoid|) (QUOTE NILADIC) T)
```

21.38 MONOID-.lsp BOOTSTRAP

MONOID- depends on **MONOID**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **MONOID-** category which we can write into the **MID** directory. We compile the lisp code and copy the **MONOID-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*MONOID-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |MONOID-;^;SNniS;1| (|x| |n| |$|)
  (SPADCALL |x| |n| (QREFELT |$| 8)))

(DEFUN |MONOID-;one?;SB;2| (|x| |$|)
  (SPADCALL |x| (|spadConstant| |$| 10) (QREFELT |$| 12)))

(DEFUN |MONOID-;sample;S;3| (|$|)
  (|spadConstant| |$| 10))

(DEFUN |MONOID-;recip;SU;4| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 15)) (CONS 0 |x|))
    ((QUOTE T) (CONS 1 "failed"))))

(DEFUN |MONOID-;**;SNniS;5| (|x| |n| |$|)
  (COND
    ((ZEROP |n|) (|spadConstant| |$| 10))
    ((QUOTE T) (SPADCALL |x| |n| (QREFELT |$| 20)))))

(DEFUN |Monoid&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|Monoid&|))
        (LETT |dv$| (LIST (QUOTE |Monoid&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 22) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(MAKEPROP
```

```
(QUOTE |Monoid&|)
(QUOTE |infovec|)
(LIST
  (QUOTE
    #(NIL NIL NIL NIL NIL NIL
      (|local| |#1|)
      (|NonNegativeInteger|)
      (0 . |**|)
      |MONOID-;^;SNniS;1|
      (6 . |One|)
      (|Boolean|)
      (10 . |=|)
      |MONOID-;one?;SB;2|
      |MONOID-;sample;S;3|
      (16 . |one?|)
      (|Union| |$| (QUOTE "failed"))
      |MONOID-;recip;SU;4|
      (|PositiveInteger|)
      (|RepeatedSquaring| 6)
      (21 . |expt|)
      |MONOID-;**;SNniS;5|))
  (QUOTE #(|sample| 27 |recip| 31 |one?| 36 |^| 41 |**| 47))
  (QUOTE NIL)
  (CONS
    (|makeByteWordVec2| 1 (QUOTE NIL))
    (CONS
      (QUOTE #())
      (CONS
        (QUOTE #())
        (|makeByteWordVec2| 21
          (QUOTE
            (2 6 0 0 7 8 0 6 0 10 2 6 11 0 0 12 1 6 11 0 15 2 19 6 6 18 20
            0 0 0 14 1 0 16 0 17 1 0 11 0 13 2 0 0 0 7 9 2 0 0 0 7 21))))))
(QUOTE |lookupComplete|)))
```

21.39 MTSCAT.lsp BOOTSTRAP

MTSCAT depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **MTSCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **MTSCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*MTSCAT.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |MultivariateTaylorSeriesCategory;CAT| (QUOTE NIL))

(SETQ |MultivariateTaylorSeriesCategory;AL| (QUOTE NIL))

(DEFUN |MultivariateTaylorSeriesCategory|
 (|&REST| #1=#:G83334 |&AUX| #2=#:G83332)
   (DSETQ #2# #1#)
   (LET (#3=#:G83333)
    (COND
     ((SETQ #3#
       (|assoc| (|devaluateList| #2#) |MultivariateTaylorSeriesCategory;AL|))
        (CDR #3#))
     (T
      (SETQ |MultivariateTaylorSeriesCategory;AL|
       (|cons5|
        (CONS
         (|devaluateList| #2#)
         (SETQ #3# (APPLY (FUNCTION |MultivariateTaylorSeriesCategory;|) #2#)))
        |MultivariateTaylorSeriesCategory;AL|))
       #3#))))

(DEFUN |MultivariateTaylorSeriesCategory;| (|t#1| |t#2|)
 (PROG (#1=#:G83331)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1| |t#2|)) (LIST (|devaluate| |t#1|) (|devaluate| |t#2|)))
      (|sublisV|
       (PAIR (QUOTE (#2=#:G83330)) (LIST (QUOTE (|IndexedExponents| |t#2|))))
       (COND
        (|MultivariateTaylorSeriesCategory;CAT|)
        ((QUOTE T)
```

```
(LETT |MultivariateTaylorSeriesCategory;CAT|
 (|Join|
  (|PartialDifferentialRing| (QUOTE |t#2|))
  (|PowerSeriesCategory| (QUOTE |t#1|) (QUOTE #2#) (QUOTE |t#2|))
  (|InnerEvalable| (QUOTE |t#2|) (QUOTE |$|))
  (|Evalable| (QUOTE |$|))
  (|mkCategory|
   (QUOTE |domain|)
   (QUOTE (
    ((|coefficient| (|$| |$| |t#2| (|NonNegativeInteger|))) T)
    ((|coefficient|
        (|$| |$| (|List| |t#2|) (|List| (|NonNegativeInteger|))))
        T)
    ((|extend| (|$| |$| (|NonNegativeInteger|))) T)
    ((|monomial| (|$| |$| |t#2| (|NonNegativeInteger|))) T)
    ((|monomial|
       (|$| |$| (|List| |t#2|) (|List| (|NonNegativeInteger|))))
        T)
    ((|order| ((|NonNegativeInteger|) |$| |t#2|)) T)
    ((|order|
       ((|NonNegativeInteger|) |$| |t#2| (|NonNegativeInteger|)))
        T)
    ((|polynomial|
      ((|Polynomial| |t#1|) |$| (|NonNegativeInteger|)))
       T)
    ((|polynomial|
      ((|Polynomial| |t#1|)
       |$|
       (|NonNegativeInteger|)
       (|NonNegativeInteger|)))
        T)
    ((|integrate| (|$| |$| |t#2|))
       (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))))
   (QUOTE (
    ((|RadicalCategory|)
       (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))
    ((|TranscendentalFunctionCategory|)
       (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))))
   (QUOTE
    ((|Polynomial| |t#1|)
     (|NonNegativeInteger|)
     (|List| |t#2|)
     (|List| (|NonNegativeInteger|))))
    NIL))
 . #3=(|MultivariateTaylorSeriesCategory|))))))
. #3#)
```

```
    (SETELT #1# 0
     (LIST
      (QUOTE |MultivariateTaylorSeriesCategory|)
      (|devaluate| |t#1|)
      (|devaluate| |t#2|)))))))
```

21.40 OINTDOM.lsp BOOTSTRAP

OINTDOM depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **OINTDOM** category which we can write into the **MID** directory. We compile the lisp code and copy the **OINTDOM.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*OINTDOM.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |OrderedIntegralDomain;AL| (QUOTE NIL))

(DEFUN |OrderedIntegralDomain| NIL
  (LET (#:G84531)
    (COND
      (|OrderedIntegralDomain;AL|)
      (T (SETQ |OrderedIntegralDomain;AL| (|OrderedIntegralDomain;|))))))

(DEFUN |OrderedIntegralDomain;| NIL
  (PROG (#1=#:G84529)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join| (|IntegralDomain|) (|OrderedRing|)) |OrderedIntegralDomain|)
        (SETELT #1# 0 (QUOTE (|OrderedIntegralDomain|)))))))

(MAKEPROP (QUOTE |OrderedIntegralDomain|) (QUOTE NILADIC) T)
```

21.41 ORDRING.lsp BOOTSTRAP

ORDRING depends on **INT**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ORDRING** category which we can write into the **MID** directory. We compile the lisp code and copy the **ORDRING.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Technically I can't justify this bootstrap stanza based on the lattice since **INT** is already bootstrapped. However using **INT** naked generates a "value stack overflow" error suggesting an infinite recursive loop. This code is here to experiment with breaking that loop.

Note that this code is not included in the generated catdef.spad file.

⟨*ORDRING.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |OrderedRing;AL| (QUOTE NIL))

(DEFUN |OrderedRing| NIL
  (LET (#:G84457)
    (COND
      (|OrderedRing;AL|)
      (T (SETQ |OrderedRing;AL| (|OrderedRing;|))))))

(DEFUN |OrderedRing;| NIL
  (PROG (#1=#:G84455)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|OrderedAbelianGroup|)
            (|Ring|)
            (|Monoid|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|positive?| ((|Boolean|) |$|)) T)
                ((|negative?| ((|Boolean|) |$|)) T)
                ((|sign| ((|Integer|) |$|)) T)
                ((|abs| (|$| |$|)) T)))
              NIL
              (QUOTE ((|Integer|) (|Boolean|)))
              NIL))
           |OrderedRing|)
```

```
          (SETELT #1# 0 (QUOTE (|OrderedRing|))))))))

  (MAKEPROP (QUOTE |OrderedRing|) (QUOTE NILADIC) T)
```

21.42 ORDRING-.lsp BOOTSTRAP

ORDRING- depends on **ORDRING**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ORDRING-** category which we can write into the **MID** directory. We compile the lisp code and copy the **ORDRING-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ORDRING-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |ORDRING-;positive?;SB;1| (|x| |$|)
  (SPADCALL (|spadConstant| |$| 7) |x| (QREFELT |$| 9)))

(DEFUN |ORDRING-;negative?;SB;2| (|x| |$|)
  (SPADCALL |x| (|spadConstant| |$| 7) (QREFELT |$| 9)))

(DEFUN |ORDRING-;sign;SI;3| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 12)) 1)
    ((SPADCALL |x| (QREFELT |$| 13)) -1)
    ((SPADCALL |x| (QREFELT |$| 15)) 0)
    ((QUOTE T)
      (|error| "x satisfies neither positive?, negative? or zero?"))))

(DEFUN |ORDRING-;abs;2S;4| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 12)) |x|)
    ((SPADCALL |x| (QREFELT |$| 13)) (SPADCALL |x| (QREFELT |$| 18)))
    ((SPADCALL |x| (QREFELT |$| 15)) (|spadConstant| |$| 7))
    ((QUOTE T)
      (|error| "x satisfies neither positive?, negative? or zero?"))))

(DEFUN |OrderedRing&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|OrderedRing&|))
        (LETT |dv$| (LIST (QUOTE |OrderedRing&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 20) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
```

```
        |$|)))))

  (MAKEPROP
    (QUOTE |OrderedRing&|)
    (QUOTE |infovec|)
    (LIST
      (QUOTE
        #(NIL NIL NIL NIL NIL NIL
          (|local| |#1|)
          (0 . |Zero|)
          (|Boolean|)
          (4 . |<|)
          |ORDRING-;positive?;SB;1|
          |ORDRING-;negative?;SB;2|
          (10 . |positive?|)
          (15 . |negative?|)
          (20 . |One|)
          (24 . |zero?|)
          (|Integer|)
          |ORDRING-;sign;SI;3|
          (29 . |-|)
          |ORDRING-;abs;2S;4|))
      (QUOTE #(|sign| 34 |positive?| 39 |negative?| 44 |abs| 49))
      (QUOTE NIL)
      (CONS
        (|makeByteWordVec2| 1 (QUOTE NIL))
        (CONS
          (QUOTE #())
          (CONS
            (QUOTE #())
            (|makeByteWordVec2| 19
              (QUOTE
                (0 6 0 7 2 6 8 0 0 9 1 6 8 0 12 1 6 8 0 13 0 6 0 14 1 6 8 0 15
                 1 6 0 0 18 1 0 16 0 17 1 0 8 0 10 1 0 8 0 11 1 0 0 0 19))))))
      (QUOTE |lookupComplete|)))
```

21.43 POLYCAT.lsp BOOTSTRAP

POLYCAT depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **POLYCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **POLYCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*POLYCAT.lsp BOOTSTRAP*⟩≡

```
(/VERSIONCHECK 2)

(SETQ |PolynomialCategory;CAT| (QUOTE NIL))

(SETQ |PolynomialCategory;AL| (QUOTE NIL))

(DEFUN |PolynomialCategory| (&REST #0=#:G1430 &AUX #1=#:G1428)
 (DSETQ #1# #0#)
 (LET (#2=#:G1429)
  (COND
   ((SETQ #2# (|assoc| (|devaluateList| #1#) |PolynomialCategory;AL|))
     (CDR #2#))
   (T
     (SETQ |PolynomialCategory;AL|
      (|cons5|
       (CONS (|devaluateList| #1#)
        (SETQ #2# (APPLY (FUNCTION |PolynomialCategory;|) #1#)))
       |PolynomialCategory;AL|))
     #2#))))

(DEFUN |PolynomialCategory;| (|t#1| |t#2| |t#3|)
 (PROG (#0=#:G1427)
  (RETURN
   (PROG1
    (LETT #0#
     (|sublisV|
      (PAIR (QUOTE (|t#1| |t#2| |t#3|)) (LIST (|devaluate| |t#1|) (|devaluate| |t#2|) (|deva
      (COND
       (|PolynomialCategory;CAT|)
       ((QUOTE T)
        (LETT |PolynomialCategory;CAT|
         (|Join|
          (|PartialDifferentialRing| (QUOTE |t#3|))
          (|FiniteAbelianMonoidRing| (QUOTE |t#1|) (QUOTE |t#2|))
          (|Evalable| (QUOTE $))
```

```
(|InnerEvalable| (QUOTE |t#3|) (QUOTE |t#1|))
(|InnerEvalable| (QUOTE |t#3|) (QUOTE $))
(|RetractableTo| (QUOTE |t#3|))
(|FullyLinearlyExplicitRingOver| (QUOTE |t#1|))
(|mkCategory| (QUOTE |domain|)
 (QUOTE
  ((((|degree| ((|NonNegativeInteger|) $ |t#3|)) T)
   ((|degree| ((|List| (|NonNegativeInteger|)) $ (|List| |t#3|))) T)
   ((|coefficient| ($ $ |t#3| (|NonNegativeInteger|))) T)
   ((|coefficient| ($ $ (|List| |t#3|)
     (|List| (|NonNegativeInteger|)))) T)
   ((|monomials| ((|List| $) $)) T)
   ((|univariate| ((|SparseUnivariatePolynomial| $) $ |t#3|)) T)
   ((|univariate| ((|SparseUnivariatePolynomial| |t#1|) $)) T)
   ((|mainVariable| ((|Union| |t#3| "failed") $)) T)
   ((|minimumDegree| ((|NonNegativeInteger|) $ |t#3|)) T)
   ((|minimumDegree| ((|List| (|NonNegativeInteger|)) $
     (|List| |t#3|))) T)
   ((|monicDivide|
     ((|Record| (|:| |quotient| $) (|:| |remainder| $)) $ $ |t#3|))
     T)
   ((|monomial| ($ $ |t#3| (|NonNegativeInteger|))) T)
   ((|monomial| ($ $ (|List| |t#3|) (|List| (|NonNegativeInteger|))))
     T)
   ((|multivariate| ($ (|SparseUnivariatePolynomial| |t#1|) |t#3|))
     T)
   ((|multivariate| ($ (|SparseUnivariatePolynomial| $) |t#3|)) T)
   ((|isPlus| ((|Union| (|List| $) "failed") $)) T)
   ((|isTimes| ((|Union| (|List| $) "failed") $)) T)
   ((|isExpt|
     ((|Union|
      (|Record| (|:| |var| |t#3|)
                (|:| |exponent| (|NonNegativeInteger|)))
      "failed") $))
     T)
   ((|totalDegree| ((|NonNegativeInteger|) $)) T)
   ((|totalDegree| ((|NonNegativeInteger|) $ (|List| |t#3|))) T)
   ((|variables| ((|List| |t#3|) $)) T)
   ((|primitiveMonomials| ((|List| $) $)) T)
   ((|resultant| ($ $ $ |t#3|)) (|has| |t#1| (|CommutativeRing|)))
   ((|discriminant| ($ $ |t#3|)) (|has| |t#1| (|CommutativeRing|)))
   ((|content| ($ $ |t#3|)) (|has| |t#1| (|GcdDomain|)))
   ((|primitivePart| ($ $)) (|has| |t#1| (|GcdDomain|)))
   ((|primitivePart| ($ $ |t#3|)) (|has| |t#1| (|GcdDomain|)))
   ((|squareFree| ((|Factored| $) $)) (|has| |t#1| (|GcdDomain|)))
   ((|squareFreePart| ($ $)) (|has| |t#1| (|GcdDomain|)))))
```

```
        (QUOTE
         ((((|OrderedSet|) (|has| |t#1| (|OrderedSet|)))
          ((|ConvertibleTo| (|InputForm|))
           (AND (|has| |t#3| (|ConvertibleTo| (|InputForm|)))
                (|has| |t#1| (|ConvertibleTo| (|InputForm|)))))
          ((|ConvertibleTo| (|Pattern| (|Integer|)))
           (AND (|has| |t#3| (|ConvertibleTo| (|Pattern| (|Integer|))))
                (|has| |t#1| (|ConvertibleTo| (|Pattern| (|Integer|))))))
          ((|ConvertibleTo| (|Pattern| (|Float|)))
           (AND (|has| |t#3| (|ConvertibleTo| (|Pattern| (|Float|))))
                (|has| |t#1| (|ConvertibleTo| (|Pattern| (|Float|))))))
          ((|PatternMatchable| (|Integer|))
           (AND
            (|has| |t#3| (|PatternMatchable| (|Integer|)))
            (|has| |t#1| (|PatternMatchable| (|Integer|)))))
          ((|PatternMatchable| (|Float|))
           (AND
            (|has| |t#3| (|PatternMatchable| (|Float|)))
            (|has| |t#1| (|PatternMatchable| (|Float|)))))
          ((|GcdDomain|) (|has| |t#1| (|GcdDomain|)))
          (|canonicalUnitNormal|
            (|has| |t#1| (ATTRIBUTE |canonicalUnitNormal|)))
          ((|PolynomialFactorizationExplicit|)
            (|has| |t#1| (|PolynomialFactorizationExplicit|)))))
        (QUOTE
         ((|Factored| $)
          (|List| $)
          (|List| |t#3|)
          (|NonNegativeInteger|)
          (|SparseUnivariatePolynomial| $)
          (|SparseUnivariatePolynomial| |t#1|)
          (|List| (|NonNegativeInteger|))))
         NIL))
       . #1=(|PolynomialCategory|)))))
  . #1#)
(SETELT #0# 0
 (LIST (QUOTE |PolynomialCategory|)
  (|devaluate| |t#1|) (|devaluate| |t#2|) (|devaluate| |t#3|)))))))
```

21.44 POLYCAT-.lsp BOOTSTRAP

POLYCAT- depends on **POLYCAT**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **POLYCAT-** category which we can write into the **MID** directory. We compile the lisp code and copy the **POLYCAT-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*POLYCAT-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)


(/VERSIONCHECK 2)

(DEFUN |POLYCAT-;eval;SLS;1| (|p| |l| $)
 (PROG (#0=#:G1444 #1=#:G1438 #2=#:G1445 #3=#:G1446 |lvar| #4=#:G1447
        |e| #5=#:G1448)
  (RETURN
   (SEQ
    (COND
     ((NULL |l|) |p|)
     ((QUOTE T)
      (SEQ
       (SEQ
        (EXIT
         (SEQ
          (LETT |e| NIL |POLYCAT-;eval;SLS;1|)
          (LETT #0# |l| |POLYCAT-;eval;SLS;1|)
          G190
          (COND
           ((OR (ATOM #0#)
                (PROGN (LETT |e| (CAR #0#) |POLYCAT-;eval;SLS;1|) NIL))
            (GO G191)))
          (SEQ
           (EXIT
            (COND
             ((QEQCAR
               (SPADCALL (SPADCALL |e| (QREFELT $ 11)) (QREFELT $ 13)) 1)
              (PROGN
               (LETT #1#
                (|error| "cannot find a variable to evaluate")
                |POLYCAT-;eval;SLS;1|)
               (GO #1#))))))
          (LETT #0# (CDR #0#) |POLYCAT-;eval;SLS;1|)
```

```
    (GO G190)
    G191
    (EXIT NIL)))
  #1# (EXIT #1#))
 (LETT |lvar|
  (PROGN
   (LETT #2# NIL |POLYCAT-;eval;SLS;1|)
   (SEQ
    (LETT |e| NIL |POLYCAT-;eval;SLS;1|)
    (LETT #3# |l| |POLYCAT-;eval;SLS;1|)
    G190
    (COND
     ((OR (ATOM #3#)
          (PROGN (LETT |e| (CAR #3#) |POLYCAT-;eval;SLS;1|) NIL))
      (GO G191)))
    (SEQ
     (EXIT
      (LETT #2#
       (CONS (SPADCALL (SPADCALL |e| (QREFELT $ 11)) (QREFELT $ 14))
             #2#)
       |POLYCAT-;eval;SLS;1|)))
    (LETT #3# (CDR #3#) |POLYCAT-;eval;SLS;1|)
    (GO G190)
    G191
    (EXIT (NREVERSE0 #2#))))
   |POLYCAT-;eval;SLS;1|)
 (EXIT
  (SPADCALL |p| |lvar|
   (PROGN
    (LETT #4# NIL |POLYCAT-;eval;SLS;1|)
    (SEQ
     (LETT |e| NIL |POLYCAT-;eval;SLS;1|)
     (LETT #5# |l| |POLYCAT-;eval;SLS;1|)
     G190
     (COND
      ((OR (ATOM #5#)
           (PROGN (LETT |e| (CAR #5#) |POLYCAT-;eval;SLS;1|) NIL))
       (GO G191)))
     (SEQ
      (EXIT
       (LETT #4# (CONS (SPADCALL |e| (QREFELT $ 15)) #4#)
        |POLYCAT-;eval;SLS;1|)))
     (LETT #5# (CDR #5#) |POLYCAT-;eval;SLS;1|)
     (GO G190)
     G191
     (EXIT (NREVERSE0 #4#))))
```

```
          (QREFELT $ 18)))))))))))

  (DEFUN |POLYCAT-;monomials;SL;2| (|p| $)
   (PROG (|ml|)
    (RETURN
     (SEQ
      (LETT |ml| NIL |POLYCAT-;monomials;SL;2|)
      (SEQ G190
       (COND
        ((NULL
          (COND
           ((SPADCALL |p| (|spadConstant| $ 21) (QREFELT $ 24)) (QUOTE NIL))
           ((QUOTE T) (QUOTE T))))
         (GO G191)))
       (SEQ
        (LETT |ml|
         (CONS (SPADCALL |p| (QREFELT $ 25)) |ml|)
         |POLYCAT-;monomials;SL;2|)
        (EXIT
         (LETT |p| (SPADCALL |p| (QREFELT $ 26)) |POLYCAT-;monomials;SL;2|)))
       NIL
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT (REVERSE |ml|))))))

  (DEFUN |POLYCAT-;isPlus;SU;3| (|p| $)
   (PROG (|l|)
    (RETURN
     (COND
      ((NULL
         (CDR (LETT |l| (SPADCALL |p| (QREFELT $ 28)) |POLYCAT-;isPlus;SU;3|)))
        (CONS 1 "failed"))
      ((QUOTE T) (CONS 0 |l|))))))

  (DEFUN |POLYCAT-;isTimes;SU;4| (|p| $)
   (PROG (|lv| #0=#:G1470 |v| #1=#:G1471 |l| |r|)
    (RETURN
     (SEQ
      (COND
       ((OR (NULL
             (LETT |lv| (SPADCALL |p| (QREFELT $ 31)) |POLYCAT-;isTimes;SU;4|))
            (NULL (SPADCALL |p| (QREFELT $ 32))))
        (CONS 1 "failed"))
       ((QUOTE T)
        (SEQ
```

```
         (LETT |l|
          (PROGN
           (LETT #0# NIL |POLYCAT-;isTimes;SU;4|)
           (SEQ
            (LETT |v| NIL |POLYCAT-;isTimes;SU;4|)
            (LETT #1# |lv| |POLYCAT-;isTimes;SU;4|)
            G190
            (COND
             ((OR (ATOM #1#)
                  (PROGN (LETT |v| (CAR #1#) |POLYCAT-;isTimes;SU;4|) NIL))
              (GO G191)))
            (SEQ
             (EXIT
              (LETT #0#
               (CONS
                (SPADCALL (|spadConstant| $ 33) |v|
                 (SPADCALL |p| |v| (QREFELT $ 36)) (QREFELT $ 37))
                #0#)
               |POLYCAT-;isTimes;SU;4|)))
            (LETT #1# (CDR #1#) |POLYCAT-;isTimes;SU;4|)
            (GO G190)
            G191
            (EXIT (NREVERSE0 #0#))))
          |POLYCAT-;isTimes;SU;4|)
         (LETT |r| (SPADCALL |p| (QREFELT $ 38)) |POLYCAT-;isTimes;SU;4|)
         (EXIT
          (COND
           ((SPADCALL |r| (|spadConstant| $ 34) (QREFELT $ 39))
             (COND
              ((NULL (CDR |lv|)) (CONS 1 "failed"))
              ((QUOTE T) (CONS 0 |l|))))
           ((QUOTE T)
             (CONS 0 (CONS (SPADCALL |r| (QREFELT $ 40)) |l|)))))))))

(DEFUN |POLYCAT-;isExpt;SU;5| (|p| $)
 (PROG (|u| |d|)
  (RETURN
   (SEQ
    (LETT |u| (SPADCALL |p| (QREFELT $ 42)) |POLYCAT-;isExpt;SU;5|)
    (EXIT
     (COND
      ((OR (QEQCAR |u| 1)
           (NULL
            (SPADCALL |p|
             (SPADCALL (|spadConstant| $ 33)
              (QCDR |u|)
```

```
               (LETT |d| (SPADCALL |p| (QCDR |u|) (QREFELT $ 36))
                |POLYCAT-;isExpt;SU;5|)
               (QREFELT $ 37))
              (QREFELT $ 24))))
        (CONS 1 "failed"))
       ((QUOTE T) (CONS 0 (CONS (QCDR |u|) |d|))))))))))

(DEFUN |POLYCAT-;coefficient;SVarSetNniS;6| (|p| |v| |n| $)
 (SPADCALL (SPADCALL |p| |v| (QREFELT $ 47)) |n| (QREFELT $ 49)))

(DEFUN |POLYCAT-;coefficient;SLLS;7| (|p| |lv| |ln| $)
 (COND
  ((NULL |lv|)
   (COND
    ((NULL |ln|) |p|)
    ((QUOTE T) (|error| "mismatched lists in coefficient"))))
  ((NULL |ln|) (|error| "mismatched lists in coefficient"))
  ((QUOTE T)
   (SPADCALL
    (SPADCALL
     (SPADCALL |p| (|SPADfirst| |lv|) (QREFELT $ 47))
     (|SPADfirst| |ln|)
     (QREFELT $ 49))
    (CDR |lv|)
    (CDR |ln|)
    (QREFELT $ 52)))))

(DEFUN |POLYCAT-;monomial;SLLS;8| (|p| |lv| |ln| $)
 (COND
  ((NULL |lv|)
   (COND
    ((NULL |ln|) |p|)
    ((QUOTE T) (|error| "mismatched lists in monomial"))))
  ((NULL |ln|) (|error| "mismatched lists in monomial"))
  ((QUOTE T)
   (SPADCALL
    (SPADCALL |p| (|SPADfirst| |lv|) (|SPADfirst| |ln|) (QREFELT $ 37))
    (CDR |lv|)
    (CDR |ln|)
    (QREFELT $ 54)))))

(DEFUN |POLYCAT-;retract;SVarSet;9| (|p| $)
 (PROG (#0=#:G1496 |q|)
  (RETURN
   (SEQ
    (LETT |q|
```

```
       (PROG2
        (LETT #0# (SPADCALL |p| (QREFELT $ 42)) |POLYCAT-;retract;SVarSet;9|)
        (QCDR #0#)
        (|check-union| (QEQCAR #0# 0) (QREFELT $ 9) #0#))
       |POLYCAT-;retract;SVarSet;9|)
      (EXIT
       (COND
        ((SPADCALL (SPADCALL |q| (QREFELT $ 56)) |p| (QREFELT $ 24)) |q|)
        ((QUOTE T) (|error| "Polynomial is not a single variable")))))))

(DEFUN |POLYCAT-;retractIfCan;SU;10| (|p| $)
 (PROG (|q| #0=#:G1504)
  (RETURN
   (SEQ
    (EXIT
     (SEQ
      (SEQ
       (LETT |q| (SPADCALL |p| (QREFELT $ 42)) |POLYCAT-;retractIfCan;SU;10|)
       (EXIT
        (COND
         ((QEQCAR |q| 0)
          (COND
           ((SPADCALL (SPADCALL (QCDR |q|) (QREFELT $ 56)) |p| (QREFELT $ 24))
             (PROGN
              (LETT #0# |q| |POLYCAT-;retractIfCan;SU;10|)
              (GO #0#)))))))))
      (EXIT (CONS 1 "failed"))))
     #0#
     (EXIT #0#)))))

(DEFUN |POLYCAT-;mkPrim| (|p| $)
 (SPADCALL
  (|spadConstant| $ 34)
  (SPADCALL |p| (QREFELT $ 59))
  (QREFELT $ 60)))

(DEFUN |POLYCAT-;primitiveMonomials;SL;12| (|p| $)
 (PROG (#0=#:G1509 |q| #1=#:G1510)
  (RETURN
   (SEQ
    (PROGN
     (LETT #0# NIL |POLYCAT-;primitiveMonomials;SL;12|)
     (SEQ
      (LETT |q| NIL |POLYCAT-;primitiveMonomials;SL;12|)
      (LETT #1# (SPADCALL |p| (QREFELT $ 28)) |POLYCAT-;primitiveMonomials;SL;12|)
      G190
```

```
        (COND
         ((OR (ATOM #1#)
              (PROGN
               (LETT |q| (CAR #1#) |POLYCAT-;primitiveMonomials;SL;12|)
               NIL))
          (GO G191)))
        (SEQ
         (EXIT
          (LETT #0# (CONS (|POLYCAT-;mkPrim| |q| $) #0#)
           |POLYCAT-;primitiveMonomials;SL;12|)))
        (LETT #1# (CDR #1#) |POLYCAT-;primitiveMonomials;SL;12|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #0#))))))))

  (DEFUN |POLYCAT-;totalDegree;SNni;13| (|p| $)
   (PROG (#0=#:G1512 |d| |u|)
    (RETURN
     (SEQ
      (COND
       ((SPADCALL |p| (QREFELT $ 62)) 0)
       ((QUOTE T)
        (SEQ
         (LETT |u|
          (SPADCALL |p|
           (PROG2
            (LETT #0#
             (SPADCALL |p| (QREFELT $ 42))
             |POLYCAT-;totalDegree;SNni;13|)
            (QCDR #0#)
            (|check-union| (QEQCAR #0# 0) (QREFELT $ 9) #0#))
           (QREFELT $ 47))
          |POLYCAT-;totalDegree;SNni;13|)
         (LETT |d| 0 |POLYCAT-;totalDegree;SNni;13|)
         (SEQ G190
          (COND
           ((NULL
             (COND
              ((SPADCALL |u| (|spadConstant| $ 63) (QREFELT $ 64)) (QUOTE NIL))
              ((QUOTE T) (QUOTE T)))) (GO G191)))
          (SEQ
           (LETT |d|
            (MAX |d|
             (+
              (SPADCALL |u| (QREFELT $ 65))
              (SPADCALL (SPADCALL |u| (QREFELT $ 66)) (QREFELT $ 67))))
```

```
            |POLYCAT-;totalDegree;SNni;13|)
           (EXIT
            (LETT |u|
             (SPADCALL |u| (QREFELT $ 68)) |POLYCAT-;totalDegree;SNni;13|)))
          NIL
          (GO G190)
          G191
          (EXIT NIL))
         (EXIT |d|))))))))

(DEFUN |POLYCAT-;totalDegree;SLNni;14| (|p| |lv| $)
 (PROG (#0=#:G1520 |v| |w| |d| |u|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |p| (QREFELT $ 62)) 0)
     ((QUOTE T)
      (SEQ
       (LETT |u|
        (SPADCALL |p|
         (LETT |v|
          (PROG2
           (LETT #0#
            (SPADCALL |p| (QREFELT $ 42))
            |POLYCAT-;totalDegree;SLNni;14|)
           (QCDR #0#)
           (|check-union| (QEQCAR #0# 0) (QREFELT $ 9) #0#))
          |POLYCAT-;totalDegree;SLNni;14|)
         (QREFELT $ 47))
        |POLYCAT-;totalDegree;SLNni;14|)
       (LETT |d| 0 |POLYCAT-;totalDegree;SLNni;14|)
       (LETT |w| 0 |POLYCAT-;totalDegree;SLNni;14|)
       (COND
        ((SPADCALL |v| |lv| (QREFELT $ 70))
         (LETT |w| 1 |POLYCAT-;totalDegree;SLNni;14|)))
       (SEQ G190
        (COND
         ((NULL
          (COND
           ((SPADCALL |u| (|spadConstant| $ 63) (QREFELT $ 64)) (QUOTE NIL))
           ((QUOTE T) (QUOTE T)))) (GO G191)))
        (SEQ
         (LETT |d|
          (MAX |d|
           (+
            (* |w| (SPADCALL |u| (QREFELT $ 65)))
```

```
             (SPADCALL (SPADCALL |u| (QREFELT $ 66)) |lv| (QREFELT $ 71))))
            |POLYCAT-;totalDegree;SLNni;14|)
           (EXIT
            (LETT |u|
             (SPADCALL |u| (QREFELT $ 68))
             |POLYCAT-;totalDegree;SLNni;14|)))
           NIL
           (GO G190)
           G191
           (EXIT NIL))
          (EXIT |d|)))))))))

(DEFUN |POLYCAT-;resultant;2SVarSetS;15| (|p1| |p2| |mvar| $)
 (SPADCALL
  (SPADCALL |p1| |mvar| (QREFELT $ 47))
  (SPADCALL |p2| |mvar| (QREFELT $ 47))
  (QREFELT $ 73)))

(DEFUN |POLYCAT-;discriminant;SVarSetS;16| (|p| |var| $)
 (SPADCALL (SPADCALL |p| |var| (QREFELT $ 47)) (QREFELT $ 75)))

(DEFUN |POLYCAT-;allMonoms| (|l| $)
 (PROG (#0=#:G1532 |p| #1=#:G1533)
  (RETURN
   (SEQ
    (SPADCALL
     (SPADCALL
      (PROGN
       (LETT #0# NIL |POLYCAT-;allMonoms|)
       (SEQ
        (LETT |p| NIL |POLYCAT-;allMonoms|)
        (LETT #1# |l| |POLYCAT-;allMonoms|)
        G190
        (COND
         ((OR (ATOM #1#) (PROGN (LETT |p| (CAR #1#) |POLYCAT-;allMonoms|) NIL))
          (GO G191)))
        (SEQ
         (EXIT
          (LETT #0#
           (CONS (SPADCALL |p| (QREFELT $ 77)) #0#)
           |POLYCAT-;allMonoms|)))
        (LETT #1# (CDR #1#) |POLYCAT-;allMonoms|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #0#))))
      (QREFELT $ 79))
```

```
      (QREFELT $ 80))))))

(DEFUN |POLYCAT-;P2R| (|p| |b| |n| $)
 (PROG (|w| |bj| #0=#:G1538 |i| #1=#:G1537)
  (RETURN
   (SEQ
    (LETT |w|
     (SPADCALL |n| (|spadConstant| $ 22) (QREFELT $ 82))
     |POLYCAT-;P2R|)
    (SEQ
     (LETT |bj| NIL |POLYCAT-;P2R|)
     (LETT #0# |b| |POLYCAT-;P2R|)
     (LETT |i| (SPADCALL |w| (QREFELT $ 84)) |POLYCAT-;P2R|)
     (LETT #1# (QVSIZE |w|) |POLYCAT-;P2R|)
     G190
     (COND
      ((OR (> |i| #1#)
           (ATOM #0#)
           (PROGN (LETT |bj| (CAR #0#) |POLYCAT-;P2R|) NIL))
       (GO G191)))
     (SEQ
      (EXIT
       (SPADCALL |w| |i| (SPADCALL |p| |bj| (QREFELT $ 85)) (QREFELT $ 86))))
     (LETT |i|
      (PROG1 (+ |i| 1) (LETT #0# (CDR #0#) |POLYCAT-;P2R|)) |POLYCAT-;P2R|)
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |w|)))))

(DEFUN |POLYCAT-;eq2R| (|l| |b| $)
 (PROG (#0=#:G1542 |bj| #1=#:G1543 #2=#:G1544 |p| #3=#:G1545)
  (RETURN
   (SEQ
    (SPADCALL
     (PROGN
      (LETT #0# NIL |POLYCAT-;eq2R|)
      (SEQ
       (LETT |bj| NIL |POLYCAT-;eq2R|)
       (LETT #1# |b| |POLYCAT-;eq2R|)
       G190
       (COND
        ((OR (ATOM #1#)
             (PROGN (LETT |bj| (CAR #1#) |POLYCAT-;eq2R|) NIL)) (GO G191)))
       (SEQ
        (EXIT
```

```
          (LETT #0#
           (CONS
            (PROGN
             (LETT #2# NIL |POLYCAT-;eq2R|)
             (SEQ
              (LETT |p| NIL |POLYCAT-;eq2R|)
              (LETT #3# |l| |POLYCAT-;eq2R|)
              G190
              (COND
               ((OR (ATOM #3#) (PROGN (LETT |p| (CAR #3#) |POLYCAT-;eq2R|) NIL))
                (GO G191)))
              (SEQ
               (EXIT
                (LETT #2#
                 (CONS (SPADCALL |p| |bj| (QREFELT $ 85)) #2#)
                 |POLYCAT-;eq2R|)))
              (LETT #3# (CDR #3#) |POLYCAT-;eq2R|)
              (GO G190)
              G191
              (EXIT (NREVERSE0 #2#))))
             #0#)
            |POLYCAT-;eq2R|)))
        (LETT #1# (CDR #1#) |POLYCAT-;eq2R|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #0#))))
     (QREFELT $ 89))))))

(DEFUN |POLYCAT-;reducedSystem;MM;20| (|m| $)
 (PROG (#0=#:G1555 |r| #1=#:G1556 |b| #2=#:G1557 |bj| #3=#:G1558 |d| |mm| |l|)
  (RETURN
   (SEQ
    (LETT |l| (SPADCALL |m| (QREFELT $ 92)) |POLYCAT-;reducedSystem;MM;20|)
    (LETT |b|
     (SPADCALL
      (SPADCALL
       (PROGN
        (LETT #0# NIL |POLYCAT-;reducedSystem;MM;20|)
        (SEQ
         (LETT |r| NIL |POLYCAT-;reducedSystem;MM;20|)
         (LETT #1# |l| |POLYCAT-;reducedSystem;MM;20|)
         G190
         (COND
          ((OR (ATOM #1#)
               (PROGN (LETT |r| (CAR #1#) |POLYCAT-;reducedSystem;MM;20|) NIL))
           (GO G191)))
```

```
    (SEQ
     (EXIT
      (LETT #0#
       (CONS (|POLYCAT-;allMonoms| |r| $) #0#)
       |POLYCAT-;reducedSystem;MM;20|)))
    (LETT #1# (CDR #1#) |POLYCAT-;reducedSystem;MM;20|)
    (GO G190)
    G191
    (EXIT (NREVERSE0 #0#))))
   (QREFELT $ 79))
  (QREFELT $ 80))
 |POLYCAT-;reducedSystem;MM;20|)
(LETT |d|
 (PROGN
  (LETT #2# NIL |POLYCAT-;reducedSystem;MM;20|)
  (SEQ
   (LETT |bj| NIL |POLYCAT-;reducedSystem;MM;20|)
   (LETT #3# |b| |POLYCAT-;reducedSystem;MM;20|)
   G190
   (COND
    ((OR (ATOM #3#)
         (PROGN (LETT |bj| (CAR #3#) |POLYCAT-;reducedSystem;MM;20|) NIL))
     (GO G191)))
   (SEQ
    (EXIT
     (LETT #2#
      (CONS (SPADCALL |bj| (QREFELT $ 59)) #2#)
      |POLYCAT-;reducedSystem;MM;20|)))
   (LETT #3# (CDR #3#) |POLYCAT-;reducedSystem;MM;20|)
   (GO G190)
   G191
   (EXIT (NREVERSE0 #2#))))
  |POLYCAT-;reducedSystem;MM;20|)
(LETT |mm|
 (|POLYCAT-;eq2R| (|SPADfirst| |l|) |d| $) |POLYCAT-;reducedSystem;MM;20|)
(LETT |l| (CDR |l|) |POLYCAT-;reducedSystem;MM;20|)
(SEQ G190
 (COND
  ((NULL (COND ((NULL |l|) (QUOTE NIL)) ((QUOTE T) (QUOTE T)))) (GO G191)))
 (SEQ
  (LETT |mm|
   (SPADCALL |mm| (|POLYCAT-;eq2R| (|SPADfirst| |l|) |d| $) (QREFELT $ 93))
   |POLYCAT-;reducedSystem;MM;20|)
  (EXIT (LETT |l| (CDR |l|) |POLYCAT-;reducedSystem;MM;20|)))
 NIL
 (GO G190)
```

```
      G191
      (EXIT NIL))
     (EXIT |mm|)))))

(DEFUN |POLYCAT-;reducedSystem;MVR;21| (|m| |v| $)
 (PROG (#0=#:G1570 |s| #1=#:G1571 |b| #2=#:G1572 |bj| #3=#:G1573 |d| |n|
        |mm| |w| |l| |r|)
  (RETURN
   (SEQ
    (LETT |l| (SPADCALL |m| (QREFELT $ 92)) |POLYCAT-;reducedSystem;MVR;21|)
    (LETT |r| (SPADCALL |v| (QREFELT $ 97)) |POLYCAT-;reducedSystem;MVR;21|)
    (LETT |b|
     (SPADCALL
      (SPADCALL
       (|POLYCAT-;allMonoms| |r| $)
       (SPADCALL
        (PROGN
         (LETT #0# NIL |POLYCAT-;reducedSystem;MVR;21|)
         (SEQ
          (LETT |s| NIL |POLYCAT-;reducedSystem;MVR;21|)
          (LETT #1# |l| |POLYCAT-;reducedSystem;MVR;21|)
          G190
          (COND
           ((OR (ATOM #1#)
                (PROGN
                 (LETT |s| (CAR #1#) |POLYCAT-;reducedSystem;MVR;21|)
                 NIL))
            (GO G191)))
          (SEQ
           (EXIT
            (LETT #0#
             (CONS (|POLYCAT-;allMonoms| |s| $) #0#)
             |POLYCAT-;reducedSystem;MVR;21|)))
          (LETT #1# (CDR #1#) |POLYCAT-;reducedSystem;MVR;21|)
          (GO G190)
          G191
          (EXIT (NREVERSE0 #0#))))
        (QREFELT $ 79))
       (QREFELT $ 98))
      (QREFELT $ 80))
     |POLYCAT-;reducedSystem;MVR;21|)
    (LETT |d|
     (PROGN
      (LETT #2# NIL |POLYCAT-;reducedSystem;MVR;21|)
      (SEQ
       (LETT |bj| NIL |POLYCAT-;reducedSystem;MVR;21|)
```

```
    (LETT #3# |b| |POLYCAT-;reducedSystem;MVR;21|)
    G190
    (COND
     ((OR (ATOM #3#)
          (PROGN (LETT |bj| (CAR #3#) |POLYCAT-;reducedSystem;MVR;21|) NIL))
      (GO G191)))
    (SEQ
     (EXIT
      (LETT #2#
       (CONS (SPADCALL |bj| (QREFELT $ 59)) #2#)
       |POLYCAT-;reducedSystem;MVR;21|)))
    (LETT #3# (CDR #3#) |POLYCAT-;reducedSystem;MVR;21|)
    (GO G190)
    G191
    (EXIT (NREVERSE0 #2#))))
   |POLYCAT-;reducedSystem;MVR;21|)
(LETT |n| (LENGTH |d|) |POLYCAT-;reducedSystem;MVR;21|)
(LETT |mm|
 (|POLYCAT-;eq2R| (|SPADfirst| |l|) |d| $)
 |POLYCAT-;reducedSystem;MVR;21|)
(LETT |w|
 (|POLYCAT-;P2R| (|SPADfirst| |r|) |d| |n| $)
 |POLYCAT-;reducedSystem;MVR;21|)
(LETT |l| (CDR |l|) |POLYCAT-;reducedSystem;MVR;21|)
(LETT |r| (CDR |r|) |POLYCAT-;reducedSystem;MVR;21|)
(SEQ G190
 (COND
  ((NULL (COND ((NULL |l|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
   (GO G191)))
 (SEQ
  (LETT |mm|
   (SPADCALL |mm| (|POLYCAT-;eq2R| (|SPADfirst| |l|) |d| $) (QREFELT $ 93))
   |POLYCAT-;reducedSystem;MVR;21|)
  (LETT |w|
   (SPADCALL |w|
    (|POLYCAT-;P2R| (|SPADfirst| |r|) |d| |n| $)
    (QREFELT $ 99))
   |POLYCAT-;reducedSystem;MVR;21|)
  (LETT |l| (CDR |l|) |POLYCAT-;reducedSystem;MVR;21|)
  (EXIT (LETT |r| (CDR |r|) |POLYCAT-;reducedSystem;MVR;21|)))
  NIL
  (GO G190)
  G191
  (EXIT NIL))
(EXIT (CONS |mm| |w|))))))
```

```
(DEFUN |POLYCAT-;gcdPolynomial;3Sup;22| (|pp| |qq| $)
 (SPADCALL |pp| |qq| (QREFELT $ 104)))

(DEFUN |POLYCAT-;solveLinearPolynomialEquation;LSupU;23| (|lpp| |pp| $)
 (SPADCALL |lpp| |pp| (QREFELT $ 109)))

(DEFUN |POLYCAT-;factorPolynomial;SupF;24| (|pp| $)
 (SPADCALL |pp| (QREFELT $ 114)))

(DEFUN |POLYCAT-;factorSquareFreePolynomial;SupF;25| (|pp| $)
 (SPADCALL |pp| (QREFELT $ 117)))

(DEFUN |POLYCAT-;factor;SF;26| (|p| $)
 (PROG (|v| |ansR| #0=#:G1615 |w| #1=#:G1616 |up| |ansSUP| #2=#:G1617
        |ww| #3=#:G1618)
  (RETURN
   (SEQ
    (LETT |v| (SPADCALL |p| (QREFELT $ 42)) |POLYCAT-;factor;SF;26|)
    (EXIT
     (COND
      ((QEQCAR |v| 1)
       (SEQ
        (LETT |ansR|
         (SPADCALL (SPADCALL |p| (QREFELT $ 38)) (QREFELT $ 120))
         |POLYCAT-;factor;SF;26|)
        (EXIT
         (SPADCALL
          (SPADCALL (SPADCALL |ansR| (QREFELT $ 122)) (QREFELT $ 40))
          (PROGN
           (LETT #0# NIL |POLYCAT-;factor;SF;26|)
           (SEQ
            (LETT |w| NIL |POLYCAT-;factor;SF;26|)
            (LETT #1#
             (SPADCALL |ansR| (QREFELT $ 126))
             |POLYCAT-;factor;SF;26|)
            G190
            (COND
             ((OR (ATOM #1#)
                  (PROGN (LETT |w| (CAR #1#) |POLYCAT-;factor;SF;26|) NIL))
              (GO G191)))
            (SEQ
             (EXIT
              (LETT #0#
               (CONS
                (VECTOR (QVELT |w| 0)
                  (SPADCALL (QVELT |w| 1) (QREFELT $ 40)) (QVELT |w| 2))
```

```
                #0#)
               |POLYCAT-;factor;SF;26|)))
            (LETT #1# (CDR #1#) |POLYCAT-;factor;SF;26|)
            (GO G190)
            G191
            (EXIT (NREVERSE0 #0#))))
          (QREFELT $ 130)))))
       ((QUOTE T)
        (SEQ
         (LETT |up|
          (SPADCALL |p| (QCDR |v|) (QREFELT $ 47)) |POLYCAT-;factor;SF;26|)
         (LETT |ansSUP| (SPADCALL |up| (QREFELT $ 114)) |POLYCAT-;factor;SF;26|)
         (EXIT
          (SPADCALL
           (SPADCALL
            (SPADCALL |ansSUP| (QREFELT $ 131)) (QCDR |v|) (QREFELT $ 132))
           (PROGN
            (LETT #2# NIL |POLYCAT-;factor;SF;26|)
            (SEQ
             (LETT |ww| NIL |POLYCAT-;factor;SF;26|)
             (LETT #3#
              (SPADCALL |ansSUP| (QREFELT $ 135))
              |POLYCAT-;factor;SF;26|)
             G190
             (COND
              ((OR (ATOM #3#)
                   (PROGN (LETT |ww| (CAR #3#) |POLYCAT-;factor;SF;26|) NIL))
               (GO G191)))
             (SEQ
              (EXIT
               (LETT #2#
                (CONS
                 (VECTOR (QVELT |ww| 0) (SPADCALL (QVELT |ww| 1) (QCDR |v|)
                         (QREFELT $ 132)) (QVELT |ww| 2))
                 #2#)
                |POLYCAT-;factor;SF;26|)))
             (LETT #3# (CDR #3#) |POLYCAT-;factor;SF;26|)
             (GO G190)
             G191
             (EXIT (NREVERSE0 #2#))))
           (QREFELT $ 130)))))))))))

(DEFUN |POLYCAT-;conditionP;MU;27| (|mat| $)
 (PROG (|ll| #0=#:G1653 |z| #1=#:G1654 |ch| |l| #2=#:G1655 #3=#:G1656
        #4=#:G1625 #5=#:G1623 #6=#:G1624 #7=#:G1657 |vars| |degs|
        #8=#:G1658 |d| #9=#:G1659 |nd| #10=#:G1652 #11=#:G1632 |deg1|
```

```
        |redmons| #12=#:G1660 |v| #13=#:G1662 |u| #14=#:G1661 |llR|
        |monslist| |ans| #15=#:G1663 #16=#:G1664 |mons| #17=#:G1665 |m|
        #18=#:G1666 |i| #19=#:G1648 #20=#:G1646 #21=#:G1647)
   (RETURN
    (SEQ
     (EXIT
      (SEQ
       (LETT |ll|
        (SPADCALL (SPADCALL |mat| (QREFELT $ 137)) (QREFELT $ 92))
        |POLYCAT-;conditionP;MU;27|)
       (LETT |llR|
        (PROGN
         (LETT #0# NIL |POLYCAT-;conditionP;MU;27|)
         (SEQ
          (LETT |z| NIL |POLYCAT-;conditionP;MU;27|)
          (LETT #1# (|SPADfirst| |ll|) |POLYCAT-;conditionP;MU;27|)
          G190
          (COND
           ((OR (ATOM #1#)
                (PROGN (LETT |z| (CAR #1#) |POLYCAT-;conditionP;MU;27|) NIL))
            (GO G191)))
          (SEQ (EXIT (LETT #0# (CONS NIL #0#) |POLYCAT-;conditionP;MU;27|)))
          (LETT #1# (CDR #1#) |POLYCAT-;conditionP;MU;27|)
          (GO G190)
          G191
          (EXIT (NREVERSE0 #0#))))
        |POLYCAT-;conditionP;MU;27|)
       (LETT |monslist| NIL |POLYCAT-;conditionP;MU;27|)
       (LETT |ch| (SPADCALL (QREFELT $ 138)) |POLYCAT-;conditionP;MU;27|)
       (SEQ
        (LETT |l| NIL |POLYCAT-;conditionP;MU;27|)
        (LETT #2# |ll| |POLYCAT-;conditionP;MU;27|)
        G190
        (COND
         ((OR (ATOM #2#)
              (PROGN (LETT |l| (CAR #2#) |POLYCAT-;conditionP;MU;27|) NIL))
          (GO G191)))
        (SEQ
         (LETT |mons|
          (PROGN
           (LETT #6# NIL |POLYCAT-;conditionP;MU;27|)
           (SEQ
            (LETT |u| NIL |POLYCAT-;conditionP;MU;27|)
            (LETT #3# |l| |POLYCAT-;conditionP;MU;27|)
            G190
            (COND
```

```
      ((OR (ATOM #3#)
           (PROGN (LETT |u| (CAR #3#) |POLYCAT-;conditionP;MU;27|) NIL))
       (GO G191)))
     (SEQ
      (EXIT
       (PROGN
        (LETT #4#
         (SPADCALL |u| (QREFELT $ 77))
         |POLYCAT-;conditionP;MU;27|)
        (COND
         (#6#
          (LETT #5#
           (SPADCALL #5# #4# (QREFELT $ 139))
           |POLYCAT-;conditionP;MU;27|))
         ((QUOTE T)
          (PROGN
           (LETT #5# #4# |POLYCAT-;conditionP;MU;27|)
           (LETT #6# (QUOTE T) |POLYCAT-;conditionP;MU;27|)))))))
     (LETT #3# (CDR #3#) |POLYCAT-;conditionP;MU;27|)
     (GO G190)
     G191
     (EXIT NIL))
    (COND (#6# #5#) ((QUOTE T) (|IdentityError| (QUOTE |setUnion|)))))
   |POLYCAT-;conditionP;MU;27|)
  (LETT |redmons| NIL |POLYCAT-;conditionP;MU;27|)
  (SEQ
   (LETT |m| NIL |POLYCAT-;conditionP;MU;27|)
   (LETT #7# |mons| |POLYCAT-;conditionP;MU;27|)
   G190
   (COND
    ((OR (ATOM #7#)
         (PROGN (LETT |m| (CAR #7#) |POLYCAT-;conditionP;MU;27|) NIL))
     (GO G191)))
   (SEQ
    (LETT |vars|
     (SPADCALL |m| (QREFELT $ 31))
     |POLYCAT-;conditionP;MU;27|)
    (LETT |degs|
     (SPADCALL |m| |vars| (QREFELT $ 140))
     |POLYCAT-;conditionP;MU;27|)
    (LETT |deg1|
     (PROGN
      (LETT #8# NIL |POLYCAT-;conditionP;MU;27|)
      (SEQ
       (LETT |d| NIL |POLYCAT-;conditionP;MU;27|)
       (LETT #9# |degs| |POLYCAT-;conditionP;MU;27|)
```

```
   G190
   (COND
    ((OR (ATOM #9#)
         (PROGN
          (LETT |d| (CAR #9#) |POLYCAT-;conditionP;MU;27|)
          NIL))
     (GO G191)))
   (SEQ
    (EXIT
     (LETT #8#
      (CONS
       (SEQ
        (LETT |nd|
         (SPADCALL |d| |ch| (QREFELT $ 142))
         |POLYCAT-;conditionP;MU;27|)
        (EXIT
         (COND
          ((QEQCAR |nd| 1)
            (PROGN
              (LETT #10#
               (CONS 1 "failed") |POLYCAT-;conditionP;MU;27|)
              (GO #10#)))
          ((QUOTE T)
            (PROG1
             (LETT #11# (QCDR |nd|) |POLYCAT-;conditionP;MU;27|)
             (|check-subtype|
              (>= #11# 0) (QUOTE (|NonNegativeInteger|)) #11#))))))
       #8#)
      |POLYCAT-;conditionP;MU;27|)))
    (LETT #9# (CDR #9#) |POLYCAT-;conditionP;MU;27|)
    (GO G190)
    G191
    (EXIT (NREVERSE0 #8#))))
  |POLYCAT-;conditionP;MU;27|)
 (LETT |redmons|
  (CONS
   (SPADCALL (|spadConstant| $ 33) |vars| |deg1| (QREFELT $ 54))
   |redmons|)
  |POLYCAT-;conditionP;MU;27|)
 (EXIT
  (LETT |llR|
   (PROGN
    (LETT #12# NIL |POLYCAT-;conditionP;MU;27|)
    (SEQ
     (LETT |v| NIL |POLYCAT-;conditionP;MU;27|)
     (LETT #13# |llR| |POLYCAT-;conditionP;MU;27|)
```

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            (LETT |u| NIL |POLYCAT-;conditionP;MU;27|)
            (LETT #14# |l| |POLYCAT-;conditionP;MU;27|)
            G190
            (COND
             ((OR (ATOM #14#)
                  (PROGN
                   (LETT |u| (CAR #14#) |POLYCAT-;conditionP;MU;27|)
                    NIL)
                  (ATOM #13#)
                  (PROGN (LETT |v| (CAR #13#) |POLYCAT-;conditionP;MU;27|) NIL))
              (GO G191)))
            (SEQ
             (EXIT
              (LETT #12#
               (CONS
                (CONS
                 (SPADCALL
                  (SPADCALL |u| |vars| |degs| (QREFELT $ 52))
                  (QREFELT $ 143))
                 |v|)
                #12#)
               |POLYCAT-;conditionP;MU;27|)))
            (LETT #14#
             (PROG1
              (CDR #14#)
              (LETT #13# (CDR #13#) |POLYCAT-;conditionP;MU;27|))
             |POLYCAT-;conditionP;MU;27|)
            (GO G190)
            G191
            (EXIT (NREVERSE0 #12#))))
          |POLYCAT-;conditionP;MU;27|)))
      (LETT #7# (CDR #7#) |POLYCAT-;conditionP;MU;27|)
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT
      (LETT |monslist|
       (CONS |redmons| |monslist|)
       |POLYCAT-;conditionP;MU;27|)))
    (LETT #2# (CDR #2#) |POLYCAT-;conditionP;MU;27|)
    (GO G190)
    G191
    (EXIT NIL))
   (LETT |ans|
    (SPADCALL (SPADCALL (SPADCALL |llR| (QREFELT $ 89)) (QREFELT $ 144))
              (QREFELT $ 146))
```

```
 |POLYCAT-;conditionP;MU;27|)
(EXIT
 (COND
  ((QEQCAR |ans| 1) (CONS 1 "failed"))
  ((QUOTE T)
   (SEQ
    (LETT |i| 0 |POLYCAT-;conditionP;MU;27|)
    (EXIT
     (CONS 0
      (PRIMVEC2ARR
       (PROGN
        (LETT #15# (GETREFV (SIZE |monslist|))
         |POLYCAT-;conditionP;MU;27|)
        (SEQ
         (LETT #16# 0 |POLYCAT-;conditionP;MU;27|)
         (LETT |mons| NIL |POLYCAT-;conditionP;MU;27|)
         (LETT #17# |monslist| |POLYCAT-;conditionP;MU;27|)
         G190
         (COND
          ((OR (ATOM #17#)
               (PROGN
                (LETT |mons| (CAR #17#) |POLYCAT-;conditionP;MU;27|)
                NIL))
           (GO G191)))
         (SEQ
          (EXIT
           (SETELT #15# #16#
            (PROGN
             (LETT #21# NIL |POLYCAT-;conditionP;MU;27|)
             (SEQ
              (LETT |m| NIL |POLYCAT-;conditionP;MU;27|)
              (LETT #18# |mons| |POLYCAT-;conditionP;MU;27|)
              G190
              (COND
               ((OR (ATOM #18#)
                    (PROGN
                     (LETT |m| (CAR #18#) |POLYCAT-;conditionP;MU;27|)
                     NIL))
                (GO G191)))
              (SEQ
               (EXIT
                (PROGN
                 (LETT #19#
                  (SPADCALL |m|
                   (SPADCALL
                    (SPADCALL
```

```
                              (QCDR |ans|)
                              (LETT |i| (+ |i| 1) |POLYCAT-;conditionP;MU;27|)
                              (QREFELT $ 147))
                             (QREFELT $ 40))
                            (QREFELT $ 148))
                           |POLYCAT-;conditionP;MU;27|)
                          (COND
                           (#21#
                            (LETT #20#
                             (SPADCALL #20# #19# (QREFELT $ 149))
                             |POLYCAT-;conditionP;MU;27|))
                           ((QUOTE T)
                             (PROGN
                              (LETT #20# #19# |POLYCAT-;conditionP;MU;27|)
                              (LETT #21#
                               (QUOTE T)
                               |POLYCAT-;conditionP;MU;27|)))))))
                       (LETT #18# (CDR #18#) |POLYCAT-;conditionP;MU;27|)
                       (GO G190)
                       G191
                       (EXIT NIL))
                      (COND (#21# #20#) ((QUOTE T) (|spadConstant| $ 21)))))))
                 (LETT #17#
                  (PROG1
                   (CDR #17#)
                   (LETT #16# (QSADD1 #16#) |POLYCAT-;conditionP;MU;27|))
                  |POLYCAT-;conditionP;MU;27|)
                 (GO G190)
                 G191
                 (EXIT NIL))
                #15#)))))))))
    #10#
    (EXIT #10#)))))

(DEFUN |POLYCAT-;charthRoot;SU;28| (|p| $)
 (PROG (|vars| |ans| |ch|)
  (RETURN
   (SEQ
    (LETT |vars| (SPADCALL |p| (QREFELT $ 31)) |POLYCAT-;charthRoot;SU;28|)
    (EXIT
     (COND
      ((NULL |vars|)
        (SEQ
         (LETT |ans|
          (SPADCALL (SPADCALL |p| (QREFELT $ 143)) (QREFELT $ 151))
          |POLYCAT-;charthRoot;SU;28|)
```

```
          (EXIT
           (COND
            ((QEQCAR |ans| 1) (CONS 1 "failed"))
            ((QUOTE T) (CONS 0 (SPADCALL (QCDR |ans|) (QREFELT $ 40))))))))
       ((QUOTE T)
        (SEQ
         (LETT |ch| (SPADCALL (QREFELT $ 138)) |POLYCAT-;charthRoot;SU;28|)
         (EXIT (|POLYCAT-;charthRootlv| |p| |vars| |ch| $)))))))))

(DEFUN |POLYCAT-;charthRootlv| (|p| |vars| |ch| $)
 (PROG (|v| |dd| |cp| |d| #0=#:G1687 |ans| |ansx| #1=#:G1694)
  (RETURN
   (SEQ
    (EXIT
     (COND
      ((NULL |vars|)
       (SEQ
        (LETT |ans|
         (SPADCALL (SPADCALL |p| (QREFELT $ 143)) (QREFELT $ 151))
         |POLYCAT-;charthRootlv|)
        (EXIT
         (COND
          ((QEQCAR |ans| 1) (CONS 1 "failed"))
          ((QUOTE T) (CONS 0 (SPADCALL (QCDR |ans|) (QREFELT $ 40))))))))
      ((QUOTE T)
       (SEQ
        (LETT |v| (|SPADfirst| |vars|) |POLYCAT-;charthRootlv|)
        (LETT |vars| (CDR |vars|) |POLYCAT-;charthRootlv|)
        (LETT |d| (SPADCALL |p| |v| (QREFELT $ 36)) |POLYCAT-;charthRootlv|)
        (LETT |ans| (|spadConstant| $ 21) |POLYCAT-;charthRootlv|)
        (SEQ G190
         (COND ((NULL (< 0 |d|)) (GO G191)))
         (SEQ
          (LETT |dd|
           (SPADCALL |d| |ch| (QREFELT $ 142))
           |POLYCAT-;charthRootlv|)
          (EXIT
           (COND
            ((QEQCAR |dd| 1)
              (PROGN
               (LETT #1# (CONS 1 "failed") |POLYCAT-;charthRootlv|)
               (GO #1#)))
             ((QUOTE T)
              (SEQ
               (LETT |cp|
                (SPADCALL |p| |v| |d| (QREFELT $ 154))
```

```
          |POLYCAT-;charthRootlv|)
         (LETT |p|
          (SPADCALL |p|
           (SPADCALL |cp| |v| |d| (QREFELT $ 37))
           (QREFELT $ 155))
          |POLYCAT-;charthRootlv|)
         (LETT |ansx|
          (|POLYCAT-;charthRootlv| |cp| |vars| |ch| $)
          |POLYCAT-;charthRootlv|)
         (EXIT
          (COND
           ((QEQCAR |ansx| 1)
             (PROGN
              (LETT #1# (CONS 1 "failed") |POLYCAT-;charthRootlv|)
              (GO #1#)))
           ((QUOTE T)
            (SEQ
             (LETT |d|
              (SPADCALL |p| |v| (QREFELT $ 36))
              |POLYCAT-;charthRootlv|)
             (EXIT
              (LETT |ans|
               (SPADCALL |ans|
                (SPADCALL (QCDR |ansx|) |v|
                 (PROG1
                  (LETT #0# (QCDR |dd|) |POLYCAT-;charthRootlv|)
                  (|check-subtype| (>= #0# 0)
                   (QUOTE (|NonNegativeInteger|)) #0#))
                 (QREFELT $ 37))
                (QREFELT $ 149))
               |POLYCAT-;charthRootlv|))))))))))
 NIL
 (GO G190)
 G191
 (EXIT NIL))
(LETT |ansx|
 (|POLYCAT-;charthRootlv| |p| |vars| |ch| $)
 |POLYCAT-;charthRootlv|)
(EXIT
 (COND
  ((QEQCAR |ansx| 1)
   (PROGN
    (LETT #1# (CONS 1 "failed") |POLYCAT-;charthRootlv|)
    (GO #1#)))
  ((QUOTE T)
   (PROGN
```

```
              (LETT #1#
               (CONS 0 (SPADCALL |ans| (QCDR |ansx|) (QREFELT $ 149)))
               |POLYCAT-;charthRootlv|)
              (GO #1#)))))))))
     #1#
     (EXIT #1#)))))

(DEFUN |POLYCAT-;monicDivide;2SVarSetR;30| (|p1| |p2| |mvar| $)
 (PROG (|result|)
  (RETURN
   (SEQ
    (LETT |result|
     (SPADCALL
      (SPADCALL |p1| |mvar| (QREFELT $ 47))
      (SPADCALL |p2| |mvar| (QREFELT $ 47))
      (QREFELT $ 157))
     |POLYCAT-;monicDivide;2SVarSetR;30|)
    (EXIT
     (CONS
      (SPADCALL (QCAR |result|) |mvar| (QREFELT $ 132))
      (SPADCALL (QCDR |result|) |mvar| (QREFELT $ 132)))))))))

(DEFUN |POLYCAT-;squareFree;SF;31| (|p| $)
 (SPADCALL |p| (QREFELT $ 160)))

(DEFUN |POLYCAT-;squareFree;SF;32| (|p| $)
 (SPADCALL |p| (QREFELT $ 163)))

(DEFUN |POLYCAT-;squareFree;SF;33| (|p| $)
 (SPADCALL |p| (QREFELT $ 163)))

(DEFUN |POLYCAT-;squareFreePart;2S;34| (|p| $)
 (PROG (|s| |f| #0=#:G1710 #1=#:G1708 #2=#:G1706 #3=#:G1707)
  (RETURN
   (SEQ
    (SPADCALL
     (SPADCALL
      (LETT |s| (SPADCALL |p| (QREFELT $ 164)) |POLYCAT-;squareFreePart;2S;34|)
      (QREFELT $ 165))
     (PROGN
      (LETT #3# NIL |POLYCAT-;squareFreePart;2S;34|)
      (SEQ
       (LETT |f| NIL |POLYCAT-;squareFreePart;2S;34|)
       (LETT #0# (SPADCALL |s| (QREFELT $ 168)) |POLYCAT-;squareFreePart;2S;34|)
       G190
       (COND
```

```
          ((OR (ATOM #0#)
               (PROGN (LETT |f| (CAR #0#) |POLYCAT-;squareFreePart;2S;34|) NIL))
           (GO G191)))
         (SEQ
          (EXIT
           (PROGN
            (LETT #1# (QCAR |f|) |POLYCAT-;squareFreePart;2S;34|)
            (COND
             (#3#
              (LETT #2#
               (SPADCALL #2# #1# (QREFELT $ 148))
               |POLYCAT-;squareFreePart;2S;34|))
             ((QUOTE T)
              (PROGN
               (LETT #2# #1# |POLYCAT-;squareFreePart;2S;34|)
               (LETT #3# (QUOTE T) |POLYCAT-;squareFreePart;2S;34|)))))))
          (LETT #0# (CDR #0#) |POLYCAT-;squareFreePart;2S;34|)
          (GO G190)
          G191
          (EXIT NIL))
         (COND (#3# #2#) ((QUOTE T) (|spadConstant| $ 33))))
        (QREFELT $ 148))))))

(DEFUN |POLYCAT-;content;SVarSetS;35| (|p| |v| $)
 (SPADCALL (SPADCALL |p| |v| (QREFELT $ 47)) (QREFELT $ 170)))

(DEFUN |POLYCAT-;primitivePart;2S;36| (|p| $)
 (PROG (#0=#:G1713)
  (RETURN
   (COND
    ((SPADCALL |p| (QREFELT $ 172)) |p|)
    ((QUOTE T)
     (QVELT
      (SPADCALL
       (PROG2
        (LETT #0#
         (SPADCALL |p| (SPADCALL |p| (QREFELT $ 173)) (QREFELT $ 174))
         |POLYCAT-;primitivePart;2S;36|)
        (QCDR #0#)
        (|check-union| (QEQCAR #0# 0) (QREFELT $ 6) #0#))
       (QREFELT $ 176))
      1))))))

(DEFUN |POLYCAT-;primitivePart;SVarSetS;37| (|p| |v| $)
 (PROG (#0=#:G1720)
  (RETURN
```

```
    (COND
     ((SPADCALL |p| (QREFELT $ 172)) |p|)
     ((QUOTE T)
      (QVELT
       (SPADCALL
        (PROG2
         (LETT #0#
          (SPADCALL |p| (SPADCALL |p| |v| (QREFELT $ 178)) (QREFELT $ 179))
          |POLYCAT-;primitivePart;SVarSetS;37|)
         (QCDR #0#)
         (|check-union| (QEQCAR #0# 0) (QREFELT $ 6) #0#))
        (QREFELT $ 176))
       1))))))

(DEFUN |POLYCAT-;<;2SB;38| (|p| |q| $)
 (PROG (|dp| |dq|)
  (RETURN
   (SEQ
    (LETT |dp| (SPADCALL |p| (QREFELT $ 59)) |POLYCAT-;<;2SB;38|)
    (LETT |dq| (SPADCALL |q| (QREFELT $ 59)) |POLYCAT-;<;2SB;38|)
    (EXIT
     (COND
      ((SPADCALL |dp| |dq| (QREFELT $ 181))
        (SPADCALL
         (|spadConstant| $ 22)
         (SPADCALL |q| (QREFELT $ 38))
         (QREFELT $ 182)))
      ((SPADCALL |dq| |dp| (QREFELT $ 181))
        (SPADCALL
         (SPADCALL |p| (QREFELT $ 38))
         (|spadConstant| $ 22)
         (QREFELT $ 182)))
      ((QUOTE T)
       (SPADCALL
        (SPADCALL (SPADCALL |p| |q| (QREFELT $ 155)) (QREFELT $ 38))
        (|spadConstant| $ 22)
        (QREFELT $ 182)))))))))

(DEFUN |POLYCAT-;patternMatch;SP2Pmr;39| (|p| |pat| |l| $)
 (SPADCALL |p| |pat| |l| (QREFELT $ 187)))

(DEFUN |POLYCAT-;patternMatch;SP2Pmr;40| (|p| |pat| |l| $)
 (SPADCALL |p| |pat| |l| (QREFELT $ 193)))

(DEFUN |POLYCAT-;convert;SP;41| (|x| $)
 (SPADCALL (ELT $ 196) (ELT $ 197) |x| (QREFELT $ 201)))
```

```
(DEFUN |POLYCAT-;convert;SP;42| (|x| $)
 (SPADCALL (ELT $ 203) (ELT $ 204) |x| (QREFELT $ 208)))

(DEFUN |POLYCAT-;convert;SIf;43| (|p| $)
 (SPADCALL (ELT $ 211) (ELT $ 212) |p| (QREFELT $ 216)))

(DEFUN |PolynomialCategory&| (|#1| |#2| |#3| |#4|)
 (PROG (DV$1 DV$2 DV$3 DV$4 |dv$| $ |pv$|)
  (RETURN
   (PROGN
    (LETT DV$1 (|devaluate| |#1|) . #0=(|PolynomialCategory&|))
    (LETT DV$2 (|devaluate| |#2|) . #0#)
    (LETT DV$3 (|devaluate| |#3|) . #0#)
    (LETT DV$4 (|devaluate| |#4|) . #0#)
    (LETT |dv$| (LIST (QUOTE |PolynomialCategory&|) DV$1 DV$2 DV$3 DV$4) . #0#)
    (LETT $ (GETREFV 226) . #0#)
    (QSETREFV $ 0 |dv$|)
    (QSETREFV $ 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST
        (|HasCategory| |#2| (QUOTE (|PolynomialFactorizationExplicit|)))
        (|HasAttribute| |#2| (QUOTE |canonicalUnitNormal|))
        (|HasCategory| |#2| (QUOTE (|GcdDomain|)))
        (|HasCategory| |#2| (QUOTE (|CommutativeRing|)))
        (|HasCategory| |#4| (QUOTE (|PatternMatchable| (|Float|))))
        (|HasCategory| |#2| (QUOTE (|PatternMatchable| (|Float|))))
        (|HasCategory| |#4| (QUOTE (|PatternMatchable| (|Integer|))))
        (|HasCategory| |#2| (QUOTE (|PatternMatchable| (|Integer|))))
        (|HasCategory| |#4| (QUOTE (|ConvertibleTo| (|Pattern| (|Float|)))))
        (|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|Pattern| (|Float|)))))
        (|HasCategory| |#4| (QUOTE (|ConvertibleTo| (|Pattern| (|Integer|)))))
        (|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|Pattern| (|Integer|)))))
        (|HasCategory| |#4| (QUOTE (|ConvertibleTo| (|InputForm|))))
        (|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|InputForm|))))
        (|HasCategory| |#2| (QUOTE (|OrderedSet|)))))
      . #0#))
    (|stuffDomainSlots| $)
    (QSETREFV $ 6 |#1|)
    (QSETREFV $ 7 |#2|)
    (QSETREFV $ 8 |#3|)
    (QSETREFV $ 9 |#4|)
    (COND
     ((|testBitVector| |pv$| 4)
      (PROGN
```

```
   (QSETREFV $ 74
    (CONS (|dispatchFunction| |POLYCAT-;resultant;2SVarSetS;15|) $))
   (QSETREFV $ 76
    (CONS (|dispatchFunction| |POLYCAT-;discriminant;SVarSetS;16|) $)))))
(COND
 ((|HasCategory| |#2| (QUOTE (|IntegralDomain|)))
  (PROGN
   (QSETREFV $ 95
    (CONS (|dispatchFunction| |POLYCAT-;reducedSystem;MM;20|) $))
   (QSETREFV $ 102
    (CONS (|dispatchFunction| |POLYCAT-;reducedSystem;MVR;21|) $)))))
(COND
 ((|testBitVector| |pv$| 1)
  (PROGN
   (QSETREFV $ 105
    (CONS (|dispatchFunction| |POLYCAT-;gcdPolynomial;3Sup;22|) $))
   (QSETREFV $ 112
    (CONS
     (|dispatchFunction|
       |POLYCAT-;solveLinearPolynomialEquation;LSupU;23|)
     $))
   (QSETREFV $ 116
    (CONS (|dispatchFunction| |POLYCAT-;factorPolynomial;SupF;24|) $))
   (QSETREFV $ 118
    (CONS
     (|dispatchFunction| |POLYCAT-;factorSquareFreePolynomial;SupF;25|)
     $))
   (QSETREFV $ 136 (CONS (|dispatchFunction| |POLYCAT-;factor;SF;26|) $))
   (COND
    ((|HasCategory| |#2| (QUOTE (|CharacteristicNonZero|)))
     (PROGN
      (QSETREFV $ 150
       (CONS (|dispatchFunction| |POLYCAT-;conditionP;MU;27|) $)))))))
(COND
 ((|HasCategory| |#2| (QUOTE (|CharacteristicNonZero|)))
  (PROGN
   (QSETREFV $ 152
    (CONS (|dispatchFunction| |POLYCAT-;charthRoot;SU;28|) $)))))
(COND
 ((|testBitVector| |pv$| 3)
  (PROGN
   (COND
    ((|HasCategory| |#2| (QUOTE (|EuclideanDomain|)))
     (COND
      ((|HasCategory| |#2| (QUOTE (|CharacteristicZero|)))
       (QSETREFV $ 161
```

```
          (CONS (|dispatchFunction| |POLYCAT-;squareFree;SF;31|) $)))
        ((QUOTE T)
         (QSETREFV $ 161
          (CONS (|dispatchFunction| |POLYCAT-;squareFree;SF;32|) $)))))
      ((QUOTE T)
       (QSETREFV $ 161
        (CONS (|dispatchFunction| |POLYCAT-;squareFree;SF;33|) $))))
    (QSETREFV $ 169
     (CONS (|dispatchFunction| |POLYCAT-;squareFreePart;2S;34|) $))
    (QSETREFV $ 171
     (CONS (|dispatchFunction| |POLYCAT-;content;SVarSetS;35|) $))
    (QSETREFV $ 177
     (CONS (|dispatchFunction| |POLYCAT-;primitivePart;2S;36|) $))
    (QSETREFV $ 180
     (CONS (|dispatchFunction| |POLYCAT-;primitivePart;SVarSetS;37|) $)))))
  (COND
   ((|testBitVector| |pv$| 15)
    (PROGN
     (QSETREFV $ 183 (CONS (|dispatchFunction| |POLYCAT-;<;2SB;38|) $))
     (COND
      ((|testBitVector| |pv$| 8)
       (COND
        ((|testBitVector| |pv$| 7)
         (QSETREFV $ 189
          (CONS
           (|dispatchFunction| |POLYCAT-;patternMatch;SP2Pmr;39|)
           $))))))
     (COND
      ((|testBitVector| |pv$| 6)
       (COND
        ((|testBitVector| |pv$| 5)
         (QSETREFV $ 195
          (CONS
           (|dispatchFunction| |POLYCAT-;patternMatch;SP2Pmr;40|)
           $)))))))))
  (COND
   ((|testBitVector| |pv$| 12)
    (COND
     ((|testBitVector| |pv$| 11)
      (QSETREFV $ 202
       (CONS (|dispatchFunction| |POLYCAT-;convert;SP;41|) $))))))
  (COND
   ((|testBitVector| |pv$| 10)
    (COND
     ((|testBitVector| |pv$| 9)
      (QSETREFV $ 209
```

```
          (CONS (|dispatchFunction| |POLYCAT-;convert;SP;42|) $))))))
     (COND
      ((|testBitVector| |pv$| 14)
       (COND
        ((|testBitVector| |pv$| 13)
         (QSETREFV $ 217
          (CONS (|dispatchFunction| |POLYCAT-;convert;SIf;43|) $))))))
     $))))

(MAKEPROP
 (QUOTE |PolynomialCategory&|)
 (QUOTE |infovec|)
 (LIST (QUOTE
  #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|local| |#3|)
    (|local| |#4|) (|Equation| 6) (0 . |lhs|) (|Union| 9 (QUOTE "failed"))
    (5 . |retractIfCan|) (10 . |retract|) (15 . |rhs|) (|List| 9) (|List| $)
    (20 . |eval|) (|List| 221) |POLYCAT-;eval;SLS;1| (27 . |Zero|)
    (31 . |Zero|) (|Boolean|) (35 . =) (41 . |leadingMonomial|)
    (46 . |reductum|) |POLYCAT-;monomials;SL;2| (51 . |monomials|)
    (|Union| 17 (QUOTE "failed")) |POLYCAT-;isPlus;SU;3| (56 . |variables|)
    (61 . |monomial?|) (66 . |One|) (70 . |One|) (|NonNegativeInteger|)
    (74 . |degree|) (80 . |monomial|) (87 . |leadingCoefficient|) (92 . =)
    (98 . |coerce|) |POLYCAT-;isTimes;SU;4| (103 . |mainVariable|)
    (|Record| (|:| |var| 9) (|:| |exponent| 35))
    (|Union| 43 (QUOTE "failed")) |POLYCAT-;isExpt;SU;5|
    (|SparseUnivariatePolynomial| $) (108 . |univariate|)
    (|SparseUnivariatePolynomial| 6) (114 . |coefficient|)
    |POLYCAT-;coefficient;SVarSetNniS;6| (|List| 35) (120 . |coefficient|)
    |POLYCAT-;coefficient;SLLS;7| (127 . |monomial|)
    |POLYCAT-;monomial;SLLS;8| (134 . |coerce|)
    |POLYCAT-;retract;SVarSet;9| |POLYCAT-;retractIfCan;SU;10|
    (139 . |degree|) (144 . |monomial|) |POLYCAT-;primitiveMonomials;SL;12|
    (150 . |ground?|) (155 . |Zero|) (159 . =) (165 . |degree|)
    (170 . |leadingCoefficient|) (175 . |totalDegree|) (180 . |reductum|)
    |POLYCAT-;totalDegree;SNni;13| (185 . |member?|) (191 . |totalDegree|)
    |POLYCAT-;totalDegree;SLNni;14| (197 . |resultant|) (203 . |resultant|)
    (210 . |discriminant|) (215 . |discriminant|) (221 . |primitiveMonomials|)
    (|List| 6) (226 . |concat|) (231 . |removeDuplicates!|) (|Vector| 7)
    (236 . |new|) (|Integer|) (242 . |minIndex|) (247 . |coefficient|)
    (253 . |qsetelt!|) (|List| 220) (|Matrix| 7) (260 . |matrix|)
    (|List| 78) (|Matrix| 6) (265 . |listOfLists|) (270 . |vertConcat|)
    (|Matrix| $) (276 . |reducedSystem|) (|Vector| 6) (281 . |entries|)
    (286 . |concat|) (292 . |concat|)
    (|Record| (|:| |mat| 88) (|:| |vec| 81)) (|Vector| $)
    (298 . |reducedSystem|) (|GeneralPolynomialGcdPackage| 8 9 7 6)
    (304 . |gcdPolynomial|) (310 . |gcdPolynomial|)
```

```
(|Union| 107 (QUOTE "failed")) (|List| 48)
(|PolynomialFactorizationByRecursion| 7 8 9 6)
(316 . |solveLinearPolynomialEquationByRecursion|)
(|Union| 111 (QUOTE "failed")) (|List| 46)
(322 . |solveLinearPolynomialEquation|) (|Factored| 48)
(328 . |factorByRecursion|) (|Factored| 46) (333 . |factorPolynomial|)
(338 . |factorSquareFreeByRecursion|)
(343 . |factorSquareFreePolynomial|) (|Factored| $) (348 . |factor|)
(|Factored| 7) (353 . |unit|)
(|Union| (QUOTE "nil") (QUOTE "sqfr") (QUOTE "irred") (QUOTE "prime"))
(|Record| (|:| |flg| 123) (|:| |fctr| 7) (|:| |xpnt| 83))
(|List| 124) (358 . |factorList|)
(|Record| (|:| |flg| 123) (|:| |fctr| 6) (|:| |xpnt| 83))
(|List| 127) (|Factored| 6) (363 . |makeFR|) (369 . |unit|)
(374 . |multivariate|)
(|Record| (|:| |flg| 123) (|:| |fctr| 48) (|:| |xpnt| 83))
(|List| 133) (380 . |factorList|) (385 . |factor|) (390 . |transpose|)
(395 . |characteristic|) (399 . |setUnion|) (405 . |degree|)
(|Union| $ (QUOTE "failed")) (411 . |exquo|) (417 . |ground|)
(422 . |transpose|) (|Union| 101 (QUOTE "failed")) (427 . |conditionP|)
(432 . |elt|) (438 . *) (444 . +) (450 . |conditionP|)
(455 . |charthRoot|) (460 . |charthRoot|) (465 . |Zero|)
(469 . |coefficient|) (476 . -)
(|Record| (|:| |quotient| $) (|:| |remainder| $))
(482 . |monicDivide|) |POLYCAT-;monicDivide;2SVarSetR;30|
(|MultivariateSquareFree| 8 9 7 6) (488 . |squareFree|)
(493 . |squareFree|) (|PolynomialSquareFree| 9 8 7 6)
(498 . |squareFree|) (503 . |squareFree|) (508 . |unit|)
(|Record| (|:| |factor| 6) (|:| |exponent| 83)) (|List| 166)
(513 . |factors|) (518 . |squareFreePart|) (523 . |content|)
(528 . |content|) (534 . |zero?|) (539 . |content|) (544 . |exquo|)
(|Record| (|:| |unit| $) (|:| |canonical| $) (|:| |associate| $))
(550 . |unitNormal|) (555 . |primitivePart|) (560 . |content|)
(566 . |exquo|) (572 . |primitivePart|) (578 . <) (584 . <) (590 . <)
(|PatternMatchResult| 83 6) (|Pattern| 83)
(|PatternMatchPolynomialCategory| 83 8 9 7 6) (596 . |patternMatch|)
(|PatternMatchResult| 83 $) (603 . |patternMatch|)
(|PatternMatchResult| (|Float|) 6) (|Pattern| (|Float|))
(|PatternMatchPolynomialCategory| (|Float|) 8 9 7 6)
(610 . |patternMatch|) (|PatternMatchResult| (|Float|) $)
(617 . |patternMatch|) (624 . |convert|) (629 . |convert|)
(|Mapping| 185 9) (|Mapping| 185 7)
(|PolynomialCategoryLifting| 8 9 7 6 185) (634 . |map|)
(641 . |convert|) (646 . |convert|) (651 . |convert|) (|Mapping| 191 9)
(|Mapping| 191 7) (|PolynomialCategoryLifting| 8 9 7 6 191)
(656 . |map|) (663 . |convert|) (|InputForm|) (668 . |convert|)
```

```
  (673 . |convert|) (|Mapping| 210 9) (|Mapping| 210 7)
  (|PolynomialCategoryLifting| 8 9 7 6 210) (678 . |map|)
  (685 . |convert|) (|Record| (|:| |mat| 219) (|:| |vec| (|Vector| 83)))
  (|Matrix| 83) (|List| 7) (|Equation| $) (|Union| 83 (QUOTE "failed"))
  (|Union| 224 (QUOTE "failed")) (|Fraction| 83)
  (|Union| 7 (QUOTE "failed"))))
(QUOTE #(|totalDegree| 690 |squareFreePart| 701 |squareFree| 706
 |solveLinearPolynomialEquation| 711 |retractIfCan| 717 |retract| 722
 |resultant| 727 |reducedSystem| 734 |primitivePart| 745
 |primitiveMonomials| 756 |patternMatch| 761 |monomials| 775
 |monomial| 780 |monicDivide| 787 |isTimes| 794 |isPlus| 799
 |isExpt| 804 |gcdPolynomial| 809 |factorSquareFreePolynomial| 815
 |factorPolynomial| 820 |factor| 825 |eval| 830 |discriminant| 836
 |convert| 842 |content| 857 |conditionP| 863 |coefficient| 868
 |charthRoot| 882 < 887))
(QUOTE NIL)
(CONS (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS (QUOTE #())
  (CONS (QUOTE #())
   (|makeByteWordVec2| 217 (QUOTE
    (1 10 6 0 11 1 6 12 0 13 1 6 9 0 14 1 10 6 0 15 3 6 0 0 16 17 18 0 6 0
     21 0 7 0 22 2 6 23 0 0 24 1 6 0 0 25 1 6 0 0 26 1 6 17 0 28 1 6 16 0
     31 1 6 23 0 32 0 6 0 33 0 7 0 34 2 6 35 0 9 36 3 6 0 0 9 35 37 1 6 7
     0 38 2 7 23 0 0 39 1 6 0 7 40 1 6 12 0 42 2 6 46 0 9 47 2 48 6 0 35
     49 3 6 0 0 16 51 52 3 6 0 0 16 51 54 1 6 0 9 56 1 6 8 0 59 2 6 0 7 8
     60 1 6 23 0 62 0 48 0 63 2 48 23 0 0 64 1 48 35 0 65 1 48 6 0 66 1 6
     35 0 67 1 48 0 0 68 2 16 23 9 0 70 2 6 35 0 16 71 2 48 6 0 0 73 3 0
     0 0 0 9 74 1 48 6 0 75 2 0 0 0 9 76 1 6 17 0 77 1 78 0 17 79 1 78 0
     0 80 2 81 0 35 7 82 1 81 83 0 84 2 6 7 0 8 85 3 81 7 0 83 7 86 1 88
     0 87 89 1 91 90 0 92 2 88 0 0 0 93 1 0 88 94 95 1 96 78 0 97 2 78 0
     0 0 98 2 81 0 0 0 99 2 0 100 94 101 102 2 103 48 48 48 104 2 0 46 46
     46 105 2 108 106 107 48 109 2 0 110 111 46 112 1 108 113 48 114 1 0
     115 46 116 1 108 113 48 117 1 0 115 46 118 1 7 119 0 120 1 121 7 0
     122 1 121 125 0 126 2 129 0 6 128 130 1 113 48 0 131 2 6 0 46 9 132
     1 113 134 0 135 1 0 119 0 136 1 91 0 0 137 0 6 35 138 2 78 0 0 0 139
     2 6 51 0 16 140 2 83 141 0 0 142 1 6 7 0 143 1 88 0 0 144 1 7 145 94
     146 2 81 7 0 83 147 2 6 0 0 0 148 2 6 0 0 0 149 1 0 145 94 150 1 7
     141 0 151 1 0 141 0 152 0 8 0 153 3 6 0 0 9 35 154 2 6 0 0 0 155 2
     48 156 0 0 157 1 159 129 6 160 1 0 119 0 161 1 162 129 6 163 1 6 119
     0 164 1 129 6 0 165 1 129 167 0 168 1 0 0 0 169 1 48 6 0 170 2 0 0 0
     9 171 1 6 23 0 172 1 6 7 0 173 2 6 141 0 7 174 1 6 175 0 176 1 0 0 0
     177 2 6 0 0 9 178 2 6 141 0 0 179 2 0 0 0 9 180 2 8 23 0 0 181 2 7 23
     0 0 182 2 0 23 0 0 183 3 186 184 6 185 184 187 3 0 188 0 185 188 189
     3 192 190 6 191 190 193 3 0 194 0 191 194 195 1 9 185 0 196 1 7 185
     0 197 3 200 185 198 199 6 201 1 0 185 0 202 1 9 191 0 203 1 7 191 0
     204 3 207 191 205 206 6 208 1 0 191 0 209 1 9 210 0 211 1 7 210 0
```

```
      212 3 215 210 213 214 6 216 1 0 210 0 217 2 0 35 0 16 72 1 0 35 0 69
      1 0 0 0 169 1 0 119 0 161 2 0 110 111 46 112 1 0 12 0 58 1 0 9 0 57
      3 0 0 0 0 9 74 1 0 88 94 95 2 0 100 94 101 102 2 0 0 0 9 180 1 0 0 0
      177 1 0 17 0 61 3 0 188 0 185 188 189 3 0 194 0 191 194 195 1 0 17 0
      27 3 0 0 0 16 51 55 3 0 156 0 0 9 158 1 0 29 0 41 1 0 29 0 30 1 0 44
      0 45 2 0 46 46 46 105 1 0 115 46 118 1 0 115 46 116 1 0 119 0 136 2
      0 0 0 19 20 2 0 0 0 9 76 1 0 210 0 217 1 0 185 0 202 1 0 191 0 209 2
      0 0 0 9 171 1 0 145 94 150 3 0 0 0 16 51 53 3 0 0 0 9 35 50 1 0 141
      0 152 2 0 23 0 0 183))))))
   (QUOTE |lookupComplete|)))
```

21.45 PSETCAT.lsp BOOTSTRAP

PSETCAT depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **PSETCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **PSETCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*PSETCAT.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |PolynomialSetCategory;CAT| (QUOTE NIL))

(SETQ |PolynomialSetCategory;AL| (QUOTE NIL))

(DEFUN |PolynomialSetCategory| (|&REST| #1=#:G82375 |&AUX| #2=#:G82373)
 (DSETQ #2# #1#)
 (LET (#3=#:G82374)
  (COND
   ((SETQ #3# (|assoc| (|devaluateList| #2#) |PolynomialSetCategory;AL|))
     (CDR #3#))
   (T
    (SETQ |PolynomialSetCategory;AL|
     (|cons5|
      (CONS
       (|devaluateList| #2#)
       (SETQ #3# (APPLY (FUNCTION |PolynomialSetCategory;|) #2#)))
      |PolynomialSetCategory;AL|))
    #3#))))

(DEFUN |PolynomialSetCategory;| (|t#1| |t#2| |t#3| |t#4|)
 (PROG (#1=#:G82372)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR
       (QUOTE (|t#1| |t#2| |t#3| |t#4|))
       (LIST
        (|devaluate| |t#1|)
        (|devaluate| |t#2|)
        (|devaluate| |t#3|)
        (|devaluate| |t#4|)))
      (|sublisV|
```

```
(PAIR (QUOTE (#2=#:G82371)) (LIST (QUOTE (|List| |t#4|))))
(COND
 (|PolynomialSetCategory;CAT|)
 ((QUOTE T)
  (LETT |PolynomialSetCategory;CAT|
   (|Join|
    (|SetCategory|)
    (|Collection| (QUOTE |t#4|))
    (|CoercibleTo| (QUOTE #2#))
    (|mkCategory|
     (QUOTE |domain|)
     (QUOTE (
      ((|retractIfCan| ((|Union| |$| "failed") (|List| |t#4|))) T)
      ((|retract| (|$| (|List| |t#4|))) T)
      ((|mvar| (|t#3| |$|)) T)
      ((|variables| ((|List| |t#3|) |$|)) T)
      ((|mainVariables| ((|List| |t#3|) |$|)) T)
      ((|mainVariable?| ((|Boolean|) |t#3| |$|)) T)
      ((|collectUnder| (|$| |$| |t#3|)) T)
      ((|collect| (|$| |$| |t#3|)) T)
      ((|collectUpper| (|$| |$| |t#3|)) T)
      ((|sort|
       ((|Record|
         (|:| |under| |$|)
         (|:| |floor| |$|)
         (|:| |upper| |$|)) |$| |t#3|))
        T)
      ((|trivialIdeal?| ((|Boolean|) |$|)) T)
      ((|roughBase?| ((|Boolean|) |$|))
        (|has| |t#1| (|IntegralDomain|)))
      ((|roughSubIdeal?| ((|Boolean|) |$| |$|))
        (|has| |t#1| (|IntegralDomain|)))
      ((|roughEqualIdeals?| ((|Boolean|) |$| |$|))
        (|has| |t#1| (|IntegralDomain|)))
      ((|roughUnitIdeal?| ((|Boolean|) |$|))
        (|has| |t#1| (|IntegralDomain|)))
      ((|headRemainder|
        ((|Record|
          (|:| |num| |t#4|)
          (|:| |den| |t#1|)) |t#4| |$|))
         (|has| |t#1| (|IntegralDomain|)))
      ((|remainder|
        ((|Record|
          (|:| |rnum| |t#1|)
          (|:| |polnum| |t#4|)
          (|:| |den| |t#1|)) |t#4| |$|))
```

```
              (|has| |t#1| (|IntegralDomain|)))
             ((|rewriteIdealWithHeadRemainder|
              ((|List| |t#4|) (|List| |t#4|) |$|))
               (|has| |t#1| (|IntegralDomain|)))
             ((|rewriteIdealWithRemainder| ((|List| |t#4|) (|List| |t#4|) |$|))
               (|has| |t#1| (|IntegralDomain|)))
             ((|triangular?| ((|Boolean|) |$|))
               (|has| |t#1| (|IntegralDomain|)))))
            (QUOTE ((|finiteAggregate| T)))
            (QUOTE ((|Boolean|) (|List| |t#4|) (|List| |t#3|)))
            NIL))
    . #3=(|PolynomialSetCategory|))))))
  . #3#)
 (SETELT #1# 0
  (LIST
   (QUOTE |PolynomialSetCategory|)
   (|devaluate| |t#1|)
   (|devaluate| |t#2|)
   (|devaluate| |t#3|)
   (|devaluate| |t#4|)))))))
```

21.46 PSETCAT-.lsp BOOTSTRAP

PSETCAT- depends on **PSETCAT**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **PSETCAT-** category which we can write into the **MID** directory. We compile the lisp code and copy the **PSETCAT-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*PSETCAT-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |PSETCAT-;elements| (|ps| |$|)
 (PROG (|lp|)
  (RETURN
   (LETT |lp| (SPADCALL |ps| (QREFELT |$| 12)) |PSETCAT-;elements|))))

(DEFUN |PSETCAT-;variables1| (|lp| |$|)
 (PROG (#1=#:G82392 |p| #2=#:G82393 |lvars|)
  (RETURN
   (SEQ
    (LETT |lvars|
     (PROGN
      (LETT #1# NIL |PSETCAT-;variables1|)
      (SEQ
       (LETT |p| NIL |PSETCAT-;variables1|)
       (LETT #2# |lp| |PSETCAT-;variables1|)
       G190
       (COND
        ((OR (ATOM #2#) (PROGN (LETT |p| (CAR #2#) |PSETCAT-;variables1|) NIL))
          (GO G191)))
       (SEQ
        (EXIT
         (LETT #1#
          (CONS (SPADCALL |p| (QREFELT |$| 14)) #1#) |PSETCAT-;variables1|)))
       (LETT #2# (CDR #2#) |PSETCAT-;variables1|)
       (GO G190)
       G191
       (EXIT (NREVERSE0 #1#))))
     |PSETCAT-;variables1|)
    (EXIT
     (SPADCALL
      (CONS (FUNCTION |PSETCAT-;variables1!0|) |$|)
      (SPADCALL (SPADCALL |lvars| (QREFELT |$| 18)) (QREFELT |$| 19))
      (QREFELT |$| 21)))))))
```

```
(DEFUN |PSETCAT-;variables1!0| (|#1| |#2| |$|)
 (SPADCALL |#2| |#1| (QREFELT |$| 16)))

(DEFUN |PSETCAT-;variables2| (|lp| |$|)
 (PROG (#1=#:G82397 |p| #2=#:G82398 |lvars|)
  (RETURN
   (SEQ
    (LETT |lvars|
     (PROGN
      (LETT #1# NIL |PSETCAT-;variables2|)
      (SEQ
       (LETT |p| NIL |PSETCAT-;variables2|)
       (LETT #2# |lp| |PSETCAT-;variables2|)
       G190
       (COND
        ((OR (ATOM #2#) (PROGN (LETT |p| (CAR #2#) |PSETCAT-;variables2|) NIL))
          (GO G191)))
       (SEQ
        (EXIT
         (LETT #1#
          (CONS (SPADCALL |p| (QREFELT |$| 22)) #1#) |PSETCAT-;variables2|)))
       (LETT #2# (CDR #2#) |PSETCAT-;variables2|)
       (GO G190)
       G191
       (EXIT (NREVERSE0 #1#))))
     |PSETCAT-;variables2|)
    (EXIT
     (SPADCALL
      (CONS (FUNCTION |PSETCAT-;variables2!0|) |$|)
      (SPADCALL |lvars| (QREFELT |$| 19))
      (QREFELT |$| 21)))))))

(DEFUN |PSETCAT-;variables2!0| (|#1| |#2| |$|)
 (SPADCALL |#2| |#1| (QREFELT |$| 16)))

(DEFUN |PSETCAT-;variables;SL;4| (|ps| |$|)
 (|PSETCAT-;variables1| (|PSETCAT-;elements| |ps| |$|) |$|))

(DEFUN |PSETCAT-;mainVariables;SL;5| (|ps| |$|)
 (|PSETCAT-;variables2|
  (SPADCALL (ELT |$| 24) (|PSETCAT-;elements| |ps| |$|) (QREFELT |$| 26))
  |$|))

(DEFUN |PSETCAT-;mainVariable?;VarSetSB;6| (|v| |ps| |$|)
 (PROG (|lp|)
```

```
    (RETURN
     (SEQ
      (LETT |lp|
       (SPADCALL (ELT |$| 24) (|PSETCAT-;elements| |ps| |$|) (QREFELT |$| 26))
       |PSETCAT-;mainVariable?;VarSetSB;6|)
      (SEQ
       G190
       (COND
        ((NULL
          (COND
           ((OR
             (NULL |lp|)
             (SPADCALL
              (SPADCALL (|SPADfirst| |lp|) (QREFELT |$| 22))
              |v|
              (QREFELT |$| 28)))
            (QUOTE NIL))
           ((QUOTE T) (QUOTE T))))
         (GO G191)))
       (SEQ (EXIT (LETT |lp| (CDR |lp|) |PSETCAT-;mainVariable?;VarSetSB;6|)))
       NIL
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))))))

  (DEFUN |PSETCAT-;collectUnder;SVarSetS;7| (|ps| |v| |$|)
   (PROG (|p| |lp| |lq|)
    (RETURN
     (SEQ
      (LETT |lp|
       (|PSETCAT-;elements| |ps| |$|)
       |PSETCAT-;collectUnder;SVarSetS;7|)
      (LETT |lq| NIL |PSETCAT-;collectUnder;SVarSetS;7|)
      (SEQ
       G190
       (COND
        ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
          (GO G191)))
       (SEQ
        (LETT |p| (|SPADfirst| |lp|) |PSETCAT-;collectUnder;SVarSetS;7|)
        (LETT |lp| (CDR |lp|) |PSETCAT-;collectUnder;SVarSetS;7|)
        (EXIT
         (COND
          ((OR
            (SPADCALL |p| (QREFELT |$| 24))
```

```
           (SPADCALL (SPADCALL |p| (QREFELT |$| 22)) |v| (QREFELT |$| 16)))
          (LETT |lq| (CONS |p| |lq|) |PSETCAT-;collectUnder;SVarSetS;7|)))))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (SPADCALL |lq| (QREFELT |$| 30)))))))

(DEFUN |PSETCAT-;collectUpper;SVarSetS;8| (|ps| |v| |$|)
 (PROG (|p| |lp| |lq|)
  (RETURN
   (SEQ
    (LETT |lp|
     (|PSETCAT-;elements| |ps| |$|)
     |PSETCAT-;collectUpper;SVarSetS;8|)
    (LETT |lq| NIL |PSETCAT-;collectUpper;SVarSetS;8|)
    (SEQ
     G190
     (COND
      ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
        (GO G191)))
     (SEQ
      (LETT |p| (|SPADfirst| |lp|) |PSETCAT-;collectUpper;SVarSetS;8|)
      (LETT |lp| (CDR |lp|) |PSETCAT-;collectUpper;SVarSetS;8|)
      (EXIT
       (COND
        ((NULL (SPADCALL |p| (QREFELT |$| 24)))
         (COND
          ((SPADCALL |v| (SPADCALL |p| (QREFELT |$| 22)) (QREFELT |$| 16))
           (LETT |lq| (CONS |p| |lq|) |PSETCAT-;collectUpper;SVarSetS;8|)))))))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (SPADCALL |lq| (QREFELT |$| 30)))))))

(DEFUN |PSETCAT-;collect;SVarSetS;9| (|ps| |v| |$|)
 (PROG (|p| |lp| |lq|)
  (RETURN
   (SEQ
    (LETT |lp| (|PSETCAT-;elements| |ps| |$|) |PSETCAT-;collect;SVarSetS;9|)
    (LETT |lq| NIL |PSETCAT-;collect;SVarSetS;9|)
    (SEQ
     G190
     (COND
      ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
```

```
          (GO G191)))
        (SEQ
         (LETT |p| (|SPADfirst| |lp|) |PSETCAT-;collect;SVarSetS;9|)
         (LETT |lp| (CDR |lp|) |PSETCAT-;collect;SVarSetS;9|)
         (EXIT
          (COND
           ((NULL (SPADCALL |p| (QREFELT |$| 24)))
            (COND
             ((SPADCALL (SPADCALL |p| (QREFELT |$| 22)) |v| (QREFELT |$| 28))
               (LETT |lq| (CONS |p| |lq|) |PSETCAT-;collect;SVarSetS;9|)))))))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT (SPADCALL |lq| (QREFELT |$| 30)))))))

(DEFUN |PSETCAT-;sort;SVarSetR;10| (|ps| |v| |$|)
 (PROG (|p| |lp| |us| |vs| |ws|)
  (RETURN
   (SEQ
    (LETT |lp| (|PSETCAT-;elements| |ps| |$|) |PSETCAT-;sort;SVarSetR;10|)
    (LETT |us| NIL |PSETCAT-;sort;SVarSetR;10|)
    (LETT |vs| NIL |PSETCAT-;sort;SVarSetR;10|)
    (LETT |ws| NIL |PSETCAT-;sort;SVarSetR;10|)
    (SEQ
     G190
     (COND
      ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (LETT |p| (|SPADfirst| |lp|) |PSETCAT-;sort;SVarSetR;10|)
      (LETT |lp| (CDR |lp|) |PSETCAT-;sort;SVarSetR;10|)
      (EXIT
       (COND
        ((OR
           (SPADCALL |p| (QREFELT |$| 24))
           (SPADCALL (SPADCALL |p| (QREFELT |$| 22)) |v| (QREFELT |$| 16)))
          (LETT |us| (CONS |p| |us|) |PSETCAT-;sort;SVarSetR;10|))
        ((SPADCALL (SPADCALL |p| (QREFELT |$| 22)) |v| (QREFELT |$| 28))
          (LETT |vs| (CONS |p| |vs|) |PSETCAT-;sort;SVarSetR;10|))
        ((QUOTE T) (LETT |ws| (CONS |p| |ws|) |PSETCAT-;sort;SVarSetR;10|)))))))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT
```

```
      (VECTOR
       (SPADCALL |us| (QREFELT |$| 30))
       (SPADCALL |vs| (QREFELT |$| 30))
       (SPADCALL |ws| (QREFELT |$| 30))))))))

(DEFUN |PSETCAT-;=;2SB;11| (|ps1| |ps2| |$|)
 (PROG (#1=#:G82439 #2=#:G82440 #3=#:G82437 |p| #4=#:G82438)
  (RETURN
   (SEQ
    (SPADCALL
     (SPADCALL
      (PROGN
       (LETT #1# NIL |PSETCAT-;=;2SB;11|)
       (SEQ
        (LETT |p| NIL |PSETCAT-;=;2SB;11|)
        (LETT #2# (|PSETCAT-;elements| |ps1| |$|) |PSETCAT-;=;2SB;11|)
        G190
        (COND
         ((OR (ATOM #2#) (PROGN (LETT |p| (CAR #2#) |PSETCAT-;=;2SB;11|) NIL))
           (GO G191)))
        (SEQ (EXIT (LETT #1# (CONS |p| #1#) |PSETCAT-;=;2SB;11|)))
        (LETT #2# (CDR #2#) |PSETCAT-;=;2SB;11|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #1#))))
      (QREFELT |$| 37))
     (SPADCALL
      (PROGN
       (LETT #3# NIL |PSETCAT-;=;2SB;11|)
       (SEQ
        (LETT |p| NIL |PSETCAT-;=;2SB;11|)
        (LETT #4# (|PSETCAT-;elements| |ps2| |$|) |PSETCAT-;=;2SB;11|)
        G190
        (COND
         ((OR (ATOM #4#) (PROGN (LETT |p| (CAR #4#) |PSETCAT-;=;2SB;11|) NIL))
           (GO G191)))
        (SEQ (EXIT (LETT #3# (CONS |p| #3#) |PSETCAT-;=;2SB;11|)))
        (LETT #4# (CDR #4#) |PSETCAT-;=;2SB;11|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #3#))))
      (QREFELT |$| 37))
     (QREFELT |$| 38))))))

(DEFUN |PSETCAT-;localInf?| (|p| |q| |$|)
 (SPADCALL
```

```
    (SPADCALL |p| (QREFELT |$| 40))
    (SPADCALL |q| (QREFELT |$| 40))
    (QREFELT |$| 41)))

  (DEFUN |PSETCAT-;localTriangular?| (|lp| |$|)
   (PROG (|q| |p|)
    (RETURN
     (SEQ
      (LETT |lp|
       (SPADCALL (ELT |$| 42) |lp| (QREFELT |$| 26)) |PSETCAT-;localTriangular?|)
      (EXIT
       (COND
        ((NULL |lp|) (QUOTE T))
        ((SPADCALL (ELT |$| 24) |lp| (QREFELT |$| 43)) (QUOTE NIL))
        ((QUOTE T)
         (SEQ
          (LETT |lp|
           (SPADCALL
            (CONS (FUNCTION |PSETCAT-;localTriangular?!0|) |$|)
            |lp|
            (QREFELT |$| 45))
           |PSETCAT-;localTriangular?|)
          (LETT |p| (|SPADfirst| |lp|) |PSETCAT-;localTriangular?|)
          (LETT |lp| (CDR |lp|) |PSETCAT-;localTriangular?|)
          (SEQ
           G190
           (COND
            ((NULL
              (COND
               ((NULL |lp|) (QUOTE NIL))
               ((QUOTE T)
                (SPADCALL
                 (SPADCALL
                  (LETT |q|
                   (|SPADfirst| |lp|)
                   |PSETCAT-;localTriangular?|)
                  (QREFELT |$| 22))
                 (SPADCALL |p| (QREFELT |$| 22)) (QREFELT |$| 16)))))
             (GO G191)))
           (SEQ
            (LETT |p| |q| |PSETCAT-;localTriangular?|)
            (EXIT (LETT |lp| (CDR |lp|) |PSETCAT-;localTriangular?|)))
           NIL
           (GO G190)
           G191
           (EXIT NIL))
```

```
         (EXIT (NULL |lp|))))))))))

(DEFUN |PSETCAT-;localTriangular?!0| (|#1| |#2| |$|)
 (SPADCALL
  (SPADCALL |#2| (QREFELT |$| 22))
  (SPADCALL |#1| (QREFELT |$| 22))
  (QREFELT |$| 16)))

(DEFUN |PSETCAT-;triangular?;SB;14| (|ps| |$|)
 (|PSETCAT-;localTriangular?| (|PSETCAT-;elements| |ps| |$|) |$|))

(DEFUN |PSETCAT-;trivialIdeal?;SB;15| (|ps| |$|)
 (NULL
  (SPADCALL (ELT |$| 42) (|PSETCAT-;elements| |ps| |$|) (QREFELT |$| 26))))

(DEFUN |PSETCAT-;roughUnitIdeal?;SB;16| (|ps| |$|)
 (SPADCALL
  (ELT |$| 24)
  (SPADCALL (ELT |$| 42) (|PSETCAT-;elements| |ps| |$|) (QREFELT |$| 26))
  (QREFELT |$| 43)))

(DEFUN |PSETCAT-;relativelyPrimeLeadingMonomials?| (|p| |q| |$|)
 (PROG (|dp| |dq|)
  (RETURN
   (SEQ
    (LETT |dp|
     (SPADCALL |p| (QREFELT |$| 40))
     |PSETCAT-;relativelyPrimeLeadingMonomials?|)
    (LETT |dq|
     (SPADCALL |q| (QREFELT |$| 40))
     |PSETCAT-;relativelyPrimeLeadingMonomials?|)
    (EXIT
     (SPADCALL
      (SPADCALL |dp| |dq| (QREFELT |$| 49))
      (SPADCALL |dp| |dq| (QREFELT |$| 50))
      (QREFELT |$| 51)))))))

(DEFUN |PSETCAT-;roughBase?;SB;18| (|ps| |$|)
 (PROG (|p| |lp| |rB?| |copylp|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL (ELT |$| 42) (|PSETCAT-;elements| |ps| |$|) (QREFELT |$| 26))
     |PSETCAT-;roughBase?;SB;18|)
    (EXIT
     (COND
```

```
        ((NULL |lp|) (QUOTE T))
        ((QUOTE T)
         (SEQ
          (LETT |rB?| (QUOTE T) |PSETCAT-;roughBase?;SB;18|)
          (SEQ
           G190
           (COND
            ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) |rB?|)))
             (GO G191)))
           (SEQ
            (LETT |p| (|SPADfirst| |lp|) |PSETCAT-;roughBase?;SB;18|)
            (LETT |lp| (CDR |lp|) |PSETCAT-;roughBase?;SB;18|)
            (LETT |copylp| |lp| |PSETCAT-;roughBase?;SB;18|)
            (EXIT
             (SEQ
              G190
              (COND
               ((NULL (COND ((NULL |copylp|) (QUOTE NIL)) ((QUOTE T) |rB?|)))
                (GO G191)))
              (SEQ
               (LETT |rB?|
                (|PSETCAT-;relativelyPrimeLeadingMonomials?| |p|
                 (|SPADfirst| |copylp|) |$|)
                |PSETCAT-;roughBase?;SB;18|)
               (EXIT (LETT |copylp| (CDR |copylp|) |PSETCAT-;roughBase?;SB;18|)))
              NIL
              (GO G190)
              G191
              (EXIT NIL))))
           NIL
           (GO G190)
           G191
           (EXIT NIL))
          (EXIT |rB?|)))))))))

(DEFUN |PSETCAT-;roughSubIdeal?;2SB;19| (|ps1| |ps2| |$|)
 (PROG (|lp|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL (|PSETCAT-;elements| |ps1| |$|) |ps2| (QREFELT |$| 53))
     |PSETCAT-;roughSubIdeal?;2SB;19|)
    (EXIT (NULL (SPADCALL (ELT |$| 42) |lp| (QREFELT |$| 26)))))))))

(DEFUN |PSETCAT-;roughEqualIdeals?;2SB;20| (|ps1| |ps2| |$|)
 (COND
```

```
    ((SPADCALL |ps1| |ps2| (QREFELT |$| 55)) (QUOTE T))
    ((SPADCALL |ps1| |ps2| (QREFELT |$| 56))
      (SPADCALL |ps2| |ps1| (QREFELT |$| 56)))
    ((QUOTE T) (QUOTE NIL))))

  (DEFUN |PSETCAT-;exactQuo| (|r| |s| |$|)
   (SPADCALL |r| |s| (QREFELT |$| 58)))

  (DEFUN |PSETCAT-;exactQuo| (|r| |s| |$|)
   (PROG (#1=#:G82473)
    (RETURN
     (PROG2
      (LETT #1# (SPADCALL |r| |s| (QREFELT |$| 60)) |PSETCAT-;exactQuo|)
      (QCDR #1#)
      (|check-union| (QEQCAR #1# 0) (QREFELT |$| 7) #1#)))))

  (DEFUN |PSETCAT-;headRemainder;PSR;23| (|a| |ps| |$|)
   (PROG (|lp1| |p| |e| |g| |#G47| |#G48| |lca| |lcp| |r| |lp2|)
    (RETURN
     (SEQ
      (LETT |lp1|
       (SPADCALL (ELT |$| 42) (|PSETCAT-;elements| |ps| |$|) (QREFELT |$| 26))
       |PSETCAT-;headRemainder;PSR;23|)
      (EXIT
       (COND
        ((NULL |lp1|) (CONS |a| (|spadConstant| |$| 61)))
        ((SPADCALL (ELT |$| 24) |lp1| (QREFELT |$| 43))
          (CONS (SPADCALL |a| (QREFELT |$| 62)) (|spadConstant| |$| 61)))
        ((QUOTE T)
         (SEQ
          (LETT |r| (|spadConstant| |$| 61) |PSETCAT-;headRemainder;PSR;23|)
          (LETT |lp1|
           (SPADCALL
            (CONS (|function| |PSETCAT-;localInf?|) |$|)
            (REVERSE (|PSETCAT-;elements| |ps| |$|))
            (QREFELT |$| 45))
           |PSETCAT-;headRemainder;PSR;23|)
          (LETT |lp2| |lp1| |PSETCAT-;headRemainder;PSR;23|)
          (SEQ
           G190
           (COND
            ((NULL
              (COND
               ((OR (SPADCALL |a| (QREFELT |$| 42)) (NULL |lp2|)) (QUOTE NIL))
               ((QUOTE T) (QUOTE T))))
             (GO G191)))
```

```
(SEQ
 (LETT |p| (|SPADfirst| |lp2|) |PSETCAT-;headRemainder;PSR;23|)
 (LETT |e|
  (SPADCALL
   (SPADCALL |a| (QREFELT |$| 40))
   (SPADCALL |p| (QREFELT |$| 40))
   (QREFELT |$| 63))
  |PSETCAT-;headRemainder;PSR;23|)
 (EXIT
  (COND
   ((QEQCAR |e| 0)
    (SEQ
     (LETT |g|
      (SPADCALL
       (LETT |lca|
        (SPADCALL |a| (QREFELT |$| 64))
        |PSETCAT-;headRemainder;PSR;23|)
       (LETT |lcp|
        (SPADCALL |p| (QREFELT |$| 64))
        |PSETCAT-;headRemainder;PSR;23|)
       (QREFELT |$| 65))
      |PSETCAT-;headRemainder;PSR;23|)
     (PROGN
      (LETT |#G47|
       (|PSETCAT-;exactQuo| |lca| |g| |$|)
       |PSETCAT-;headRemainder;PSR;23|)
      (LETT |#G48|
       (|PSETCAT-;exactQuo| |lcp| |g| |$|)
       |PSETCAT-;headRemainder;PSR;23|)
      (LETT |lca| |#G47| |PSETCAT-;headRemainder;PSR;23|)
      (LETT |lcp| |#G48| |PSETCAT-;headRemainder;PSR;23|))
     (LETT |a|
      (SPADCALL
       (SPADCALL |lcp|
        (SPADCALL |a| (QREFELT |$| 62))
        (QREFELT |$| 66))
       (SPADCALL
        (SPADCALL |lca| (QCDR |e|) (QREFELT |$| 67))
        (SPADCALL |p| (QREFELT |$| 62)) (QREFELT |$| 68))
       (QREFELT |$| 69))
      |PSETCAT-;headRemainder;PSR;23|)
     (LETT |r|
      (SPADCALL |r| |lcp| (QREFELT |$| 70))
      |PSETCAT-;headRemainder;PSR;23|)
     (EXIT (LETT |lp2| |lp1| |PSETCAT-;headRemainder;PSR;23|))))
   ((QUOTE T)
```

```
              (LETT |lp2| (CDR |lp2|) |PSETCAT-;headRemainder;PSR;23|)))))
           NIL
           (GO G190)
           G191
           (EXIT NIL))
          (EXIT (CONS |a| |r|))))))))))

(DEFUN |PSETCAT-;makeIrreducible!| (|frac| |$|)
 (PROG (|g|)
  (RETURN
   (SEQ
    (LETT |g|
     (SPADCALL (QCDR |frac|) (QCAR |frac|) (QREFELT |$| 73))
     |PSETCAT-;makeIrreducible!|)
    (EXIT
     (COND
      ((SPADCALL |g| (QREFELT |$| 74)) |frac|)
      ((QUOTE T)
       (SEQ
        (PROGN
         (RPLACA |frac| (SPADCALL (QCAR |frac|) |g| (QREFELT |$| 75)))
         (QCAR |frac|))
        (PROGN
         (RPLACD |frac| (|PSETCAT-;exactQuo| (QCDR |frac|) |g| |$|))
         (QCDR |frac|))
        (EXIT |frac|)))))))))

(DEFUN |PSETCAT-;remainder;PSR;25| (|a| |ps| |$|)
 (PROG (|hRa| |r| |lca| |g| |b| |c|)
  (RETURN
   (SEQ
    (LETT |hRa|
     (|PSETCAT-;makeIrreducible!| (SPADCALL |a| |ps| (QREFELT |$| 76)) |$|)
     |PSETCAT-;remainder;PSR;25|)
    (LETT |a| (QCAR |hRa|) |PSETCAT-;remainder;PSR;25|)
    (LETT |r| (QCDR |hRa|) |PSETCAT-;remainder;PSR;25|)
    (EXIT
     (COND
      ((SPADCALL |a| (QREFELT |$| 42))
        (VECTOR (|spadConstant| |$| 61) |a| |r|))
      ((QUOTE T)
       (SEQ
        (LETT |b|
         (SPADCALL
          (|spadConstant| |$| 61)
          (SPADCALL |a| (QREFELT |$| 40))
```

```
  (QREFELT |$| 67))
 |PSETCAT-;remainder;PSR;25|)
(LETT |c| (SPADCALL |a| (QREFELT |$| 64)) |PSETCAT-;remainder;PSR;25|)
(SEQ
 G190
 (COND
  ((NULL
    (COND
     ((SPADCALL
        (LETT |a|
         (SPADCALL |a| (QREFELT |$| 62))
         |PSETCAT-;remainder;PSR;25|)
        (QREFELT |$| 42))
       (QUOTE NIL))
     ((QUOTE T) (QUOTE T))))
   (GO G191)))
 (SEQ
  (LETT |hRa|
   (|PSETCAT-;makeIrreducible!|
    (SPADCALL |a| |ps| (QREFELT |$| 76))
    |$|)
   |PSETCAT-;remainder;PSR;25|)
  (LETT |a| (QCAR |hRa|) |PSETCAT-;remainder;PSR;25|)
  (LETT |r|
   (SPADCALL |r| (QCDR |hRa|) (QREFELT |$| 70))
   |PSETCAT-;remainder;PSR;25|)
  (LETT |g|
   (SPADCALL |c|
    (LETT |lca|
     (SPADCALL |a| (QREFELT |$| 64))
     |PSETCAT-;remainder;PSR;25|)
    (QREFELT |$| 65))
   |PSETCAT-;remainder;PSR;25|)
  (LETT |b|
   (SPADCALL
    (SPADCALL
     (SPADCALL
      (QCDR |hRa|)
      (|PSETCAT-;exactQuo| |c| |g| |$|)
      (QREFELT |$| 70))
     |b|
     (QREFELT |$| 66))
    (SPADCALL
     (|PSETCAT-;exactQuo| |lca| |g| |$|)
     (SPADCALL |a| (QREFELT |$| 40))
     (QREFELT |$| 67))
```

```
             (QREFELT |$| 77))
            |PSETCAT-;remainder;PSR;25|)
           (EXIT (LETT |c| |g| |PSETCAT-;remainder;PSR;25|)))
          NIL
          (GO G190)
          G191
          (EXIT NIL))
         (EXIT (VECTOR |c| |b| |r|))))))))))

(DEFUN |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26| (|ps| |cs| |$|)
 (PROG (|p| |rs|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |cs| (QREFELT |$| 80)) |ps|)
     ((SPADCALL |cs| (QREFELT |$| 81)) (LIST (|spadConstant| |$| 82)))
     ((QUOTE T)
      (SEQ
       (LETT |ps|
        (SPADCALL (ELT |$| 42) |ps| (QREFELT |$| 26))
        |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
       (EXIT
        (COND
         ((NULL |ps|) |ps|)
         ((SPADCALL (ELT |$| 24) |ps| (QREFELT |$| 43))
           (LIST (|spadConstant| |$| 83)))
         ((QUOTE T)
           (SEQ
            (LETT |rs| NIL |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
            (SEQ
             G190
             (COND
              ((NULL (COND ((NULL |ps|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
                (GO G191)))
             (SEQ
              (LETT |p|
               (|SPADfirst| |ps|)
               |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
              (LETT |ps|
               (CDR |ps|)
               |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
              (LETT |p|
               (QCAR (SPADCALL |p| |cs| (QREFELT |$| 76)))
               |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
              (EXIT
               (COND
```

```
                ((NULL (SPADCALL |p| (QREFELT |$| 42)))
                 (COND
                  ((SPADCALL |p| (QREFELT |$| 24))
                   (SEQ
                    (LETT |ps| NIL
                     |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
                    (EXIT
                     (LETT |rs|
                      (LIST (|spadConstant| |$| 83))
                      |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|))))
                  ((QUOTE T)
                   (SEQ
                    (SPADCALL |p| (QREFELT |$| 84))
                    (EXIT
                     (LETT |rs|
                      (CONS |p| |rs|)
                      |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)))))))
               NIL
               (GO G190)
               G191
               (EXIT NIL))
              (EXIT (SPADCALL |rs| (QREFELT |$| 85)))))))))))))

(DEFUN |PSETCAT-;rewriteIdealWithRemainder;LSL;27| (|ps| |cs| |$|)
 (PROG (|p| |rs|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |cs| (QREFELT |$| 80)) |ps|)
     ((SPADCALL |cs| (QREFELT |$| 81)) (LIST (|spadConstant| |$| 82)))
     ((QUOTE T)
      (SEQ
       (LETT |ps|
        (SPADCALL (ELT |$| 42) |ps| (QREFELT |$| 26))
        |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
       (EXIT
        (COND
         ((NULL |ps|) |ps|)
         ((SPADCALL (ELT |$| 24) |ps| (QREFELT |$| 43))
           (LIST (|spadConstant| |$| 83)))
         ((QUOTE T)
          (SEQ
           (LETT |rs| NIL |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
           (SEQ
            G190
            (COND
```

```
                ((NULL (COND ((NULL |ps|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
                  (GO G191)))
               (SEQ
                (LETT |p|
                 (|SPADfirst| |ps|)
                 |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
                (LETT |ps| (CDR |ps|) |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
                (LETT |p|
                 (QVELT (SPADCALL |p| |cs| (QREFELT |$| 87)) 1)
                 |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
                (EXIT
                 (COND
                  ((NULL (SPADCALL |p| (QREFELT |$| 42)))
                   (COND
                    ((SPADCALL |p| (QREFELT |$| 24))
                     (SEQ
                      (LETT |ps| NIL |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
                      (EXIT
                       (LETT |rs|
                        (LIST (|spadConstant| |$| 83))
                        |PSETCAT-;rewriteIdealWithRemainder;LSL;27|))))
                    ((QUOTE T)
                     (LETT |rs|
                      (CONS (SPADCALL |p| (QREFELT |$| 88)) |rs|)
                      |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)))))))
                NIL
                (GO G190)
                G191
                (EXIT NIL))
               (EXIT (SPADCALL |rs| (QREFELT |$| 85)))))))))))))

(DEFUN |PolynomialSetCategory&| (|#1| |#2| |#3| |#4| |#5|)
 (PROG (|DV$1| |DV$2| |DV$3| |DV$4| |DV$5| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|PolynomialSetCategory&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |DV$3| (|devaluate| |#3|) . #1#)
    (LETT |DV$4| (|devaluate| |#4|) . #1#)
    (LETT |DV$5| (|devaluate| |#5|) . #1#)
    (LETT |dv$|
     (LIST
      (QUOTE |PolynomialSetCategory&|)
      |DV$1| |DV$2| |DV$3| |DV$4| |DV$5|) . #1#)
    (LETT |$| (GETREFV 90) . #1#)
    (QSETREFV |$| 0 |dv$|)
```

```
      (QSETREFV |$| 3
       (LETT |pv$|
        (|buildPredVector| 0 0
         (LIST (|HasCategory| |#2| (QUOTE (|IntegralDomain|))))) . #1#))
      (|stuffDomainSlots| |$|)
      (QSETREFV |$| 6 |#1|)
      (QSETREFV |$| 7 |#2|)
      (QSETREFV |$| 8 |#3|)
      (QSETREFV |$| 9 |#4|)
      (QSETREFV |$| 10 |#5|)
      (COND
       ((|testBitVector| |pv$| 1)
        (PROGN
         (QSETREFV |$| 48
          (CONS (|dispatchFunction| |PSETCAT-;roughUnitIdeal?;SB;16|) |$|))
         (QSETREFV |$| 52
          (CONS (|dispatchFunction| |PSETCAT-;roughBase?;SB;18|) |$|))
         (QSETREFV |$| 54
          (CONS (|dispatchFunction| |PSETCAT-;roughSubIdeal?;2SB;19|) |$|))
         (QSETREFV |$| 57
          (CONS (|dispatchFunction| |PSETCAT-;roughEqualIdeals?;2SB;20|) |$|)))))
      (COND
       ((|HasCategory| |#2| (QUOTE (|GcdDomain|)))
       (COND
        ((|HasCategory| |#4| (QUOTE (|ConvertibleTo| (|Symbol|))))
         (PROGN
          (QSETREFV |$| 72
           (CONS (|dispatchFunction| |PSETCAT-;headRemainder;PSR;23|) |$|))
          (QSETREFV |$| 79
           (CONS (|dispatchFunction| |PSETCAT-;remainder;PSR;25|) |$|))
          (QSETREFV |$| 86
           (CONS
            (|dispatchFunction| |PSETCAT-;rewriteIdealWithHeadRemainder;LSL;26|)
            |$|))
          (QSETREFV |$| 89
           (CONS
            (|dispatchFunction| |PSETCAT-;rewriteIdealWithRemainder;LSL;27|)
            |$|)))))))
      |$|))))

  (MAKEPROP
   (QUOTE |PolynomialSetCategory&|)
   (QUOTE |infovec|)
   (LIST
    (QUOTE
     #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|local| |#3|)
```

```
   (|local| |#4|) (|local| |#5|) (|List| 10) (0 . |members|) (|List| 9)
   (5 . |variables|) (|Boolean|) (10 . |<|) (|List| |$|) (16 . |concat|)
   (21 . |removeDuplicates|) (|Mapping| 15 9 9) (26 . |sort|)
   (32 . |mvar|) |PSETCAT-;variables;SL;4| (37 . |ground?|)
   (|Mapping| 15 10) (42 . |remove|) |PSETCAT-;mainVariables;SL;5|
   (48 . |=|) |PSETCAT-;mainVariable?;VarSetSB;6| (54 . |construct|)
   |PSETCAT-;collectUnder;SVarSetS;7| |PSETCAT-;collectUpper;SVarSetS;8|
   |PSETCAT-;collect;SVarSetS;9| (|Record| (|:| |under| |$|)
   (|:| |floor| |$|) (|:| |upper| |$|)) |PSETCAT-;sort;SVarSetR;10|
   (|Set| 10) (59 . |brace|) (64 . |=|) |PSETCAT-;=;2SB;11|
   (70 . |degree|) (75 . |<|) (81 . |zero?|) (86 . |any?|)
   (|Mapping| 15 10 10) (92 . |sort|) |PSETCAT-;triangular?;SB;14|
   |PSETCAT-;trivialIdeal?;SB;15| (98 . |roughUnitIdeal?|)
   (103 . |sup|) (109 . |+|) (115 . |=|) (121 . |roughBase?|)
   (126 . |rewriteIdealWithRemainder|) (132 . |roughSubIdeal?|)
   (138 . |=|) (144 . |roughSubIdeal?|) (150 . |roughEqualIdeals?|)
   (156 . |quo|) (|Union| |$| (QUOTE "failed")) (162 . |exquo|)
   (168 . |One|) (172 . |reductum|) (177 . |subtractIfCan|)
   (183 . |leadingCoefficient|) (188 . |gcd|) (194 . |*|)
   (200 . |monomial|) (206 . |*|) (212 . |-|) (218 . |*|)
   (|Record| (|:| |num| 10) (|:| |den| 7)) (224 . |headRemainder|)
   (230 . |gcd|) (236 . |one?|) (241 . |exactQuotient!|)
   (247 . |headRemainder|) (253 . |+|) (|Record| (|:| |rnum| 7)
   (|:| |polnum| 10) (|:| |den| 7)) (259 . |remainder|)
   (265 . |trivialIdeal?|) (270 . |roughUnitIdeal?|)
   (275 . |Zero|) (279 . |One|) (283 . |primitivePart!|)
   (288 . |removeDuplicates|) (293 . |rewriteIdealWithHeadRemainder|)
   (299 . |remainder|) (305 . |unitCanonical|)
   (310 . |rewriteIdealWithRemainder|)))
(QUOTE #(|variables| 316 |trivialIdeal?| 321 |triangular?| 326
   |sort| 331 |roughUnitIdeal?| 337 |roughSubIdeal?| 342
   |roughEqualIdeals?| 348 |roughBase?| 354 |rewriteIdealWithRemainder|
   359 |rewriteIdealWithHeadRemainder| 365 |remainder| 371 |mainVariables|
   377 |mainVariable?| 382 |headRemainder| 388 |collectUpper| 394
   |collectUnder| 400 |collect| 406 |=| 412))
(QUOTE NIL)
(CONS
 (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS
  (QUOTE #())
  (CONS
   (QUOTE #())
   (|makeByteWordVec2| 89 (QUOTE (1 6 11 0 12 1 10 13 0 14 2 9 15 0
    0 16 1 13 0 17 18 1 13 0 0 19 2 13 0 20 0 21 1 10 9 0 22 1 10 15 0 24
    2 11 0 25 0 26 2 9 15 0 0 28 1 6 0 11 30 1 36 0 11 37 2 36 15 0 0 38 1
    10 8 0 40 2 8 15 0 0 41 1 10 15 0 42 2 11 15 25 0 43 2 11 0 44 0 45 1
```

```
    0 15 0 48 2 8 0 0 0 49 2 8 0 0 0 50 2 8 15 0 0 51 1 0 15 0 52 2 6 11 11
    0 53 2 0 15 0 0 54 2 6 15 0 0 55 2 6 15 0 0 56 2 0 15 0 0 57 2 7 0 0 0
    58 2 7 59 0 0 60 0 7 0 61 1 10 0 0 62 2 8 59 0 0 63 1 10 7 0 64 2 7 0
    0 0 65 2 10 0 7 0 66 2 10 0 7 8 67 2 10 0 0 0 68 2 10 0 0 0 69 2 7 0
    0 0 70 2 0 71 10 0 72 2 10 7 7 0 73 1 7 15 0 74 2 10 0 0 7 75 2 6 71
    10 0 76 2 10 0 0 0 77 2 0 78 10 0 79 1 6 15 0 80 1 6 15 0 81 0 10 0
    82 0 10 0 83 1 10 0 0 84 1 11 0 0 85 2 0 11 11 0 86 2 6 78 10 0 87 1
    10 0 0 88 2 0 11 11 0 89 1 0 13 0 23 1 0 15 0 47 1 0 15 0 46 2 0 34
    0 9 35 1 0 15 0 48 2 0 15 0 0 54 2 0 15 0 0 57 1 0 15 0 52 2 0 11 11
    0 89 2 0 11 11 0 86 2 0 78 10 0 79 1 0 13 0 27 2 0 15 9 0 29 2 0 71
    10 0 72 2 0 0 0 9 32 2 0 0 0 9 31 2 0 0 0 9 33 2 0 15 0 0 39))))))
(QUOTE |lookupComplete|)))
```

21.47 QFCAT.lsp BOOTSTRAP

QFCAT depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **QFCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **QFCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*QFCAT.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |QuotientFieldCategory;CAT| (QUOTE NIL))

(SETQ |QuotientFieldCategory;AL| (QUOTE NIL))

(DEFUN |QuotientFieldCategory| (#1=#:G103631)
 (LET (#2=#:G103632)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |QuotientFieldCategory;AL|))
     (CDR #2#))
   (T
    (SETQ |QuotientFieldCategory;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|QuotientFieldCategory;| #1#)))
      |QuotientFieldCategory;AL|))
     #2#))))

(DEFUN |QuotientFieldCategory;| (|t#1|)
 (PROG (#1=#:G103630)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|QuotientFieldCategory;CAT|)
       ((QUOTE T)
        (LETT |QuotientFieldCategory;CAT|
         (|Join|
          (|Field|)
          (|Algebra| (QUOTE |t#1|))
          (|RetractableTo| (QUOTE |t#1|))
          (|FullyEvalableOver| (QUOTE |t#1|))
          (|DifferentialExtension| (QUOTE |t#1|))
```

```
        (|FullyLinearlyExplicitRingOver| (QUOTE |t#1|))
        (|Patternable| (QUOTE |t#1|))
        (|FullyPatternMatchable| (QUOTE |t#1|))
        (|mkCategory|
         (QUOTE |domain|)
         (QUOTE (
          ((|/| (|$| |t#1| |t#1|)) T)
          ((|numer| (|t#1| |$|)) T)
          ((|denom| (|t#1| |$|)) T)
          ((|numerator| (|$| |$|)) T)
          ((|denominator| (|$| |$|)) T)
          ((|wholePart| (|t#1| |$|)) (|has| |t#1| (|EuclideanDomain|)))
          ((|fractionPart| (|$| |$|)) (|has| |t#1| (|EuclideanDomain|)))
          ((|random| (|$|)) (|has| |t#1| (|IntegerNumberSystem|)))
          ((|ceiling| (|t#1| |$|)) (|has| |t#1| (|IntegerNumberSystem|)))
          ((|floor| (|t#1| |$|)) (|has| |t#1| (|IntegerNumberSystem|)))))
        (QUOTE (
          ((|StepThrough|) (|has| |t#1| (|StepThrough|)))
          ((|RetractableTo| (|Integer|))
             (|has| |t#1| (|RetractableTo| (|Integer|))))
          ((|RetractableTo| (|Fraction| (|Integer|)))
             (|has| |t#1| (|RetractableTo| (|Integer|))))
          ((|OrderedSet|) (|has| |t#1| (|OrderedSet|)))
          ((|OrderedIntegralDomain|) (|has| |t#1| (|OrderedIntegralDomain|)))
          ((|RealConstant|) (|has| |t#1| (|RealConstant|)))
          ((|ConvertibleTo| (|InputForm|))
             (|has| |t#1| (|ConvertibleTo| (|InputForm|))))
          ((|CharacteristicZero|) (|has| |t#1| (|CharacteristicZero|)))
          ((|CharacteristicNonZero|) (|has| |t#1| (|CharacteristicNonZero|)))
          ((|RetractableTo| (|Symbol|))
             (|has| |t#1| (|RetractableTo| (|Symbol|))))
          ((|PolynomialFactorizationExplicit|)
             (|has| |t#1| (|PolynomialFactorizationExplicit|)))))
        (QUOTE NIL) NIL)) . #2=(|QuotientFieldCategory|))))) . #2#)
    (SETELT #1# 0
      (LIST (QUOTE |QuotientFieldCategory|) (|devaluate| |t#1|)))))))
```

21.48 QFCAT-.lsp BOOTSTRAP

QFCAT- depends on **QFCAT**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **QFCAT-** category which we can write into the **MID** directory. We compile the lisp code and copy the **QFCAT-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*QFCAT-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |QFCAT-;numerator;2A;1| (|x| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) (QREFELT |$| 9)))

(DEFUN |QFCAT-;denominator;2A;2| (|x| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) (QREFELT |$| 9)))

(DEFUN |QFCAT-;init;A;3| (|$|)
 (SPADCALL (|spadConstant| |$| 13) (|spadConstant| |$| 14) (QREFELT |$| 15)))

(DEFUN |QFCAT-;nextItem;AU;4| (|n| |$|)
 (PROG (|m|)
  (RETURN
   (SEQ
    (LETT |m|
     (SPADCALL
      (SPADCALL |n| (QREFELT |$| 8))
      (QREFELT |$| 18))
     |QFCAT-;nextItem;AU;4|)
    (EXIT
     (COND
      ((QEQCAR |m| 1)
       (|error| "We seem to have a Fraction of a finite object"))
      ((QUOTE T)
       (CONS 0
        (SPADCALL (QCDR |m|) (|spadConstant| |$| 14) (QREFELT |$| 15))))))))))

(DEFUN |QFCAT-;map;M2A;5| (|fn| |x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) |fn|)
  (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) |fn|)
  (QREFELT |$| 15)))

(DEFUN |QFCAT-;reducedSystem;MM;6| (|m| |$|)
```

```
 (SPADCALL |m| (QREFELT |$| 26)))

(DEFUN |QFCAT-;characteristic;Nni;7| (|$|)
 (SPADCALL (QREFELT |$| 30)))

(DEFUN |QFCAT-;differentiate;AMA;8| (|x| |deriv| |$|)
 (PROG (|n| |d|)
  (RETURN
   (SEQ
    (LETT |n| (SPADCALL |x| (QREFELT |$| 8)) |QFCAT-;differentiate;AMA;8|)
    (LETT |d| (SPADCALL |x| (QREFELT |$| 11)) |QFCAT-;differentiate;AMA;8|)
    (EXIT
     (SPADCALL
      (SPADCALL
       (SPADCALL (SPADCALL |n| |deriv|) |d| (QREFELT |$| 32))
       (SPADCALL |n| (SPADCALL |d| |deriv|) (QREFELT |$| 32))
       (QREFELT |$| 33))
      (SPADCALL |d| 2 (QREFELT |$| 35)) (QREFELT |$| 15)))))))

(DEFUN |QFCAT-;convert;AIf;9| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) (QREFELT |$| 38))
  (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) (QREFELT |$| 38))
  (QREFELT |$| 39)))

(DEFUN |QFCAT-;convert;AF;10| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) (QREFELT |$| 42))
  (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) (QREFELT |$| 42))
  (QREFELT |$| 43)))

(DEFUN |QFCAT-;convert;ADf;11| (|x| |$|)
 (|/|
  (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) (QREFELT |$| 46))
  (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) (QREFELT |$| 46))))

(DEFUN |QFCAT-;<;2AB;12| (|x| |y| |$|)
 (SPADCALL
  (SPADCALL
   (SPADCALL |x| (QREFELT |$| 8))
   (SPADCALL |y| (QREFELT |$| 11))
   (QREFELT |$| 32))
  (SPADCALL
   (SPADCALL |y| (QREFELT |$| 8))
   (SPADCALL |x| (QREFELT |$| 11))
   (QREFELT |$| 32))
```

```
    (QREFELT |$| 49)))

  (DEFUN |QFCAT-;<;2AB;13| (|x| |y| |$|)
   (PROG (|#G19| |#G20| |#G21| |#G22|)
    (RETURN
     (SEQ
      (COND
       ((SPADCALL
         (SPADCALL |x| (QREFELT |$| 11))
         (|spadConstant| |$| 51)
         (QREFELT |$| 49))
        (PROGN
         (LETT |#G19| |y| |QFCAT-;<;2AB;13|)
         (LETT |#G20| |x| |QFCAT-;<;2AB;13|)
         (LETT |x| |#G19| |QFCAT-;<;2AB;13|)
         (LETT |y| |#G20| |QFCAT-;<;2AB;13|))))
      (COND
       ((SPADCALL
          (SPADCALL |y| (QREFELT |$| 11))
          (|spadConstant| |$| 51)
          (QREFELT |$| 49))
         (PROGN
          (LETT |#G21| |y| |QFCAT-;<;2AB;13|)
          (LETT |#G22| |x| |QFCAT-;<;2AB;13|)
          (LETT |x| |#G21| |QFCAT-;<;2AB;13|)
          (LETT |y| |#G22| |QFCAT-;<;2AB;13|))))
      (EXIT
       (SPADCALL
        (SPADCALL
         (SPADCALL |x| (QREFELT |$| 8))
         (SPADCALL |y| (QREFELT |$| 11))
         (QREFELT |$| 32))
        (SPADCALL
         (SPADCALL |y| (QREFELT |$| 8))
         (SPADCALL |x| (QREFELT |$| 11))
         (QREFELT |$| 32))
        (QREFELT |$| 49)))))))

  (DEFUN |QFCAT-;<;2AB;14| (|x| |y| |$|)
   (SPADCALL
    (SPADCALL
     (SPADCALL |x| (QREFELT |$| 8))
     (SPADCALL |y| (QREFELT |$| 11))
     (QREFELT |$| 32))
    (SPADCALL
     (SPADCALL |y| (QREFELT |$| 8))
```

```
    (SPADCALL |x| (QREFELT |$| 11))
    (QREFELT |$| 32))
   (QREFELT |$| 49)))

(DEFUN |QFCAT-;fractionPart;2A;15| (|x| |$|)
 (SPADCALL |x|
  (SPADCALL (SPADCALL |x| (QREFELT |$| 52)) (QREFELT |$| 9))
  (QREFELT |$| 53)))

(DEFUN |QFCAT-;coerce;SA;16| (|s| |$|)
 (SPADCALL (SPADCALL |s| (QREFELT |$| 56)) (QREFELT |$| 9)))

(DEFUN |QFCAT-;retract;AS;17| (|x| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 58)) (QREFELT |$| 59)))

(DEFUN |QFCAT-;retractIfCan;AU;18| (|x| |$|)
 (PROG (|r|)
  (RETURN
   (SEQ
    (LETT |r| (SPADCALL |x| (QREFELT |$| 62)) |QFCAT-;retractIfCan;AU;18|)
    (EXIT
     (COND
      ((QEQCAR |r| 1) (CONS 1 "failed"))
      ((QUOTE T) (SPADCALL (QCDR |r|) (QREFELT |$| 64)))))))))

(DEFUN |QFCAT-;convert;AP;19| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) (QREFELT |$| 67))
  (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) (QREFELT |$| 67))
  (QREFELT |$| 68)))

(DEFUN |QFCAT-;patternMatch;AP2Pmr;20| (|x| |p| |l| |$|)
 (SPADCALL |x| |p| |l| (QREFELT |$| 72)))

(DEFUN |QFCAT-;convert;AP;21| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 8)) (QREFELT |$| 76))
  (SPADCALL (SPADCALL |x| (QREFELT |$| 11)) (QREFELT |$| 76))
  (QREFELT |$| 77)))

(DEFUN |QFCAT-;patternMatch;AP2Pmr;22| (|x| |p| |l| |$|)
 (SPADCALL |x| |p| |l| (QREFELT |$| 81)))

(DEFUN |QFCAT-;coerce;FA;23| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 86)) (QREFELT |$| 87))
```

```
   (SPADCALL (SPADCALL |x| (QREFELT |$| 88)) (QREFELT |$| 87))
   (QREFELT |$| 89)))

 (DEFUN |QFCAT-;retract;AI;24| (|x| |$|)
  (SPADCALL (SPADCALL |x| (QREFELT |$| 58)) (QREFELT |$| 91)))

 (DEFUN |QFCAT-;retractIfCan;AU;25| (|x| |$|)
  (PROG (|u|)
   (RETURN
    (SEQ
     (LETT |u| (SPADCALL |x| (QREFELT |$| 62)) |QFCAT-;retractIfCan;AU;25|)
     (EXIT
      (COND
       ((QEQCAR |u| 1) (CONS 1 "failed"))
       ((QUOTE T) (SPADCALL (QCDR |u|) (QREFELT |$| 94)))))))))

 (DEFUN |QFCAT-;random;A;26| (|$|)
  (PROG (|d|)
   (RETURN
    (SEQ
     (SEQ
       G190
       (COND
         ((NULL
           (SPADCALL
            (LETT |d| (SPADCALL (QREFELT |$| 96)) |QFCAT-;random;A;26|)
            (QREFELT |$| 97)))
          (GO G191)))
       (SEQ (EXIT |d|))
       NIL
       (GO G190)
       G191
      (EXIT NIL))
     (EXIT (SPADCALL (SPADCALL (QREFELT |$| 96)) |d| (QREFELT |$| 15)))))))

 (DEFUN |QFCAT-;reducedSystem;MVR;27| (|m| |v| |$|)
  (PROG (|n|)
   (RETURN
    (SEQ
     (LETT |n|
      (SPADCALL
       (SPADCALL (SPADCALL |v| (QREFELT |$| 100)) |m| (QREFELT |$| 101))
       (QREFELT |$| 102))
      |QFCAT-;reducedSystem;MVR;27|)
     (EXIT
      (CONS
```

```
        (SPADCALL |n|
         (SPADCALL |n| (QREFELT |$| 103))
         (SPADCALL |n| (QREFELT |$| 104))
         (|+| 1 (SPADCALL |n| (QREFELT |$| 105)))
         (SPADCALL |n| (QREFELT |$| 106))
         (QREFELT |$| 107))
        (SPADCALL |n| (SPADCALL |n| (QREFELT |$| 105)) (QREFELT |$| 109))))))))

(DEFUN |QuotientFieldCategory&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|QuotientFieldCategory&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |QuotientFieldCategory&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (GETREFV 119) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0 (LIST
       (|HasCategory| |#2| (QUOTE (|PolynomialFactorizationExplicit|)))
       (|HasCategory| |#2| (QUOTE (|IntegerNumberSystem|)))
       (|HasCategory| |#2| (QUOTE (|EuclideanDomain|)))
       (|HasCategory| |#2| (QUOTE (|RetractableTo| (|Symbol|))))
       (|HasCategory| |#2| (QUOTE (|CharacteristicNonZero|)))
       (|HasCategory| |#2| (QUOTE (|CharacteristicZero|)))
       (|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|InputForm|))))
       (|HasCategory| |#2| (QUOTE (|RealConstant|)))
       (|HasCategory| |#2| (QUOTE (|OrderedIntegralDomain|)))
       (|HasCategory| |#2| (QUOTE (|OrderedSet|)))
       (|HasCategory| |#2| (QUOTE (|RetractableTo| (|Integer|))))
       (|HasCategory| |#2| (QUOTE (|StepThrough|))))) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|testBitVector| |pv$| 12)
       (PROGN
        (QSETREFV |$| 16 (CONS (|dispatchFunction| |QFCAT-;init;A;3|) |$|))
        (QSETREFV |$| 20
         (CONS (|dispatchFunction| |QFCAT-;nextItem;AU;4|) |$|)))))
    (COND
     ((|testBitVector| |pv$| 7)
       (QSETREFV |$| 40
        (CONS (|dispatchFunction| |QFCAT-;convert;AIf;9|) |$|))))
    (COND
```

```
 ((|testBitVector| |pv$| 8)
   (PROGN
    (QSETREFV |$| 44
     (CONS (|dispatchFunction| |QFCAT-;convert;AF;10|) |$|))
    (QSETREFV |$| 47
     (CONS (|dispatchFunction| |QFCAT-;convert;ADf;11|) |$|)))))
(COND
 ((|testBitVector| |pv$| 9)
  (COND
   ((|HasAttribute| |#2| (QUOTE |canonicalUnitNormal|))
     (QSETREFV |$| 50 (CONS (|dispatchFunction| |QFCAT-;<;2AB;12|) |$|)))
   ((QUOTE T)
     (QSETREFV |$| 50 (CONS (|dispatchFunction| |QFCAT-;<;2AB;13|) |$|)))))
 ((|testBitVector| |pv$| 10)
  (QSETREFV |$| 50 (CONS (|dispatchFunction| |QFCAT-;<;2AB;14|) |$|))))
(COND
 ((|testBitVector| |pv$| 3)
   (QSETREFV |$| 54
    (CONS (|dispatchFunction| |QFCAT-;fractionPart;2A;15|) |$|))))
(COND
 ((|testBitVector| |pv$| 4)
  (PROGN
   (QSETREFV |$| 57 (CONS (|dispatchFunction| |QFCAT-;coerce;SA;16|) |$|))
   (QSETREFV |$| 60 (CONS (|dispatchFunction| |QFCAT-;retract;AS;17|) |$|))
   (QSETREFV |$| 65
    (CONS (|dispatchFunction| |QFCAT-;retractIfCan;AU;18|) |$|)))))
(COND
 ((|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|Pattern| (|Integer|)))))
  (PROGN
   (QSETREFV |$| 69 (CONS (|dispatchFunction| |QFCAT-;convert;AP;19|) |$|))
   (COND
    ((|HasCategory| |#2| (QUOTE (|PatternMatchable| (|Integer|))))
      (QSETREFV |$| 74
       (CONS
        (|dispatchFunction| |QFCAT-;patternMatch;AP2Pmr;20|) |$|)))))))
(COND
 ((|HasCategory| |#2| (QUOTE (|ConvertibleTo| (|Pattern| (|Float|)))))
  (PROGN
   (QSETREFV |$| 78 (CONS (|dispatchFunction| |QFCAT-;convert;AP;21|) |$|))
   (COND
    ((|HasCategory| |#2| (QUOTE (|PatternMatchable| (|Float|))))
     (QSETREFV |$| 83
      (CONS (|dispatchFunction| |QFCAT-;patternMatch;AP2Pmr;22|) |$|)))))))
(COND
 ((|testBitVector| |pv$| 11)
  (PROGN
```

```
        (QSETREFV |$| 90 (CONS (|dispatchFunction| |QFCAT-;coerce;FA;23|) |$|))
        (COND
         ((|domainEqual| |#2| (|Integer|)))
         ((QUOTE T)
          (PROGN
           (QSETREFV |$| 92
            (CONS (|dispatchFunction| |QFCAT-;retract;AI;24|) |$|))
           (QSETREFV |$| 95
            (CONS (|dispatchFunction| |QFCAT-;retractIfCan;AU;25|) |$|)))))))
      (COND
       ((|testBitVector| |pv$| 2)
        (QSETREFV |$| 98 (CONS (|dispatchFunction| |QFCAT-;random;A;26|) |$|))))
      |$|))))

  (MAKEPROP
   (QUOTE |QuotientFieldCategory&|)
   (QUOTE |infovec|)
   (LIST
    (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
     (0 . |numer|) (5 . |coerce|) |QFCAT-;numerator;2A;1| (10 . |denom|)
     |QFCAT-;denominator;2A;2| (15 . |init|) (19 . |One|) (23 . |/|)
     (29 . |init|) (|Union| |$| (QUOTE "failed")) (33 . |nextItem|)
     (38 . |One|) (42 . |nextItem|) (|Mapping| 7 7) |QFCAT-;map;M2A;5|
     (|Matrix| 7) (|Matrix| 6) (|MatrixCommonDenominator| 7 6)
     (47 . |clearDenominator|) (|Matrix| |$|) |QFCAT-;reducedSystem;MM;6|
     (|NonNegativeInteger|) (52 . |characteristic|)
     |QFCAT-;characteristic;Nni;7| (56 . |*|) (62 . |-|)
     (|PositiveInteger|) (68 . |**|) |QFCAT-;differentiate;AMA;8|
     (|InputForm|) (74 . |convert|) (79 . |/|) (85 . |convert|)
     (|Float|) (90 . |convert|) (95 . |/|) (101 . |convert|)
     (|DoubleFloat|) (106 . |convert|) (111 . |convert|) (|Boolean|)
     (116 . |<|) (122 . |<|) (128 . |Zero|) (132 . |wholePart|)
     (137 . |-|) (143 . |fractionPart|) (|Symbol|) (148 . |coerce|)
     (153 . |coerce|) (158 . |retract|) (163 . |retract|)
     (168 . |retract|) (|Union| 7 (QUOTE "failed")) (173 . |retractIfCan|)
     (|Union| 55 (QUOTE "failed")) (178 . |retractIfCan|)
     (183 . |retractIfCan|) (|Pattern| 84) (188 . |convert|)
     (193 . |/|) (199 . |convert|) (|PatternMatchResult| 84 6)
     (|PatternMatchQuotientFieldCategory| 84 7 6) (204 . |patternMatch|)
     (|PatternMatchResult| 84 |$|) (211 . |patternMatch|) (|Pattern| 41)
     (218 . |convert|) (223 . |/|) (229 . |convert|)
     (|PatternMatchResult| 41 6)
     (|PatternMatchQuotientFieldCategory| 41 7 6) (234 . |patternMatch|)
     (|PatternMatchResult| 41 |$|) (241 . |patternMatch|) (|Integer|)
     (|Fraction| 84) (248 . |numer|) (253 . |coerce|) (258 . |denom|)
     (263 . |/|) (269 . |coerce|) (274 . |retract|) (279 . |retract|)
```

```
 (|Union| 84 (QUOTE "failed")) (284 . |retractIfCan|)
 (289 . |retractIfCan|) (294 . |random|) (298 . |zero?|)
 (303 . |random|) (|Vector| 6) (307 . |coerce|) (312 . |horizConcat|)
 (318 . |reducedSystem|) (323 . |minRowIndex|) (328 . |maxRowIndex|)
 (333 . |minColIndex|) (338 . |maxColIndex|) (343 . |subMatrix|)
 (|Vector| 7) (352 . |column|) (|Record| (|:| |mat| 23)
 (|:| |vec| 108)) (|Vector| |$|) |QFCAT-;reducedSystem;MVR;27|
 (|Union| 85 (QUOTE "failed")) (|Record| (|:| |mat| 115)
 (|:| |vec| (|Vector| 84))) (|Matrix| 84) (|List| 55) (|List| 29)
 (|OutputForm|)))
(QUOTE #(|retractIfCan| 358 |retract| 368 |reducedSystem| 378
 |random| 389 |patternMatch| 393 |numerator| 407 |nextItem| 412
 |map| 417 |init| 423 |fractionPart| 427 |differentiate| 432
 |denominator| 438 |convert| 443 |coerce| 468 |characteristic| 478
 |<| 482))
(QUOTE NIL)
(CONS
 (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS
  (QUOTE #())
  (CONS (QUOTE #())
   (|makeByteWordVec2| 112 (QUOTE
    (1 6 7 0 8 1 6 0 7 9 1 6 7 0 11 0 7 0 13 0 7 0 14 2 6 0 7 7 15 0
     0 0 16 1 7 17 0 18 0 6 0 19 1 0 17 0 20 1 25 23 24 26 0 7 29 30 2
     7 0 0 0 32 2 7 0 0 0 33 2 7 0 0 34 35 1 7 37 0 38 2 37 0 0 0 39 1
     0 37 0 40 1 7 41 0 42 2 41 0 0 0 43 1 0 41 0 44 1 7 45 0 46 1 0 45
     0 47 2 7 48 0 0 49 2 0 48 0 0 50 0 7 0 51 1 6 7 0 52 2 6 0 0 0 53 1
     0 0 0 54 1 7 0 55 56 1 0 0 55 57 1 6 7 0 58 1 7 55 0 59 1 0 55 0 60
     1 6 61 0 62 1 7 63 0 64 1 0 63 0 65 1 7 66 0 67 2 66 0 0 0 68 1 0
     66 0 69 3 71 70 6 66 70 72 3 0 73 0 66 73 74 1 7 75 0 76 2 75 0 0
     0 77 1 0 75 0 78 3 80 79 6 75 79 81 3 0 82 0 75 82 83 1 85 84 0 86
     1 6 0 84 87 1 85 84 0 88 2 6 0 0 0 89 1 0 0 85 90 1 7 84 0 91 1 0
     84 0 92 1 7 93 0 94 1 0 93 0 95 0 7 0 96 1 7 48 0 97 0 0 0 98 1 24
     0 99 100 2 24 0 0 0 101 1 6 23 27 102 1 23 84 0 103 1 23 84 0 104
     1 23 84 0 105 1 23 84 0 106 5 23 0 0 84 84 84 84 107 2 23 108 0 84
     109 1 0 93 0 95 1 0 63 0 65 1 0 84 0 92 1 0 55 0 60 2 0 110 27 111
     112 1 0 23 27 28 0 0 0 98 3 0 82 0 75 82 83 3 0 73 0 66 73 74 1 0
     0 0 10 1 0 17 0 20 2 0 0 21 0 22 0 0 0 16 1 0 0 0 54 2 0 0 0 21 36
     1 0 0 0 12 1 0 45 0 47 1 0 37 0 40 1 0 41 0 44 1 0 66 0 69 1 0 75 0
     78 1 0 0 55 57 1 0 0 85 90 0 0 29 31 2 0 48 0 0 50))))))
(QUOTE |lookupComplete|)))
```

21.49 RCAGG.lsp BOOTSTRAP

RCAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RCAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **RCAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*RCAGG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |RecursiveAggregate;CAT| (QUOTE NIL))

(SETQ |RecursiveAggregate;AL| (QUOTE NIL))

(DEFUN |RecursiveAggregate| (#1=#:G84501)
 (LET (#2=#:G84502)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |RecursiveAggregate;AL|)) (CDR #2#))
   (T
    (SETQ |RecursiveAggregate;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|RecursiveAggregate;| #1#)))
      |RecursiveAggregate;AL|))
    #2#))))

(DEFUN |RecursiveAggregate;| (|t#1|)
 (PROG (#1=#:G84500)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|RecursiveAggregate;CAT|)
       ((QUOTE T)
        (LETT |RecursiveAggregate;CAT|
         (|Join|
          (|HomogeneousAggregate| (QUOTE |t#1|))
          (|mkCategory|
           (QUOTE |domain|)
           (QUOTE (
            ((|children| ((|List| |$|) |$|)) T)
            ((|nodes| ((|List| |$|) |$|)) T)
```

```
            ((|leaf?| ((|Boolean|) |$|)) T)
            ((|value| (|t#1| |$|)) T)
            ((|elt| (|t#1| |$| "value")) T)
            ((|cyclic?| ((|Boolean|) |$|)) T)
            ((|leaves| ((|List| |t#1|) |$|)) T)
            ((|distance| ((|Integer|) |$| |$|)) T)
            ((|child?| ((|Boolean|) |$| |$|)) (|has| |t#1| (|SetCategory|)))
            ((|node?| ((|Boolean|) |$| |$|)) (|has| |t#1| (|SetCategory|)))
            ((|setchildren!| (|$| |$| (|List| |$|)))
              (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
            ((|setelt| (|t#1| |$| "value" |t#1|))
              (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
            ((|setvalue!| (|t#1| |$| |t#1|))
              (|has| |$| (ATTRIBUTE |shallowlyMutable|)))))
          NIL
          (QUOTE ((|List| |$|) (|Boolean|) (|Integer|) (|List| |t#1|)))
          NIL))
       . #2=(|RecursiveAggregate|))))))
  . #2#)
(SETELT #1# 0 (LIST (QUOTE |RecursiveAggregate|) (|devaluate| |t#1|)))))))
```

21.50 RCAGG-.lsp BOOTSTRAP

RCAGG- depends on **RCAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RCAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **RCAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*RCAGG-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |RCAGG-;elt;AvalueS;1| (|x| G84515 |$|)
 (SPADCALL |x| (QREFELT |$| 8)))

(DEFUN |RCAGG-;setelt;Avalue2S;2| (|x| G84517 |y| |$|)
 (SPADCALL |x| |y| (QREFELT |$| 11)))

(DEFUN |RCAGG-;child?;2AB;3| (|x| |l| |$|)
 (SPADCALL |x| (SPADCALL |l| (QREFELT |$| 14)) (QREFELT |$| 17)))

(DEFUN |RecursiveAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|RecursiveAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |RecursiveAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (GETREFV 19) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST
        (|HasAttribute| |#1| (QUOTE |shallowlyMutable|))
        (|HasCategory| |#2| (QUOTE (|SetCategory|)))))
      . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|testBitVector| |pv$| 1)
      (QSETREFV |$| 12
       (CONS (|dispatchFunction| |RCAGG-;setelt;Avalue2S;2|) |$|))))
    (COND
```

```
      ((|testBitVector| |pv$| 2)
       (QSETREFV |$| 18 (CONS (|dispatchFunction| |RCAGG-;child?;2AB;3|) |$|))))
     |$|)))))

(MAKEPROP
 (QUOTE |RecursiveAggregate&|)
 (QUOTE |infovec|)
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
   (0 . |value|) (QUOTE "value") |RCAGG-;elt;AvalueS;1| (5 . |setvalue!|)
   (11 . |setelt|) (|List| |$|) (18 . |children|) (|Boolean|) (|List| 6)
   (23 . |member?|) (29 . |child?|)))
  (QUOTE #(|setelt| 35 |elt| 42 |child?| 48))
  (QUOTE NIL)
  (CONS (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS
  (QUOTE #())
  (CONS
   (QUOTE #())
   (|makeByteWordVec2| 18 (QUOTE (1 6 7 0 8 2 6 7 0 7 11 3 0 7 0 9 7 12
     1 6 13 0 14 2 16 15 6 0 17 2 0 15 0 0 18 3 0 7 0 9 7 12 2 0 7 0 9
     10 2 0 15 0 0 18))))))
  (QUOTE |lookupComplete|)))
```

21.51 RING.lsp BOOTSTRAP

RING depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RING** category which we can write into the **MID** directory. We compile the lisp code and copy the **RING.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*RING.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |Ring;AL| (QUOTE NIL))

(DEFUN |Ring| NIL
  (LET (#:G82789)
    (COND
      (|Ring;AL|)
      (T (SETQ |Ring;AL| (|Ring;|))))))

(DEFUN |Ring;| NIL
  (PROG (#1=#:G82787)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|Rng|)
            (|Monoid|)
            (|LeftModule| (QUOTE |$|))
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|characteristic| ((|NonNegativeInteger|))) T)
                ((|coerce| (|$| (|Integer|))) T)))
              (QUOTE ((|unitsKnown| T)))
              (QUOTE ((|Integer|) (|NonNegativeInteger|)))
              NIL))
           |Ring|)
        (SETELT #1# 0 (QUOTE (|Ring|)))))))

(MAKEPROP (QUOTE |Ring|) (QUOTE NILADIC) T)
```

21.52 RING-.lsp BOOTSTRAP

RING- depends on **RING**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RING-** category which we can write into the **MID** directory. We compile the lisp code and copy the **RING-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*RING-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |RING-;coerce;IS;1| (|n| |$|)
  (SPADCALL |n| (|spadConstant| |$| 7) (QREFELT |$| 9)))

(DEFUN |Ring&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|Ring&|))
        (LETT |dv$| (LIST (QUOTE |Ring&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 12) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(MAKEPROP
  (QUOTE |Ring&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (0 . |One|)
        (|Integer|)
        (4 . |*|)
        |RING-;coerce;IS;1|
        (|OutputForm|)))
  (QUOTE #(|coerce| 10))
  (QUOTE NIL)
  (CONS
    (|makeByteWordVec2| 1 (QUOTE NIL))
    (CONS
```

```
        (QUOTE #())
        (CONS
          (QUOTE #())
          (|makeByteWordVec2| 10 (QUOTE (0 6 0 7 2 6 0 8 0 9 1 0 0 8 10))))))
    (QUOTE |lookupComplete|)))
```

21.53 RNG.lsp BOOTSTRAP

RNG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RNG** category which we can write into the **MID** directory. We compile the lisp code and copy the **RNG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*RNG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |Rng;AL| (QUOTE NIL))

(DEFUN |Rng| NIL
  (LET (#:G82722)
    (COND
      (|Rng;AL|)
      (T (SETQ |Rng;AL| (|Rng;|))))))

(DEFUN |Rng;| NIL
  (PROG (#1=#:G82720)
    (RETURN
      (PROG1
        (LETT #1# (|Join| (|AbelianGroup|) (|SemiGroup|)) |Rng|)
        (SETELT #1# 0 (QUOTE (|Rng|)))))))

(MAKEPROP (QUOTE |Rng|) (QUOTE NILADIC) T)
```

21.54 RNS.lsp BOOTSTRAP

RNS depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RNS** category which we can write into the **MID** directory. We compile the lisp code and copy the **RNS.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*RNS.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |RealNumberSystem;AL| (QUOTE NIL))

(DEFUN |RealNumberSystem| NIL
  (LET (#:G105478)
    (COND
      (|RealNumberSystem;AL|)
      (T (SETQ |RealNumberSystem;AL| (|RealNumberSystem;|))))))

(DEFUN |RealNumberSystem;| NIL
  (PROG (#1=#:G105476)
    (RETURN
      (PROG1
        (LETT #1#
          (|sublisV|
            (PAIR
              (QUOTE (#2=#:G105472 #3=#:G105473 #4=#:G105474 #5=#:G105475))
              (LIST
                (QUOTE (|Integer|))
                (QUOTE (|Fraction| (|Integer|)))
                (QUOTE (|Pattern| (|Float|)))
                (QUOTE (|Float|))))
            (|Join|
              (|Field|)
              (|OrderedRing|)
              (|RealConstant|)
              (|RetractableTo| (QUOTE #2#))
              (|RetractableTo| (QUOTE #3#))
              (|RadicalCategory|)
              (|ConvertibleTo| (QUOTE #4#))
              (|PatternMatchable| (QUOTE #5#))
              (|CharacteristicZero|)
              (|mkCategory|
                (QUOTE |domain|)
```

```
                (QUOTE (
                  ((|norm| (|$| |$|)) T)
                  ((|ceiling| (|$| |$|)) T)
                  ((|floor| (|$| |$|)) T)
                  ((|wholePart| ((|Integer|) |$|)) T)
                  ((|fractionPart| (|$| |$|)) T)
                  ((|truncate| (|$| |$|)) T)
                  ((|round| (|$| |$|)) T)
                  ((|abs| (|$| |$|)) T)))
                NIL
                (QUOTE ((|Integer|)))
                NIL)))
          |RealNumberSystem|)
        (SETELT #1# 0 (QUOTE (|RealNumberSystem|)))))))

(MAKEPROP (QUOTE |RealNumberSystem|) (QUOTE NILADIC) T)
```

21.55 RNS-.lsp BOOTSTRAP

RNS- depends **RNS**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **RNS-** category which we can write into the **MID** directory. We compile the lisp code and copy the **RNS.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*RNS-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(PUT
  (QUOTE |RNS-;characteristic;Nni;1|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM NIL 0)))

(DEFUN |RNS-;characteristic;Nni;1| (|$|) 0)

(DEFUN |RNS-;fractionPart;2S;2| (|x| |$|)
  (SPADCALL |x| (SPADCALL |x| (QREFELT |$| 9)) (QREFELT |$| 10)))

(DEFUN |RNS-;truncate;2S;3| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 13))
      (SPADCALL
        (SPADCALL
          (SPADCALL |x| (QREFELT |$| 14))
          (QREFELT |$| 15))
        (QREFELT |$| 14)))
    ((QUOTE T) (SPADCALL |x| (QREFELT |$| 15)))))

(DEFUN |RNS-;round;2S;4| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 13))
      (SPADCALL
        (SPADCALL |x|
          (SPADCALL
            (|spadConstant| |$| 17)
            (SPADCALL 2 (QREFELT |$| 19))
            (QREFELT |$| 20))
          (QREFELT |$| 10))
        (QREFELT |$| 9)))
    ((QUOTE T)
      (SPADCALL
        (SPADCALL |x|
```

```
            (SPADCALL
              (|spadConstant| |$| 17)
              (SPADCALL 2 (QREFELT |$| 19))
              (QREFELT |$| 20))
            (QREFELT |$| 21))
          (QREFELT |$| 9)))))

(DEFUN |RNS-;norm;2S;5| (|x| |$|)
  (SPADCALL |x| (QREFELT |$| 23)))

(DEFUN |RNS-;coerce;FS;6| (|x| |$|)
  (SPADCALL
    (SPADCALL
      (SPADCALL |x| (QREFELT |$| 26))
      (QREFELT |$| 19))
    (SPADCALL
      (SPADCALL |x| (QREFELT |$| 27))
      (QREFELT |$| 19))
    (QREFELT |$| 20)))

(DEFUN |RNS-;convert;SP;7| (|x| |$|)
  (SPADCALL (SPADCALL |x| (QREFELT |$| 30)) (QREFELT |$| 32)))

(DEFUN |RNS-;floor;2S;8| (|x| |$|)
  (PROG (|x1|)
    (RETURN
      (SEQ
        (LETT |x1|
          (SPADCALL (SPADCALL |x| (QREFELT |$| 34)) (QREFELT |$| 19))
          |RNS-;floor;2S;8|)
        (EXIT
          (COND
            ((SPADCALL |x| |x1| (QREFELT |$| 35)) |x|)
            ((SPADCALL |x| (|spadConstant| |$| 36) (QREFELT |$| 37))
              (SPADCALL |x1| (|spadConstant| |$| 17) (QREFELT |$| 10)))
            ((QUOTE T) |x1|)))))))

(DEFUN |RNS-;ceiling;2S;9| (|x| |$|)
  (PROG (|x1|)
    (RETURN
      (SEQ
        (LETT |x1|
          (SPADCALL (SPADCALL |x| (QREFELT |$| 34)) (QREFELT |$| 19))
          |RNS-;ceiling;2S;9|)
        (EXIT
          (COND
```

```
            ((SPADCALL |x| |x1| (QREFELT |$| 35)) |x|)
            ((SPADCALL |x| (|spadConstant| |$| 36) (QREFELT |$| 37)) |x1|)
            ((QUOTE T)
              (SPADCALL |x1| (|spadConstant| |$| 17) (QREFELT |$| 21)))))))))

(DEFUN |RNS-;patternMatch;SP2Pmr;10| (|x| |p| |l| |$|)
  (PROG (|r|)
    (RETURN
      (SEQ
        (COND
          ((SPADCALL |p| (QREFELT |$| 40))
            (SPADCALL |p| |x| |l| (QREFELT |$| 42)))
          ((SPADCALL |p| (QREFELT |$| 43))
            (SEQ
              (LETT |r|
                (SPADCALL |p| (QREFELT |$| 45))
                |RNS-;patternMatch;SP2Pmr;10|)
              (EXIT
                (COND
                  ((QEQCAR |r| 0)
                    (COND
                      ((SPADCALL
                         (SPADCALL |x| (QREFELT |$| 30))
                         (QCDR |r|)
                         (QREFELT |$| 46))
                        |l|)
                       ((QUOTE T) (SPADCALL (QREFELT |$| 47)))))
                  ((QUOTE T) (SPADCALL (QREFELT |$| 47)))))))
          ((QUOTE T) (SPADCALL (QREFELT |$| 47)))))))

(DEFUN |RealNumberSystem&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|RealNumberSystem&|))
        (LETT |dv$| (LIST (QUOTE |RealNumberSystem&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 52) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(MAKEPROP
  (QUOTE |RealNumberSystem&|)
  (QUOTE |infovec|)
```

```
(LIST
  (QUOTE
    #(NIL NIL NIL NIL NIL NIL
      (|local| |#1|)
      (|NonNegativeInteger|)
      |RNS-;characteristic;Nni;1|
      (0 . |truncate|)
      (5 . |-|)
      |RNS-;fractionPart;2S;2|
      (|Boolean|)
      (11 . |negative?|)
      (16 . |-|)
      (21 . |floor|)
      |RNS-;truncate;2S;3|
      (26 . |One|)
      (|Integer|)
      (30 . |coerce|)
      (35 . |/|)
      (41 . |+|)
      |RNS-;round;2S;4|
      (47 . |abs|)
      |RNS-;norm;2S;5|
      (|Fraction| 18)
      (52 . |numer|)
      (57 . |denom|)
      |RNS-;coerce;FS;6|
      (|Float|)
      (62 . |convert|)
      (|Pattern| 29)
      (67 . |coerce|)
      |RNS-;convert;SP;7|
      (72 . |wholePart|)
      (77 . |=|)
      (83 . |Zero|)
      (87 . |<|)
      |RNS-;floor;2S;8|
      |RNS-;ceiling;2S;9|
      (93 . |generic?|)
      (|PatternMatchResult| 29 6)
      (98 . |addMatch|)
      (105 . |constant?|)
      (|Union| 29 (QUOTE "failed"))
      (110 . |retractIfCan|)
      (115 . |=|)
      (121 . |failed|)
      (|PatternMatchResult| 29 |$|)
```

```
    |RNS-;patternMatch;SP2Pmr;10|
    (|DoubleFloat|)
    (|OutputForm|)))
(QUOTE
  #(|truncate| 125 |round| 130 |patternMatch| 135 |norm| 142
    |fractionPart| 147 |floor| 152 |convert| 157 |coerce| 162
    |characteristic| 172 |ceiling| 176))
(QUOTE NIL)
(CONS
  (|makeByteWordVec2| 1 (QUOTE NIL))
  (CONS
    (QUOTE #())
    (CONS
      (QUOTE #())
      (|makeByteWordVec2| 49
        (QUOTE
          (1 6 0 0 9 2 6 0 0 0 10 1 6 12 0 13 1 6 0 0 14 1 6 0 0 15 0 6 0
           17 1 6 0 18 19 2 6 0 0 0 20 2 6 0 0 0 21 1 6 0 0 23 1 25 18 0
           26 1 25 18 0 27 1 6 29 0 30 1 31 0 29 32 1 6 18 0 34 2 6 12 0
           0 35 0 6 0 36 2 6 12 0 0 37 1 31 12 0 40 3 41 0 31 6 0 42 1 31
           12 0 43 1 31 44 0 45 2 29 12 0 0 46 0 41 0 47 1 0 0 0 16 1 0 0
           0 22 3 0 48 0 31 48 49 1 0 0 0 24 1 0 0 0 11 1 0 0 0 38 1 0 31
           0 33 1 0 0 25 28 1 0 0 25 28 0 0 7 8 1 0 0 0 39))))))
(QUOTE |lookupComplete|)))
```

21.56 SETAGG.lsp BOOTSTRAP

SETAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **SETAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **SETAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*SETAGG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |SetAggregate;CAT| (QUOTE NIL))

(SETQ |SetAggregate;AL| (QUOTE NIL))

(DEFUN |SetAggregate| (#1=#:G83200)
 (LET (#2=#:G83201)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |SetAggregate;AL|)) (CDR #2#))
   (T
    (SETQ |SetAggregate;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|SetAggregate;| #1#)))
      |SetAggregate;AL|))
    #2#))))

(DEFUN |SetAggregate;| (|t#1|)
 (PROG (#1=#:G83199)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|SetAggregate;CAT|)
       ((QUOTE T)
        (LETT |SetAggregate;CAT|
         (|Join|
          (|SetCategory|)
          (|Collection| (QUOTE |t#1|))
          (|mkCategory|
           (QUOTE |domain|)
           (QUOTE (
            ((|<| ((|Boolean|) |$| |$|)) T)
```

```
            ((|brace| (|$|)) T)
            ((|brace| (|$| (|List| |t#1|))) T)
            ((|set| (|$|)) T)
            ((|set| (|$| (|List| |t#1|))) T)
            ((|intersect| (|$| |$| |$|)) T)
            ((|difference| (|$| |$| |$|)) T)
            ((|difference| (|$| |$| |t#1|)) T)
            ((|symmetricDifference| (|$| |$| |$|)) T)
            ((|subset?| ((|Boolean|) |$| |$|)) T)
            ((|union| (|$| |$| |$|)) T)
            ((|union| (|$| |$| |t#1|)) T)
            ((|union| (|$| |t#1| |$|)) T)))
          (QUOTE ((|partiallyOrderedSet| T)))
          (QUOTE ((|Boolean|) (|List| |t#1|)))
          NIL))
        . #2=(|SetAggregate|)))))
    . #2#)
  (SETELT #1# 0 (LIST (QUOTE |SetAggregate|) (|devaluate| |t#1|)))))))
```

21.57 SETAGG-.lsp BOOTSTRAP

SETAGG- depends on **SETAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **SETAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **SETAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*SETAGG-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |SETAGG-;symmetricDifference;3A;1| (|x| |y| |$|)
 (SPADCALL
  (SPADCALL |x| |y| (QREFELT |$| 8))
  (SPADCALL |y| |x| (QREFELT |$| 8))
  (QREFELT |$| 9)))

(DEFUN |SETAGG-;union;ASA;2| (|s| |x| |$|)
 (SPADCALL |s| (SPADCALL (LIST |x|) (QREFELT |$| 12)) (QREFELT |$| 9)))

(DEFUN |SETAGG-;union;S2A;3| (|x| |s| |$|)
 (SPADCALL |s| (SPADCALL (LIST |x|) (QREFELT |$| 12)) (QREFELT |$| 9)))

(DEFUN |SETAGG-;difference;ASA;4| (|s| |x| |$|)
 (SPADCALL |s| (SPADCALL (LIST |x|) (QREFELT |$| 12)) (QREFELT |$| 8)))

(DEFUN |SetAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|SetAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |SetAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (GETREFV 16) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    |$|))))

(MAKEPROP
 (QUOTE |SetAggregate&|)
 (QUOTE |infovec|)
```

```
(LIST
 (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
   (0 . |difference|) (6 . |union|) |SETAGG-;symmetricDifference;3A;1|
   (|List| 7) (12 . |brace|) |SETAGG-;union;ASA;2| |SETAGG-;union;S2A;3|
   |SETAGG-;difference;ASA;4|))
 (QUOTE #(|union| 17 |symmetricDifference| 29 |difference| 35))
 (QUOTE NIL)
 (CONS
  (|makeByteWordVec2| 1 (QUOTE NIL))
  (CONS
   (QUOTE #())
   (CONS
    (QUOTE #())
    (|makeByteWordVec2| 15 (QUOTE (2 6 0 0 0 8 2 6 0 0 0 9 1 6 0 11 12 2
      0 0 7 0 14 2 0 0 0 7 13 2 0 0 0 0 10 2 0 0 0 7 15))))))
 (QUOTE |lookupComplete|)))
```

21.58 SETCAT.lsp BOOTSTRAP

SETCAT needs **SINT** which needs **UFD** which needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON** which needs **ABELSG** which needs **SETCAT**. We break this chain with **SETCAT.lsp** which we cache here. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **SETCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **SETCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*SETCAT.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |SetCategory;AL| (QUOTE NIL))

(DEFUN |SetCategory| NIL
  (LET (#:G82359)
    (COND
      (|SetCategory;AL|)
      (T (SETQ |SetCategory;AL| (|SetCategory;|))))))

(DEFUN |SetCategory;| NIL
  (PROG (#1=#:G82357)
    (RETURN
      (PROG1
        (LETT #1#
          (|sublisV|
            (PAIR
              (QUOTE (#2=#:G82356))
              (LIST (QUOTE (|OutputForm|))))
            (|Join|
              (|BasicType|)
              (|CoercibleTo| (QUOTE #2#))
              (|mkCategory|
                (QUOTE |domain|)
                (QUOTE (
                  ((|hash| ((|SingleInteger|) |$|)) T)
                  ((|latex| ((|String|) |$|)) T)))
                NIL
                (QUOTE ((|String|) (|SingleInteger|)))
                NIL)))
            |SetCategory|)
          (SETELT #1# 0 (QUOTE (|SetCategory|)))))))
```

```
(MAKEPROP (QUOTE |SetCategory|) (QUOTE NILADIC) T)
```

21.59 SETCAT-.lsp BOOTSTRAP

SETCAT- is the implementation of the operations exported by **SETCAT**. It comes into existance whenever **SETCAT** gets compiled by Axiom. However this will not happen at the lisp level so we also cache this information here. See the explanation under the **SETCAT.lsp** section for more details.

Note that this code is not included in the generated catdef.spad file.

⟨*SETCAT-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(PUT
  (QUOTE |SETCAT-;hash;SSi;1|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|s|) 0)))

(DEFUN |SETCAT-;hash;SSi;1| (|s| |$|) 0)

(PUT
  (QUOTE |SETCAT-;latex;SS;2|)
  (QUOTE |SPADreplace|)
  (QUOTE (XLAM (|s|) "\\mbox{\\bf Unimplemented}")))

(DEFUN |SETCAT-;latex;SS;2| (|s| |$|)
  "\\mbox{\\bf Unimplemented}")

(DEFUN |SetCategory&| (|#1|)
 (PROG (|DV$1| |dv$| |$| |pv$|)
   (RETURN
     (PROGN
       (LETT |DV$1| (|devaluate| |#1|) . #1=(|SetCategory&|))
       (LETT |dv$| (LIST (QUOTE |SetCategory&|) |DV$1|) . #1#)
       (LETT |$| (GETREFV 11) . #1#)
       (QSETREFV |$| 0 |dv$|)
       (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
       (|stuffDomainSlots| |$|)
       (QSETREFV |$| 6 |#1|)
       |$|))))

(MAKEPROP
  (QUOTE |SetCategory&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
```

```
    (|local| |#1|)
    (|SingleInteger|)
    |SETCAT-;hash;SSi;1|
    (|String|)
    |SETCAT-;latex;SS;2|))
(QUOTE
  #(|latex| 0 |hash| 5))
(QUOTE NIL)
(CONS
  (|makeByteWordVec2| 1 (QUOTE NIL))
  (CONS
    (QUOTE #())
    (CONS
      (QUOTE #())
      (|makeByteWordVec2|
        10
        (QUOTE (1 0 9 0 10 1 0 7 0 8))))))
(QUOTE |lookupComplete|)))
```

21.60 STAGG.lsp BOOTSTRAP

STAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **STAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **STAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*STAGG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |StreamAggregate;CAT| (QUOTE NIL))

(SETQ |StreamAggregate;AL| (QUOTE NIL))

(DEFUN |StreamAggregate| (#1=#:G87035)
 (LET (#2=#:G87036)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |StreamAggregate;AL|)) (CDR #2#))
   (T
    (SETQ |StreamAggregate;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|StreamAggregate;| #1#)))
      |StreamAggregate;AL|))
    #2#))))
v
(DEFUN |StreamAggregate;| (|t#1|)
 (PROG (#1=#:G87034)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|StreamAggregate;CAT|)
       ((QUOTE T)
        (LETT |StreamAggregate;CAT|
         (|Join|
          (|UnaryRecursiveAggregate| (QUOTE |t#1|))
          (|LinearAggregate| (QUOTE |t#1|))
          (|mkCategory|
           (QUOTE |domain|)
           (QUOTE (
            ((|explicitlyFinite?| ((|Boolean|) |$|)) T)
```

```
          ((|possiblyInfinite?| ((|Boolean|) |$|)) T)))
         NIL
         (QUOTE ((|Boolean|)))
         NIL))
       . #2=(|StreamAggregate|)))))
  . #2#)
(SETELT #1# 0 (LIST (QUOTE |StreamAggregate|) (|devaluate| |t#1|)))))))
```

21.61 STAGG-.lsp BOOTSTRAP

STAGG- depends on **STAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **STAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **STAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*STAGG-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |STAGG-;explicitlyFinite?;AB;1| (|x| |$|)
 (COND ((SPADCALL |x| (QREFELT |$| 9)) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))

(DEFUN |STAGG-;possiblyInfinite?;AB;2| (|x| |$|)
 (SPADCALL |x| (QREFELT |$| 9)))

(DEFUN |STAGG-;first;ANniA;3| (|x| |n| |$|)
 (PROG (#1=#:G87053 |i|)
  (RETURN
   (SEQ
    (SPADCALL
     (PROGN
      (LETT #1# NIL |STAGG-;first;ANniA;3|)
      (SEQ
       (LETT |i| 1 |STAGG-;first;ANniA;3|)
       G190
       (COND ((QSGREATERP |i| |n|) (GO G191)))
       (SEQ
        (EXIT
         (LETT #1#
          (CONS
           (|STAGG-;c2| |x|
            (LETT |x| (SPADCALL |x| (QREFELT |$| 12)) |STAGG-;first;ANniA;3|)
            |$|)
           #1#)
          |STAGG-;first;ANniA;3|)))
        (LETT |i| (QSADD1 |i|) |STAGG-;first;ANniA;3|)
        (GO G190)
        G191
        (EXIT (NREVERSE0 #1#))))
      (QREFELT |$| 14))))))

(DEFUN |STAGG-;c2| (|x| |r| |$|)
```

```
   (COND
     ((SPADCALL |x| (QREFELT |$| 17)) (|error| "Index out of range"))
     ((QUOTE T) (SPADCALL |x| (QREFELT |$| 18)))))

  (DEFUN |STAGG-;elt;AIS;5| (|x| |i| |$|)
   (PROG (#1=#:G87056)
    (RETURN
     (SEQ
      (LETT |i| (|-| |i| (SPADCALL |x| (QREFELT |$| 20))) |STAGG-;elt;AIS;5|)
      (COND
       ((OR
         (|<| |i| 0)
         (SPADCALL
          (LETT |x|
           (SPADCALL |x|
            (PROG1
             (LETT #1# |i| |STAGG-;elt;AIS;5|)
             (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
            (QREFELT |$| 21))
           |STAGG-;elt;AIS;5|)
          (QREFELT |$| 17)))
        (EXIT (|error| "index out of range"))))
      (EXIT (SPADCALL |x| (QREFELT |$| 18)))))))

  (DEFUN |STAGG-;elt;AUsA;6| (|x| |i| |$|)
   (PROG (|l| #1=#:G87060 |h| #2=#:G87062 #3=#:G87063)
    (RETURN
     (SEQ
      (LETT |l|
       (|-| (SPADCALL |i| (QREFELT |$| 24)) (SPADCALL |x| (QREFELT |$| 20)))
       |STAGG-;elt;AUsA;6|)
      (EXIT
       (COND
        ((|<| |l| 0) (|error| "index out of range"))
        ((NULL (SPADCALL |i| (QREFELT |$| 25)))
          (SPADCALL
           (SPADCALL |x|
            (PROG1
             (LETT #1# |l| |STAGG-;elt;AUsA;6|)
             (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
            (QREFELT |$| 21))
           (QREFELT |$| 26)))
        ((QUOTE T)
         (SEQ
          (LETT |h|
           (|-| (SPADCALL |i| (QREFELT |$| 27)) (SPADCALL |x| (QREFELT |$| 20)))
```

```
           |STAGG-;elt;AUsA;6|)
          (EXIT
           (COND
            ((|<| |h| |l|) (SPADCALL (QREFELT |$| 28)))
            ((QUOTE T)
             (SPADCALL
              (SPADCALL |x|
               (PROG1
                (LETT #2# |l| |STAGG-;elt;AUsA;6|)
                (|check-subtype|
                  (|>=| #2# 0) (QUOTE (|NonNegativeInteger|)) #2#))
               (QREFELT |$| 21))
              (PROG1
               (LETT #3# (|+| (|-| |h| |l|) 1) |STAGG-;elt;AUsA;6|)
               (|check-subtype| (|>=| #3# 0) (QUOTE (|NonNegativeInteger|)) #3#))
              (QREFELT |$| 29)))))))))))))

(DEFUN |STAGG-;concat;3A;7| (|x| |y| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 26)) |y| (QREFELT |$| 31)))

(DEFUN |STAGG-;concat;LA;8| (|l| |$|)
 (COND
  ((NULL |l|) (SPADCALL (QREFELT |$| 28)))
  ((QUOTE T)
   (SPADCALL
    (SPADCALL (|SPADfirst| |l|) (QREFELT |$| 26))
    (SPADCALL (CDR |l|) (QREFELT |$| 34))
    (QREFELT |$| 31)))))

(DEFUN |STAGG-;map!;M2A;9| (|f| |l| |$|)
 (PROG (|y|)
  (RETURN
   (SEQ
    (LETT |y| |l| |STAGG-;map!;M2A;9|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |l| (QREFELT |$| 17)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (SPADCALL |l|
       (SPADCALL (SPADCALL |l| (QREFELT |$| 18)) |f|) (QREFELT |$| 36))
      (EXIT (LETT |l| (SPADCALL |l| (QREFELT |$| 12)) |STAGG-;map!;M2A;9|)))
```

```
      NIL
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT |y|)))))

(DEFUN |STAGG-;fill!;ASA;10| (|x| |s| |$|)
 (PROG (|y|)
  (RETURN
   (SEQ
    (LETT |y| |x| |STAGG-;fill!;ASA;10|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |y| (QREFELT |$| 17)) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (SPADCALL |y| |s| (QREFELT |$| 36))
      (EXIT (LETT |y| (SPADCALL |y| (QREFELT |$| 12)) |STAGG-;fill!;ASA;10|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |x|)))))

(DEFUN |STAGG-;setelt;AI2S;11| (|x| |i| |s| |$|)
 (PROG (#1=#:G87081)
  (RETURN
   (SEQ
    (LETT |i|
     (|-| |i| (SPADCALL |x| (QREFELT |$| 20))) |STAGG-;setelt;AI2S;11|)
    (COND
     ((OR
       (|<| |i| 0)
       (SPADCALL
        (LETT |x|
         (SPADCALL |x|
          (PROG1
           (LETT #1# |i| |STAGG-;setelt;AI2S;11|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 21))
         |STAGG-;setelt;AI2S;11|)
        (QREFELT |$| 17)))
      (EXIT (|error| "index out of range"))))
```

```
      (EXIT (SPADCALL |x| |s| (QREFELT |$| 36)))))))

  (DEFUN |STAGG-;setelt;AUs2S;12| (|x| |i| |s| |$|)
   (PROG (|l| |h| #1=#:G87086 #2=#:G87087 |z| |y|)
    (RETURN
     (SEQ
      (LETT |l|
       (|-| (SPADCALL |i| (QREFELT |$| 24)) (SPADCALL |x| (QREFELT |$| 20)))
        |STAGG-;setelt;AUs2S;12|)
      (EXIT
       (COND
        ((|<| |l| 0) (|error| "index out of range"))
        ((QUOTE T)
         (SEQ
          (LETT |h|
           (COND
            ((SPADCALL |i| (QREFELT |$| 25))
              (|-|
               (SPADCALL |i| (QREFELT |$| 27))
               (SPADCALL |x| (QREFELT |$| 20))))
            ((QUOTE T) (SPADCALL |x| (QREFELT |$| 41))))
           |STAGG-;setelt;AUs2S;12|)
          (EXIT
           (COND
            ((|<| |h| |l|) |s|)
            ((QUOTE T)
              (SEQ
               (LETT |y|
                (SPADCALL |x|
                 (PROG1
                  (LETT #1# |l| |STAGG-;setelt;AUs2S;12|)
                  (|check-subtype|
                   (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
                 (QREFELT |$| 21))
                |STAGG-;setelt;AUs2S;12|)
               (LETT |z|
                (SPADCALL |y|
                 (PROG1
                  (LETT #2# (|+| (|-| |h| |l|) 1) |STAGG-;setelt;AUs2S;12|)
                  (|check-subtype|
                    (|>=| #2# 0) (QUOTE (|NonNegativeInteger|)) #2#))
                 (QREFELT |$| 21))
                |STAGG-;setelt;AUs2S;12|)
               (SEQ
                 G190
                 (COND
```

```
                ((NULL
                  (COND
                   ((SPADCALL |y| |z| (QREFELT |$| 42)) (QUOTE NIL))
                   ((QUOTE T) (QUOTE T))))
                 (GO G191)))
                (SEQ
                 (SPADCALL |y| |s| (QREFELT |$| 36))
                 (EXIT
                  (LETT |y|
                   (SPADCALL |y| (QREFELT |$| 12))
                   |STAGG-;setelt;AUs2S;12|)))
                NIL
                (GO G190)
                G191
                (EXIT NIL))
               (EXIT |s|)))))))))))))

(DEFUN |STAGG-;concat!;3A;13| (|x| |y| |$|)
 (SEQ
  (COND
   ((SPADCALL |x| (QREFELT |$| 17)) |y|)
   ((QUOTE T)
     (SEQ
      (SPADCALL (SPADCALL |x| (QREFELT |$| 44)) |y| (QREFELT |$| 45))
      (EXIT |x|))))))

(DEFUN |StreamAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|StreamAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |StreamAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (GETREFV 51) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
    (QSETREFV |$| 7 |#2|)
    (COND
     ((|HasAttribute| |#1| (QUOTE |shallowlyMutable|))
      (PROGN
       (QSETREFV |$| 32 (CONS (|dispatchFunction| |STAGG-;concat;3A;7|) |$|))
       (QSETREFV |$| 35 (CONS (|dispatchFunction| |STAGG-;concat;LA;8|) |$|))
       (QSETREFV |$| 38 (CONS (|dispatchFunction| |STAGG-;map!;M2A;9|) |$|))
       (QSETREFV |$| 39 (CONS (|dispatchFunction| |STAGG-;fill!;ASA;10|) |$|))
```

```
        (QSETREFV |$| 40
          (CONS (|dispatchFunction| |STAGG-;setelt;AI2S;11|) |$|))
        (QSETREFV |$| 43
          (CONS (|dispatchFunction| |STAGG-;setelt;AUs2S;12|) |$|))
        (QSETREFV |$| 46
          (CONS (|dispatchFunction| |STAGG-;concat!;3A;13|) |$|)))))
      |$|))))

(MAKEPROP
 (QUOTE |StreamAggregate&|)
 (QUOTE |infovec|)
 (LIST
  (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
   (|Boolean|) (0 . |cyclic?|) |STAGG-;explicitlyFinite?;AB;1|
   |STAGG-;possiblyInfinite?;AB;2| (5 . |rest|) (|List| 7)
   (10 . |construct|) (|NonNegativeInteger|) |STAGG-;first;ANniA;3|
   (15 . |empty?|) (20 . |first|) (|Integer|) (25 . |minIndex|)
   (30 . |rest|) |STAGG-;elt;AIS;5| (|UniversalSegment| 19) (36 . |lo|)
   (41 . |hasHi|) (46 . |copy|) (51 . |hi|) (56 . |empty|) (60 . |first|)
   |STAGG-;elt;AUsA;6| (66 . |concat!|) (72 . |concat|) (|List| |$|)
   (78 . |concat|) (83 . |concat|) (88 . |setfirst!|) (|Mapping| 7 7)
   (94 . |map!|) (100 . |fill!|) (106 . |setelt|) (113 . |maxIndex|)
   (118 . |eq?|) (124 . |setelt|) (131 . |tail|) (136 . |setrest!|)
   (142 . |concat!|) (QUOTE "rest") (QUOTE "last") (QUOTE "first")
   (QUOTE "value")))
  (QUOTE #(|setelt| 148 |possiblyInfinite?| 162 |map!| 167 |first| 173
    |fill!| 179 |explicitlyFinite?| 185 |elt| 190 |concat!| 202 |concat| 208))
  (QUOTE NIL)
  (CONS
   (|makeByteWordVec2| 1 (QUOTE NIL))
   (CONS
    (QUOTE #())
    (CONS
     (QUOTE #())
     (|makeByteWordVec2| 46 (QUOTE (1 6 8 0 9 1 6 0 0 12 1 6 0 13 14 1 6
       8 0 17 1 6 7 0 18 1 6 19 0 20 2 6 0 0 15 21 1 23 19 0 24 1 23 8
       0 25 1 6 0 0 26 1 23 19 0 27 0 6 0 28 2 6 0 0 15 29 2 6 0 0 0 31
       2 0 0 0 0 32 1 6 0 33 34 1 0 0 33 35 2 6 7 0 7 36 2 0 0 37 0 38 2
       0 0 0 7 39 3 0 7 0 19 7 40 1 6 19 0 41 2 6 8 0 0 42 3 0 7 0 23 7 43
       1 6 0 0 44 2 6 0 0 0 45 2 0 0 0 0 46 3 0 7 0 19 7 40 3 0 7 0 23 7 43
       1 0 8 0 11 2 0 0 37 0 38 2 0 0 0 15 16 2 0 0 0 7 39 1 0 8 0 10 2 0 7
       0 19 22 2 0 0 0 23 30 2 0 0 0 0 46 1 0 0 33 35 2 0 0 0 0 32))))))
  (QUOTE |lookupComplete|)))
```

21.62 TSETCAT.lsp BOOTSTRAP

TSETCAT depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **TSETCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **TSETCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*TSETCAT.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |TriangularSetCategory;CAT| (QUOTE NIL))

(SETQ |TriangularSetCategory;AL| (QUOTE NIL))

(DEFUN |TriangularSetCategory| (|&REST| #1=#:G82394 |&AUX| #2=#:G82392)
 (DSETQ #2# #1#)
 (LET (#3=#:G82393)
  (COND
   ((SETQ #3# (|assoc| (|devaluateList| #2#) |TriangularSetCategory;AL|))
     (CDR #3#))
   (T
    (SETQ |TriangularSetCategory;AL|
     (|cons5|
      (CONS
       (|devaluateList| #2#)
       (SETQ #3# (APPLY (FUNCTION |TriangularSetCategory;|) #2#)))
      |TriangularSetCategory;AL|))
     #3#))))

(DEFUN |TriangularSetCategory;| (|t#1| |t#2| |t#3| |t#4|)
 (PROG (#1=#:G82391)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR
       (QUOTE (|t#1| |t#2| |t#3| |t#4|))
       (LIST
        (|devaluate| |t#1|)
        (|devaluate| |t#2|)
        (|devaluate| |t#3|)
        (|devaluate| |t#4|)))
      (COND
```

```
(|TriangularSetCategory;CAT|)
((QUOTE T)
 (LETT |TriangularSetCategory;CAT|
  (|Join|
   (|PolynomialSetCategory|
    (QUOTE |t#1|)
    (QUOTE |t#2|)
    (QUOTE |t#3|)
    (QUOTE |t#4|))
   (|mkCategory|
    (QUOTE |domain|)
    (QUOTE (
     ((|infRittWu?| ((|Boolean|) |$| |$|)) T)
     ((|basicSet| (
        (|Union|
         (|Record| (|:| |bas| |$|) (|:| |top| (|List| |t#4|))) "failed")
        (|List| |t#4|)
        (|Mapping| (|Boolean|) |t#4| |t#4|)))
       T)
     ((|basicSet| (
        (|Union|
         (|Record| (|:| |bas| |$|) (|:| |top| (|List| |t#4|))) "failed")
        (|List| |t#4|)
        (|Mapping| (|Boolean|) |t#4|)
        (|Mapping| (|Boolean|) |t#4| |t#4|)))
       T)
     ((|initials| ((|List| |t#4|) |$|)) T)
     ((|degree| ((|NonNegativeInteger|) |$|)) T)
     ((|quasiComponent| (
        (|Record|
         (|:| |close| (|List| |t#4|))
         (|:| |open| (|List| |t#4|)))
        |$|))
       T)
     ((|normalized?| ((|Boolean|) |t#4| |$|)) T)
     ((|normalized?| ((|Boolean|) |$|)) T)
     ((|reduced?| (
        (|Boolean|)
        |t#4|
        |$|
        (|Mapping| (|Boolean|) |t#4| |t#4|)))
       T)
     ((|stronglyReduced?| ((|Boolean|) |t#4| |$|)) T)
     ((|headReduced?| ((|Boolean|) |t#4| |$|)) T)
     ((|initiallyReduced?| ((|Boolean|) |t#4| |$|)) T)
     ((|autoReduced?| (
```

```
    (|Boolean|)
    |$|
    (|Mapping| (|Boolean|) |t#4| (|List| |t#4|)))
    T)
  ((|stronglyReduced?| ((|Boolean|) |$|)) T)
  ((|headReduced?| ((|Boolean|) |$|)) T)
  ((|initiallyReduced?| ((|Boolean|) |$|)) T)
  ((|reduce| (
     |t#4|
     |t#4|
     |$|
     (|Mapping| |t#4| |t#4| |t#4|)
     (|Mapping| (|Boolean|) |t#4| |t#4|)))
    T)
  ((|rewriteSetWithReduction| (
    (|List| |t#4|)
    (|List| |t#4|)
    |$|
    (|Mapping| |t#4| |t#4| |t#4|)
    (|Mapping| (|Boolean|) |t#4| |t#4|)))
    T)
  ((|stronglyReduce| (|t#4| |t#4| |$|)) T)
  ((|headReduce| (|t#4| |t#4| |$|)) T)
  ((|initiallyReduce| (|t#4| |t#4| |$|)) T)
  ((|removeZero| (|t#4| |t#4| |$|)) T)
  ((|collectQuasiMonic| (|$| |$|)) T)
  ((|reduceByQuasiMonic| (|t#4| |t#4| |$|)) T)
  ((|zeroSetSplit| ((|List| |$|) (|List| |t#4|))) T)
  ((|zeroSetSplitIntoTriangularSystems|
    ((|List|
      (|Record| (|:| |close| |$|) (|:| |open| (|List| |t#4|))))
     (|List| |t#4|)))
    T)
  ((|first| ((|Union| |t#4| "failed") |$|)) T)
  ((|last| ((|Union| |t#4| "failed") |$|)) T)
  ((|rest| ((|Union| |$| "failed") |$|)) T)
  ((|algebraicVariables| ((|List| |t#3|) |$|)) T)
  ((|algebraic?| ((|Boolean|) |t#3| |$|)) T)
  ((|select| ((|Union| |t#4| "failed") |$| |t#3|)) T)
  ((|extendIfCan| ((|Union| |$| "failed") |$| |t#4|)) T)
  ((|extend| (|$| |$| |t#4|)) T)
  ((|coHeight| ((|NonNegativeInteger|) |$|))
    (|has| |t#3| (|Finite|)))))
 (QUOTE (
  (|finiteAggregate| T)
  (|shallowlyMutable| T)))
```

```
        (QUOTE ((|NonNegativeInteger|)
         (|Boolean|)
         (|List| |t#3|)
         (|List| (|Record| (|:| |close| |$|) (|:| |open| (|List| |t#4|))))
         (|List| |t#4|)
         (|List| |$|)))
        NIL))
  . #2=(|TriangularSetCategory|)))))
 . #2#)
(SETELT #1# 0
 (LIST
  (QUOTE |TriangularSetCategory|)
  (|devaluate| |t#1|)
  (|devaluate| |t#2|)
  (|devaluate| |t#3|)
  (|devaluate| |t#4|)))))))
```

21.63 TSETCAT-.lsp BOOTSTRAP

TSETCAT depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **TSETCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **TSETCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*TSETCAT-.lsp BOOTSTRAP*⟩≡

```
(/VERSIONCHECK 2)

(DEFUN |TSETCAT-;=;2SB;1| (|ts| |us| $)
 (PROG (#0=#:G1475 #1=#:G1481)
  (RETURN
   (COND
    ((SPADCALL |ts| (QREFELT $ 12)) (SPADCALL |us| (QREFELT $ 12)))
    ((OR
      (SPADCALL |us| (QREFELT $ 12))
      (NULL
       (SPADCALL
        (PROG2
         (LETT #0# (SPADCALL |ts| (QREFELT $ 14)) |TSETCAT-;=;2SB;1|)
         (QCDR #0#)
         (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
        (PROG2
         (LETT #0# (SPADCALL |us| (QREFELT $ 14)) |TSETCAT-;=;2SB;1|)
         (QCDR #0#)
         (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
        (QREFELT $ 15))))
      (QUOTE NIL))
     ((QUOTE T)
      (SPADCALL
       (PROG2
        (LETT #1# (SPADCALL |ts| (QREFELT $ 17)) |TSETCAT-;=;2SB;1|)
        (QCDR #1#)
        (|check-union| (QEQCAR #1# 0) (QREFELT $ 6) #1#))
       (PROG2
        (LETT #1# (SPADCALL |us| (QREFELT $ 17)) |TSETCAT-;=;2SB;1|)
        (QCDR #1#)
        (|check-union| (QEQCAR #1# 0) (QREFELT $ 6) #1#))
       (QREFELT $ 18)))))))

(DEFUN |TSETCAT-;infRittWu?;2SB;2| (|ts| |us| $)
 (PROG (|p| #0=#:G1489 |q| |v|)
```

```
    (RETURN
     (SEQ
      (COND
       ((SPADCALL |us| (QREFELT $ 12))
         (COND
          ((SPADCALL |ts| (QREFELT $ 12)) (QUOTE NIL))
          ((QUOTE T) (QUOTE T))))
       ((SPADCALL |ts| (QREFELT $ 12)) (QUOTE NIL))
       ((QUOTE T)
        (SEQ
         (LETT |p|
          (PROG2
           (LETT #0# (SPADCALL |ts| (QREFELT $ 20)) |TSETCAT-;infRittWu?;2SB;2|)
           (QCDR #0#)
           (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
          |TSETCAT-;infRittWu?;2SB;2|)
         (LETT |q|
          (PROG2
           (LETT #0# (SPADCALL |us| (QREFELT $ 20)) |TSETCAT-;infRittWu?;2SB;2|)
           (QCDR #0#)
           (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
          |TSETCAT-;infRittWu?;2SB;2|)
         (EXIT
          (COND
           ((SPADCALL |p| |q| (QREFELT $ 21)) (QUOTE T))
           ((SPADCALL |p| |q| (QREFELT $ 22)) (QUOTE NIL))
           ((QUOTE T)
            (SEQ
             (LETT |v| (SPADCALL |p| (QREFELT $ 23)) |TSETCAT-;infRittWu?;2SB;2|)
             (EXIT
              (SPADCALL
               (SPADCALL |ts| |v| (QREFELT $ 24))
               (SPADCALL |us| |v| (QREFELT $ 24)) (QREFELT $ 25))))))))))))

  (DEFUN |TSETCAT-;reduced?;PSMB;3| (|p| |ts| |redOp?| $)
   (PROG (|lp|)
    (RETURN
     (SEQ
      (LETT |lp| (SPADCALL |ts| (QREFELT $ 28)) |TSETCAT-;reduced?;PSMB;3|)
      (SEQ
       G190
       (COND
        ((NULL
          (COND
           ((NULL |lp|) (QUOTE NIL))
           ((QUOTE T) (SPADCALL |p| (|SPADfirst| |lp|) |redOp?|))))
```

```
        (GO G191)))
      (SEQ (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;reduced?;PSMB;3|)))
      NIL
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT (NULL |lp|))))))

(DEFUN |TSETCAT-;basicSet;LMU;4| (|ps| |redOp?| $)
 (PROG (|b| |bs| |p| |ts|)
  (RETURN
   (SEQ
    (LETT |ps|
     (SPADCALL (ELT $ 31) |ps| (QREFELT $ 33))
     |TSETCAT-;basicSet;LMU;4|)
    (EXIT
     (COND
      ((SPADCALL (ELT $ 34) |ps| (QREFELT $ 35)) (CONS 1 "failed"))
      ((QUOTE T)
       (SEQ
        (LETT |ps|
         (SPADCALL (ELT $ 21) |ps| (QREFELT $ 36))
         |TSETCAT-;basicSet;LMU;4|)
        (LETT |bs| (SPADCALL (QREFELT $ 37)) |TSETCAT-;basicSet;LMU;4|)
        (LETT |ts| NIL |TSETCAT-;basicSet;LMU;4|)
        (SEQ
         G190
         (COND
          ((NULL (COND ((NULL |ps|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
            (GO G191)))
          (SEQ
           (LETT |b| (|SPADfirst| |ps|) |TSETCAT-;basicSet;LMU;4|)
           (LETT |bs|
            (SPADCALL |bs| |b| (QREFELT $ 38))
            |TSETCAT-;basicSet;LMU;4|)
           (LETT |ps| (CDR |ps|) |TSETCAT-;basicSet;LMU;4|)
           (EXIT
            (SEQ
             G190
             (COND
              ((NULL
               (COND
                ((OR
                  (NULL |ps|)
                  (SPADCALL
                   (LETT |p| (|SPADfirst| |ps|) |TSETCAT-;basicSet;LMU;4|)
```

```
                    |bs| |redOp?| (QREFELT $ 39)))
                   (QUOTE NIL))
                  ((QUOTE T) (QUOTE T))))
                 (GO G191)))
               (SEQ
                (LETT |ts| (CONS |p| |ts|) |TSETCAT-;basicSet;LMU;4|)
                (EXIT (LETT |ps| (CDR |ps|) |TSETCAT-;basicSet;LMU;4|)))
               NIL
               (GO G190)
               G191
               (EXIT NIL))))
             NIL
             (GO G190)
             G191
             (EXIT NIL))
            (EXIT (CONS 0 (CONS |bs| |ts|))))))))))))

  (DEFUN |TSETCAT-;basicSet;LMMU;5| (|ps| |pred?| |redOp?| $)
   (PROG (|bps| |b| |bs| |p| |gps| |ts|)
    (RETURN
     (SEQ
      (LETT |ps|
       (SPADCALL (ELT $ 31) |ps| (QREFELT $ 33))
       |TSETCAT-;basicSet;LMMU;5|)
      (EXIT
       (COND
        ((SPADCALL (ELT $ 34) |ps| (QREFELT $ 35)) (CONS 1 "failed"))
        ((QUOTE T)
         (SEQ
          (LETT |gps| NIL |TSETCAT-;basicSet;LMMU;5|)
          (LETT |bps| NIL |TSETCAT-;basicSet;LMMU;5|)
          (SEQ
           G190
           (COND
            ((NULL (COND ((NULL |ps|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
              (GO G191)))
            (SEQ
             (LETT |p| (|SPADfirst| |ps|) |TSETCAT-;basicSet;LMMU;5|)
             (LETT |ps| (CDR |ps|) |TSETCAT-;basicSet;LMMU;5|)
             (EXIT
              (COND
               ((SPADCALL |p| |pred?|)
                (LETT |gps| (CONS |p| |gps|) |TSETCAT-;basicSet;LMMU;5|))
               ((QUOTE T)
                (LETT |bps| (CONS |p| |bps|) |TSETCAT-;basicSet;LMMU;5|)))))
            NIL
```

```
  (GO G190)
  G191
  (EXIT NIL))
(LETT |gps|
 (SPADCALL (ELT $ 21) |gps| (QREFELT $ 36)) |TSETCAT-;basicSet;LMMU;5|)
(LETT |bs| (SPADCALL (QREFELT $ 37)) |TSETCAT-;basicSet;LMMU;5|)
(LETT |ts| NIL |TSETCAT-;basicSet;LMMU;5|)
(SEQ
 G190
 (COND
  ((NULL (COND ((NULL |gps|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
   (GO G191)))
 (SEQ
  (LETT |b| (|SPADfirst| |gps|) |TSETCAT-;basicSet;LMMU;5|)
  (LETT |bs|
   (SPADCALL |bs| |b| (QREFELT $ 38)) |TSETCAT-;basicSet;LMMU;5|)
  (LETT |gps| (CDR |gps|) |TSETCAT-;basicSet;LMMU;5|)
  (EXIT
   (SEQ
    G190
    (COND
     ((NULL
      (COND
       ((OR
         (NULL |gps|)
         (SPADCALL
          (LETT |p| (|SPADfirst| |gps|) |TSETCAT-;basicSet;LMMU;5|)
          |bs| |redOp?| (QREFELT $ 39)))
        (QUOTE NIL))
       ((QUOTE T) (QUOTE T))))
      (GO G191)))
    (SEQ
     (LETT |ts| (CONS |p| |ts|) |TSETCAT-;basicSet;LMMU;5|)
     (EXIT (LETT |gps| (CDR |gps|) |TSETCAT-;basicSet;LMMU;5|)))
    NIL
    (GO G190)
    G191
    (EXIT NIL))))
 NIL
 (GO G190)
 G191
 (EXIT NIL))
(LETT |ts|
 (SPADCALL
  (ELT $ 21)
  (SPADCALL |ts| |bps| (QREFELT $ 43))
```

```
           (QREFELT $ 36))
          |TSETCAT-;basicSet;LMMU;5|)
         (EXIT (CONS 0 (CONS |bs| |ts|))))))))))))

(DEFUN |TSETCAT-;initials;SL;6| (|ts| $)
 (PROG (|p| |ip| |lip| |lp|)
  (RETURN
   (SEQ
    (LETT |lip| NIL |TSETCAT-;initials;SL;6|)
    (EXIT
     (COND
      ((SPADCALL |ts| (QREFELT $ 12)) |lip|)
      ((QUOTE T)
       (SEQ
        (LETT |lp| (SPADCALL |ts| (QREFELT $ 28)) |TSETCAT-;initials;SL;6|)
        (SEQ
         G190
         (COND
          ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
            (GO G191)))
         (SEQ
          (LETT |p| (|SPADfirst| |lp|) |TSETCAT-;initials;SL;6|)
          (COND
           ((NULL
             (SPADCALL
              (LETT |ip|
               (SPADCALL |p| (QREFELT $ 45))
               |TSETCAT-;initials;SL;6|)
              (QREFELT $ 34)))
            (LETT |lip|
             (CONS (SPADCALL |ip| (QREFELT $ 46)) |lip|)
             |TSETCAT-;initials;SL;6|)))
          (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;initials;SL;6|)))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT (SPADCALL |lip| (QREFELT $ 47))))))))))))

(DEFUN |TSETCAT-;degree;SNni;7| (|ts| $)
 (PROG (|lp| |d|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |ts| (QREFELT $ 12)) 0)
     ((QUOTE T)
```

```
        (SEQ
         (LETT |lp| (SPADCALL |ts| (QREFELT $ 28)) |TSETCAT-;degree;SNni;7|)
         (LETT |d|
          (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 50))
          |TSETCAT-;degree;SNni;7|)
         (SEQ
          G190
          (COND
           ((NULL
             (COND
              ((NULL
                (LETT |lp| (CDR |lp|) |TSETCAT-;degree;SNni;7|)) (QUOTE NIL))
               ((QUOTE T) (QUOTE T))))
            (GO G191)))
          (SEQ
           (EXIT
            (LETT |d|
             (* |d| (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 50)))
             |TSETCAT-;degree;SNni;7|)))
          NIL
          (GO G190)
          G191
          (EXIT NIL))
         (EXIT |d|)))))))

(DEFUN |TSETCAT-;quasiComponent;SR;8| (|ts| $)
 (CONS (SPADCALL |ts| (QREFELT $ 28)) (SPADCALL |ts| (QREFELT $ 52))))

(DEFUN |TSETCAT-;normalized?;PSB;9| (|p| |ts| $)
 (SPADCALL |p| (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 56)))

(DEFUN |TSETCAT-;stronglyReduced?;PSB;10| (|p| |ts| $)
 (SPADCALL |p| (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 58)))

(DEFUN |TSETCAT-;headReduced?;PSB;11| (|p| |ts| $)
 (SPADCALL (SPADCALL |p| (QREFELT $ 60)) |ts| (QREFELT $ 61)))

(DEFUN |TSETCAT-;initiallyReduced?;PSB;12| (|p| |ts| $)
 (PROG (|lp| |red|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL |ts| (QREFELT $ 28)) |TSETCAT-;initiallyReduced?;PSB;12|)
    (LETT |red| (QUOTE T) |TSETCAT-;initiallyReduced?;PSB;12|)
    (SEQ
     G190
```

```
(COND
 ((NULL
   (COND
    ((OR (NULL |lp|) (SPADCALL |p| (QREFELT $ 34))) (QUOTE NIL))
    ((QUOTE T) |red|)))
  (GO G191)))
(SEQ
 (SEQ
  G190
  (COND
   ((NULL
    (COND
     ((NULL |lp|) (QUOTE NIL))
     ((QUOTE T)
      (SPADCALL
       (SPADCALL |p| (QREFELT $ 23))
       (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 23))
       (QREFELT $ 63)))))
    (GO G191)))
  (SEQ (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;initiallyReduced?;PSB;12|)))
  NIL
  (GO G190)
  G191
  (EXIT NIL))
 (EXIT
  (COND
   ((NULL (NULL |lp|))
    (COND
     ((SPADCALL
       (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 23))
       (SPADCALL |p| (QREFELT $ 23))
       (QREFELT $ 64))
      (COND
       ((SPADCALL |p| (|SPADfirst| |lp|) (QREFELT $ 65))
        (SEQ
         (LETT |lp| (CDR |lp|) |TSETCAT-;initiallyReduced?;PSB;12|)
         (EXIT
          (LETT |p|
           (SPADCALL |p| (QREFELT $ 45))
           |TSETCAT-;initiallyReduced?;PSB;12|))))
       ((QUOTE T)
        (LETT |red| (QUOTE NIL) |TSETCAT-;initiallyReduced?;PSB;12|))))
     ((QUOTE T)
      (LETT |p|
       (SPADCALL |p| (QREFELT $ 45))
       |TSETCAT-;initiallyReduced?;PSB;12|)))))))
```

```
      NIL
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT |red|)))))

(DEFUN |TSETCAT-;reduce;PSMMP;13| (|p| |ts| |redOp| |redOp?| $)
 (PROG (|ts0| #0=#:G1572 |reductor| #1=#:G1575)
  (RETURN
   (SEQ
    (COND
     ((OR (SPADCALL |ts| (QREFELT $ 12)) (SPADCALL |p| (QREFELT $ 34))) |p|)
     ((QUOTE T)
      (SEQ
       (LETT |ts0| |ts| |TSETCAT-;reduce;PSMMP;13|)
       (SEQ
        G190
        (COND
         ((NULL
           (COND
            ((OR
              (SPADCALL |ts| (QREFELT $ 12))
              (SPADCALL |p| (QREFELT $ 34)))
             (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (LETT |reductor|
           (PROG2
            (LETT #0# (SPADCALL |ts| (QREFELT $ 14)) |TSETCAT-;reduce;PSMMP;13|)
            (QCDR #0#)
            (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
           |TSETCAT-;reduce;PSMMP;13|)
          (LETT |ts|
           (PROG2
            (LETT #1# (SPADCALL |ts| (QREFELT $ 17)) |TSETCAT-;reduce;PSMMP;13|)
            (QCDR #1#)
            (|check-union| (QEQCAR #1# 0) (QREFELT $ 6) #1#))
           |TSETCAT-;reduce;PSMMP;13|)
          (EXIT
           (COND
            ((NULL (SPADCALL |p| |reductor| |redOp?|))
             (SEQ
              (LETT |p|
               (SPADCALL |p| |reductor| |redOp|) |TSETCAT-;reduce;PSMMP;13|)
              (EXIT (LETT |ts| |ts0| |TSETCAT-;reduce;PSMMP;13|)))))))))))))
```

```
          NIL
          (GO G190)
          G191
          (EXIT NIL))
         (EXIT |p|)))))))))

(DEFUN |TSETCAT-;rewriteSetWithReduction;LSMML;14|
    (|lp| |ts| |redOp| |redOp?| $)
 (PROG (|p| |rs|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |ts| (QREFELT $ 69)) |lp|)
     ((QUOTE T)
      (SEQ
       (LETT |lp|
        (SPADCALL (ELT $ 31) |lp| (QREFELT $ 33))
        |TSETCAT-;rewriteSetWithReduction;LSMML;14|)
       (EXIT
        (COND
         ((NULL |lp|) |lp|)
         ((SPADCALL (ELT $ 34) |lp| (QREFELT $ 35))
           (LIST (|spadConstant| $ 70)))
         ((QUOTE T)
          (SEQ
           (LETT |rs| NIL |TSETCAT-;rewriteSetWithReduction;LSMML;14|)
           (SEQ
            G190
            (COND
             ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
              (GO G191)))
            (SEQ
             (LETT |p|
              (|SPADfirst| |lp|)
              |TSETCAT-;rewriteSetWithReduction;LSMML;14|)
             (LETT |lp| (CDR |lp|) |TSETCAT-;rewriteSetWithReduction;LSMML;14|)
             (LETT |p|
              (SPADCALL
               (SPADCALL |p| |ts| |redOp| |redOp?| (QREFELT $ 71))
               (QREFELT $ 46))
              |TSETCAT-;rewriteSetWithReduction;LSMML;14|)
             (EXIT
              (COND
               ((NULL (SPADCALL |p| (QREFELT $ 31)))
                 (COND
                  ((SPADCALL |p| (QREFELT $ 34))
```

```
                    (SEQ
                     (LETT |lp| NIL |TSETCAT-;rewriteSetWithReduction;LSMML;14|)
                     (EXIT
                      (LETT |rs|
                       (LIST (|spadConstant| $ 70))
                       |TSETCAT-;rewriteSetWithReduction;LSMML;14|))))
                   ((QUOTE T)
                    (LETT |rs|
                     (CONS |p| |rs|)
                     |TSETCAT-;rewriteSetWithReduction;LSMML;14|)))))))
              NIL
              (GO G190)
              G191
              (EXIT NIL))
             (EXIT (SPADCALL |rs| (QREFELT $ 47))))))))))))))

(DEFUN |TSETCAT-;stronglyReduce;PSP;15| (|p| |ts| $)
 (SPADCALL |p| |ts| (ELT $ 73) (ELT $ 65) (QREFELT $ 71)))

(DEFUN |TSETCAT-;headReduce;PSP;16| (|p| |ts| $)
 (SPADCALL |p| |ts| (ELT $ 75) (ELT $ 76) (QREFELT $ 71)))

(DEFUN |TSETCAT-;initiallyReduce;PSP;17| (|p| |ts| $)
 (SPADCALL |p| |ts| (ELT $ 78) (ELT $ 79) (QREFELT $ 71)))

(DEFUN |TSETCAT-;removeZero;PSP;18| (|p| |ts| $)
 (PROG (|v| |tsv-| #0=#:G1599 #1=#:G1608 |q|)
  (RETURN
   (SEQ
    (EXIT
     (COND
      ((OR (SPADCALL |p| (QREFELT $ 34)) (SPADCALL |ts| (QREFELT $ 12))) |p|)
      ((QUOTE T)
       (SEQ
        (LETT |v| (SPADCALL |p| (QREFELT $ 23)) |TSETCAT-;removeZero;PSP;18|)
        (LETT |tsv-|
         (SPADCALL |ts| |v| (QREFELT $ 81))
         |TSETCAT-;removeZero;PSP;18|)
        (COND
         ((SPADCALL |v| |ts| (QREFELT $ 82))
          (SEQ
           (LETT |q|
            (SPADCALL |p|
             (PROG2
              (LETT #0#
               (SPADCALL |ts| |v| (QREFELT $ 83))
```

```
          |TSETCAT-;removeZero;PSP;18|)
         (QCDR #0#)
         (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
        (QREFELT $ 73))
       |TSETCAT-;removeZero;PSP;18|)
      (EXIT
       (COND
        ((SPADCALL |q| (QREFELT $ 31))
          (PROGN (LETT #1# |q| |TSETCAT-;removeZero;PSP;18|) (GO #1#)))
         ((SPADCALL (SPADCALL |q| |tsv-| (QREFELT $ 84)) (QREFELT $ 31))
           (PROGN
            (LETT #1#
             (|spadConstant| $ 85)
             |TSETCAT-;removeZero;PSP;18|) (GO #1#)))))))
(EXIT
 (COND
  ((SPADCALL |tsv-| (QREFELT $ 12)) |p|)
  ((QUOTE T)
   (SEQ
    (LETT |q| (|spadConstant| $ 85) |TSETCAT-;removeZero;PSP;18|)
    (SEQ
     G190
     (COND
      ((NULL
        (SPADCALL (SPADCALL |p| |v| (QREFELT $ 86)) (QREFELT $ 88)))
       (GO G191)))
     (SEQ
      (LETT |q|
       (SPADCALL
        (SPADCALL
         (SPADCALL
          (SPADCALL |p| (QREFELT $ 45))
           |tsv-| (QREFELT $ 84))
          (SPADCALL |p| (QREFELT $ 89))
         (QREFELT $ 90))
        |q|
        (QREFELT $ 91))
       |TSETCAT-;removeZero;PSP;18|)
      (EXIT
       (LETT |p|
        (SPADCALL |p| (QREFELT $ 92))
        |TSETCAT-;removeZero;PSP;18|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
```

```
            (EXIT
             (SPADCALL |q|
              (SPADCALL |p| |tsv-| (QREFELT $ 84))
              (QREFELT $ 91)))))))))))
    #1#
    (EXIT #1#)))))

(DEFUN |TSETCAT-;reduceByQuasiMonic;PSP;19| (|p| |ts| $)
 (COND
  ((OR (SPADCALL |p| (QREFELT $ 34)) (SPADCALL |ts| (QREFELT $ 12))) |p|)
  ((QUOTE T)
    (QVELT (SPADCALL |p| (SPADCALL |ts| (QREFELT $ 94)) (QREFELT $ 96)) 1))))

(DEFUN |TSETCAT-;autoReduced?;SMB;20| (|ts| |redOp?| $)
 (PROG (|p| |lp|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |ts| (QREFELT $ 12)) (QUOTE T))
     ((QUOTE T)
      (SEQ
       (LETT |lp|
        (SPADCALL |ts| (QREFELT $ 28)) |TSETCAT-;autoReduced?;SMB;20|)
       (LETT |p| (|SPADfirst| |lp|) |TSETCAT-;autoReduced?;SMB;20|)
       (LETT |lp| (CDR |lp|) |TSETCAT-;autoReduced?;SMB;20|)
       (SEQ
        G190
        (COND
         ((NULL
           (COND
            ((NULL |lp|) (QUOTE NIL))
            ((QUOTE T) (SPADCALL |p| |lp| |redOp?|))))
          (GO G191)))
        (SEQ
         (LETT |p| (|SPADfirst| |lp|) |TSETCAT-;autoReduced?;SMB;20|)
         (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;autoReduced?;SMB;20|)))
        NIL
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT (NULL |lp|)))))))))

(DEFUN |TSETCAT-;stronglyReduced?;SB;21| (|ts| $)
 (SPADCALL |ts| (ELT $ 58) (QREFELT $ 100)))

(DEFUN |TSETCAT-;normalized?;SB;22| (|ts| $)
```

```
 (SPADCALL |ts| (ELT $ 56) (QREFELT $ 100)))

(DEFUN |TSETCAT-;headReduced?;SB;23| (|ts| $)
 (SPADCALL |ts| (ELT $ 103) (QREFELT $ 100)))

(DEFUN |TSETCAT-;initiallyReduced?;SB;24| (|ts| $)
 (SPADCALL |ts| (ELT $ 105) (QREFELT $ 100)))

(DEFUN |TSETCAT-;mvar;SV;25| (|ts| $)
 (PROG (#0=#:G1627)
  (RETURN
   (COND
    ((SPADCALL |ts| (QREFELT $ 12))
      (|error| "Error from TSETCAT in mvar : #1 is empty"))
    ((QUOTE T)
      (SPADCALL
       (PROG2
        (LETT #0# (SPADCALL |ts| (QREFELT $ 14)) |TSETCAT-;mvar;SV;25|)
        (QCDR #0#)
        (|check-union| (QEQCAR #0# 0) (QREFELT $ 10) #0#))
       (QREFELT $ 23)))))))

(DEFUN |TSETCAT-;first;SU;26| (|ts| $)
 (PROG (|lp|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |ts| (QREFELT $ 12)) (CONS 1 "failed"))
     ((QUOTE T)
      (SEQ
       (LETT |lp|
        (SPADCALL (ELT $ 22) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36))
        |TSETCAT-;first;SU;26|)
       (EXIT (CONS 0 (|SPADfirst| |lp|)))))))))))

(DEFUN |TSETCAT-;last;SU;27| (|ts| $)
 (PROG (|lp|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |ts| (QREFELT $ 12)) (CONS 1 "failed"))
     ((QUOTE T)
      (SEQ
       (LETT |lp|
        (SPADCALL (ELT $ 21) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36))
        |TSETCAT-;last;SU;27|)
```

```
        (EXIT (CONS 0 (|SPADfirst| |lp|))))))))))

 (DEFUN |TSETCAT-;rest;SU;28| (|ts| $)
  (PROG (|lp|)
   (RETURN
    (SEQ
     (COND
      ((SPADCALL |ts| (QREFELT $ 12)) (CONS 1 "failed"))
      ((QUOTE T)
       (SEQ
        (LETT |lp|
         (SPADCALL (ELT $ 22) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36))
         |TSETCAT-;rest;SU;28|)
        (EXIT (CONS 0 (SPADCALL (CDR |lp|) (QREFELT $ 110))))))))))

 (DEFUN |TSETCAT-;coerce;SL;29| (|ts| $)
  (SPADCALL (ELT $ 22) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36)))

 (DEFUN |TSETCAT-;algebraicVariables;SL;30| (|ts| $)
  (PROG (#0=#:G1652 |p| #1=#:G1653)
   (RETURN
    (SEQ
     (PROGN
      (LETT #0# NIL |TSETCAT-;algebraicVariables;SL;30|)
      (SEQ
       (LETT |p| NIL |TSETCAT-;algebraicVariables;SL;30|)
       (LETT #1#
        (SPADCALL |ts| (QREFELT $ 28)) |TSETCAT-;algebraicVariables;SL;30|)
       G190
       (COND
        ((OR
          (ATOM #1#)
          (PROGN (LETT |p| (CAR #1#) |TSETCAT-;algebraicVariables;SL;30|) NIL))
         (GO G191)))
       (SEQ
        (EXIT
         (LETT #0#
          (CONS (SPADCALL |p| (QREFELT $ 23)) #0#)
          |TSETCAT-;algebraicVariables;SL;30|)))
       (LETT #1# (CDR #1#) |TSETCAT-;algebraicVariables;SL;30|)
       (GO G190)
       G191
       (EXIT (NREVERSE0 #0#))))))))

 (DEFUN |TSETCAT-;algebraic?;VSB;31| (|v| |ts| $)
  (SPADCALL |v| (SPADCALL |ts| (QREFELT $ 115)) (QREFELT $ 116)))
```

```
(DEFUN |TSETCAT-;select;SVU;32| (|ts| |v| $)
 (PROG (|lp|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL (ELT $ 22) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36))
     |TSETCAT-;select;SVU;32|)
    (SEQ
     G190
     (COND
      ((NULL
       (COND
        ((OR
          (NULL |lp|)
          (SPADCALL |v|
           (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 23))
           (QREFELT $ 64)))
          (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;select;SVU;32|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT
     (COND
      ((NULL |lp|) (CONS 1 "failed"))
      ((QUOTE T) (CONS 0 (|SPADfirst| |lp|)))))))))

(DEFUN |TSETCAT-;collectQuasiMonic;2S;33| (|ts| $)
 (PROG (|newlp| |lp|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL |ts| (QREFELT $ 28))
     |TSETCAT-;collectQuasiMonic;2S;33|)
    (LETT |newlp| NIL |TSETCAT-;collectQuasiMonic;2S;33|)
    (SEQ
     G190
     (COND
      ((NULL (COND ((NULL |lp|) (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
        (GO G191)))
     (SEQ
      (COND
```

```
        ((SPADCALL (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 45)) (QREFELT $ 34))
         (LETT |newlp|
          (CONS (|SPADfirst| |lp|) |newlp|)
          |TSETCAT-;collectQuasiMonic;2S;33|)))
       (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;collectQuasiMonic;2S;33|)))
      NIL
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT (SPADCALL |newlp| (QREFELT $ 110)))))))

(DEFUN |TSETCAT-;collectUnder;SVS;34| (|ts| |v| $)
 (PROG (|lp|)
  (RETURN
   (SEQ
    (LETT |lp|
     (SPADCALL (ELT $ 22) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36))
     |TSETCAT-;collectUnder;SVS;34|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((OR
           (NULL |lp|)
           (SPADCALL
            (SPADCALL (|SPADfirst| |lp|) (QREFELT $ 23))
            |v|
            (QREFELT $ 63)))
          (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ (EXIT (LETT |lp| (CDR |lp|) |TSETCAT-;collectUnder;SVS;34|)))
     NIL
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT (SPADCALL |lp| (QREFELT $ 110)))))))

(DEFUN |TSETCAT-;collectUpper;SVS;35| (|ts| |v| $)
 (PROG (|lp2| |lp1|)
  (RETURN
   (SEQ
    (LETT |lp1|
     (SPADCALL (ELT $ 22) (SPADCALL |ts| (QREFELT $ 28)) (QREFELT $ 36))
     |TSETCAT-;collectUpper;SVS;35|)
```

```
      (LETT |lp2| NIL |TSETCAT-;collectUpper;SVS;35|)
      (SEQ
       G190
       (COND
        ((NULL
          (COND
           ((NULL |lp1|) (QUOTE NIL))
           ((QUOTE T)
            (SPADCALL |v|
             (SPADCALL (|SPADfirst| |lp1|) (QREFELT $ 23))
             (QREFELT $ 63)))))
         (GO G191)))
       (SEQ
        (LETT |lp2|
         (CONS (|SPADfirst| |lp1|) |lp2|)
         |TSETCAT-;collectUpper;SVS;35|)
        (EXIT (LETT |lp1| (CDR |lp1|) |TSETCAT-;collectUpper;SVS;35|)))
       NIL
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT (SPADCALL (REVERSE |lp2|) (QREFELT $ 110)))))))

  (DEFUN |TSETCAT-;construct;LS;36| (|lp| $)
   (PROG (|rif|)
    (RETURN
     (SEQ
      (LETT |rif| (SPADCALL |lp| (QREFELT $ 122)) |TSETCAT-;construct;LS;36|)
      (EXIT
       (COND
        ((QEQCAR |rif| 0) (QCDR |rif|))
        ((QUOTE T)
         (|error| "in construct : LP -> $ from TSETCAT : bad arg")))))))))

  (DEFUN |TSETCAT-;retractIfCan;LU;37| (|lp| $)
   (PROG (|rif|)
    (RETURN
     (SEQ
      (COND
       ((NULL |lp|) (CONS 0 (SPADCALL (QREFELT $ 37))))
       ((QUOTE T)
        (SEQ
         (LETT |lp|
          (SPADCALL (ELT $ 22) |lp| (QREFELT $ 36))
          |TSETCAT-;retractIfCan;LU;37|)
         (LETT |rif|
```

```
         (SPADCALL (CDR |lp|) (QREFELT $ 122))
         |TSETCAT-;retractIfCan;LU;37|)
        (EXIT
         (COND
          ((QEQCAR |rif| 0)
            (SPADCALL (QCDR |rif|) (|SPADfirst| |lp|) (QREFELT $ 124)))
          ((QUOTE T)
            (|error| "in retractIfCan : LP -> ... from TSETCAT : bad arg")
  ))))))))))

(DEFUN |TSETCAT-;extend;SPS;38| (|ts| |p| $)
 (PROG (|eif|)
  (RETURN
   (SEQ
    (LETT |eif| (SPADCALL |ts| |p| (QREFELT $ 124)) |TSETCAT-;extend;SPS;38|)
    (EXIT
     (COND
      ((QEQCAR |eif| 0) (QCDR |eif|))
      ((QUOTE T)
       (|error| "in extend : ($,P) -> $ from TSETCAT : bad ars")))))))))

(DEFUN |TSETCAT-;coHeight;SNni;39| (|ts| $)
 (PROG (|n| |m| #0=#:G1696)
  (RETURN
   (SEQ
    (LETT |n| (SPADCALL (QREFELT $ 127)) |TSETCAT-;coHeight;SNni;39|)
    (LETT |m|
     (LENGTH (SPADCALL |ts| (QREFELT $ 28)))
     |TSETCAT-;coHeight;SNni;39|)
    (EXIT
     (PROG2
      (LETT #0# (SPADCALL |n| |m| (QREFELT $ 128)) |TSETCAT-;coHeight;SNni;39|)
      (QCDR #0#)
      (|check-union| (QEQCAR #0# 0) (|NonNegativeInteger|) #0#)))))))

(DEFUN |TriangularSetCategory&| (|#1| |#2| |#3| |#4| |#5|)
 (PROG (DV$1 DV$2 DV$3 DV$4 DV$5 |dv$| $ |pv$|)
  (RETURN
   (PROGN
    (LETT DV$1 (|devaluate| |#1|) . #0=(|TriangularSetCategory&|))
    (LETT DV$2 (|devaluate| |#2|) . #0#)
    (LETT DV$3 (|devaluate| |#3|) . #0#)
    (LETT DV$4 (|devaluate| |#4|) . #0#)
    (LETT DV$5 (|devaluate| |#5|) . #0#)
    (LETT |dv$|
     (LIST (QUOTE |TriangularSetCategory&|) DV$1 DV$2 DV$3 DV$4 DV$5) . #0#)
```

```
      (LETT $ (GETREFV 131) . #0#)
      (QSETREFV $ 0 |dv$|)
      (QSETREFV $ 3
       (LETT |pv$|
        (|buildPredVector| 0 0 (LIST (|HasCategory| |#4| (QUOTE (|Finite|)))))
        . #0#))
      (|stuffDomainSlots| $)
      (QSETREFV $ 6 |#1|)
      (QSETREFV $ 7 |#2|)
      (QSETREFV $ 8 |#3|)
      (QSETREFV $ 9 |#4|)
      (QSETREFV $ 10 |#5|)
      (COND
       ((|testBitVector| |pv$| 1)
         (QSETREFV $ 129
          (CONS (|dispatchFunction| |TSETCAT-;coHeight;SNni;39|) $))))
       $))))

  (MAKEPROP
   (QUOTE |TriangularSetCategory&|)
   (QUOTE |infovec|)
   (LIST (QUOTE
    #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|) (|local| |#3|)
      (|local| |#4|) (|local| |#5|) (|Boolean|) (0 . |empty?|)
      (|Union| 10 (QUOTE "failed")) (5 . |first|) (10 . =)
      (|Union| $ (QUOTE "failed")) (16 . |rest|) (21 . =) |TSETCAT-;=;2SB;1|
      (27 . |last|) (32 . |infRittWu?|) (38 . |supRittWu?|) (44 . |mvar|)
      (49 . |collectUpper|) (55 . |infRittWu?|) |TSETCAT-;infRittWu?;2SB;2|
      (|List| 10) (61 . |members|) (|Mapping| 11 10 10)
      |TSETCAT-;reduced?;PSMB;3| (66 . |zero?|) (|Mapping| 11 10)
      (71 . |remove|) (77 . |ground?|) (82 . |any?|) (88 . |sort|)
      (94 . |empty|) (98 . |extend|) (104 . |reduced?|)
      (|Record| (|:| |bas| $) (|:| |top| 27))
      (|Union| 40 (QUOTE "failed")) |TSETCAT-;basicSet;LMU;4|
      (111 . |concat|) |TSETCAT-;basicSet;LMMU;5| (117 . |init|)
      (122 . |primPartElseUnitCanonical|) (127 . |removeDuplicates|)
      |TSETCAT-;initials;SL;6| (|NonNegativeInteger|) (132 . |mdeg|)
      |TSETCAT-;degree;SNni;7| (137 . |initials|)
      (|Record| (|:| |close| 27) (|:| |open| 27))
      |TSETCAT-;quasiComponent;SR;8| (|List| $) (142 . |normalized?|)
      |TSETCAT-;normalized?;PSB;9| (148 . |reduced?|)
      |TSETCAT-;stronglyReduced?;PSB;10| (154 . |head|)
      (159 . |stronglyReduced?|) |TSETCAT-;headReduced?;PSB;11|
      (165 . <) (171 . =) (177 . |reduced?|)
      |TSETCAT-;initiallyReduced?;PSB;12| (|Mapping| 10 10 10)
      |TSETCAT-;reduce;PSMMP;13| (183 . |trivialIdeal?|) (188 . |One|)
```

```
   (192 . |reduce|) |TSETCAT-;rewriteSetWithReduction;LSMML;14|
   (200 . |lazyPrem|) |TSETCAT-;stronglyReduce;PSP;15|
   (206 . |headReduce|) (212 . |headReduced?|)
   |TSETCAT-;headReduce;PSP;16| (218 . |initiallyReduce|)
   (224 . |initiallyReduced?|) |TSETCAT-;initiallyReduce;PSP;17|
   (230 . |collectUnder|) (236 . |algebraic?|) (242 . |select|)
   (248 . |removeZero|) (254 . |Zero|) (258 . |degree|) (|Integer|)
   (264 . |positive?|) (269 . |mainMonomial|) (274 . *) (280 . +)
   (286 . |tail|) |TSETCAT-;removeZero;PSP;18| (291 . |collectQuasiMonic|)
   (|Record| (|:| |rnum| 7) (|:| |polnum| 10) (|:| |den| 7))
   (296 . |remainder|) |TSETCAT-;reduceByQuasiMonic;PSP;19|
   (|Mapping| 11 10 27) |TSETCAT-;autoReduced?;SMB;20|
   (302 . |autoReduced?|) |TSETCAT-;stronglyReduced?;SB;21|
   |TSETCAT-;normalized?;SB;22| (308 . |headReduced?|)
   |TSETCAT-;headReduced?;SB;23| (314 . |initiallyReduced?|)
   |TSETCAT-;initiallyReduced?;SB;24| |TSETCAT-;mvar;SV;25|
   |TSETCAT-;first;SU;26| |TSETCAT-;last;SU;27| (320 . |construct|)
   |TSETCAT-;rest;SU;28| |TSETCAT-;coerce;SL;29| (|List| 9)
   |TSETCAT-;algebraicVariables;SL;30| (325 . |algebraicVariables|)
   (330 . |member?|) |TSETCAT-;algebraic?;VSB;31|
   |TSETCAT-;select;SVU;32| |TSETCAT-;collectQuasiMonic;2S;33|
   |TSETCAT-;collectUnder;SVS;34| |TSETCAT-;collectUpper;SVS;35|
   (336 . |retractIfCan|) |TSETCAT-;construct;LS;36|
   (341 . |extendIfCan|) |TSETCAT-;retractIfCan;LU;37|
   |TSETCAT-;extend;SPS;38| (347 . |size|) (351 . |subtractIfCan|)
   (357 . |coHeight|) (|OutputForm|)))
(QUOTE #(|stronglyReduced?| 362 |stronglyReduce| 373 |select| 379
   |rewriteSetWithReduction| 385 |retractIfCan| 393 |rest| 398 |removeZero|
   403 |reduced?| 409 |reduceByQuasiMonic| 416 |reduce| 422
   |quasiComponent| 430 |normalized?| 435 |mvar| 446 |last| 451
   |initials| 456 |initiallyReduced?| 461 |initiallyReduce| 472
   |infRittWu?| 478 |headReduced?| 484 |headReduce| 495 |first| 501
   |extend| 506 |degree| 512 |construct| 517 |collectUpper| 522
   |collectUnder| 528 |collectQuasiMonic| 534 |coerce| 539 |coHeight|
   544 |basicSet| 549 |autoReduced?| 562 |algebraicVariables| 568
   |algebraic?| 573 = 579))
(QUOTE NIL)
(CONS
 (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS (QUOTE #()) (CONS (QUOTE #())
  (|makeByteWordVec2| 129 (QUOTE (1 6 11 0 12 1 6 13 0 14 2 10 11 0 0 15
    1 6 16 0 17 2 6 11 0 0 18 1 6 13 0 20 2 10 11 0 0 21 2 10 11 0 0 22
    1 10 9 0 23 2 6 0 0 9 24 2 6 11 0 0 25 1 6 27 0 28 1 10 11 0 31 2 27
    0 32 0 33 1 10 11 0 34 2 27 11 32 0 35 2 27 0 29 0 36 0 6 0 37 2 6 0
    0 10 38 3 6 11 10 0 29 39 2 27 0 0 0 43 1 10 0 0 45 1 10 0 0 46 1 27
    0 0 47 1 10 49 0 50 1 6 27 0 52 2 10 11 0 55 56 2 10 11 0 55 58 1 10
```

```
   0 0 60 2 6 11 10 0 61 2 9 11 0 0 63 2 9 11 0 0 64 2 10 11 0 0 65 1 6
   11 0 69 0 10 0 70 4 6 10 10 0 67 29 71 2 10 0 0 0 73 2 10 0 0 0 75 2
   10 11 0 0 76 2 10 0 0 0 78 2 10 11 0 0 79 2 6 0 0 9 81 2 6 11 9 0 82
   2 6 13 0 9 83 2 6 10 10 0 84 0 10 0 85 2 10 49 0 9 86 1 87 11 0 88 1
   10 0 0 89 2 10 0 0 0 90 2 10 0 0 0 91 1 10 0 0 92 1 6 0 0 94 2 6 95
   10 0 96 2 6 11 0 98 100 2 10 11 0 55 103 2 10 11 0 55 105 1 6 0 27
   110 1 6 113 0 115 2 113 11 9 0 116 1 6 16 27 122 2 6 16 0 10 124 0
   9 49 127 2 49 16 0 0 128 1 0 49 0 129 1 0 11 0 101 2 0 11 10 0 59 2
   0 10 10 0 74 2 0 13 0 9 118 4 0 27 27 0 67 29 72 1 0 16 27 125 1 0
   16 0 111 2 0 10 10 0 93 3 0 11 10 0 29 30 2 0 10 10 0 97 4 0 10 10 0
   67 29 68 1 0 53 0 54 1 0 11 0 102 2 0 11 10 0 57 1 0 9 0 107 1 0 13
   0 109 1 0 27 0 48 1 0 11 0 106 2 0 11 10 0 66 2 0 10 10 0 80 2 0 11
   0 0 26 1 0 11 0 104 2 0 11 10 0 62 2 0 10 10 0 77 1 0 13 0 108 2 0 0
   0 10 126 1 0 49 0 51 1 0 0 27 123 2 0 0 0 9 121 2 0 0 0 9 120 1 0 0
   0 119 1 0 27 0 112 1 0 49 0 129 3 0 41 27 32 29 44 2 0 41 27 29 42 2
   0 11 0 98 99 1 0 113 0 114 2 0 11 9 0 117 2 0 11 0 0 19))))))
(QUOTE |lookupComplete|)))
```

21.64 UFD.lsp BOOTSTRAP

UFD needs **GCDDOM** which needs **COMRING** which needs **RING** which needs **RNG** which needs **ABELGRP** which needs **CABMON** which needs **ABELMON** which needs **ABELSG** which needs **SETCAT** which needs **SINT** which needs **UFD**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **UFD** category which we can write into the **MID** directory. We compile the lisp code and copy the **UFD.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*UFD.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |UniqueFactorizationDomain;AL| (QUOTE NIL))

(DEFUN |UniqueFactorizationDomain| NIL
  (LET (#:G83334)
    (COND
      (|UniqueFactorizationDomain;AL|)
      (T
        (SETQ
          |UniqueFactorizationDomain;AL|
          (|UniqueFactorizationDomain;|)))))))

(DEFUN |UniqueFactorizationDomain;| NIL
  (PROG (#1=#:G83332)
    (RETURN
      (PROG1
        (LETT #1#
          (|Join|
            (|GcdDomain|)
            (|mkCategory|
              (QUOTE |domain|)
              (QUOTE (
                ((|prime?| ((|Boolean|) |$|)) T)
                ((|squareFree| ((|Factored| |$|) |$|)) T)
                ((|squareFreePart| (|$| |$|)) T)
                ((|factor| ((|Factored| |$|) |$|)) T)))
              NIL
              (QUOTE ((|Factored| |$|) (|Boolean|)))
              NIL))
            |UniqueFactorizationDomain|)
        (SETELT #1# 0 (QUOTE (|UniqueFactorizationDomain|)))))))
```

```
(MAKEPROP (QUOTE |UniqueFactorizationDomain|) (QUOTE NILADIC) T)
```

21.65 UFD-.lsp BOOTSTRAP

UFD- needs **UFD**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **UFD-** category which we can write into the **MID** directory. We compile the lisp code and copy the **UFD-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*UFD-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |UFD-;squareFreePart;2S;1| (|x| |$|)
  (PROG (|s| |f| #1=#:G83349 #2=#:G83347 #3=#:G83345 #4=#:G83346)
    (RETURN
      (SEQ
        (SPADCALL
          (SPADCALL
            (LETT |s|
              (SPADCALL |x| (QREFELT |$| 8))
              |UFD-;squareFreePart;2S;1|)
            (QREFELT |$| 10))
          (PROGN
            (LETT #4# NIL |UFD-;squareFreePart;2S;1|)
            (SEQ
              (LETT |f| NIL |UFD-;squareFreePart;2S;1|)
              (LETT #1#
                (SPADCALL |s| (QREFELT |$| 13))
                |UFD-;squareFreePart;2S;1|)
              G190
              (COND
                ((OR
                   (ATOM #1#)
                   (PROGN
                     (LETT |f| (CAR #1#) |UFD-;squareFreePart;2S;1|)
                     NIL))
                 (GO G191)))
              (SEQ
                (EXIT
                  (PROGN
                    (LETT #2# (QCAR |f|) |UFD-;squareFreePart;2S;1|)
                    (COND
                      (#4#
                        (LETT #3#
                          (SPADCALL #3# #2# (QREFELT |$| 14))
                          |UFD-;squareFreePart;2S;1|))
```

```
                        ((QUOTE T)
                          (PROGN
                            (LETT #3# #2# |UFD-;squareFreePart;2S;1|)
                            (LETT #4# (QUOTE T)
                              |UFD-;squareFreePart;2S;1|)))))))
                (LETT #1# (CDR #1#) |UFD-;squareFreePart;2S;1|)
                (GO G190)
                G191
                (EXIT NIL))
              (COND
                (#4# #3#)
                ((QUOTE T) (|spadConstant| |$| 15))))
          (QREFELT |$| 14))))))

(DEFUN |UFD-;prime?;SB;2| (|x| |$|)
  (EQL
    (LENGTH (SPADCALL (SPADCALL |x| (QREFELT |$| 17)) (QREFELT |$| 21))) 1))

(DEFUN |UniqueFactorizationDomain&| (|#1|)
  (PROG (|DV$1| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|UniqueFactorizationDomain&|))
        (LETT |dv$| (LIST (QUOTE |UniqueFactorizationDomain&|) |DV$1|) . #1#)
        (LETT |$| (GETREFV 24) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #1#))
        (|stuffDomainSlots| |$|)
        (QSETREFV |$| 6 |#1|)
        |$|))))

(MAKEPROP
  (QUOTE |UniqueFactorizationDomain&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (|Factored| |$|)
        (0 . |squareFree|)
        (|Factored| 6)
        (5 . |unit|)
        (|Record| (|:| |factor| 6) (|:| |exponent| (|Integer|)))
        (|List| 11)
        (10 . |factors|)
        (15 . |*|)
```

```
   (21 . |One|)
   |UFD-;squareFreePart;2S;1|
   (25 . |factor|)
   (|Union|
     (QUOTE "nil") (QUOTE "sqfr") (QUOTE "irred") (QUOTE "prime"))
   (|Record| (|:| |flg| 18) (|:| |fctr| 6) (|:| |xpnt| (|Integer|)))
   (|List| 19)
   (30 . |factorList|)
   (|Boolean|)
   |UFD-;prime?;SB;2|))
(QUOTE #(|squareFreePart| 35 |prime?| 40))
(QUOTE NIL)
(CONS
  (|makeByteWordVec2| 1 (QUOTE NIL))
  (CONS
    (QUOTE #())
    (CONS
      (QUOTE #())
      (|makeByteWordVec2| 23
        (QUOTE
          (1 6 7 0 8 1 9 6 0 10 1 9 12 0 13 2 6 0 0 0 14 0 6 0 15 1 6 7
           0 17 1 9 20 0 21 1 0 0 0 16 1 0 22 0 23))))))
 (QUOTE |lookupComplete|)))
```

21.66 ULSCAT.lsp BOOTSTRAP

ULSCAT depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **ULSCAT** category which we can write into the **MID** directory. We compile the lisp code and copy the **ULSCAT.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*ULSCAT.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |UnivariateLaurentSeriesCategory;CAT| (QUOTE NIL))

(SETQ |UnivariateLaurentSeriesCategory;AL| (QUOTE NIL))

(DEFUN |UnivariateLaurentSeriesCategory| (#1=#:G83278)
 (LET (#2=#:G83279)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |UnivariateLaurentSeriesCategory;AL|))
     (CDR #2#))
   (T
    (SETQ |UnivariateLaurentSeriesCategory;AL|
     (|cons5|
      (CONS
       (|devaluate| #1#)
       (SETQ #2# (|UnivariateLaurentSeriesCategory;| #1#)))
      |UnivariateLaurentSeriesCategory;AL|))
     #2#))))

(DEFUN |UnivariateLaurentSeriesCategory;| (|t#1|)
 (PROG (#1=#:G83277)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (|sublisV|
       (PAIR (QUOTE (#2=#:G83276)) (LIST (QUOTE (|Integer|))))
       (COND
        (|UnivariateLaurentSeriesCategory;CAT|)
        ((QUOTE T)
         (LETT |UnivariateLaurentSeriesCategory;CAT|
          (|Join|
           (|UnivariatePowerSeriesCategory| (QUOTE |t#1|) (QUOTE #2#))
```

```
            (|mkCategory| (QUOTE |domain|)
             (QUOTE (
              ((|series|
                (|$|
                 (|Stream| (|Record| (|:| |k| (|Integer|)) (|:| |c| |t#1|)))))
                 T)
              ((|multiplyCoefficients| (|$| (|Mapping| |t#1| (|Integer|)) |$|))
                 T)
              ((|rationalFunction|
                ((|Fraction| (|Polynomial| |t#1|)) |$| (|Integer|)))
                 (|has| |t#1| (|IntegralDomain|)))
              ((|rationalFunction|
                ((|Fraction| (|Polynomial| |t#1|)) |$| (|Integer|) (|Integer|)))
                 (|has| |t#1| (|IntegralDomain|)))
              ((|integrate| (|$| |$|))
                (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))
              ((|integrate| (|$| |$| (|Symbol|)))
                (AND
                 (|has| |t#1|
                   (SIGNATURE |variables| ((|List| (|Symbol|)) |t#1|)))
                 (|has| |t#1| (SIGNATURE |integrate| (|t#1| |t#1| (|Symbol|))))
                 (|has| |t#1| (|Algebra| (|Fraction| (|Integer|))))))
              ((|integrate| (|$| |$| (|Symbol|)))
                (AND
                 (|has| |t#1| (|AlgebraicallyClosedFunctionSpace| (|Integer|)))
                 (|has| |t#1| (|PrimitiveFunctionCategory|))
                 (|has| |t#1| (|TranscendentalFunctionCategory|))
                 (|has| |t#1| (|Algebra| (|Fraction| (|Integer|))))))))
            (QUOTE (
             ((|RadicalCategory|)
               (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))
             ((|TranscendentalFunctionCategory|)
               (|has| |t#1| (|Algebra| (|Fraction| (|Integer|)))))
             ((|Field|) (|has| |t#1| (|Field|)))))
            (QUOTE (
             (|Symbol|)
             (|Fraction| (|Polynomial| |t#1|))
             (|Integer|)
             (|Stream| (|Record| (|:| |k| (|Integer|)) (|:| |c| |t#1|)))))
             NIL))
            . #3=(|UnivariateLaurentSeriesCategory|))))))
           . #3#)
      (SETELT #1# 0
       (LIST (QUOTE |UnivariateLaurentSeriesCategory|) (|devaluate| |t#1|)))))))
```

21.67 UPOLYC.lsp BOOTSTRAP

UPOLYC depends on itself. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **UPOLYC** category which we can write into the **MID** directory. We compile the lisp code and copy the **UPOLYC.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*UPOLYC.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |UnivariatePolynomialCategory;CAT| (QUOTE NIL))

(SETQ |UnivariatePolynomialCategory;AL| (QUOTE NIL))

(DEFUN |UnivariatePolynomialCategory| (#1=#:G103214)
  (LET (#2=#:G103215)
    (COND
      ((SETQ #2# (|assoc| (|devaluate| #1#) |UnivariatePolynomialCategory;AL|))
         (CDR #2#))
      (T
        (SETQ |UnivariatePolynomialCategory;AL|
          (|cons5|
            (CONS
              (|devaluate| #1#)
              (SETQ #2# (|UnivariatePolynomialCategory;| #1#)))
            |UnivariatePolynomialCategory;AL|))
         #2#))))

(DEFUN |UnivariatePolynomialCategory;| (|t#1|)
  (PROG (#1=#:G103213)
    (RETURN
      (PROG1
        (LETT #1#
          (|sublisV|
            (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
            (|sublisV|
              (PAIR
                (QUOTE (#2=#:G103211 #3=#:G103212))
                (LIST
                  (QUOTE (|NonNegativeInteger|))
                  (QUOTE (|SingletonAsOrderedSet|))))
              (COND
                (|UnivariatePolynomialCategory;CAT|)
```

```
((QUOTE T)
  (LETT |UnivariatePolynomialCategory;CAT|
    (|Join|
      (|PolynomialCategory|
        (QUOTE |t#1|) (QUOTE #2#) (QUOTE #3#))
      (|Eltable| (QUOTE |t#1|) (QUOTE |t#1|))
      (|Eltable| (QUOTE |$|) (QUOTE |$|))
      (|DifferentialRing|)
      (|DifferentialExtension| (QUOTE |t#1|))
      (|mkCategory|
        (QUOTE |domain|)
        (QUOTE (
          ((|vectorise|
            ((|Vector| |t#1|) |$| (|NonNegativeInteger|))) T)
          ((|makeSUP|
            ((|SparseUnivariatePolynomial| |t#1|) |$|)) T)
          ((|unmakeSUP|
            (|$| (|SparseUnivariatePolynomial| |t#1|))) T)
          ((|multiplyExponents|
            (|$| |$| (|NonNegativeInteger|))) T)
          ((|divideExponents|
            ((|Union| |$| "failed")
             |$|
             (|NonNegativeInteger|))) T)
          ((|monicDivide|
            ((|Record|
               (|:| |quotient| |$|)
               (|:| |remainder| |$|))
             |$|
             |$|)) T)
          ((|karatsubaDivide|
            ((|Record|
               (|:| |quotient| |$|)
               (|:| |remainder| |$|))
             |$|
             (|NonNegativeInteger|))) T)
          ((|shiftRight| (|$| |$| (|NonNegativeInteger|))) T)
          ((|shiftLeft| (|$| |$| (|NonNegativeInteger|))) T)
          ((|pseudoRemainder| (|$| |$| |$|)) T)
          ((|differentiate|
            (|$| |$| (|Mapping| |t#1| |t#1|) |$|)) T)
          ((|discriminant| (|t#1| |$|))
            (|has| |t#1| (|CommutativeRing|)))
          ((|resultant| (|t#1| |$| |$|))
            (|has| |t#1| (|CommutativeRing|)))
          ((|elt|
```

```
    ((|Fraction| |$|)
     (|Fraction| |$|)
     (|Fraction| |$|)))
   (|has| |t#1| (|IntegralDomain|)))
 ((|order| ((|NonNegativeInteger|) |$| |$|))
   (|has| |t#1| (|IntegralDomain|)))
 ((|subResultantGcd| (|$| |$| |$|))
   (|has| |t#1| (|IntegralDomain|)))
 ((|composite| ((|Union| |$| "failed") |$| |$|))
   (|has| |t#1| (|IntegralDomain|)))
 ((|composite|
    ((|Union| (|Fraction| |$|) "failed")
     (|Fraction| |$|)
     |$|))
   (|has| |t#1| (|IntegralDomain|)))
 ((|pseudoQuotient| (|$| |$| |$|))
   (|has| |t#1| (|IntegralDomain|)))
 ((|pseudoDivide|
    ((|Record|
      (|:| |coef| |t#1|)
      (|:| |quotient| |$|)
      (|:| |remainder| |$|))
    |$|
    |$|))
   (|has| |t#1| (|IntegralDomain|)))
 ((|separate|
    ((|Record|
      (|:| |primePart| |$|)
      (|:| |commonPart| |$|))
    |$|
    |$|))
   (|has| |t#1| (|GcdDomain|)))
 ((|elt| (|t#1| (|Fraction| |$|) |t#1|))
   (|has| |t#1| (|Field|)))
 ((|integrate| (|$| |$|))
   (|has| |t#1|
     (|Algebra| (|Fraction| (|Integer|)))))))
(QUOTE (
  ((|StepThrough|) (|has| |t#1| (|StepThrough|)))
  ((|Eltable|
     (|Fraction| |$|)
     (|Fraction| |$|))
    (|has| |t#1| (|IntegralDomain|)))
  ((|EuclideanDomain|) (|has| |t#1| (|Field|)))
  (|additiveValuation| (|has| |t#1| (|Field|)))))
(QUOTE (
```

```
                        (|Fraction| |$|)
                        (|NonNegativeInteger|)
                        (|SparseUnivariatePolynomial| |t#1|)
                        (|Vector| |t#1|)))
                      NIL))
              . #4=(|UnivariatePolynomialCategory|))))))
    . #4#)
 (SETELT #1# 0
   (LIST
     (QUOTE |UnivariatePolynomialCategory|)
     (|devaluate| |t#1|))))))
```

21.68 UPOLYC-.lsp BOOTSTRAP

UPOLYC- depends on **UPOLYC**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **UPOLYC-** category which we can write into the **MID** directory. We compile the lisp code and copy the **UPOLYC-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*UPOLYC-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |UPOLYC-;variables;SL;1| (|p| |$|)
  (COND
    ((OR
      (SPADCALL |p| (QREFELT |$| 9))
      (ZEROP (SPADCALL |p| (QREFELT |$| 11))))
     NIL)
    ((QUOTE T) (LIST (SPADCALL (QREFELT |$| 13))))))

(DEFUN |UPOLYC-;degree;SSaosNni;2| (|p| |v| |$|)
  (SPADCALL |p| (QREFELT |$| 11)))

(DEFUN |UPOLYC-;totalDegree;SLNni;3| (|p| |lv| |$|)
  (COND ((NULL |lv|) 0) ((QUOTE T) (SPADCALL |p| (QREFELT |$| 17)))))

(DEFUN |UPOLYC-;degree;SLL;4| (|p| |lv| |$|)
  (COND ((NULL |lv|) NIL) ((QUOTE T) (LIST (SPADCALL |p| (QREFELT |$| 11))))))

(DEFUN |UPOLYC-;eval;SLLS;5| (|p| |lv| |lq| |$|)
  (COND
    ((NULL |lv|) |p|)
    ((NULL (NULL (CDR |lv|)))
      (|error| "can only eval a univariate polynomial once"))
    ((QUOTE T)
      (SPADCALL |p| (|SPADfirst| |lv|) (|SPADfirst| |lq|) (QREFELT |$| 21)))))

(DEFUN |UPOLYC-;eval;SSaos2S;6| (|p| |v| |q| |$|)
  (SPADCALL |p| |q| (QREFELT |$| 24)))

(DEFUN |UPOLYC-;eval;SLLS;7| (|p| |lv| |lr| |$|)
  (COND
    ((NULL |lv|) |p|)
    ((NULL (NULL (CDR |lv|)))
      (|error| "can only eval a univariate polynomial once"))
```

```
      ((QUOTE T)
        (SPADCALL |p| (|SPADfirst| |lv|) (|SPADfirst| |lr|) (QREFELT |$| 26)))))

  (DEFUN |UPOLYC-;eval;SSaosRS;8| (|p| |v| |r| |$|)
    (SPADCALL (SPADCALL |p| |r| (QREFELT |$| 29)) (QREFELT |$| 30)))

  (DEFUN |UPOLYC-;eval;SLS;9| (|p| |le| |$|)
    (COND
      ((NULL |le|) |p|)
      ((NULL (NULL (CDR |le|)))
        (|error| "can only eval a univariate polynomial once"))
      ((QUOTE T)
        (COND
          ((QEQCAR
             (SPADCALL
               (SPADCALL (|SPADfirst| |le|) (QREFELT |$| 33))
               (QREFELT |$| 35))
             1)
            |p|)
          ((QUOTE T)
            (SPADCALL |p|
              (SPADCALL (|SPADfirst| |le|) (QREFELT |$| 36))
              (QREFELT |$| 24)))))))

  (DEFUN |UPOLYC-;mainVariable;SU;10| (|p| |$|)
    (COND
      ((ZEROP (SPADCALL |p| (QREFELT |$| 11))) (CONS 1 "failed"))
      ((QUOTE T) (CONS 0 (SPADCALL (QREFELT |$| 13))))))

  (DEFUN |UPOLYC-;minimumDegree;SSaosNni;11| (|p| |v| |$|)
    (SPADCALL |p| (QREFELT |$| 40)))

  (DEFUN |UPOLYC-;minimumDegree;SLL;12| (|p| |lv| |$|)
    (COND ((NULL |lv|) NIL) ((QUOTE T) (LIST (SPADCALL |p| (QREFELT |$| 40))))))

  (DEFUN |UPOLYC-;monomial;SSaosNniS;13| (|p| |v| |n| |$|)
    (SPADCALL
      (CONS (FUNCTION |UPOLYC-;monomial;SSaosNniS;13!0|) (VECTOR |$| |n|))
      |p|
      (QREFELT |$| 45)))

  (DEFUN |UPOLYC-;monomial;SSaosNniS;13!0| (|#1| |$$|)
    (SPADCALL |#1| (QREFELT |$$| 1) (QREFELT (QREFELT |$$| 0) 43)))

  (DEFUN |UPOLYC-;coerce;SaosS;14| (|v| |$|)
    (SPADCALL (|spadConstant| |$| 48) 1 (QREFELT |$| 49)))
```

```
(DEFUN |UPOLYC-;makeSUP;SSup;15| (|p| |$|)
  (COND
    ((SPADCALL |p| (QREFELT |$| 9)) (|spadConstant| |$| 52))
    ((QUOTE T)
      (SPADCALL
        (SPADCALL
          (SPADCALL |p| (QREFELT |$| 53))
          (SPADCALL |p| (QREFELT |$| 11))
          (QREFELT |$| 54))
        (SPADCALL
          (SPADCALL |p| (QREFELT |$| 55))
          (QREFELT |$| 56))
        (QREFELT |$| 57)))))

(DEFUN |UPOLYC-;unmakeSUP;SupS;16| (|sp| |$|)
  (COND
    ((SPADCALL |sp| (QREFELT |$| 59)) (|spadConstant| |$| 60))
    ((QUOTE T)
      (SPADCALL
        (SPADCALL
          (SPADCALL |sp| (QREFELT |$| 61))
          (SPADCALL |sp| (QREFELT |$| 62))
          (QREFELT |$| 49))
        (SPADCALL (SPADCALL |sp| (QREFELT |$| 63)) (QREFELT |$| 64))
      (QREFELT |$| 65)))))

(DEFUN |UPOLYC-;karatsubaDivide;SNniR;17| (|p| |n| |$|)
  (SPADCALL |p|
    (SPADCALL (|spadConstant| |$| 48) |n| (QREFELT |$| 49))
    (QREFELT |$| 68)))

(DEFUN |UPOLYC-;shiftRight;SNniS;18| (|p| |n| |$|)
  (QCAR
    (SPADCALL |p|
      (SPADCALL (|spadConstant| |$| 48) |n| (QREFELT |$| 49))
      (QREFELT |$| 68))))

(DEFUN |UPOLYC-;shiftLeft;SNniS;19| (|p| |n| |$|)
  (SPADCALL |p|
    (SPADCALL (|spadConstant| |$| 48) |n| (QREFELT |$| 49)) (QREFELT |$| 71)))

(DEFUN |UPOLYC-;solveLinearPolynomialEquation;LSupU;20| (|lpp| |pp| |$|)
  (SPADCALL |lpp| |pp| (QREFELT |$| 77)))

(DEFUN |UPOLYC-;factorPolynomial;SupF;21| (|pp| |$|)
```

```
    (SPADCALL |pp| (QREFELT |$| 83)))

  (DEFUN |UPOLYC-;factorSquareFreePolynomial;SupF;22| (|pp| |$|)
    (SPADCALL |pp| (QREFELT |$| 86)))

  (DEFUN |UPOLYC-;factor;SF;23| (|p| |$|)
    (PROG (|ansR| #1=#:G103310 |w| #2=#:G103311)
      (RETURN
        (SEQ
          (COND
            ((ZEROP (SPADCALL |p| (QREFELT |$| 11)))
              (SEQ
                (LETT |ansR|
                  (SPADCALL
                    (SPADCALL |p| (QREFELT |$| 53))
                    (QREFELT |$| 89))
                  |UPOLYC-;factor;SF;23|)
                (EXIT
                  (SPADCALL
                    (SPADCALL
                      (SPADCALL |ansR| (QREFELT |$| 91))
                      (QREFELT |$| 30))
                    (PROGN
                      (LETT #1# NIL |UPOLYC-;factor;SF;23|)
                      (SEQ
                        (LETT |w| NIL |UPOLYC-;factor;SF;23|)
                        (LETT #2#
                          (SPADCALL |ansR| (QREFELT |$| 95))
                          |UPOLYC-;factor;SF;23|)
                        G190
                        (COND
                          ((OR
                            (ATOM #2#)
                            (PROGN
                              (LETT |w| (CAR #2#) |UPOLYC-;factor;SF;23|)
                              NIL))
                           (GO G191)))
                        (SEQ
                          (EXIT
                            (LETT #1#
                              (CONS
                                (VECTOR
                                  (QVELT |w| 0)
                                  (SPADCALL (QVELT |w| 1) (QREFELT |$| 30))
                                  (QVELT |w| 2))
                                #1#)
```

```
                              |UPOLYC-;factor;SF;23|)))
                        (LETT #2# (CDR #2#) |UPOLYC-;factor;SF;23|)
                        (GO G190)
                        G191
                        (EXIT (NREVERSE0 #1#))))
                  (QREFELT |$| 99)))))
            ((QUOTE T)
              (SPADCALL
                (ELT |$| 64)
                (SPADCALL (SPADCALL |p| (QREFELT |$| 56)) (QREFELT |$| 100))
                (QREFELT |$| 104))))))))

(DEFUN |UPOLYC-;vectorise;SNniV;24| (|p| |n| |$|)
  (PROG (|v| |m| |i| #1=#:G103316 #2=#:G103312)
    (RETURN
      (SEQ
        (LETT |m|
          (SPADCALL
            (LETT |v|
              (SPADCALL |n| (|spadConstant| |$| 106) (QREFELT |$| 108))
              |UPOLYC-;vectorise;SNniV;24|)
            (QREFELT |$| 110))
          |UPOLYC-;vectorise;SNniV;24|)
        (SEQ
          (LETT |i|
            (SPADCALL |v| (QREFELT |$| 110))
            |UPOLYC-;vectorise;SNniV;24|)
          (LETT #1# (QVSIZE |v|) |UPOLYC-;vectorise;SNniV;24|)
          G190
          (COND ((|>| |i| #1#) (GO G191)))
          (SEQ
            (EXIT
              (SPADCALL |v| |i|
                (SPADCALL |p|
                  (PROG1
                    (LETT #2# (|-| |i| |m|) |UPOLYC-;vectorise;SNniV;24|)
                    (|check-subtype|
                      (|>=| #2# 0)
                      (QUOTE (|NonNegativeInteger|))
                      #2#))
                  (QREFELT |$| 111))
                (QREFELT |$| 112))))
          (LETT |i| (|+| |i| 1) |UPOLYC-;vectorise;SNniV;24|)
          (GO G190)
          G191
          (EXIT NIL))
```

```
         (EXIT |v|)))))

  (DEFUN |UPOLYC-;retract;SR;25| (|p| |$|)
    (COND
      ((SPADCALL |p| (QREFELT |$| 9)) (|spadConstant| |$| 106))
      ((ZEROP (SPADCALL |p| (QREFELT |$| 11))) (SPADCALL |p| (QREFELT |$| 53)))
      ((QUOTE T) (|error| "Polynomial is not of degree 0"))))

  (DEFUN |UPOLYC-;retractIfCan;SU;26| (|p| |$|)
    (COND
      ((SPADCALL |p| (QREFELT |$| 9)) (CONS 0 (|spadConstant| |$| 106)))
      ((ZEROP (SPADCALL |p| (QREFELT |$| 11)))
         (CONS 0 (SPADCALL |p| (QREFELT |$| 53))))
      ((QUOTE T) (CONS 1 "failed"))))

  (DEFUN |UPOLYC-;init;S;27| (|$|)
    (SPADCALL (|spadConstant| |$| 117) (QREFELT |$| 30)))

  (DEFUN |UPOLYC-;nextItemInner| (|n| |$|)
    (PROG (|nn| |n1| |n2| #1=#:G103337 |n3|)
      (RETURN
        (SEQ
          (COND
            ((SPADCALL |n| (QREFELT |$| 9))
              (CONS
                0
                (SPADCALL
                  (PROG2
                    (LETT #1#
                      (SPADCALL (|spadConstant| |$| 106) (QREFELT |$| 120))
                      |UPOLYC-;nextItemInner|)
                    (QCDR #1#)
                    (|check-union| (QEQCAR #1# 0) (QREFELT |$| 7) #1#))
                  (QREFELT |$| 30))))
            ((ZEROP (SPADCALL |n| (QREFELT |$| 11)))
              (SEQ
                (LETT |nn|
                  (SPADCALL (SPADCALL |n| (QREFELT |$| 53)) (QREFELT |$| 120))
                  |UPOLYC-;nextItemInner|)
                (EXIT
                  (COND
                    ((QEQCAR |nn| 1) (CONS 1 "failed"))
                    ((QUOTE T)
                      (CONS 0 (SPADCALL (QCDR |nn|) (QREFELT |$| 30))))))))
            ((QUOTE T)
              (SEQ
```

```
(LETT |n1|
  (SPADCALL |n| (QREFELT |$| 55))
  |UPOLYC-;nextItemInner|)
(LETT |n2|
  (|UPOLYC-;nextItemInner| |n1| |$|)
  |UPOLYC-;nextItemInner|)
(EXIT
  (COND
    ((QEQCAR |n2| 0)
      (CONS
        0
        (SPADCALL
          (SPADCALL
            (SPADCALL |n| (QREFELT |$| 53))
            (SPADCALL |n| (QREFELT |$| 11))
            (QREFELT |$| 49))
          (QCDR |n2|)
          (QREFELT |$| 65))))
    ((|<|
       (|+| 1 (SPADCALL |n1| (QREFELT |$| 11)))
       (SPADCALL |n| (QREFELT |$| 11)))
      (CONS
        0
        (SPADCALL
          (SPADCALL
            (SPADCALL |n| (QREFELT |$| 53))
            (SPADCALL |n| (QREFELT |$| 11))
            (QREFELT |$| 49))
          (SPADCALL
            (PROG2
              (LETT #1#
                (SPADCALL
                  (|spadConstant| |$| 117)
                  (QREFELT |$| 120))
                |UPOLYC-;nextItemInner|)
              (QCDR #1#)
              (|check-union| (QEQCAR #1# 0) (QREFELT |$| 7) #1#))
            (|+| 1 (SPADCALL |n1| (QREFELT |$| 11)))
            (QREFELT |$| 49))
          (QREFELT |$| 65))))
    ((QUOTE T)
      (SEQ
        (LETT |n3|
          (SPADCALL
            (SPADCALL |n| (QREFELT |$| 53))
            (QREFELT |$| 120))
```

```
                          |UPOLYC-;nextItemInner|)
                        (EXIT
                          (COND
                            ((QEQCAR |n3| 1) (CONS 1 "failed"))
                            ((QUOTE T)
                              (CONS
                                0
                                (SPADCALL
                                  (QCDR |n3|)
                                  (SPADCALL |n| (QREFELT |$| 11))
                                  (QREFELT |$| 49))))))))))))))))))

(DEFUN |UPOLYC-;nextItem;SU;29| (|n| |$|)
  (PROG (|n1| #1=#:G103350)
    (RETURN
      (SEQ
        (LETT |n1| (|UPOLYC-;nextItemInner| |n| |$|) |UPOLYC-;nextItem;SU;29|)
        (EXIT
          (COND
            ((QEQCAR |n1| 1)
              (CONS
                0
                (SPADCALL
                  (PROG2
                    (LETT #1#
                      (SPADCALL (|spadConstant| |$| 117) (QREFELT |$| 120))
                      |UPOLYC-;nextItem;SU;29|)
                    (QCDR #1#)
                    (|check-union| (QEQCAR #1# 0) (QREFELT |$| 7) #1#))
                  (|+| 1 (SPADCALL |n| (QREFELT |$| 11)))
                  (QREFELT |$| 49))))
            ((QUOTE T) |n1|)))))))

(DEFUN |UPOLYC-;content;SSaosS;30| (|p| |v| |$|)
  (SPADCALL (SPADCALL |p| (QREFELT |$| 123)) (QREFELT |$| 30)))

(DEFUN |UPOLYC-;primeFactor| (|p| |q| |$|)
  (PROG (#1=#:G103356 |p1|)
    (RETURN
      (SEQ
        (LETT |p1|
          (PROG2
            (LETT #1#
              (SPADCALL |p|
                (SPADCALL |p| |q| (QREFELT |$| 125))
                (QREFELT |$| 126))
```

```
                |UPOLYC-;primeFactor|)
               (QCDR #1#)
               (|check-union| (QEQCAR #1# 0) (QREFELT |$| 6) #1#))
              |UPOLYC-;primeFactor|)
          (EXIT
            (COND
              ((SPADCALL |p1| |p| (QREFELT |$| 127)) |p|)
              ((QUOTE T) (|UPOLYC-;primeFactor| |p1| |q| |$|))))))))

(DEFUN |UPOLYC-;separate;2SR;32| (|p| |q| |$|)
  (PROG (|a| #1=#:G103362)
    (RETURN
      (SEQ
        (LETT |a|
          (|UPOLYC-;primeFactor| |p| |q| |$|)
          |UPOLYC-;separate;2SR;32|)
        (EXIT
          (CONS
            |a|
            (PROG2
              (LETT #1#
                (SPADCALL |p| |a| (QREFELT |$| 126))
                |UPOLYC-;separate;2SR;32|)
              (QCDR #1#)
              (|check-union| (QEQCAR #1# 0) (QREFELT |$| 6) #1#)))))))))

(DEFUN |UPOLYC-;differentiate;SM2S;33| (|x| |deriv| |x'| |$|)
  (PROG (|dg| |lc| #1=#:G103367 |d|)
    (RETURN
      (SEQ
        (LETT |d| (|spadConstant| |$| 60) |UPOLYC-;differentiate;SM2S;33|)
        (SEQ G190
          (COND
            ((NULL
               (|<| 0
                 (LETT |dg|
                   (SPADCALL |x| (QREFELT |$| 11))
                   |UPOLYC-;differentiate;SM2S;33|)))
              (GO G191)))
          (SEQ
            (LETT |lc|
              (SPADCALL |x| (QREFELT |$| 53))
              |UPOLYC-;differentiate;SM2S;33|)
            (LETT |d|
              (SPADCALL
                (SPADCALL |d|
```

```
                    (SPADCALL |x'|
                      (SPADCALL
                        (SPADCALL |dg| |lc| (QREFELT |$| 131))
                        (PROG1
                          (LETT #1# (|-| |dg| 1) |UPOLYC-;differentiate;SM2S;33|)
                          (|check-subtype|
                            (|>=| #1# 0)
                            (QUOTE (|NonNegativeInteger|))
                            #1#))
                        (QREFELT |$| 49))
                      (QREFELT |$| 71))
                    (QREFELT |$| 65))
                    (SPADCALL
                      (SPADCALL |lc| |deriv|)
                      |dg|
                      (QREFELT |$| 49))
                  (QREFELT |$| 65))
                |UPOLYC-;differentiate;SM2S;33|)
               (EXIT
                 (LETT |x|
                   (SPADCALL |x| (QREFELT |$| 55))
                   |UPOLYC-;differentiate;SM2S;33|)))
              NIL
              (GO G190)
              G191
              (EXIT NIL))
            (EXIT
              (SPADCALL |d|
                (SPADCALL
                  (SPADCALL
                    (SPADCALL |x| (QREFELT |$| 53))
                    |deriv|)
                  (QREFELT |$| 30))
                (QREFELT |$| 65)))))))

(DEFUN |UPOLYC-;ncdiff| (|n| |x'| |$|)
  (PROG (#1=#:G103385 |n1|)
    (RETURN
      (COND
        ((ZEROP |n|) (|spadConstant| |$| 60))
        ((ZEROP
          (LETT |n1|
            (PROG1
              (LETT #1# (|-| |n| 1) |UPOLYC-;ncdiff|)
              (|check-subtype|
                (|>=| #1# 0)
```

```
                (QUOTE (|NonNegativeInteger|))
                #1#))
            |UPOLYC-;ncdiff|))
          |x'|)
        ((QUOTE T)
          (SPADCALL
            (SPADCALL |x'|
              (SPADCALL (|spadConstant| |$| 48) |n1| (QREFELT |$| 49))
              (QREFELT |$| 71))
            (SPADCALL
              (SPADCALL (|spadConstant| |$| 48) 1 (QREFELT |$| 49))
              (|UPOLYC-;ncdiff| |n1| |x'| |$|)
              (QREFELT |$| 71))
            (QREFELT |$| 65)))))))

(DEFUN |UPOLYC-;differentiate;SM2S;35| (|x| |deriv| |x'| |$|)
  (PROG (|dg| |lc| |d|)
    (RETURN
      (SEQ
        (LETT |d| (|spadConstant| |$| 60) |UPOLYC-;differentiate;SM2S;35|)
        (SEQ G190
          (COND
            ((NULL
               (|<| 0
                 (LETT |dg|
                   (SPADCALL |x| (QREFELT |$| 11))
                   |UPOLYC-;differentiate;SM2S;35|)))
              (GO G191)))
            (SEQ
              (LETT |lc|
                (SPADCALL |x| (QREFELT |$| 53))
                |UPOLYC-;differentiate;SM2S;35|)
              (LETT |d|
                (SPADCALL
                  (SPADCALL |d|
                    (SPADCALL
                      (SPADCALL |lc| |deriv|)
                      |dg|
                      (QREFELT |$| 49))
                    (QREFELT |$| 65))
                  (SPADCALL |lc|
                    (|UPOLYC-;ncdiff| |dg| |x'| |$|)
                    (QREFELT |$| 134))
                  (QREFELT |$| 65))
                |UPOLYC-;differentiate;SM2S;35|)
              (EXIT
```

```
              (LETT |x|
                (SPADCALL |x| (QREFELT |$| 55))
                |UPOLYC-;differentiate;SM2S;35|)))
            NIL
            (GO G190)
            G191
            (EXIT NIL))
          (EXIT
            (SPADCALL |d|
              (SPADCALL
                (SPADCALL (SPADCALL |x| (QREFELT |$| 53)) |deriv|)
                (QREFELT |$| 30))
              (QREFELT |$| 65)))))))

(DEFUN |UPOLYC-;differentiate;SMS;36| (|x| |deriv| |$|)
  (SPADCALL |x| |deriv| (|spadConstant| |$| 47) (QREFELT |$| 135)))

(DEFUN |UPOLYC-;differentiate;2S;37| (|x| |$|)
  (PROG (|dg| #1=#:G103394 |d|)
    (RETURN
      (SEQ
        (LETT |d| (|spadConstant| |$| 60) |UPOLYC-;differentiate;2S;37|)
        (SEQ G190
          (COND
            ((NULL
              (|<| 0
                (LETT |dg|
                  (SPADCALL |x| (QREFELT |$| 11))
                  |UPOLYC-;differentiate;2S;37|)))
              (GO G191)))
            (SEQ
              (LETT |d|
                (SPADCALL |d|
                  (SPADCALL
                    (SPADCALL |dg|
                      (SPADCALL |x| (QREFELT |$| 53)) (QREFELT |$| 131))
                    (PROG1
                      (LETT #1# (|-| |dg| 1) |UPOLYC-;differentiate;2S;37|)
                      (|check-subtype|
                        (|>=| #1# 0)
                        (QUOTE (|NonNegativeInteger|))
                        #1#))
                    (QREFELT |$| 49))
                  (QREFELT |$| 65))
                |UPOLYC-;differentiate;2S;37|)
              (EXIT
```

```
              (LETT |x|
                (SPADCALL |x| (QREFELT |$| 55))
                |UPOLYC-;differentiate;2S;37|)))
            NIL
            (GO G190)
            G191
            (EXIT NIL))
          (EXIT |d|)))))

(DEFUN |UPOLYC-;differentiate;SSaosS;38| (|x| |v| |$|)
  (SPADCALL |x| (QREFELT |$| 138)))

(DEFUN |UPOLYC-;elt;3F;39| (|g| |f| |$|)
  (SPADCALL
    (SPADCALL
      (SPADCALL |g| (QREFELT |$| 141))
      |f|
      (QREFELT |$| 143))
    (SPADCALL (SPADCALL |g| (QREFELT |$| 144)) |f| (QREFELT |$| 143))
    (QREFELT |$| 145)))

(DEFUN |UPOLYC-;pseudoQuotient;3S;40| (|p| |q| |$|)
  (PROG (|n| #1=#:G103440 #2=#:G103442)
    (RETURN
      (SEQ
        (LETT |n|
          (|+|
            (|-|
              (SPADCALL |p| (QREFELT |$| 11))
              (SPADCALL |q| (QREFELT |$| 11))) 1)
           |UPOLYC-;pseudoQuotient;3S;40|)
        (EXIT
          (COND
            ((|<| |n| 1) (|spadConstant| |$| 60))
            ((QUOTE T)
              (PROG2
                (LETT #2#
                  (SPADCALL
                    (SPADCALL
                      (SPADCALL
                        (SPADCALL |q| (QREFELT |$| 53))
                        (PROG1
                          (LETT #1# |n| |UPOLYC-;pseudoQuotient;3S;40|)
                          (|check-subtype|
                            (|>=| #1# 0)
```

```
                                    (QUOTE (|NonNegativeInteger|))
                                    #1#))
                                  (QREFELT |$| 147))
                                |p|
                                (QREFELT |$| 134))
                              (SPADCALL |p| |q| (QREFELT |$| 148))
                              (QREFELT |$| 149))
                            |q|
                            (QREFELT |$| 126))
                          |UPOLYC-;pseudoQuotient;3S;40|)
                        (QCDR #2#)
                        (|check-union| (QEQCAR #2# 0) (QREFELT |$| 6) #2#)))))))))

  (DEFUN |UPOLYC-;pseudoDivide;2SR;41| (|p| |q| |$|)
    (PROG (|n| |prem| #1=#:G103448 |lc| #2=#:G103450)
      (RETURN
        (SEQ
          (LETT |n|
            (|+|
              (|-|
                (SPADCALL |p| (QREFELT |$| 11))
                (SPADCALL |q| (QREFELT |$| 11))) 1)
            |UPOLYC-;pseudoDivide;2SR;41|)
          (EXIT
            (COND
              ((|<| |n| 1)
                (VECTOR (|spadConstant| |$| 48) (|spadConstant| |$| 60) |p|))
              ((QUOTE T)
                (SEQ
                  (LETT |prem|
                    (SPADCALL |p| |q| (QREFELT |$| 148))
                    |UPOLYC-;pseudoDivide;2SR;41|)
                  (LETT |lc|
                    (SPADCALL
                      (SPADCALL |q| (QREFELT |$| 53))
                      (PROG1
                        (LETT #1# |n| |UPOLYC-;pseudoDivide;2SR;41|)
                        (|check-subtype|
                          (|>=| #1# 0)
                          (QUOTE (|NonNegativeInteger|)) #1#))
                      (QREFELT |$| 147))
                    |UPOLYC-;pseudoDivide;2SR;41|)
                  (EXIT
                    (VECTOR |lc|
                      (PROG2
                        (LETT #2#
```

```
                        (SPADCALL
                          (SPADCALL
                            (SPADCALL |lc| |p| (QREFELT |$| 134))
                            |prem|
                            (QREFELT |$| 149))
                          |q|
                          (QREFELT |$| 126))
                        |UPOLYC-;pseudoDivide;2SR;41|)
                      (QCDR #2#)
                      (|check-union| (QEQCAR #2# 0) (QREFELT |$| 6) #2#))
                    |prem|))))))))))

(DEFUN |UPOLYC-;composite;FSU;42| (|f| |q| |$|)
  (PROG (|n| |d|)
    (RETURN
      (SEQ
        (LETT |n|
          (SPADCALL (SPADCALL |f| (QREFELT |$| 141)) |q| (QREFELT |$| 153))
          |UPOLYC-;composite;FSU;42|)
        (EXIT
          (COND
            ((QEQCAR |n| 1) (CONS 1 "failed"))
            ((QUOTE T)
              (SEQ
                (LETT |d|
                  (SPADCALL
                    (SPADCALL |f| (QREFELT |$| 144)) |q| (QREFELT |$| 153))
                  |UPOLYC-;composite;FSU;42|)
                (EXIT
                  (COND
                    ((QEQCAR |d| 1) (CONS 1 "failed"))
                    ((QUOTE T)
                      (CONS
                        0
                        (SPADCALL
                          (QCDR |n|)
                          (QCDR |d|)
                          (QREFELT |$| 154))))))))))))))

(DEFUN |UPOLYC-;composite;2SU;43| (|p| |q| |$|)
  (PROG (|cqr| |v| |u| |w| #1=#:G103476)
    (RETURN
      (SEQ
        (COND
          ((SPADCALL |p| (QREFELT |$| 157)) (CONS 0 |p|))
          ((QUOTE T)
```

```
(SEQ
  (EXIT
    (SEQ
      (LETT |cqr|
        (SPADCALL |p| |q| (QREFELT |$| 158))
        |UPOLYC-;composite;2SU;43|)
      (COND
        ((SPADCALL (QVELT |cqr| 2) (QREFELT |$| 157))
          (SEQ
            (LETT |v|
              (SPADCALL
                (QVELT |cqr| 2)
                (QVELT |cqr| 0)
                (QREFELT |$| 159))
              |UPOLYC-;composite;2SU;43|)
            (EXIT
              (COND
                ((QEQCAR |v| 0)
                  (SEQ
                    (LETT |u|
                      (SPADCALL
                        (QVELT |cqr| 1)
                        |q|
                        (QREFELT |$| 153))
                      |UPOLYC-;composite;2SU;43|)
                    (EXIT
                      (COND
                        ((QEQCAR |u| 0)
                          (SEQ
                            (LETT |w|
                              (SPADCALL
                                (QCDR |u|)
                                (QVELT |cqr| 0)
                                (QREFELT |$| 159))
                              |UPOLYC-;composite;2SU;43|)
                            (EXIT
                              (COND
                                ((QEQCAR |w| 0)
                                  (PROGN
                                    (LETT #1#
                                      (CONS
                                        0
                                        (SPADCALL
                                          (QCDR |v|)
                                          (SPADCALL
                                            (SPADCALL
```

```
                                                    (|spadConstant| |$| 48)
                                                    1
                                                    (QREFELT |$| 49))
                                                  (QCDR |w|)
                                                  (QREFELT |$| 71))
                                                 (QREFELT |$| 65)))
                                               |UPOLYC-;composite;2SU;43|)
                                             (GO #1#)))))))))))))))))
                    (EXIT (CONS 1 "failed"))))
                #1#
                (EXIT #1#))))))))

(DEFUN |UPOLYC-;elt;S2F;44| (|p| |f| |$|)
  (PROG (|n| #1=#:G103483 |ans|)
    (RETURN
      (SEQ
        (COND
          ((SPADCALL |p| (QREFELT |$| 9)) (|spadConstant| |$| 161))
          ((QUOTE T)
            (SEQ
              (LETT |ans|
                (SPADCALL
                  (SPADCALL (SPADCALL |p| (QREFELT |$| 53)) (QREFELT |$| 30))
                  (QREFELT |$| 162))
                |UPOLYC-;elt;S2F;44|)
              (LETT |n| (SPADCALL |p| (QREFELT |$| 11)) |UPOLYC-;elt;S2F;44|)
              (SEQ G190
                (COND
                  ((NULL
                    (COND
                      ((SPADCALL
                        (LETT |p|
                          (SPADCALL |p| (QREFELT |$| 55))
                          |UPOLYC-;elt;S2F;44|)
                        (QREFELT |$| 9))
                       (QUOTE NIL))
                      ((QUOTE T) (QUOTE T))))
                   (GO G191)))
                (SEQ
                  (EXIT
                    (LETT |ans|
                      (SPADCALL
                        (SPADCALL |ans|
                          (SPADCALL |f|
                            (PROG1
                              (LETT #1#
```

```
                                    (|-| |n|
                                      (LETT |n|
                                        (SPADCALL |p| (QREFELT |$| 11))
                                        |UPOLYC-;elt;S2F;44|))
                                    |UPOLYC-;elt;S2F;44|)
                                  (|check-subtype|
                                    (|>=| #1# 0)
                                    (QUOTE (|NonNegativeInteger|))
                                    #1#))
                                (QREFELT |$| 163))
                              (QREFELT |$| 164))
                            (SPADCALL
                              (SPADCALL
                                (SPADCALL |p| (QREFELT |$| 53))
                                (QREFELT |$| 30))
                              (QREFELT |$| 162))
                            (QREFELT |$| 165))
                          |UPOLYC-;elt;S2F;44|)))
                    NIL
                    (GO G190)
                    G191
                    (EXIT NIL))
                  (EXIT
                    (COND
                      ((ZEROP |n|) |ans|)
                      ((QUOTE T)
                        (SPADCALL |ans|
                          (SPADCALL |f| |n| (QREFELT |$| 166))
                          (QREFELT |$| 164)))))))))))

(DEFUN |UPOLYC-;order;2SNni;45| (|p| |q| |$|)
  (PROG (|u| #1=#:G103497 |ans|)
    (RETURN
      (SEQ
        (EXIT
          (COND
            ((SPADCALL |p| (QREFELT |$| 9))
              (|error| "order: arguments must be nonzero"))
            ((|<| (SPADCALL |q| (QREFELT |$| 11)) 1)
              (|error| "order: place must be non-trivial"))
            ((QUOTE T)
              (SEQ
                (LETT |ans| 0 |UPOLYC-;order;2SNni;45|)
                (EXIT
                  (SEQ G190
                    NIL
```

```
                    (SEQ
                      (LETT |u|
                        (SPADCALL |p| |q| (QREFELT |$| 126))
                        |UPOLYC-;order;2SNni;45|)
                      (EXIT
                        (COND
                          ((QEQCAR |u| 1)
                            (PROGN
                              (LETT #1# |ans| |UPOLYC-;order;2SNni;45|)
                              (GO #1#)))
                          ((QUOTE T)
                            (SEQ
                              (LETT |p| (QCDR |u|) |UPOLYC-;order;2SNni;45|)
                              (EXIT
                                (LETT
                                  |ans|
                                  (|+| |ans| 1)
                                  |UPOLYC-;order;2SNni;45|)))))))
                    NIL
                    (GO G190)
                    G191
                    (EXIT NIL)))))))
          #1#
          (EXIT #1#)))))

(DEFUN |UPOLYC-;squareFree;SF;46| (|p| |$|)
  (SPADCALL |p| (QREFELT |$| 170)))

(DEFUN |UPOLYC-;squareFreePart;2S;47| (|p| |$|)
  (SPADCALL |p| (QREFELT |$| 172)))

(DEFUN |UPOLYC-;gcdPolynomial;3Sup;48| (|pp| |qq| |$|)
  (COND
    ((SPADCALL |pp| (QREFELT |$| 174)) (SPADCALL |qq| (QREFELT |$| 175)))
    ((SPADCALL |qq| (QREFELT |$| 174)) (SPADCALL |pp| (QREFELT |$| 175)))
    ((QUOTE T)
      (SPADCALL
        (SPADCALL
          (SPADCALL
            (SPADCALL |pp| (QREFELT |$| 176))
            (SPADCALL |qq| (QREFELT |$| 176)) (QREFELT |$| 125))
          (SPADCALL
            (SPADCALL
              (SPADCALL |pp| (QREFELT |$| 177))
              (SPADCALL |qq| (QREFELT |$| 177)) (QREFELT |$| 178))
            (QREFELT |$| 177))
```

```
          (QREFELT |$| 179))
        (QREFELT |$| 175)))))

(DEFUN |UPOLYC-;squareFreePolynomial;SupF;49| (|pp| |$|)
  (SPADCALL |pp| (QREFELT |$| 182)))

(DEFUN |UPOLYC-;elt;F2R;50| (|f| |r| |$|)
  (SPADCALL
    (SPADCALL
      (SPADCALL |f| (QREFELT |$| 141))
      |r|
      (QREFELT |$| 29))
    (SPADCALL (SPADCALL |f| (QREFELT |$| 144)) |r| (QREFELT |$| 29))
    (QREFELT |$| 184)))

(DEFUN |UPOLYC-;euclideanSize;SNni;51| (|x| |$|)
  (COND
    ((SPADCALL |x| (QREFELT |$| 9))
      (|error| "euclideanSize called on 0 in Univariate Polynomial"))
    ((QUOTE T) (SPADCALL |x| (QREFELT |$| 11)))))

(DEFUN |UPOLYC-;divide;2SR;52| (|x| |y| |$|)
  (PROG (|lc| |f| #1=#:G103510 |n| |quot|)
    (RETURN
      (SEQ
        (COND
          ((SPADCALL |y| (QREFELT |$| 9))
            (|error| "division by 0 in Univariate Polynomials"))
          ((QUOTE T)
            (SEQ
              (LETT |quot| (|spadConstant| |$| 60) |UPOLYC-;divide;2SR;52|)
              (LETT |lc|
                (SPADCALL (SPADCALL |y| (QREFELT |$| 53)) (QREFELT |$| 187))
                |UPOLYC-;divide;2SR;52|)
              (SEQ G190
                (COND
                  ((NULL
                     (COND
                       ((OR
                          (SPADCALL |x| (QREFELT |$| 9))
                          (|<|
                            (SPADCALL |x| (QREFELT |$| 11))
                            (SPADCALL |y| (QREFELT |$| 11))))
                        (QUOTE NIL)) ((QUOTE T) (QUOTE T))))
                    (GO G191)))
                (SEQ
```

```
                    (LETT |f|
                      (SPADCALL |lc|
                        (SPADCALL |x| (QREFELT |$| 53))
                        (QREFELT |$| 188))
                      |UPOLYC-;divide;2SR;52|)
                    (LETT |n|
                      (PROG1
                        (LETT #1#
                          (|-|
                            (SPADCALL |x| (QREFELT |$| 11))
                            (SPADCALL |y| (QREFELT |$| 11)))
                           |UPOLYC-;divide;2SR;52|)
                        (|check-subtype|
                          (|>=| #1# 0)
                          (QUOTE (|NonNegativeInteger|))
                          #1#))
                      |UPOLYC-;divide;2SR;52|)
                    (LETT |quot|
                      (SPADCALL |quot|
                        (SPADCALL |f| |n| (QREFELT |$| 49))
                        (QREFELT |$| 65))
                      |UPOLYC-;divide;2SR;52|)
                    (EXIT
                      (LETT |x|
                        (SPADCALL |x|
                          (SPADCALL
                            (SPADCALL |f| |n| (QREFELT |$| 49))
                            |y|
                            (QREFELT |$| 71))
                          (QREFELT |$| 149))
                        |UPOLYC-;divide;2SR;52|)))
                   NIL
                   (GO G190)
                   G191
                   (EXIT NIL))
                 (EXIT (CONS |quot| |x|)))))))))

(DEFUN |UPOLYC-;integrate;2S;53| (|p| |$|)
  (PROG (|l| |d| |ans|)
    (RETURN
      (SEQ
        (LETT |ans| (|spadConstant| |$| 60) |UPOLYC-;integrate;2S;53|)
        (SEQ G190
          (COND
            ((NULL
              (COND
```

```
                ((SPADCALL |p| (|spadConstant| |$| 60) (QREFELT |$| 127))
                  (QUOTE NIL))
                ((QUOTE T) (QUOTE T)))) (GO G191)))
            (SEQ
              (LETT |l|
                (SPADCALL |p| (QREFELT |$| 53))
                |UPOLYC-;integrate;2S;53|)
              (LETT |d|
                (|+| 1 (SPADCALL |p| (QREFELT |$| 11)))
                |UPOLYC-;integrate;2S;53|)
              (LETT |ans|
                (SPADCALL |ans|
                  (SPADCALL
                    (SPADCALL (SPADCALL |d| (QREFELT |$| 191)) (QREFELT |$| 192))
                    (SPADCALL |l| |d| (QREFELT |$| 49)) (QREFELT |$| 193))
                  (QREFELT |$| 65))
                |UPOLYC-;integrate;2S;53|)
              (EXIT
                (LETT |p|
                  (SPADCALL |p| (QREFELT |$| 55))
                  |UPOLYC-;integrate;2S;53|)))
            NIL
            (GO G190)
            G191
            (EXIT NIL))
          (EXIT |ans|)))))

(DEFUN |UnivariatePolynomialCategory&| (|#1| |#2|)
  (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
    (RETURN
      (PROGN
        (LETT |DV$1| (|devaluate| |#1|) . #1=(|UnivariatePolynomialCategory&|))
        (LETT |DV$2| (|devaluate| |#2|) . #1#)
        (LETT |dv$|
          (LIST (QUOTE |UnivariatePolynomialCategory&|) |DV$1| |DV$2|) . #1#)
        (LETT |$| (GETREFV 201) . #1#)
        (QSETREFV |$| 0 |dv$|)
        (QSETREFV |$| 3
          (LETT |pv$|
            (|buildPredVector| 0 0
              (LIST
                (|HasCategory| |#2|
                  (QUOTE (|Algebra| (|Fraction| (|Integer|)))))
                (|HasCategory| |#2| (QUOTE (|Field|)))
                (|HasCategory| |#2| (QUOTE (|GcdDomain|)))
                (|HasCategory| |#2| (QUOTE (|IntegralDomain|)))
```

```
        (|HasCategory| |#2| (QUOTE (|CommutativeRing|)))
        (|HasCategory| |#2| (QUOTE (|StepThrough|))))) . #1#))
(|stuffDomainSlots| |$|)
(QSETREFV |$| 6 |#1|)
(QSETREFV |$| 7 |#2|)
(COND
  ((|HasCategory| |#2| (QUOTE (|PolynomialFactorizationExplicit|)))
    (PROGN
      (QSETREFV |$| 81
        (CONS
          (|dispatchFunction|
             |UPOLYC-;solveLinearPolynomialEquation;LSupU;20|)
          |$|))
      (QSETREFV |$| 85
        (CONS
          (|dispatchFunction| |UPOLYC-;factorPolynomial;SupF;21|)
          |$|))
      (QSETREFV |$| 87
         (CONS
            (|dispatchFunction|
               |UPOLYC-;factorSquareFreePolynomial;SupF;22|)
            |$|))
      (QSETREFV |$| 105
        (CONS (|dispatchFunction| |UPOLYC-;factor;SF;23|) |$|)))))
(COND
  ((|testBitVector| |pv$| 6)
    (PROGN
      (QSETREFV |$| 118
        (CONS (|dispatchFunction| |UPOLYC-;init;S;27|) |$|))
      NIL
      (QSETREFV |$| 122
        (CONS (|dispatchFunction| |UPOLYC-;nextItem;SU;29|) |$|)))))
(COND
  ((|testBitVector| |pv$| 3)
    (PROGN
      (QSETREFV |$| 124
        (CONS (|dispatchFunction| |UPOLYC-;content;SSaosS;30|) |$|))
      NIL
      (QSETREFV |$| 129
        (CONS (|dispatchFunction| |UPOLYC-;separate;2SR;32|) |$|)))))
(COND
  ((|testBitVector| |pv$| 5)
    (QSETREFV |$| 133
      (CONS
        (|dispatchFunction| |UPOLYC-;differentiate;SM2S;33|)
        |$|)))
```

```
  ((QUOTE T)
    (PROGN
      (QSETREFV |$| 133
        (CONS
          (|dispatchFunction| |UPOLYC-;differentiate;SM2S;35|)
          |$|))))))
(COND
  ((|testBitVector| |pv$| 4)
    (PROGN
      (QSETREFV |$| 146
        (CONS (|dispatchFunction| |UPOLYC-;elt;3F;39|) |$|))
      (QSETREFV |$| 150
        (CONS (|dispatchFunction| |UPOLYC-;pseudoQuotient;3S;40|) |$|))
      (QSETREFV |$| 152
        (CONS (|dispatchFunction| |UPOLYC-;pseudoDivide;2SR;41|) |$|))
      (QSETREFV |$| 156
        (CONS (|dispatchFunction| |UPOLYC-;composite;FSU;42|) |$|))
      (QSETREFV |$| 160
        (CONS (|dispatchFunction| |UPOLYC-;composite;2SU;43|) |$|))
      (QSETREFV |$| 167
        (CONS (|dispatchFunction| |UPOLYC-;elt;S2F;44|) |$|))
      (QSETREFV |$| 168
        (CONS (|dispatchFunction| |UPOLYC-;order;2SNni;45|) |$|)))))
(COND
  ((|testBitVector| |pv$| 3)
    (PROGN
      (QSETREFV |$| 171
        (CONS (|dispatchFunction| |UPOLYC-;squareFree;SF;46|) |$|))
      (QSETREFV |$| 173
        (CONS
          (|dispatchFunction| |UPOLYC-;squareFreePart;2S;47|)
          |$|))))))
(COND
  ((|HasCategory| |#2| (QUOTE (|PolynomialFactorizationExplicit|)))
    (PROGN
      (QSETREFV |$| 180
        (CONS
          (|dispatchFunction| |UPOLYC-;gcdPolynomial;3Sup;48|)
          |$|))
      (QSETREFV |$| 183
        (CONS
          (|dispatchFunction| |UPOLYC-;squareFreePolynomial;SupF;49|)
          |$|))))))
(COND
  ((|testBitVector| |pv$| 2)
    (PROGN
```

```
              (QSETREFV |$| 185
                (CONS (|dispatchFunction| |UPOLYC-;elt;F2R;50|) |$|))
              (QSETREFV |$| 186
                (CONS
                  (|dispatchFunction| |UPOLYC-;euclideanSize;SNni;51|)
                  |$|))
              (QSETREFV |$| 189
                (CONS (|dispatchFunction| |UPOLYC-;divide;2SR;52|) |$|)))))
          (COND
            ((|testBitVector| |pv$| 1)
               (QSETREFV |$| 194
                 (CONS
                   (|dispatchFunction| |UPOLYC-;integrate;2S;53|)
                   |$|)))) |$|))))

(MAKEPROP
  (QUOTE |UnivariatePolynomialCategory&|)
  (QUOTE |infovec|)
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL
        (|local| |#1|)
        (|local| |#2|)
        (|Boolean|)
        (0 . |zero?|)
        (|NonNegativeInteger|)
        (5 . |degree|)
        (|SingletonAsOrderedSet|)
        (10 . |create|)
        (|List| 12)
        |UPOLYC-;variables;SL;1|
        |UPOLYC-;degree;SSaosNni;2|
        (14 . |totalDegree|)
        |UPOLYC-;totalDegree;SLNni;3|
        (|List| 10)
        |UPOLYC-;degree;SLL;4|
        (19 . |eval|)
        (|List| |$|)
        |UPOLYC-;eval;SLLS;5|
        (26 . |elt|)
        |UPOLYC-;eval;SSaos2S;6|
        (32 . |eval|)
        (|List| 7)
        |UPOLYC-;eval;SLLS;7|
        (39 . |elt|)
        (45 . |coerce|)
```

```
|UPOLYC-;eval;SSaosRS;8|
(|Equation| 6)
(50 . |lhs|)
(|Union| 12 (QUOTE "failed"))
(55 . |mainVariable|)
(60 . |rhs|)
(|List| 197)
|UPOLYC-;eval;SLS;9|
|UPOLYC-;mainVariable;SU;10|
(65 . |minimumDegree|)
|UPOLYC-;minimumDegree;SSaosNni;11|
|UPOLYC-;minimumDegree;SLL;12|
(70 . |+|)
(|Mapping| 10 10)
(76 . |mapExponents|)
|UPOLYC-;monomial;SSaosNniS;13|
(82 . |One|)
(86 . |One|)
(90 . |monomial|)
|UPOLYC-;coerce;SaosS;14|
(|SparseUnivariatePolynomial| 7)
(96 . |Zero|)
(100 . |leadingCoefficient|)
(105 . |monomial|)
(111 . |reductum|)
(116 . |makeSUP|)
(121 . |+|)
|UPOLYC-;makeSUP;SSup;15|
(127 . |zero?|)
(132 . |Zero|)
(136 . |leadingCoefficient|)
(141 . |degree|)
(146 . |reductum|)
(151 . |unmakeSUP|)
(156 . |+|)
|UPOLYC-;unmakeSUP;SupS;16|
(|Record| (|:| |quotient| |$|) (|:| |remainder| |$|))
(162 . |monicDivide|)
|UPOLYC-;karatsubaDivide;SNniR;17|
|UPOLYC-;shiftRight;SNniS;18|
(168 . |*|)
|UPOLYC-;shiftLeft;SNniS;19|
(|Union| 74 (QUOTE "failed"))
(|List| 75)
(|SparseUnivariatePolynomial| 6)
(|PolynomialFactorizationByRecursionUnivariate| 7 6)
```

```
(174 . |solveLinearPolynomialEquationByRecursion|)
(|Union| 79 (QUOTE "failed"))
(|List| 80)
(|SparseUnivariatePolynomial| |$|)
(180 . |solveLinearPolynomialEquation|)
(|Factored| 75)
(186 . |factorByRecursion|)
(|Factored| 80)
(191 . |factorPolynomial|)
(196 . |factorSquareFreeByRecursion|)
(201 . |factorSquareFreePolynomial|)
(|Factored| |$|)
(206 . |factor|)
(|Factored| 7)
(211 . |unit|)
(|Union| (QUOTE "nil") (QUOTE "sqfr") (QUOTE "irred") (QUOTE "prime"))
(|Record| (|:| |flg| 92) (|:| |fctr| 7) (|:| |xpnt| 109))
(|List| 93)
(216 . |factorList|)
(|Record| (|:| |flg| 92) (|:| |fctr| 6) (|:| |xpnt| 109))
(|List| 96)
(|Factored| 6)
(221 . |makeFR|)
(227 . |factorPolynomial|)
(|Mapping| 6 51)
(|Factored| 51)
(|FactoredFunctions2| 51 6)
(232 . |map|)
(238 . |factor|)
(243 . |Zero|)
(|Vector| 7)
(247 . |new|)
(|Integer|)
(253 . |minIndex|)
(258 . |coefficient|)
(264 . |qsetelt!|)
|UPOLYC-;vectorise;SNniV;24|
|UPOLYC-;retract;SR;25|
(|Union| 7 (QUOTE "failed"))
|UPOLYC-;retractIfCan;SU;26|
(271 . |init|)
(275 . |init|)
(|Union| |$| (QUOTE "failed"))
(279 . |nextItem|)
(284 . |One|)
(288 . |nextItem|)
```

```
(293 . |content|)
(298 . |content|)
(304 . |gcd|)
(310 . |exquo|)
(316 . |=|)
(|Record| (|:| |primePart| |$|) (|:| |commonPart| |$|))
(322 . |separate|)
(328 . |Zero|)
(332 . |*|)
(|Mapping| 7 7)
(338 . |differentiate|)
(345 . |*|)
(351 . |differentiate|)
|UPOLYC-;differentiate;SMS;36|
|UPOLYC-;differentiate;2S;37|
(358 . |differentiate|)
|UPOLYC-;differentiate;SSaosS;38|
(|Fraction| 6)
(363 . |numer|)
(|Fraction| |$|)
(368 . |elt|)
(374 . |denom|)
(379 . |/|)
(385 . |elt|)
(391 . |**|)
(397 . |pseudoRemainder|)
(403 . |-|)
(409 . |pseudoQuotient|)
(|Record| (|:| |coef| 7) (|:| |quotient| |$|) (|:| |remainder| |$|))
(415 . |pseudoDivide|)
(421 . |composite|)
(427 . |/|)
(|Union| 142 (QUOTE "failed"))
(433 . |composite|)
(439 . |ground?|)
(444 . |pseudoDivide|)
(450 . |exquo|)
(456 . |composite|)
(462 . |Zero|)
(466 . |coerce|)
(471 . |**|)
(477 . |*|)
(483 . |+|)
(489 . |**|)
(495 . |elt|)
(501 . |order|)
```

```
    (|UnivariatePolynomialSquareFree| 7 6)
    (507 . |squareFree|)
    (512 . |squareFree|)
    (517 . |squareFreePart|)
    (522 . |squareFreePart|)
    (527 . |zero?|)
    (532 . |unitCanonical|)
    (537 . |content|)
    (542 . |primitivePart|)
    (547 . |subResultantGcd|)
    (553 . |*|)
    (559 . |gcdPolynomial|)
    (|UnivariatePolynomialSquareFree| 6 75)
    (565 . |squareFree|)
    (570 . |squareFreePolynomial|)
    (575 . |/|)
    (581 . |elt|)
    (587 . |euclideanSize|)
    (592 . |inv|)
    (597 . |*|)
    (603 . |divide|)
    (|Fraction| 109)
    (609 . |coerce|)
    (614 . |inv|)
    (619 . |*|)
    (625 . |integrate|)
    (|Symbol|)
    (|List| 195)
    (|Equation| |$|)
    (|Union| 109 (QUOTE "failed"))
    (|Union| 190 (QUOTE "failed"))
    (|OutputForm|)))
(QUOTE
  #(|vectorise| 630 |variables| 636 |unmakeSUP| 641 |totalDegree| 646
    |squareFreePolynomial| 652 |squareFreePart| 657 |squareFree| 662
    |solveLinearPolynomialEquation| 667 |shiftRight| 673 |shiftLeft| 679
    |separate| 685 |retractIfCan| 691 |retract| 696 |pseudoQuotient| 701
    |pseudoDivide| 707 |order| 713 |nextItem| 719 |monomial| 724
    |minimumDegree| 731 |makeSUP| 743 |mainVariable| 748
    |karatsubaDivide| 753 |integrate| 759 |init| 764 |gcdPolynomial| 768
    |factorSquareFreePolynomial| 774 |factorPolynomial| 779 |factor| 784
    |eval| 789 |euclideanSize| 823 |elt| 828 |divide| 846
    |differentiate| 852 |degree| 876 |content| 888 |composite| 894
    |coerce| 906))
(QUOTE NIL)
(CONS
```

```
(|makeByteWordVec2| 1 (QUOTE NIL))
(CONS
  (QUOTE #())
  (CONS
    (QUOTE #())
    (|makeByteWordVec2| 194
      (QUOTE
        (1 6 8 0 9 1 6 10 0 11 0 12 0 13 1 6 10 0 17 3 6 0 0 12 0 21 2
         6 0 0 0 24 3 6 0 0 12 7 26 2 6 7 0 7 29 1 6 0 7 30 1 32 6 0 33
         1 6 34 0 35 1 32 6 0 36 1 6 10 0 40 2 10 0 0 0 43 2 6 0 44 0
         45 0 6 0 47 0 7 0 48 2 6 0 7 10 49 0 51 0 52 1 6 7 0 53 2 51
         0 7 10 54 1 6 0 0 55 1 6 51 0 56 2 51 0 0 0 57 1 51 8 0 59 0
         6 0 60 1 51 7 0 61 1 51 10 0 62 1 51 0 0 63 1 6 0 51 64 2 6 0
         0 0 65 2 6 67 0 0 68 2 6 0 0 0 71 2 76 73 74 75 77 2 0 78 79
         80 81 1 76 82 75 83 1 0 84 80 85 1 76 82 75 86 1 0 84 80 87 1
         7 88 0 89 1 90 7 0 91 1 90 94 0 95 2 98 0 6 97 99 1 7 84 80
         100 2 103 98 101 102 104 1 0 88 0 105 0 7 0 106 2 107 0 10 7
         108 1 107 109 0 110 2 6 7 0 10 111 3 107 7 0 109 7 112 0 7 0
         117 0 0 0 118 1 7 119 0 120 0 75 0 121 1 0 119 0 122 1 6 7 0
         123 2 0 0 0 12 124 2 6 0 0 0 125 2 6 119 0 0 126 2 6 8 0 0 127
         2 0 128 0 0 129 0 75 0 130 2 7 0 10 0 131 3 0 0 0 132 0 133 2
         6 0 7 0 134 3 6 0 0 132 0 135 1 6 0 0 138 1 140 6 0 141 2 6
         142 0 142 143 1 140 6 0 144 2 140 0 0 0 145 2 0 142 142 142
         146 2 7 0 0 10 147 2 6 0 0 0 148 2 6 0 0 0 149 2 0 0 0 0 150
         2 0 151 0 0 152 2 6 119 0 0 153 2 140 0 6 6 154 2 0 155 142 0
         156 1 6 8 0 157 2 6 151 0 0 158 2 6 119 0 7 159 2 0 119 0 0
         160 0 140 0 161 1 140 0 6 162 2 140 0 0 109 163 2 140 0 0 0
         164 2 140 0 0 0 165 2 140 0 0 10 166 2 0 142 0 142 167 2 0 10
         0 0 168 1 169 98 6 170 1 0 88 0 171 1 169 6 6 172 1 0 0 0 173
         1 75 8 0 174 1 75 0 0 175 1 75 6 0 176 1 75 0 0 177 2 75 0 0
         0 178 2 75 0 6 0 179 2 0 80 80 80 180 1 181 82 75 182 1 0 84
         80 183 2 7 0 0 0 184 2 0 7 142 7 185 1 0 10 0 186 1 7 0 0 187
         2 7 0 0 0 188 2 0 67 0 0 189 1 190 0 109 191 1 190 0 0 192 2
         6 0 190 0 193 1 0 0 0 194 2 0 107 0 10 113 1 0 14 0 15 1 0 0
         51 66 2 0 10 0 14 18 1 0 84 80 183 1 0 0 0 173 1 0 88 0 171 2
         0 78 79 80 81 2 0 0 0 10 70 2 0 0 0 10 72 2 0 128 0 0 129 1 0
         115 0 116 1 0 7 0 114 2 0 0 0 0 150 2 0 151 0 0 152 2 0 10 0
         0 168 1 0 119 0 122 3 0 0 0 12 10 46 2 0 19 0 14 42 2 0 10 0
         12 41 1 0 51 0 58 1 0 34 0 39 2 0 67 0 10 69 1 0 0 0 194 0 0
         0 118 2 0 80 80 80 180 1 0 84 80 87 1 0 84 80 85 1 0 88 0 105
         3 0 0 0 12 0 25 3 0 0 0 14 22 23 3 0 0 0 14 27 28 3 0 0 0 12
         7 31 2 0 0 0 37 38 1 0 10 0 186 2 0 142 0 142 167 2 0 7 142 7
         185 2 0 142 142 142 146 2 0 67 0 0 189 3 0 0 0 132 0 133 2 0
         0 0 132 136 1 0 0 0 137 2 0 0 0 12 139 2 0 10 0 12 16 2 0 19
         0 14 20 2 0 0 0 12 124 2 0 119 0 0 160 2 0 155 142 0 156 1 0
         0 12 50))))))
```

```
          (QUOTE |lookupComplete|)))
```

21.69 URAGG.lsp BOOTSTRAP

URAGG depends on a chain of files. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **URAGG** category which we can write into the **MID** directory. We compile the lisp code and copy the **URAGG.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*URAGG.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(SETQ |UnaryRecursiveAggregate;CAT| (QUOTE NIL))

(SETQ |UnaryRecursiveAggregate;AL| (QUOTE NIL))

(DEFUN |UnaryRecursiveAggregate| (#1=#:G84596)
 (LET (#2=#:G84597)
  (COND
   ((SETQ #2# (|assoc| (|devaluate| #1#) |UnaryRecursiveAggregate;AL|))
     (CDR #2#))
   (T
    (SETQ |UnaryRecursiveAggregate;AL|
     (|cons5|
      (CONS (|devaluate| #1#) (SETQ #2# (|UnaryRecursiveAggregate;| #1#)))
      |UnaryRecursiveAggregate;AL|))
    #2#))))

(DEFUN |UnaryRecursiveAggregate;| (|t#1|)
 (PROG (#1=#:G84595)
  (RETURN
   (PROG1
    (LETT #1#
     (|sublisV|
      (PAIR (QUOTE (|t#1|)) (LIST (|devaluate| |t#1|)))
      (COND
       (|UnaryRecursiveAggregate;CAT|)
       ((QUOTE T)
        (LETT |UnaryRecursiveAggregate;CAT|
         (|Join|
          (|RecursiveAggregate| (QUOTE |t#1|))
          (|mkCategory|
           (QUOTE |domain|)
           (QUOTE (
            ((|concat| (|$| |$| |$|)) T)
```

```
          ((|concat| (|$| |t#1| |$|)) T)
          ((|first| (|t#1| |$|)) T)
          ((|elt| (|t#1| |$| "first")) T)
          ((|first| (|$| |$| (|NonNegativeInteger|))) T)
          ((|rest| (|$| |$|)) T)
          ((|elt| (|$| |$| "rest")) T)
          ((|rest| (|$| |$| (|NonNegativeInteger|))) T)
          ((|last| (|t#1| |$|)) T)
          ((|elt| (|t#1| |$| "last")) T)
          ((|last| (|$| |$| (|NonNegativeInteger|))) T)
          ((|tail| (|$| |$|)) T)
          ((|second| (|t#1| |$|)) T)
          ((|third| (|t#1| |$|)) T)
          ((|cycleEntry| (|$| |$|)) T)
          ((|cycleLength| ((|NonNegativeInteger|) |$|)) T)
          ((|cycleTail| (|$| |$|)) T)
          ((|concat!| (|$| |$| |$|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
          ((|concat!| (|$| |$| |t#1|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
          ((|cycleSplit!| (|$| |$|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
          ((|setfirst!| (|t#1| |$| |t#1|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
          ((|setelt| (|t#1| |$| "first" |t#1|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
          ((|setrest!| (|$| |$| |$|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
          ((|setelt| (|$| |$| "rest" |$|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
          ((|setlast!| (|t#1| |$| |t#1|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
          ((|setelt| (|t#1| |$| "last" |t#1|))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))
          ((|split!| (|$| |$| (|Integer|)))
            (|has| |$| (ATTRIBUTE |shallowlyMutable|)))))
        NIL
        (QUOTE ((|Integer|) (|NonNegativeInteger|)))
        NIL))
      . #2=(|UnaryRecursiveAggregate|)))))
 . #2#)
(SETELT #1# 0
 (LIST (QUOTE |UnaryRecursiveAggregate|) (|devaluate| |t#1|)))))))
```

21.70 URAGG-.lsp BOOTSTRAP

URAGG- depends on **URAGG**. We need to break this cycle to build the algebra. So we keep a cached copy of the translated **URAGG-** category which we can write into the **MID** directory. We compile the lisp code and copy the **URAGG-.o** file to the **OUT** directory. This is eventually forcibly replaced by a recompiled version.

Note that this code is not included in the generated catdef.spad file.

⟨*URAGG-.lsp BOOTSTRAP*⟩≡

```
(|/VERSIONCHECK| 2)

(DEFUN |URAGG-;elt;AfirstS;1| (|x| G84610 |$|)
 (SPADCALL |x| (QREFELT |$| 8)))

(DEFUN |URAGG-;elt;AlastS;2| (|x| G84612 |$|)
 (SPADCALL |x| (QREFELT |$| 11)))

(DEFUN |URAGG-;elt;ArestA;3| (|x| G84614 |$|)
 (SPADCALL |x| (QREFELT |$| 14)))

(DEFUN |URAGG-;second;AS;4| (|x| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 14)) (QREFELT |$| 8)))

(DEFUN |URAGG-;third;AS;5| (|x| |$|)
 (SPADCALL
  (SPADCALL (SPADCALL |x| (QREFELT |$| 14)) (QREFELT |$| 14))
  (QREFELT |$| 8)))

(DEFUN |URAGG-;cyclic?;AB;6| (|x| |$|)
 (COND
  ((OR
    (SPADCALL |x| (QREFELT |$| 20))
    (SPADCALL (|URAGG-;findCycle| |x| |$|) (QREFELT |$| 20)))
     (QUOTE NIL))
   ((QUOTE T) (QUOTE T))))

(DEFUN |URAGG-;last;AS;7| (|x| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 22)) (QREFELT |$| 8)))

(DEFUN |URAGG-;nodes;AL;8| (|x| |$|)
 (PROG (|l|)
  (RETURN
   (SEQ
    (LETT |l| NIL |URAGG-;nodes;AL;8|)
```

```
      (SEQ
       G190
       (COND
        ((NULL
          (COND
            ((SPADCALL |x| (QREFELT |$| 20)) (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
          (GO G191)))
       (SEQ
        (LETT |l| (CONS |x| |l|) |URAGG-;nodes;AL;8|)
        (EXIT (LETT |x| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;nodes;AL;8|)))
       NIL
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT (NREVERSE |l|))))))

  (DEFUN |URAGG-;children;AL;9| (|x| |$|)
   (PROG (|l|)
    (RETURN
     (SEQ
      (LETT |l| NIL |URAGG-;children;AL;9|)
      (EXIT
       (COND
        ((SPADCALL |x| (QREFELT |$| 20)) |l|)
        ((QUOTE T) (CONS (SPADCALL |x| (QREFELT |$| 14)) |l|))))))))

  (DEFUN |URAGG-;leaf?;AB;10| (|x| |$|)
   (SPADCALL |x| (QREFELT |$| 20)))

  (DEFUN |URAGG-;value;AS;11| (|x| |$|)
   (COND
    ((SPADCALL |x| (QREFELT |$| 20)) (|error| "value of empty object"))
    ((QUOTE T) (SPADCALL |x| (QREFELT |$| 8)))))

  (DEFUN |URAGG-;less?;ANniB;12| (|l| |n| |$|)
   (PROG (|i|)
    (RETURN
     (SEQ
      (LETT |i| |n| |URAGG-;less?;ANniB;12|)
      (SEQ
       G190
       (COND
        ((NULL
          (COND
           ((|<| 0 |i|)
```

```
           (COND
            ((SPADCALL |l| (QREFELT |$| 20)) (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
          ((QUOTE T) (QUOTE NIL))))
        (GO G191)))
      (SEQ
       (LETT |l| (SPADCALL |l| (QREFELT |$| 14)) |URAGG-;less?;ANniB;12|)
       (EXIT (LETT |i| (|-| |i| 1) |URAGG-;less?;ANniB;12|)))
      NIL
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT (|<| 0 |i|))))))

(DEFUN |URAGG-;more?;ANniB;13| (|l| |n| |$|)
 (PROG (|i|)
  (RETURN
   (SEQ
    (LETT |i| |n| |URAGG-;more?;ANniB;13|)
    (SEQ
     G190
     (COND
      ((NULL
        (COND
         ((|<| 0 |i|)
          (COND
           ((SPADCALL |l| (QREFELT |$| 20)) (QUOTE NIL))
           ((QUOTE T) (QUOTE T))))
         ((QUOTE T) (QUOTE NIL))))
        (GO G191)))
      (SEQ
       (LETT |l| (SPADCALL |l| (QREFELT |$| 14)) |URAGG-;more?;ANniB;13|)
       (EXIT (LETT |i| (|-| |i| 1) |URAGG-;more?;ANniB;13|)))
      NIL
      (GO G190)
      G191
      (EXIT NIL))
     (EXIT
      (COND
       ((ZEROP |i|)
        (COND
         ((SPADCALL |l| (QREFELT |$| 20)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       ((QUOTE T) (QUOTE NIL))))))))

(DEFUN |URAGG-;size?;ANniB;14| (|l| |n| |$|)
```

```
  (PROG (|i|)
   (RETURN
    (SEQ
     (LETT |i| |n| |URAGG-;size?;ANniB;14|)
     (SEQ
      G190
      (COND
       ((NULL
         (COND
          ((SPADCALL |l| (QREFELT |$| 20)) (QUOTE NIL))
          ((QUOTE T) (|<| 0 |i|))))
         (GO G191)))
       (SEQ
        (LETT |l| (SPADCALL |l| (QREFELT |$| 14)) |URAGG-;size?;ANniB;14|)
        (EXIT (LETT |i| (|-| |i| 1) |URAGG-;size?;ANniB;14|)))
       NIL
       (GO G190)
       G191
       (EXIT NIL))
      (EXIT
       (COND
        ((SPADCALL |l| (QREFELT |$| 20)) (ZEROP |i|))
        ((QUOTE T) (QUOTE NIL))))))))

(DEFUN |URAGG-;#;ANni;15| (|x| |$|)
 (PROG (|k|)
  (RETURN
   (SEQ
    (SEQ
     (LETT |k| 0 |URAGG-;#;ANni;15|)
     G190
     (COND
      ((NULL
        (COND
         ((SPADCALL |x| (QREFELT |$| 20)) (QUOTE NIL))
         ((QUOTE T) (QUOTE T))))
       (GO G191)))
     (SEQ
      (COND
       ((EQL |k| 1000)
         (COND
          ((SPADCALL |x| (QREFELT |$| 33)) (EXIT (|error| "cyclic list"))))))
       (EXIT (LETT |x| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;#;ANni;15|)))
     (LETT |k| (QSADD1 |k|) |URAGG-;#;ANni;15|)
     (GO G190)
     G191
```

```
      (EXIT NIL))
     (EXIT |k|))))))

(DEFUN |URAGG-;tail;2A;16| (|x| |$|)
 (PROG (|k| |y|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |x| (QREFELT |$| 20)) (|error| "empty list"))
     ((QUOTE T)
      (SEQ
       (LETT |y| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;tail;2A;16|)
       (SEQ
        (LETT |k| 0 |URAGG-;tail;2A;16|)
        G190
        (COND
         ((NULL
           (COND
            ((SPADCALL |y| (QREFELT |$| 20)) (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
          (GO G191)))
        (SEQ
         (COND
          ((EQL |k| 1000)
           (COND
            ((SPADCALL |x| (QREFELT |$| 33))
             (EXIT (|error| "cyclic list"))))))
         (EXIT
          (LETT |y|
           (SPADCALL (LETT |x| |y| |URAGG-;tail;2A;16|) (QREFELT |$| 14))
           |URAGG-;tail;2A;16|)))
        (LETT |k| (QSADD1 |k|) |URAGG-;tail;2A;16|)
        (GO G190)
        G191
        (EXIT NIL))
       (EXIT |x|)))))))

(DEFUN |URAGG-;findCycle| (|x| |$|)
 (PROG (#1=#:G84667 |y|)
  (RETURN
   (SEQ
    (EXIT
     (SEQ
      (LETT |y| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;findCycle|)
      (SEQ
       G190
```

```
         (COND
          ((NULL
            (COND
             ((SPADCALL |y| (QREFELT |$| 20)) (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (COND
           ((SPADCALL |x| |y| (QREFELT |$| 36))
             (PROGN (LETT #1# |x| |URAGG-;findCycle|) (GO #1#))))
          (LETT |x| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;findCycle|)
          (LETT |y| (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;findCycle|)
          (COND
           ((SPADCALL |y| (QREFELT |$| 20))
             (PROGN (LETT #1# |y| |URAGG-;findCycle|) (GO #1#))))
          (COND
           ((SPADCALL |x| |y| (QREFELT |$| 36))
             (PROGN (LETT #1# |y| |URAGG-;findCycle|) (GO #1#))))
          (EXIT (LETT |y| (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;findCycle|)))
         NIL
         (GO G190)
         G191
         (EXIT NIL))
        (EXIT |y|)))
      #1#
      (EXIT #1#)))))

(DEFUN |URAGG-;cycleTail;2A;18| (|x| |$|)
 (PROG (|y| |z|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL
       (LETT |y|
        (LETT |x| (SPADCALL |x| (QREFELT |$| 37)) |URAGG-;cycleTail;2A;18|)
        |URAGG-;cycleTail;2A;18|)
       (QREFELT |$| 20))
      |x|)
     ((QUOTE T)
       (SEQ
        (LETT |z| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;cycleTail;2A;18|)
        (SEQ
         G190
         (COND
          ((NULL
            (COND
```

```
              ((SPADCALL |x| |z| (QREFELT |$| 36)) (QUOTE NIL))
              ((QUOTE T) (QUOTE T))))
            (GO G191)))
           (SEQ
            (LETT |y| |z| |URAGG-;cycleTail;2A;18|)
            (EXIT
             (LETT |z|
              (SPADCALL |z| (QREFELT |$| 14)) |URAGG-;cycleTail;2A;18|)))
           NIL
           (GO G190)
           G191
           (EXIT NIL))
          (EXIT |y|)))))))

(DEFUN |URAGG-;cycleEntry;2A;19| (|x| |$|)
 (PROG (|l| |z| |k| |y|)
  (RETURN
   (SEQ
    (COND
     ((SPADCALL |x| (QREFELT |$| 20)) |x|)
     ((SPADCALL
       (LETT |y| (|URAGG-;findCycle| |x| |$|) |URAGG-;cycleEntry;2A;19|)
       (QREFELT |$| 20))
      |y|)
     ((QUOTE T)
      (SEQ
       (LETT |z| (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;cycleEntry;2A;19|)
       (SEQ
        (LETT |l| 1 |URAGG-;cycleEntry;2A;19|)
        G190
        (COND
         ((NULL
           (COND
            ((SPADCALL |y| |z| (QREFELT |$| 36)) (QUOTE NIL))
            ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (EXIT
           (LETT |z|
            (SPADCALL |z| (QREFELT |$| 14)) |URAGG-;cycleEntry;2A;19|)))
          (LETT |l|
           (QSADD1 |l|) |URAGG-;cycleEntry;2A;19|) (GO G190) G191 (EXIT NIL))
        (LETT |y| |x| |URAGG-;cycleEntry;2A;19|)
        (SEQ
         (LETT |k| 1 |URAGG-;cycleEntry;2A;19|)
         G190
```

```
          (COND ((QSGREATERP |k| |l|) (GO G191)))
          (SEQ
           (EXIT
            (LETT |y|
             (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;cycleEntry;2A;19|)))
          (LETT |k| (QSADD1 |k|) |URAGG-;cycleEntry;2A;19|)
          (GO G190)
          G191
          (EXIT NIL))
         (SEQ
          G190
          (COND
            ((NULL
              (COND
               ((SPADCALL |x| |y| (QREFELT |$| 36)) (QUOTE NIL))
               ((QUOTE T) (QUOTE T))))
             (GO G191)))
          (SEQ
           (LETT |x| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;cycleEntry;2A;19|)
           (EXIT
            (LETT |y|
             (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;cycleEntry;2A;19|)))
          NIL
          (GO G190)
          G191
          (EXIT NIL))
         (EXIT |x|))))))))

(DEFUN |URAGG-;cycleLength;ANni;20| (|x| |$|)
 (PROG (|k| |y|)
  (RETURN
   (SEQ
    (COND
     ((OR
       (SPADCALL |x| (QREFELT |$| 20))
       (SPADCALL
        (LETT |x| (|URAGG-;findCycle| |x| |$|) |URAGG-;cycleLength;ANni;20|)
        (QREFELT |$| 20)))
      0)
     ((QUOTE T)
      (SEQ
       (LETT |y| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;cycleLength;ANni;20|)
       (SEQ
        (LETT |k| 1 |URAGG-;cycleLength;ANni;20|)
        G190
        (COND
```

```
          ((NULL
            (COND
              ((SPADCALL |x| |y| (QREFELT |$| 36)) (QUOTE NIL))
              ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (EXIT
           (LETT |y|
            (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;cycleLength;ANni;20|)))
         (LETT |k| (QSADD1 |k|) |URAGG-;cycleLength;ANni;20|)
         (GO G190)
          G191
          (EXIT NIL))
        (EXIT |k|)))))))

(DEFUN |URAGG-;rest;ANniA;21| (|x| |n| |$|)
 (PROG (|i|)
  (RETURN
   (SEQ
    (SEQ
     (LETT |i| 1 |URAGG-;rest;ANniA;21|)
     G190
     (COND ((QSGREATERP |i| |n|) (GO G191)))
     (SEQ
      (EXIT
       (COND
        ((SPADCALL |x| (QREFELT |$| 20)) (|error| "Index out of range"))
        ((QUOTE T)
          (LETT |x| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;rest;ANniA;21|)))))
     (LETT |i| (QSADD1 |i|) |URAGG-;rest;ANniA;21|)
     (GO G190)
     G191
     (EXIT NIL))
    (EXIT |x|)))))

(DEFUN |URAGG-;last;ANniA;22| (|x| |n| |$|)
 (PROG (|m| #1=#:G84694)
  (RETURN
   (SEQ
    (LETT |m| (SPADCALL |x| (QREFELT |$| 42)) |URAGG-;last;ANniA;22|)
    (EXIT
     (COND
      ((|<| |m| |n|) (|error| "index out of range"))
      ((QUOTE T)
       (SPADCALL
        (SPADCALL |x|
```

```
          (PROG1
           (LETT #1# (|-| |m| |n|) |URAGG-;last;ANniA;22|)
           (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
          (QREFELT |$| 43))
         (QREFELT |$| 44)))))))))

(DEFUN |URAGG-;=;2AB;23| (|x| |y| |$|)
 (PROG (|k| #1=#:G84705)
  (RETURN
   (SEQ
    (EXIT
     (COND
      ((SPADCALL |x| |y| (QREFELT |$| 36)) (QUOTE T))
      ((QUOTE T)
       (SEQ
        (SEQ
         (LETT |k| 0 |URAGG-;=;2AB;23|)
         G190
         (COND
          ((NULL
            (COND
             ((OR
               (SPADCALL |x| (QREFELT |$| 20))
               (SPADCALL |y| (QREFELT |$| 20)))
              (QUOTE NIL))
             ((QUOTE T) (QUOTE T))))
           (GO G191)))
         (SEQ
          (COND
           ((EQL |k| 1000)
            (COND
             ((SPADCALL |x| (QREFELT |$| 33))
              (EXIT (|error| "cyclic list"))))))
          (COND
           ((NULL
             (SPADCALL
              (SPADCALL |x| (QREFELT |$| 8))
              (SPADCALL |y| (QREFELT |$| 8))
              (QREFELT |$| 46)))
            (EXIT (PROGN (LETT #1# (QUOTE NIL) |URAGG-;=;2AB;23|) (GO #1#)))))
          (LETT |x| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;=;2AB;23|)
          (EXIT (LETT |y| (SPADCALL |y| (QREFELT |$| 14)) |URAGG-;=;2AB;23|)))
         (LETT |k| (QSADD1 |k|) |URAGG-;=;2AB;23|)
         (GO G190)
         G191
         (EXIT NIL))
```

```
          (EXIT
           (COND
            ((SPADCALL |x| (QREFELT |$| 20)) (SPADCALL |y| (QREFELT |$| 20)))
            ((QUOTE T) (QUOTE NIL))))))))
      #1#
      (EXIT #1#)))))

(DEFUN |URAGG-;node?;2AB;24| (|u| |v| |$|)
 (PROG (|k| #1=#:G84711)
  (RETURN
   (SEQ
    (EXIT
     (SEQ
      (SEQ
       (LETT |k| 0 |URAGG-;node?;2AB;24|)
       G190
       (COND
         ((NULL
          (COND
           ((SPADCALL |v| (QREFELT |$| 20)) (QUOTE NIL))
           ((QUOTE T) (QUOTE T))))
          (GO G191)))
        (SEQ
         (EXIT
          (COND
           ((SPADCALL |u| |v| (QREFELT |$| 48))
             (PROGN (LETT #1# (QUOTE T) |URAGG-;node?;2AB;24|) (GO #1#)))
           ((QUOTE T)
            (SEQ
             (COND
              ((EQL |k| 1000)
               (COND
                ((SPADCALL |v| (QREFELT |$| 33))
                  (EXIT (|error| "cyclic list"))))))
             (EXIT
              (LETT |v|
               (SPADCALL |v| (QREFELT |$| 14))
               |URAGG-;node?;2AB;24|)))))))
        (LETT |k| (QSADD1 |k|) |URAGG-;node?;2AB;24|) (GO G190) G191 (EXIT NIL))
      (EXIT (SPADCALL |u| |v| (QREFELT |$| 48)))))
    #1# (EXIT #1#)))))

(DEFUN |URAGG-;setelt;Afirst2S;25| (|x| G84713 |a| |$|)
 (SPADCALL |x| |a| (QREFELT |$| 50)))

(DEFUN |URAGG-;setelt;Alast2S;26| (|x| G84715 |a| |$|)
```

```
 (SPADCALL |x| |a| (QREFELT |$| 52)))

(DEFUN |URAGG-;setelt;Arest2A;27| (|x| G84717 |a| |$|)
 (SPADCALL |x| |a| (QREFELT |$| 54)))

(DEFUN |URAGG-;concat;3A;28| (|x| |y| |$|)
 (SPADCALL (SPADCALL |x| (QREFELT |$| 44)) |y| (QREFELT |$| 56)))

(DEFUN |URAGG-;setlast!;A2S;29| (|x| |s| |$|)
 (SEQ
  (COND
   ((SPADCALL |x| (QREFELT |$| 20)) (|error| "setlast: empty list"))
   ((QUOTE T)
    (SEQ
     (SPADCALL (SPADCALL |x| (QREFELT |$| 22)) |s| (QREFELT |$| 50))
     (EXIT |s|))))))

(DEFUN |URAGG-;setchildren!;ALA;30| (|u| |lv| |$|)
 (COND
  ((EQL (LENGTH |lv|) 1) (SPADCALL |u| (|SPADfirst| |lv|) (QREFELT |$| 54)))
  ((QUOTE T) (|error| "wrong number of children specified"))))

(DEFUN |URAGG-;setvalue!;A2S;31| (|u| |s| |$|)
 (SPADCALL |u| |s| (QREFELT |$| 50)))

(DEFUN |URAGG-;split!;AIA;32| (|p| |n| |$|)
 (PROG (#1=#:G84725 |q|)
  (RETURN
   (SEQ
    (COND
     ((|<| |n| 1) (|error| "index out of range"))
     ((QUOTE T)
      (SEQ
       (LETT |p|
        (SPADCALL |p|
         (PROG1
          (LETT #1# (|-| |n| 1) |URAGG-;split!;AIA;32|)
          (|check-subtype| (|>=| #1# 0) (QUOTE (|NonNegativeInteger|)) #1#))
         (QREFELT |$| 43))
        |URAGG-;split!;AIA;32|)
       (LETT |q| (SPADCALL |p| (QREFELT |$| 14)) |URAGG-;split!;AIA;32|)
       (SPADCALL |p| (SPADCALL (QREFELT |$| 61)) (QREFELT |$| 54))
       (EXIT |q|))))))))

(DEFUN |URAGG-;cycleSplit!;2A;33| (|x| |$|)
 (PROG (|y| |z|)
```

```
   (RETURN
    (SEQ
     (COND
      ((OR
        (SPADCALL
         (LETT |y| (SPADCALL |x| (QREFELT |$| 37)) |URAGG-;cycleSplit!;2A;33|)
         (QREFELT |$| 20))
        (SPADCALL |x| |y| (QREFELT |$| 36))) |y|)
       ((QUOTE T)
        (SEQ
         (LETT |z| (SPADCALL |x| (QREFELT |$| 14)) |URAGG-;cycleSplit!;2A;33|)
         (SEQ G190
          (COND
           ((NULL
             (COND
              ((SPADCALL |z| |y| (QREFELT |$| 36)) (QUOTE NIL))
              ((QUOTE T) (QUOTE T))))
            (GO G191)))
          (SEQ
           (LETT |x| |z| |URAGG-;cycleSplit!;2A;33|)
           (EXIT
            (LETT |z|
             (SPADCALL |z| (QREFELT |$| 14)) |URAGG-;cycleSplit!;2A;33|)))
          NIL
          (GO G190)
          G191
          (EXIT NIL))
         (SPADCALL |x|
          (SPADCALL (QREFELT |$| 61)) (QREFELT |$| 54)) (EXIT |y|))))))))

(DEFUN |UnaryRecursiveAggregate&| (|#1| |#2|)
 (PROG (|DV$1| |DV$2| |dv$| |$| |pv$|)
  (RETURN
   (PROGN
    (LETT |DV$1| (|devaluate| |#1|) . #1=(|UnaryRecursiveAggregate&|))
    (LETT |DV$2| (|devaluate| |#2|) . #1#)
    (LETT |dv$| (LIST (QUOTE |UnaryRecursiveAggregate&|) |DV$1| |DV$2|) . #1#)
    (LETT |$| (GETREFV 66) . #1#)
    (QSETREFV |$| 0 |dv$|)
    (QSETREFV |$| 3
     (LETT |pv$|
      (|buildPredVector| 0 0
       (LIST (|HasAttribute| |#1| (QUOTE |shallowlyMutable|))))
      . #1#))
    (|stuffDomainSlots| |$|)
    (QSETREFV |$| 6 |#1|)
```

```
      (QSETREFV |$| 7 |#2|)
      (COND
        ((|HasAttribute| |#1| (QUOTE |finiteAggregate|))
          (QSETREFV |$| 45
            (CONS (|dispatchFunction| |URAGG-;last;ANniA;22|) |$|))))
      (COND
       ((|HasCategory| |#2| (QUOTE (|SetCategory|)))
        (PROGN
         (QSETREFV |$| 47 (CONS (|dispatchFunction| |URAGG-;=;2AB;23|) |$|))
         (QSETREFV |$| 49
          (CONS (|dispatchFunction| |URAGG-;node?;2AB;24|) |$|)))))
      (COND
       ((|testBitVector| |pv$| 1)
        (PROGN
         (QSETREFV |$| 51
          (CONS (|dispatchFunction| |URAGG-;setelt;Afirst2S;25|) |$|))
         (QSETREFV |$| 53
          (CONS (|dispatchFunction| |URAGG-;setelt;Alast2S;26|) |$|))
         (QSETREFV |$| 55
          (CONS (|dispatchFunction| |URAGG-;setelt;Arest2A;27|) |$|))
         (QSETREFV |$| 57
          (CONS (|dispatchFunction| |URAGG-;concat;3A;28|) |$|))
         (QSETREFV |$| 58
          (CONS (|dispatchFunction| |URAGG-;setlast!;A2S;29|) |$|))
         (QSETREFV |$| 59
          (CONS (|dispatchFunction| |URAGG-;setchildren!;ALA;30|) |$|))
         (QSETREFV |$| 60
          (CONS (|dispatchFunction| |URAGG-;setvalue!;A2S;31|) |$|))
         (QSETREFV |$| 63
          (CONS (|dispatchFunction| |URAGG-;split!;AIA;32|) |$|))
         (QSETREFV |$| 64
          (CONS (|dispatchFunction| |URAGG-;cycleSplit!;2A;33|) |$|)))))
      |$|))))

  (MAKEPROP
   (QUOTE |UnaryRecursiveAggregate&|)
   (QUOTE |infovec|)
   (LIST
    (QUOTE #(NIL NIL NIL NIL NIL NIL (|local| |#1|) (|local| |#2|)
     (0 . |first|) (QUOTE "first") |URAGG-;elt;AfirstS;1| (5 . |last|)
     (QUOTE "last") |URAGG-;elt;AlastS;2| (10 . |rest|) (QUOTE "rest")
     |URAGG-;elt;ArestA;3| |URAGG-;second;AS;4| |URAGG-;third;AS;5|
     (|Boolean|) (15 . |empty?|) |URAGG-;cyclic?;AB;6| (20 . |tail|)
     |URAGG-;last;AS;7| (|List| |$|) |URAGG-;nodes;AL;8|
     |URAGG-;children;AL;9| |URAGG-;leaf?;AB;10| |URAGG-;value;AS;11|
     (|NonNegativeInteger|) |URAGG-;less?;ANniB;12| |URAGG-;more?;ANniB;13|
```

```
 |URAGG-;size?;ANniB;14| (25 . |cyclic?|) |URAGG-;#;ANni;15|
 |URAGG-;tail;2A;16| (30 . |eq?|) (36 . |cycleEntry|)
 |URAGG-;cycleTail;2A;18| |URAGG-;cycleEntry;2A;19|
 |URAGG-;cycleLength;ANni;20| |URAGG-;rest;ANniA;21| (41 . |#|)
 (46 . |rest|) (52 . |copy|) (57 . |last|) (63 . |=|) (69 . |=|)
 (75 . |=|) (81 . |node?|) (87 . |setfirst!|) (93 . |setelt|)
 (100 . |setlast!|) (106 . |setelt|) (113 . |setrest!|)
 (119 . |setelt|) (126 . |concat!|) (132 . |concat|) (138 . |setlast!|)
 (144 . |setchildren!|) (150 . |setvalue!|) (156 . |empty|) (|Integer|)
 (160 . |split!|) (166 . |cycleSplit!|) (QUOTE "value")))
(QUOTE #(|value| 171 |third| 176 |tail| 181 |split!| 186 |size?| 192
 |setvalue!| 198 |setlast!| 204 |setelt| 210 |setchildren!| 231 |second|
 237 |rest| 242 |nodes| 248 |node?| 253 |more?| 259 |less?| 265 |leaf?|
 271 |last| 276 |elt| 287 |cyclic?| 305 |cycleTail| 310 |cycleSplit!|
 315 |cycleLength| 320 |cycleEntry| 325 |concat| 330 |children| 336 |=|
 341 |#| 347))
(QUOTE NIL)
(CONS
 (|makeByteWordVec2| 1 (QUOTE NIL))
 (CONS
  (QUOTE #())
  (CONS
   (QUOTE #())
   (|makeByteWordVec2| 64 (QUOTE (1 6 7 0 8 1 6 7 0 11 1 6 0 0 14 1 6
     19 0 20 1 6 0 0 22 1 6 19 0 33 2 6 19 0 0 36 1 6 0 0 37 1 6 29 0
     42 2 6 0 0 29 43 1 6 0 0 44 2 0 0 0 29 45 2 7 19 0 0 46 2 0 19 0
     0 47 2 6 19 0 0 48 2 0 19 0 0 49 2 6 7 0 7 50 3 0 7 0 9 7 51 2 6
     7 0 7 52 3 0 7 0 12 7 53 2 6 0 0 0 54 3 0 0 0 15 0 55 2 6 0 0 0 56
     2 0 0 0 0 57 2 0 7 0 7 58 2 0 0 0 24 59 2 0 7 0 7 60 0 6 0 61 2 0
     0 0 62 63 1 0 0 0 64 1 0 7 0 28 1 0 7 0 18 1 0 0 0 35 2 0 0 0 62 63
     2 0 19 0 29 32 2 0 7 0 7 60 2 0 7 0 7 58 3 0 7 0 12 7 53 3 0 0 0 15
     0 55 3 0 7 0 9 7 51 2 0 0 0 24 59 1 0 7 0 17 2 0 0 0 29 41 1 0 24 0
     25 2 0 19 0 0 49 2 0 19 0 29 31 2 0 19 0 29 30 1 0 19 0 27 2 0 0 0
     29 45 1 0 7 0 23 2 0 7 0 12 13 2 0 0 0 15 16 2 0 7 0 9 10 1 0 19 0
     21 1 0 0 0 38 1 0 0 0 64 1 0 29 0 40 1 0 0 0 39 2 0 0 0 0 57 1 0 24
     0 26 2 0 19 0 0 47 1 0 29 0 34))))))
(QUOTE |lookupComplete|)))
```

Chapter 22

Chunk collections

⟨*algebra*⟩≡
⟨*category AHYP ArcHyperbolicFunctionCategory*⟩
⟨*category ATRIG ArcTrigonometricFunctionCategory*⟩
⟨*category ATTREG AttributeRegistry*⟩
⟨*category BASTYPE BasicType*⟩
⟨*category KOERCE CoercibleTo*⟩
⟨*category CFCAT CombinatorialFunctionCategory*⟩
⟨*category KONVERT ConvertibleTo*⟩
⟨*category ELEMFUN ElementaryFunctionCategory*⟩
⟨*category ELTAB Eltable*⟩
⟨*category HYPCAT HyperbolicFunctionCategory*⟩
⟨*category IEVALAB InnerEvalable*⟩
⟨*category OM OpenMath*⟩
⟨*category PTRANFN PartialTranscendentalFunctions*⟩
⟨*category PATAB Patternable*⟩
⟨*category PRIMCAT PrimitiveFunctionCategory*⟩
⟨*category RADCAT RadicalCategory*⟩
⟨*category RETRACT RetractableTo*⟩
⟨*category SPFCAT SpecialFunctionCategory*⟩
⟨*category TRIGCAT TrigonometricFunctionCategory*⟩
⟨*category TYPE Type*⟩
⟨*category AGG Aggregate*⟩
⟨*category COMBOPC CombinatorialOpsCategory*⟩
⟨*category ELTAGG EltableAggregate*⟩
⟨*category EVALAB Evalable*⟩
⟨*category FORTCAT FortranProgramCategory*⟩
⟨*category FRETRCT FullyRetractableTo*⟩
⟨*category FPATMAB FullyPatternMatchable*⟩
⟨*category LOGIC Logic*⟩
⟨*category PPCURVE PlottablePlaneCurveCategory*⟩

⟨*category PSCURVE PlottableSpaceCurveCategory*⟩
⟨*category REAL RealConstant*⟩
⟨*category SEGCAT SegmentCategory*⟩
⟨*category SETCAT SetCategory*⟩
⟨*category TRANFUN TranscendentalFunctionCategory*⟩
⟨*category ABELSG AbelianSemiGroup*⟩
⟨*category FORTFN FortranFunctionCategory*⟩
⟨*category FMC FortranMatrixCategory*⟩
⟨*category FMFUN FortranMatrixFunctionCategory*⟩
⟨*category FVC FortranVectorCategory*⟩
⟨*category FVFUN FortranVectorFunctionCategory*⟩
⟨*category FEVALAB FullyEvalableOver*⟩
⟨*category FILECAT FileCategory*⟩
⟨*category FINITE Finite*⟩
⟨*category FNCAT FileNameCategory*⟩
⟨*category GRMOD GradedModule*⟩
⟨*category HOAGG HomogeneousAggregate*⟩
⟨*category IDPC IndexedDirectProductCategory*⟩
⟨*category LFCAT LiouvillianFunctionCategory*⟩
⟨*category MONAD Monad*⟩
⟨*category NUMINT NumericalIntegrationCategory*⟩
⟨*category OPTCAT NumericalOptimizationCategory*⟩
⟨*category ODECAT OrdinaryDifferentialEquationsSolverCategory*⟩
⟨*category ORDSET OrderedSet*⟩
⟨*category PDECAT PartialDifferentialEquationsSolverCategory*⟩
⟨*category PATMAB PatternMatchable*⟩
⟨*category RRCC RealRootCharacterizationCategory*⟩
⟨*category SEGXCAT SegmentExpansionCategory*⟩
⟨*category SGROUP SemiGroup*⟩
⟨*category SEXCAT SExpressionCategory*⟩
⟨*category STEP StepThrough*⟩
⟨*category SPACEC ThreeSpaceCategory*⟩
⟨*category ABELMON AbelianMonoid*⟩
⟨*category BGAGG BagAggregate*⟩
⟨*category CACHSET CachableSet*⟩
⟨*category CLAGG Collection*⟩
⟨*category DVARCAT DifferentialVariableCategory*⟩
⟨*category ES ExpressionSpace*⟩
⟨*category GRALG GradedAlgebra*⟩
⟨*category IXAGG IndexedAggregate*⟩
⟨*category MONADWU MonadWithUnit*⟩
⟨*category MONOID Monoid*⟩
⟨*category ORDFIN OrderedFinite*⟩
⟨*category RCAGG RecursiveAggregate*⟩
⟨*category ARR2CAT TwoDimensionalArrayCategory*⟩
⟨*category BRAGG BinaryRecursiveAggregate*⟩

⟨*category CABMON CancellationAbelianMonoid*⟩
⟨*category DIOPS DictionaryOperations*⟩
⟨*category DLAGG DoublyLinkedAggregate*⟩
⟨*category GROUP Group*⟩
⟨*category LNAGG LinearAggregate*⟩
⟨*category OASGP OrderedAbelianSemiGroup*⟩
⟨*category ORDMON OrderedMonoid*⟩
⟨*category PSETCAT PolynomialSetCategory*⟩
⟨*category PRQAGG PriorityQueueAggregate*⟩
⟨*category QUAGG QueueAggregate*⟩
⟨*category SETAGG SetAggregate*⟩
⟨*category SKAGG StackAggregate*⟩
⟨*category URAGG UnaryRecursiveAggregate*⟩
⟨*category ABELGRP AbelianGroup*⟩
⟨*category BTCAT BinaryTreeCategory*⟩
⟨*category DIAGG Dictionary*⟩
⟨*category DQAGG DequeueAggregate*⟩
⟨*category ELAGG ExtensibleLinearAggregate*⟩
⟨*category FLAGG FiniteLinearAggregate*⟩
⟨*category FAMONC FreeAbelianMonoidCategory*⟩
⟨*category MDAGG MultiDictionary*⟩
⟨*category OAMON OrderedAbelianMonoid*⟩
⟨*category PERMCAT PermutationCategory*⟩
⟨*category STAGG StreamAggregate*⟩
⟨*category TSETCAT TriangularSetCategory*⟩
⟨*category FDIVCAT FiniteDivisorCategory*⟩
⟨*category FSAGG FiniteSetAggregate*⟩
⟨*category KDAGG KeyedDictionary*⟩
⟨*category LZSTAGG LazyStreamAggregate*⟩
⟨*category LMODULE LeftModule*⟩
⟨*category LSAGG ListAggregate*⟩
⟨*category MSETAGG MultisetAggregate*⟩
⟨*category NARNG NonAssociativeRng*⟩
⟨*category A1AGG OneDimensionalArrayAggregate*⟩
⟨*category OCAMON OrderedCancellationAbelianMonoid*⟩
⟨*category RSETCAT RegularTriangularSetCategory*⟩
⟨*category RMODULE RightModule*⟩
⟨*category RNG Rng*⟩
⟨*category BMODULE BiModule*⟩
⟨*category BTAGG BitAggregate*⟩
⟨*category NASRING NonAssociativeRing*⟩
⟨*category NTSCAT NormalizedTriangularSetCategory*⟩
⟨*category OAGROUP OrderedAbelianGroup*⟩
⟨*category OAMONS OrderedAbelianMonoidSup*⟩
⟨*category OMSAGG OrderedMultisetAggregate*⟩
⟨*category RING Ring*⟩

⟨*category SFRTCAT SquareFreeRegularTriangularSetCategory*⟩
⟨*category SRAGG StringAggregate*⟩
⟨*category TBAGG TableAggregate*⟩
⟨*category VECTCAT VectorCategory*⟩
⟨*category ALAGG AssociationListAggregate*⟩
⟨*category CHARNZ CharacteristicNonZero*⟩
⟨*category CHARZ CharacteristicZero*⟩
⟨*category COMRING CommutativeRing*⟩
⟨*category DIFRING DifferentialRing*⟩
⟨*category ENTIRER EntireRing*⟩
⟨*category FMCAT FreeModuleCat*⟩
⟨*category LALG LeftAlgebra*⟩
⟨*category LINEXP LinearlyExplicitRingOver*⟩
⟨*category MODULE Module*⟩
⟨*category ORDRING OrderedRing*⟩
⟨*category PDRING PartialDifferentialRing*⟩
⟨*category PTCAT PointCategory*⟩
⟨*category RMATCAT RectangularMatrixCategory*⟩
⟨*category SNTSCAT SquareFreeNormalizedTriangularSetCategory*⟩
⟨*category STRICAT StringCategory*⟩
⟨*category OREPCAT UnivariateSkewPolynomialCategory*⟩
⟨*category XALG XAlgebra*⟩
⟨*category ALGEBRA Algebra*⟩
⟨*category DIFEXT DifferentialExtension*⟩
⟨*category FLINEXP FullyLinearlyExplicitRingOver*⟩
⟨*category LIECAT LieAlgebra*⟩
⟨*category LODOCAT LinearOrdinaryDifferentialOperatorCategory*⟩
⟨*category NAALG NonAssociativeAlgebra*⟩
⟨*category VSPACE VectorSpace*⟩
⟨*category XFALG XFreeAlgebra*⟩
⟨*category DIVRING DivisionRing*⟩
⟨*category FINAALG FiniteRankNonAssociativeAlgebra*⟩
⟨*category FLALG FreeLieAlgebra*⟩
⟨*category INTDOM IntegralDomain*⟩
⟨*category MLO MonogenicLinearOperator*⟩
⟨*category OC OctonionCategory*⟩
⟨*category QUATCAT QuaternionCategory*⟩
⟨*category SMATCAT SquareMatrixCategory*⟩
⟨*category XPOLYC XPolynomialsCat*⟩
⟨*category AMR AbelianMonoidRing*⟩
⟨*category FMTC FortranMachineTypeCategory*⟩
⟨*category FRNAALG FramedNonAssociativeAlgebra*⟩
⟨*category GCDDOM GcdDomain*⟩
⟨*category OINTDOM OrderedIntegralDomain*⟩
⟨*category FAMR FiniteAbelianMonoidRing*⟩
⟨*category INTCAT IntervalCategory*⟩

⟨*category PSCAT PowerSeriesCategory*⟩
⟨*category PID PrincipalIdealDomain*⟩
⟨*category UFD UniqueFactorizationDomain*⟩
⟨*category EUCDOM EuclideanDomain*⟩
⟨*category MTSCAT MultivariateTaylorSeriesCategory*⟩
⟨*category PFECAT PolynomialFactorizationExplicit*⟩
⟨*category UPSCAT UnivariatePowerSeriesCategory*⟩
⟨*category FIELD Field*⟩
⟨*category INS IntegerNumberSystem*⟩
⟨*category PADICCT PAdicIntegerCategory*⟩
⟨*category POLYCAT PolynomialCategory*⟩
⟨*category UTSCAT UnivariateTaylorSeriesCategory*⟩
⟨*category ACF AlgebraicallyClosedField*⟩
⟨*category DPOLCAT DifferentialPolynomialCategory*⟩
⟨*category DIRPCAT DirectProductCategory*⟩
⟨*category FPC FieldOfPrimeCharacteristic*⟩
⟨*category FINRALG FiniteRankAlgebra*⟩
⟨*category FS FunctionSpace*⟩
⟨*category MATCAT MatrixCategory*⟩
⟨*category QFCAT QuotientFieldCategory*⟩
⟨*category RCFIELD RealClosedField*⟩
⟨*category RNS RealNumberSystem*⟩
⟨*category RPOLCAT RecursivePolynomialCategory*⟩
⟨*category ULSCAT UnivariateLaurentSeriesCategory*⟩
⟨*category UPXSCAT UnivariatePuiseuxSeriesCategory*⟩
⟨*category UPOLYC UnivariatePolynomialCategory*⟩
⟨*category ACFS AlgebraicallyClosedFunctionSpace*⟩
⟨*category XF ExtensionField*⟩
⟨*category FFIELDC FiniteFieldCategory*⟩
⟨*category FPS FloatingPointSystem*⟩
⟨*category FRAMALG FramedAlgebra*⟩
⟨*category ULSCCAT UnivariateLaurentSeriesConstructorCategory*⟩
⟨*category UPXSCCA UnivariatePuiseuxSeriesConstructorCategory*⟩
⟨*category FAXF FiniteAlgebraicExtensionField*⟩
⟨*category MONOGEN MonogenicAlgebra*⟩
⟨*category COMPCAT ComplexCategory*⟩
⟨*category FFCAT FunctionFieldCategory*⟩

⟨*dotabb*⟩≡
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digraph dotabb {
 ranksep=1.25;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];
```

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⟨*AHYP.dotabb*⟩
⟨*ATRIG.dotabb*⟩
⟨*ATTREG.dotabb*⟩
⟨*BASTYPE.dotabb*⟩
⟨*KOERCE.dotabb*⟩
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⟨*KONVERT.dotabb*⟩
⟨*ELEMFUN.dotabb*⟩
⟨*ELTAB.dotabb*⟩
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⟨*IEVALAB.dotabb*⟩
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⟨*RADCAT.dotabb*⟩
⟨*RETRACT.dotabb*⟩
⟨*SPFCAT.dotabb*⟩
⟨*TRIGCAT.dotabb*⟩
⟨*TYPE.dotabb*⟩
⟨*AGG.dotabb*⟩
⟨*COMBOPC.dotabb*⟩
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⟨*PSCURVE.dotabb*⟩
⟨*REAL.dotabb*⟩
⟨*SEGCAT.dotabb*⟩
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⟨*FMFUN.dotabb*⟩
⟨*FVC.dotabb*⟩

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⟨*FEVALAB.dotabb*⟩
⟨*FILECAT.dotabb*⟩
⟨*FINITE.dotabb*⟩
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⟨*OAGROUP.dotabb*⟩
⟨*OAMONS.dotabb*⟩
⟨*OMSAGG.dotabb*⟩
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⟨*PDRING.dotabb*⟩
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⟨*DIVRING.dotabb*⟩
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⟨*UPOLYC.dotabb*⟩
⟨*ACFS.dotabb*⟩
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⟨*FFIELDC.dotabb*⟩
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⟨*UPXSCCA.dotabb*⟩
⟨*FAXF.dotabb*⟩
⟨*MONOGEN.dotabb*⟩
⟨*COMPCAT.dotabb*⟩
⟨*FFCAT.dotabb*⟩

}

⟨*dotfull*⟩≡
```
digraph dotfull {
 ranksep=1.25;
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 fontsize=10;
 bgcolor="#FFFF66";
 node [shape=box, color=white, style=filled];
```

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⟨*AHYP.dotfull*⟩
⟨*ATRIG.dotfull*⟩
⟨*ATTREG.dotfull*⟩
⟨*BASTYPE.dotfull*⟩
⟨*KOERCE.dotfull*⟩
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⟨*FAMONC.dotfull*⟩
⟨*MDAGG.dotfull*⟩
⟨*OAMON.dotfull*⟩
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⟨*FDIVCAT.dotfull*⟩
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⟨*KDAGG.dotfull*⟩
⟨*LZSTAGG.dotfull*⟩
⟨*LMODULE.dotfull*⟩
⟨*LSAGG.dotfull*⟩
⟨*MSETAGG.dotfull*⟩
⟨*NARNG.dotfull*⟩
⟨*A1AGG.dotfull*⟩
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⟨*RSETCAT.dotfull*⟩
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⟨*RNG.dotfull*⟩
⟨*BMODULE.dotfull*⟩
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⟨*NASRING.dotfull*⟩
⟨*NTSCAT.dotfull*⟩
⟨*OAGROUP.dotfull*⟩
⟨*OAMONS.dotfull*⟩
⟨*OMSAGG.dotfull*⟩
⟨*RING.dotfull*⟩
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⟨*SRAGG.dotfull*⟩
⟨*TBAGG.dotfull*⟩
⟨*VECTCAT.dotfull*⟩
⟨*ALAGG.dotfull*⟩
⟨*CHARNZ.dotfull*⟩
⟨*CHARZ.dotfull*⟩
⟨*COMRING.dotfull*⟩

⟨*DIFRING.dotfull*⟩
⟨*ENTIRER.dotfull*⟩
⟨*FMCAT.dotfull*⟩
⟨*LALG.dotfull*⟩
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⟨*FFCAT.dotfull*⟩
}

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